INTERFEROMETRIC SENSING AND CONTROL IN LIGO

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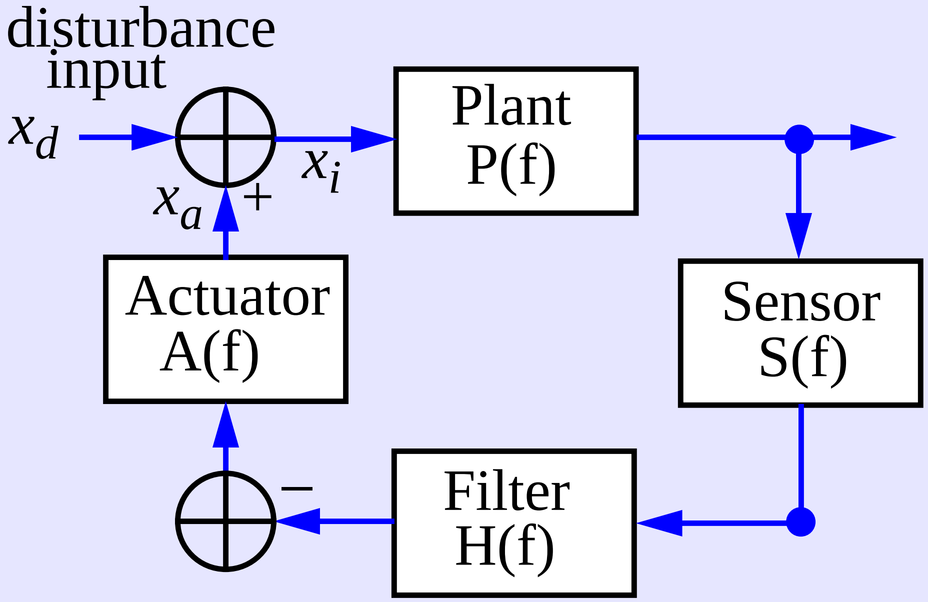
- Introduction to control systems
- Length and alignment sensing
- Noise Sensitivity
- Length control system
- Noise suppression
- More tricks? Data?

- Lock Acquisition (Brent Ware)...

Elements of a Control System



$$x_a = -PSHAx_i$$

$$x_i = x_d + x_a$$

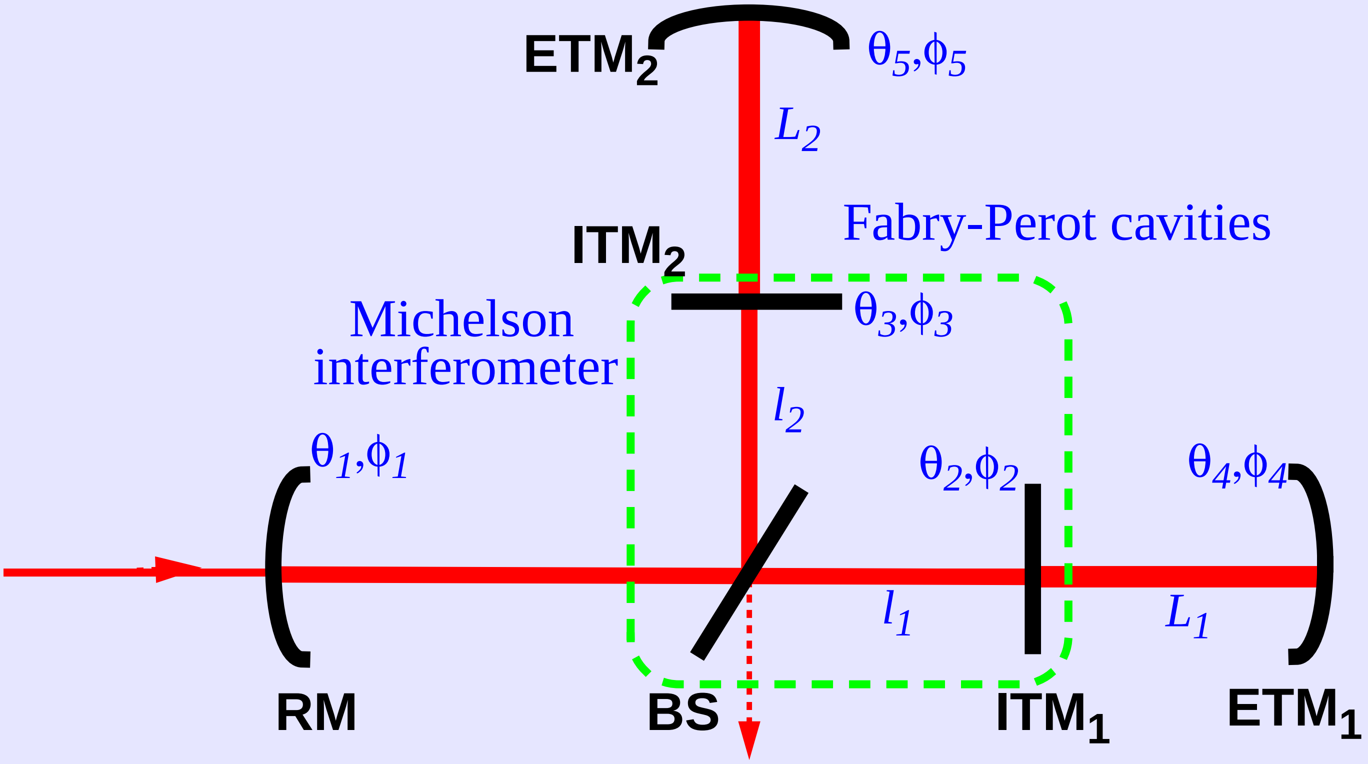
$$x_i = \frac{x_d}{1 + G(f)}$$

where

$$G(f) = PSHA$$

- **Plant: system to be controlled, e.g. cavity length**
- **Sensor: “sees” cavity length change, e.g. photodetection/demodulation**
- **Filter: designed to suppress noise as needed**
- **Actuator: “moves” cavity length, e.g. suspension**
- **Upshot: when $G(f) >> 1$ then $x_i << x_d \Rightarrow$ plant input is much smaller than the original disturbance**

INTERFEROMETER OPTICAL CONFIGURATION



- ❑ Fabry-Perot arm cavities
- ❑ Dark fringe at antisymmetric port
- ❑ Power Recycling (G_{rec} ~ 50)
- ❑ 4 longitudinal degrees of freedom
- ❑ 12 angular degrees of freedom

IFO LENGTHS: WHY CONTROL THEM?

- ❑ Cavity resonant linewidth: $\delta l \ll \frac{\lambda}{2F} \sim 10^{-8}$ m
- ❑ Deviation from perfect destructive interference ⇒ coupling of noise to GW signal
  - ❍ e.g. laser intensity noise

$$\frac{S(\delta P/P = 10^{-7})}{S(\delta L_D)} \le 0.1 \;\Rightarrow\; \delta L_D \le 10^{-13}\ \text{m}$$

- ❑ Deviation from perfect resonance ⇒ less power build-up in ifo ⇒ less GW signal
  - ❍ e.g. $\left.\frac{P(\delta L_C)}{P_{max}}\right|_{arm} \ge 0.99 \;\Rightarrow\; \delta L_C \le 10^{-12}$ m
- ❑ Ground noise excitation ~ 10^{-5} m (typical μ-seismic motion of earth)
- ❑ Sooo, need to suppress length fluctuations due to ground noise by factor $\delta L_D/\delta L_{gnd} \sim 10^{-8}$

IFO ALIGNMENT: WHY CONTROL MIRROR ANGLES?

- **Degradation of GW sensitivity**

  $\frac{(S/N)_{GW}(\delta\theta_i)}{(S/N)_{GW}(\delta\theta_i = 0)} \le 0.995 \Rightarrow \delta\theta_i \le 10^{-8}$ **rad**

- **Misalignment – input beam jitter coupling**

  $\delta\theta_{rms} \le 10^{-8}\ \mathrm{rad} \Rightarrow \alpha \sim 10^{-14}\ \mathrm{rad}/\sqrt{\mathrm{Hz}}$ **and**

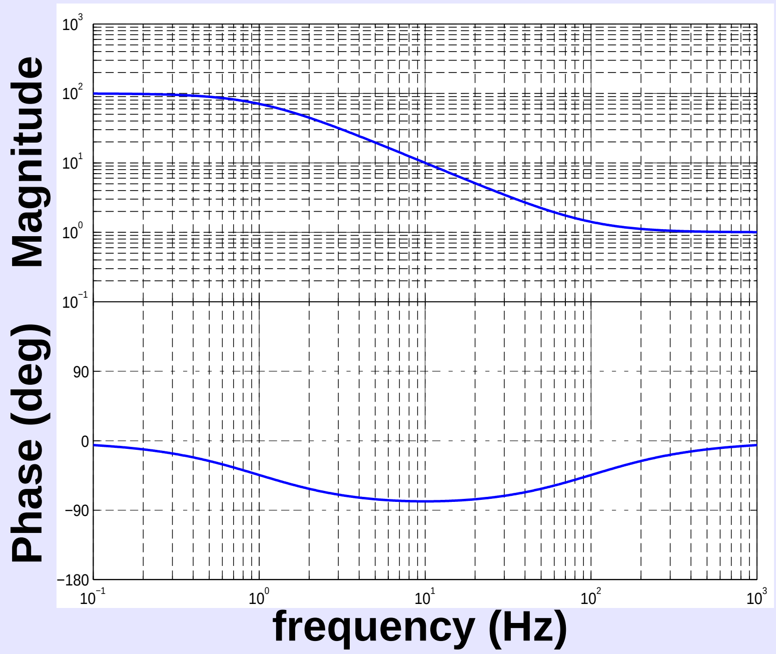
  $x \sim 10^{-9}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$

- **Mirrors drift with respect to the local frame**

  $\Rightarrow \delta\theta_i \ge 10^{-7}$ **rad**

SOME CONTROL SYSTEMS TERMINOLOGY

- **Transfer function (frequency response): magnitude and phase of output when input is a sinusoid of unit mag. and frequency *f*.**
- **Bode diagram**



$\mathrm{dB} = 20 \log_{10} G(f)$

  - **Pole: magnitude falls off with f ($f > f_0$), phase lags**
  - **Zero: magnitude increases with f ($f < f_0$), phase leads**

MORE CONTROL SYSTEMS PHENOMENA

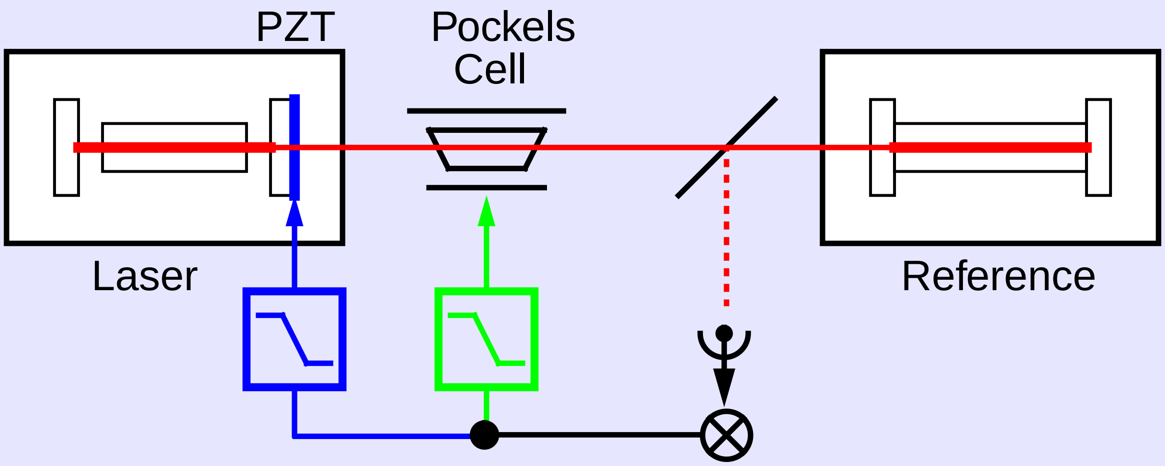
- Stability: control action must correspond to error signal throughout the control band

$x_i = \frac{x_d}{1 + G(f)}$ but $x_i \to \infty$ when $G(f) \to -1$

- Gain: gain must be high enough to adequately suppress noise input
- Bandwidth: frequency range over which control action is useful
  - limited by sensor noise, actuator response
- Dynamic range: maximum signal cf. noise of device
- Multiple actuators: “cross-overs”
  - use each actuator in frequency range where it is most effective BUT make sure it does not interfere with the other at frequencies where its control action is not good

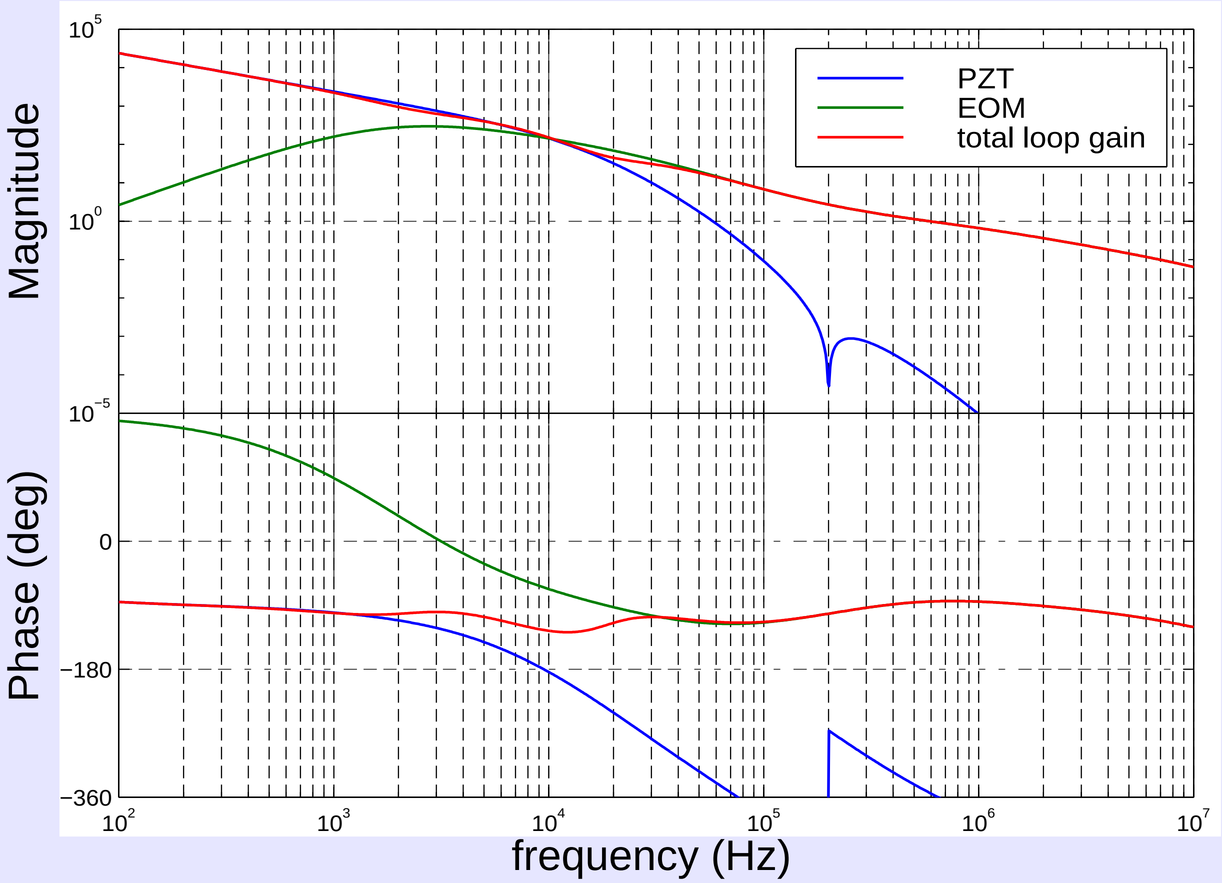
A SIMPLE(ISH) EXAMPLE: THE PRE-STABILIZED LASER

- ❑ Two frequency actuators: PZT and EOM
  - ❍ actuator 1 (PZT): large dynamic range (~400 MHz) but small bandwidth (resonances above ~200 kHz)
  - ❍ actuator 2 (EOM): small dynamic range (~200 kHz), no DC response, but works out to ~100 MHz



- ❑ Design filters so that
  - ❍ both actuation paths work together (no fighting)
  - ❍ preserve dynamic range
  - ❍ high gain where needed
  - ❍ high bandwidth where needed
  - ❍ stable!

OPEN LOOP GAINS FOR PSL FREQUENCY LOOPS

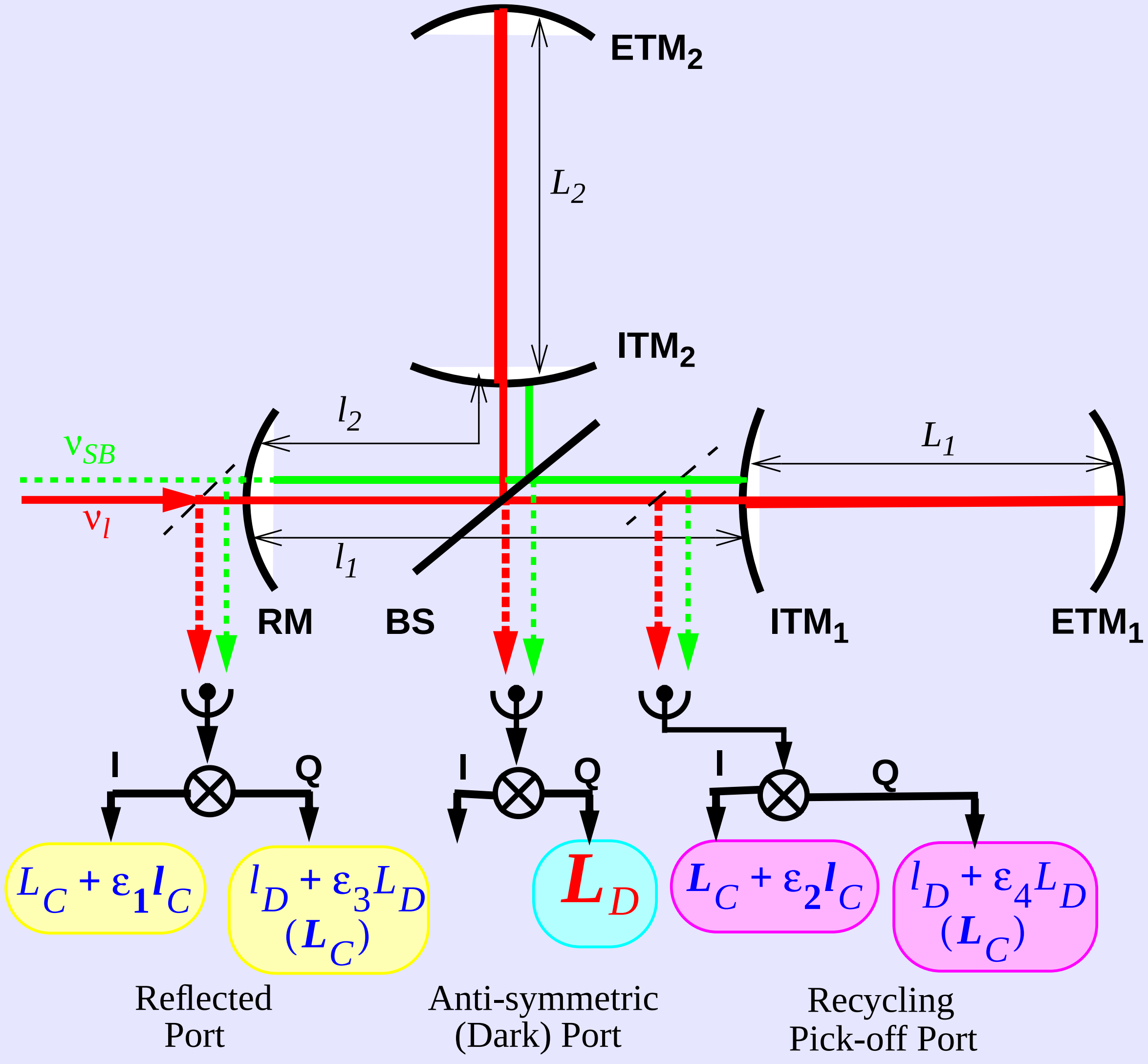


$$G_{tot} = G_{PZT} + G_{EOM}$$

LENGTH SENSING ERROR SIGNALS

- ❑ Heterodyne phase detection techniques
  - ❍ Use phase modulated light at input (*f* ~ 10–50 MHz)
  - ❍ Cavity lengths: Pound-Drever-Hall reflection locking
    (carrier that is resonant in cavity beats against rf sidebands that are not)
  - ❍ Antisymmetric port: Schnupp asymmetry / suppressed carrier scheme
    (dark port is not perfectly dark for rf sidebands)
- ❑ Signals proportional to various ifo lengths given by magnitude and phase of carrier and rf sidebands at each sensing port
  - ❍ Antisymmetric or “dark” port (GW signal!)
  - ❍ Reflection port (coupling important, i.e. sensitive to ifo loss du jour)
  - ❍ Recycling cavity pick-off port (“picked off” light is loss in rec. cav., i.e. small signal levels)

SIGNAL EXTRACTION



$L_C + \varepsilon_1 \boldsymbol{l}_C$

$l_D + \varepsilon_3 L_D$
$(\boldsymbol{L}_C)$

Reflected
Port

L_D

Anti-symmetric
(Dark) Port

$L_C + \varepsilon_2 \boldsymbol{l}_C$

$l_D + \varepsilon_4 L_D$
$(\boldsymbol{L}_C)$

Recycling
Pick-off Port

OPTICAL PLANT

❑ Antisymmetric port

$$S_{anti} \propto -g_{cr} t_{sb} r_c' k\Delta L_D \frac{1}{1+s_c} \sin\omega_m t + g_{cr} t_{sb} r_c k\Delta l_D \frac{1}{1+s_c} \sin\omega_m t$$

❑ Reflection port

$$S_{refl} \propto -g_{sb} t_{sb} r_{cr} \hat{r}_c k\Delta L_D \sin\omega_m t - g_{sb} t_{sb} r_{cr} k\Delta l_D \sin\omega_m t$$

$$+ g_{cr}^2 r_{sb} r_c' k\Delta L_C \frac{1}{1+s_{cc}} \cos\omega_m t$$

$$-\left[g_{cr}^2 r_{sb} r_c + g_{sb}^2 r_{cr} r_M\right] k\Delta l_C \frac{1+s_r}{1+s_{cc}} \cos\omega_m t$$

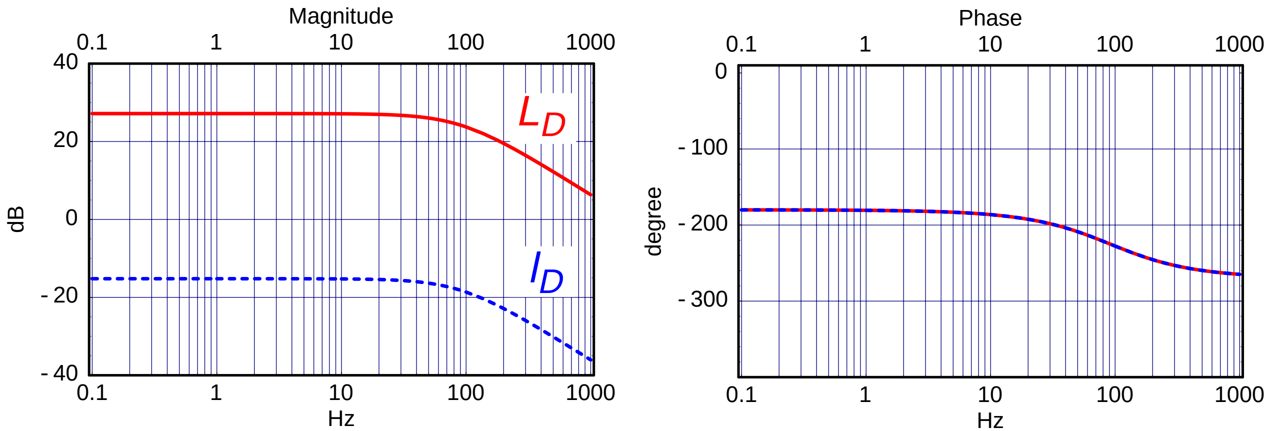
❑ Recycling cavity pick-off port

$$S_{pick\text{-}off} = \frac{g_{cr} g_{sb}}{t_5} t_{sb} \hat{r}_c k\Delta L_D \sin\omega_m t + \frac{g_{cr} g_{sb}}{t_5} t_{sb} k\Delta l_D \sin\omega_m t$$

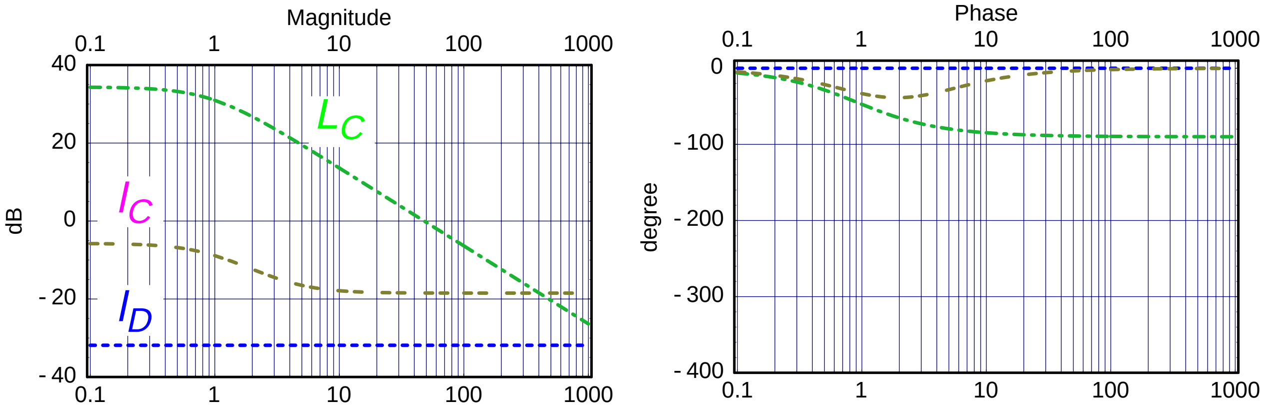
$$-\frac{g_{cr}^2 g_{sb}}{t_5} r_M r_c' k\Delta L_C \frac{1}{1+s_{cc}} \cos\omega_m t$$

$$+\frac{g_{cr} g_{sb}}{t_5} r_M r_c [g_{cr} - g_{sb}] k\Delta l_C \frac{1+s_p}{1+s_{cc}} \cos\omega_m t$$

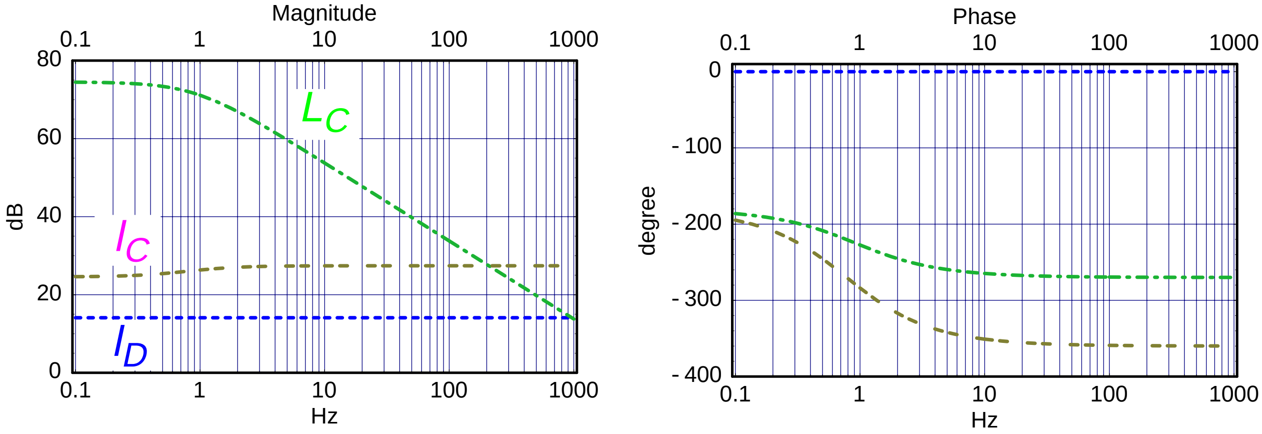
FREQUENCY DEPENDENCE



at the anti-symmetric port



in reflection of the interferometer



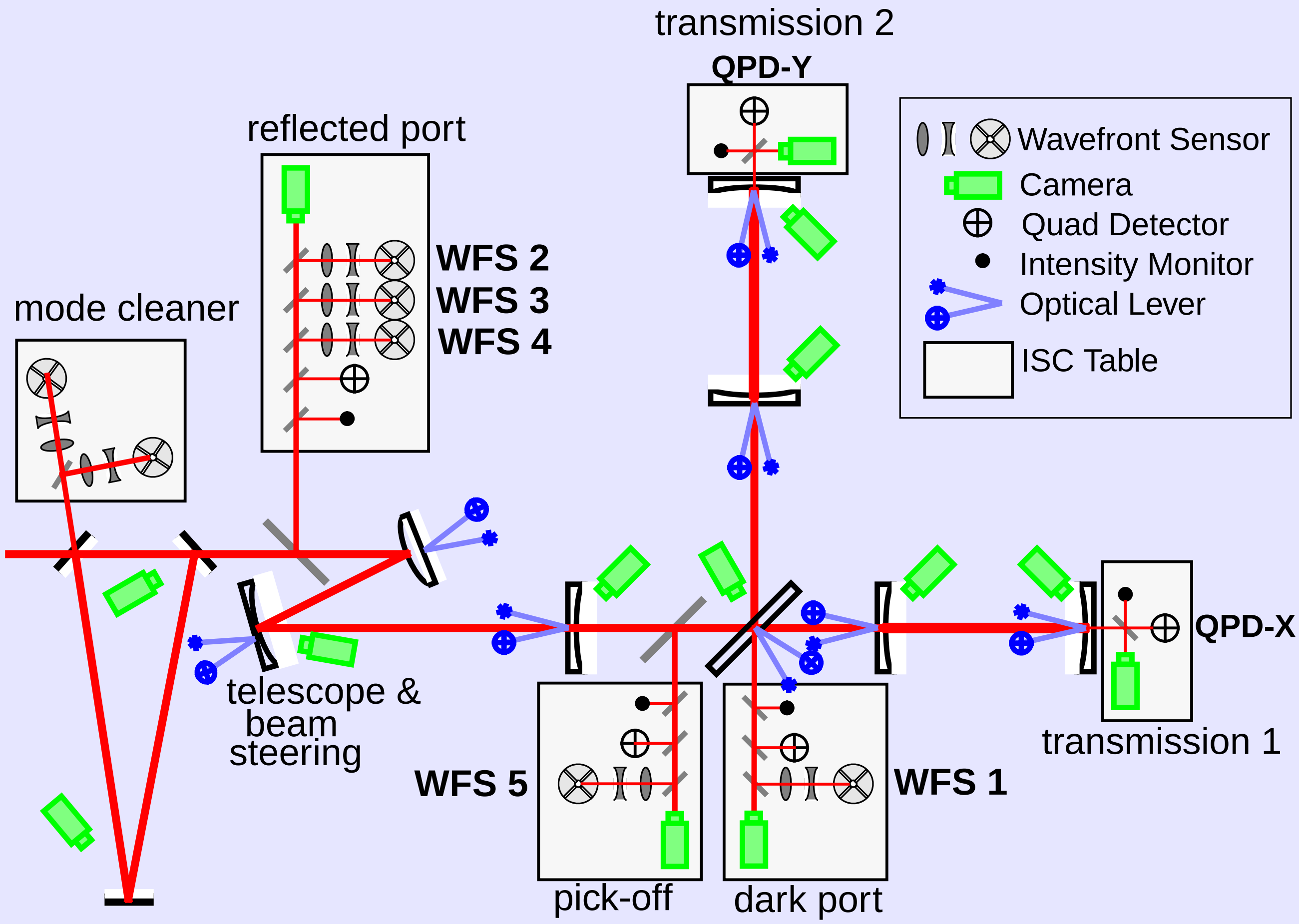
recycling cavity pick-off

ALIGNMENT SENSING

Angular misalignments excite higher-order transverse modes

- ❑ **TEM_{10} amplitude $\propto$ misalignment angle**
- ❑ **Wavefront sensor measures TEM_{10} ampl.**
    - ❍ **Length sensor signal:**
      beating of carrier TEM_{00} field against sideband TEM_{00} field
    - ❍ **Wavefront sensor signal:**
      Beating of carrier TEM_{00} field against sideband TEM_{10} field
      ➪ spatial map of this TEM_{10} mode at modulation frequency
      ➪ segmented photodetector
    - ❍ **Distinguish mirrors of the interferometer by Guoy phase shift**

ALIGNMENT OF LIGO INTERFEROMETERS



ALIGNMENT SENSING MATRIX

- **Signal at each sensor in Watts per normalized angle ($\theta_D \sim 10^{-5}$ rad)**

Wavefront Sensor				**Angular degree of freedom**				
# (port)	f_{mod}	Φ_{RF}	Φ_{Guoy}	ΔETM	ΔITM	$\overline{ETM}$	$\overline{ITM}$	***RM***
WFS 1 (dark)	f_{res}	Q	90°	-0.044	-0.02	0	0	0
WFS 2 (refl)	f_{res}	I	145°	0	0	-2.0×10^{-3}	0.026	-0.041
WFS 2b (refl)	f_{res}	Q	145°	9.6×10^{-5}	$\mathbf{-5.8 \times 10^{-3}}$	0	4.6×10^{-4}	-7.0×10^{-4}
WFS 3 (refl)	f_{NR}	I	0°	0	0	-7.0×10^{-4}	-3.2×10^{-4}	$\mathbf{7.3 \times 10^{-3}}$
WFS 4 (refl	f_{NR}	I	90°	0	0	$\mathbf{-8.0 \times 10^{-3}}$	$\mathbf{-3.7 \times 10^{-3}}$	6.4×10^{-4}
WFS 5 (pick-off)	f_{res}	Q	145°	6.5×10^{-4}	-0.039	0	3.2×10^{-3}	-4.4×10^{-3}

- **Frequency dependence given by transverse mode spacing rather than cavity storage time, i.e. flat response out to ~10 kHz**

... Last Week
This week...

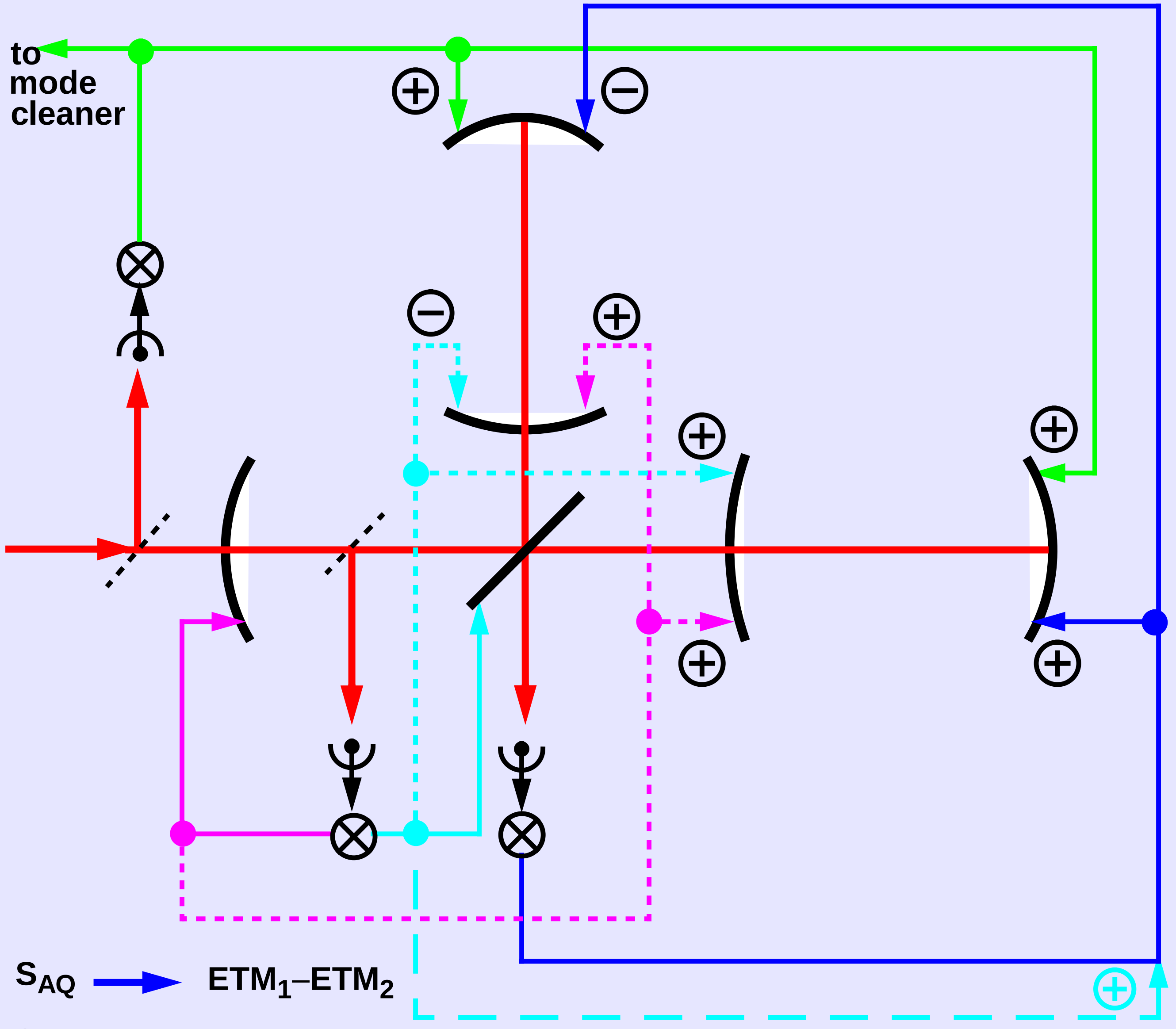
- **Review**
  - **Closed loop control system (SISO)**
    - Plant is disturbed ⇒ Sensor detects disturbance ⇒ Controller converts into control action ⇒ actuator fights disturbance
  - **Multiple actuation paths (SIMO): PSL**
    - Control action must be coordinated
  - **LIGO interferometer (MIMO)**
    - Optical plant ⇒ 4 (coupled) length degrees of freedom
frequency response of 4 sensors
- **Today**
  - **Control system requirements:**
    - Noise inputs: couplings via optical plant; sensing noise; actuator response ⇒ GAIN CONSTRAINTS
  - **Matrix solutions for MIMO system**
  - **Differential-mode (GW) loops**
    - Non-diagonal (or “crossed”) controller ⇒ feedforward path
  - **Common-mode loops**
    - Nested loops ⇒ closed loops within closed loops
  - **GW strain sensitivity: noise contributions**

LENGTH SENSING MATRIX

- **Signals in amperes of photocurrent per meter of length offset assuming current best know LIGO optical parameters using FFT model to include mirror losses**

Length Sensor		**Length Degree of Freedom**			
Port (Sensor)	Φ_{RF}	L_D	l_D	L_C	l_C
Anti (S_{AQ})	Q	$\frac{-9.7\times10^{9}}{1+s/(2\pi\times90\ \mathrm{Hz})}$	-7.4×10^{7}	0	0
Refl (S_{RQ})	Q	$\frac{6.3\times10^{4}}{1+s/(2\pi\times90\ \mathrm{Hz})}$	7.2×10^{6}	0	0
Pick-off (S_{PQ})	I	$\frac{4.5\times10^{3}}{1+s/(2\pi\times90\ \mathrm{Hz})}$	5.3×10^{5}	0	0
Refl (S_{RI})	I	0	0	$\frac{2.2\times10^{10}}{1+s/(2\pi\times0.8\ \mathrm{Hz})}$	$1.4\times10^{8}\ \frac{(1+s/(2\pi\times3.0\ \mathrm{Hz}))}{1+s/(2\pi\times0.8\ \mathrm{Hz})}$
Pick-off (S_{PI})	I	0	0	$\frac{-5.3\times10^{8}}{1+s/(2\pi\times0.8\ \mathrm{Hz})}$	$-1.6\times10^{6}\ \frac{(1-s/(2\pi\times0.9\ \mathrm{Hz}))}{1+s/(2\pi\times0.8\ \mathrm{Hz})}$

CONTROL TOPOLOGY



S_{AQ} → ETM_1-ETM_2

S_{RI} → ETM_1+ETM_2

S_{PI} → RM ···→ ITM_1+ITM_2

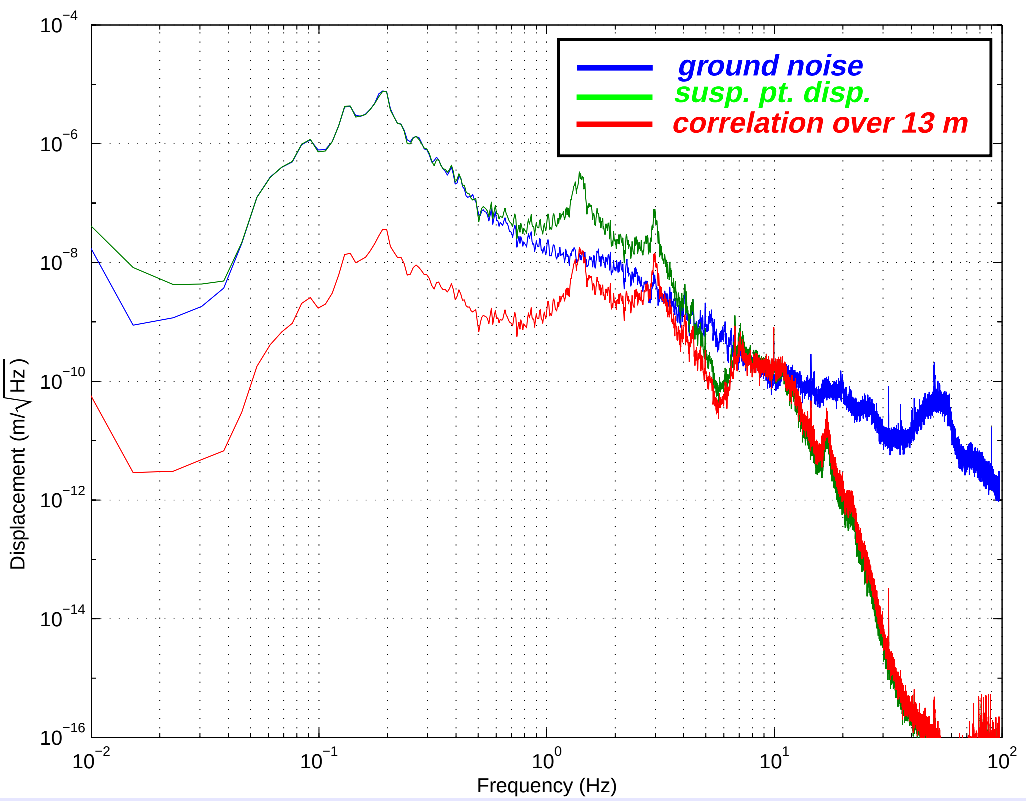
damping

S_{PQ} → BS ···→ ITM_1-ITM_2

- - → Crossed control

NOISE INPUTS: SEISMIC NOISE

❑ Correlation model



D.o.f.	***rms***	***p-p***	***ratio***
Ground	1.9 µm	12.4 µm	6.5
RM s.p.	2.0 µm	12.8 µm	6.4
Lm s.p.	3.0 µm	19.0 µm	6.3
Lp s.p.	3.3 µm	20.0 µm	6.0
lm s.p.	9 nm	63.0 nm	7.0
lp s.p.	20 nm	163 nm	8.1

SEISMIC NOISE: VERTICAL MODE

- f_V = 13 Hz (19 Hz for beamsplitter)
- Q = 2000 (may be closer to 1000 sitting on stack)
- Coupling: $s_V = \xi \cdot \tilde{z}_g(f_V) \cdot T_{zz} \cdot \sqrt{Q_V \cdot f_V}$;

  $z_g = 10^{-9}\ \mathrm{m_{rms}}$; $T_{zz} = 0.5$
- ξ: bulk propagation ($\alpha(n-1)$) + surface (β) effects

Degree of freedom	Displacement ($\mathrm{m_{rms}}$)	Coupling coefficient (ξ)	Coupling due to
Lm	6×10^{-11}	7×10^{-4}	Earth's curvature
lm	1×10^{-9}	1×10^{-2}	20 mrad wedge angle (ITM)
Lp	6×10^{-11}	7×10^{-4}	Earth's curvature
lp	2×10^{-9}	1×10^{-2} 2×10^{-2}	20 mrad wedge angle (ITM), & 20 mrad surface angle (RM)

SENSING NOISE: SHOT NOISE

- Port power levels predicted by ‘baseline’ FFT model
- Also used to compute DC plant matrix elements

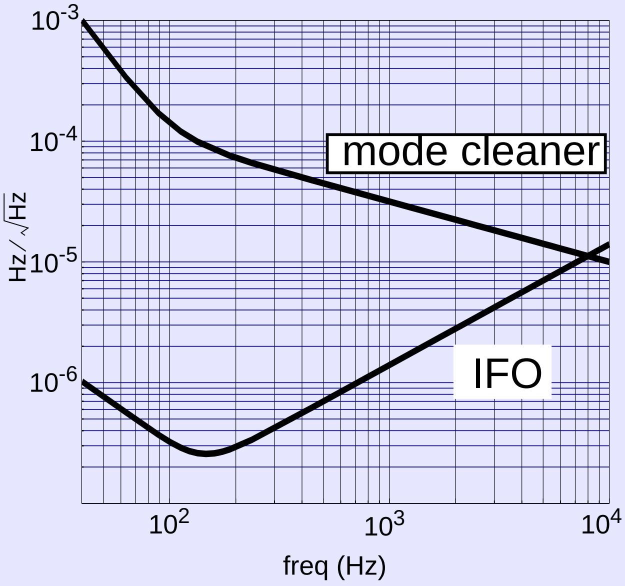
Port/Sensor	**Total Power (C + SB)**	**Noise current (A/√Hz)**	**Equivalent Length Noise (DC)**
Antisym./S_{AQ}	0.30 + 0.86 = 1.16 W	4.19×10^{-10}	$\delta\tilde{L}_m = 4.3\times10^{-20}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$
RC pickoff/S_{PI}	65 + 6 = 71 mW	9.05×10^{-11}	$\delta\tilde{l}_p = 5.6\times10^{-17}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$
RC pickoff/S_{PQ}		8.67×10^{-11}	$\delta\tilde{l}_m = 1.7\times10^{-16}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$
Reflected/S_{RI}	0.13 + 0.16 = 0.29 W	2.04×10^{-10}	$\delta\tilde{L}_p = 9.1\times10^{-21}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$
Reflected/S_{RQ}		1.53×10^{-10}	$\delta\tilde{l}_m = 2.1\times10^{-17}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$

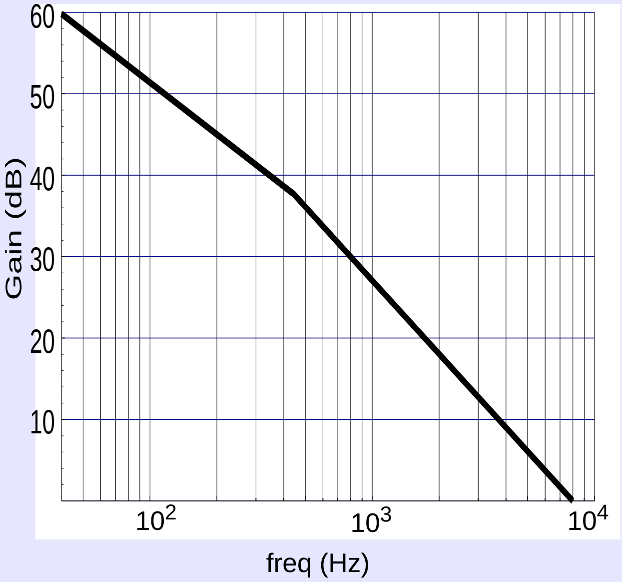
GAIN CONSTRAINTS: RESIDUAL FLUCTUATIONS

❑ IFO Length/ phase

Degree of freedom	**Residual deviation**	**Units**	**Coupling mechanism**
$\delta Lm + (\pi/(2F))\delta lm$	1×10^{-13}	m_{rms}	Amplitude noise coupling
$\delta lm + (\pi/(2F))\delta Lm$	1×10^{-9}	m_{rms}	Amplitude noise coupling
$\delta(k_l \cdot Lp)$	9×10^{-6}	rad_{rms}	Arm cavity power reduction
$\delta(k_l \cdot lp)$	7×10^{-4}	rad_{rms}	Arm cavity power reduction

❑ Laser frequency





GAIN CONSTRAINTS: MIRROR INTERNAL RESONANCES

- **Require open loop gain <1 at internal mode frequency to ensure stability**
- **Force applied at magnets is transmitted to mirror surface** (calculated by D. Coyne using FEA model with Q=1.3M)

	Mode description	*Resonant frequency (Hz)*	*Transmissibility (m/N)*	*Maximum servo gain*
Test Masses & RM	Non-axisymmetric, astigmatic mode	6595	2×10^{-10}	-6 dB
	First symmetric (drum head) mode	9206 (calc.) 9476 (meas.)	2×10^{-4}	-131 dB
	Second symmetric mode	14475	3×10^{-6}	-103 dB
Beamsplitter	Non-axisymmetric, astigmatic mode	3785	3×10^{-8}	-30 dB
	Symmetric (drum head) mode	5578	1.6×10^{-3}	-133 dB
	Second symmetric mode	14630	9×10^{-7}	-85 dB

- **Solution: 80dB stopband filter centered at 9.4kHz (+ 20 dB from anti-alias filter)**

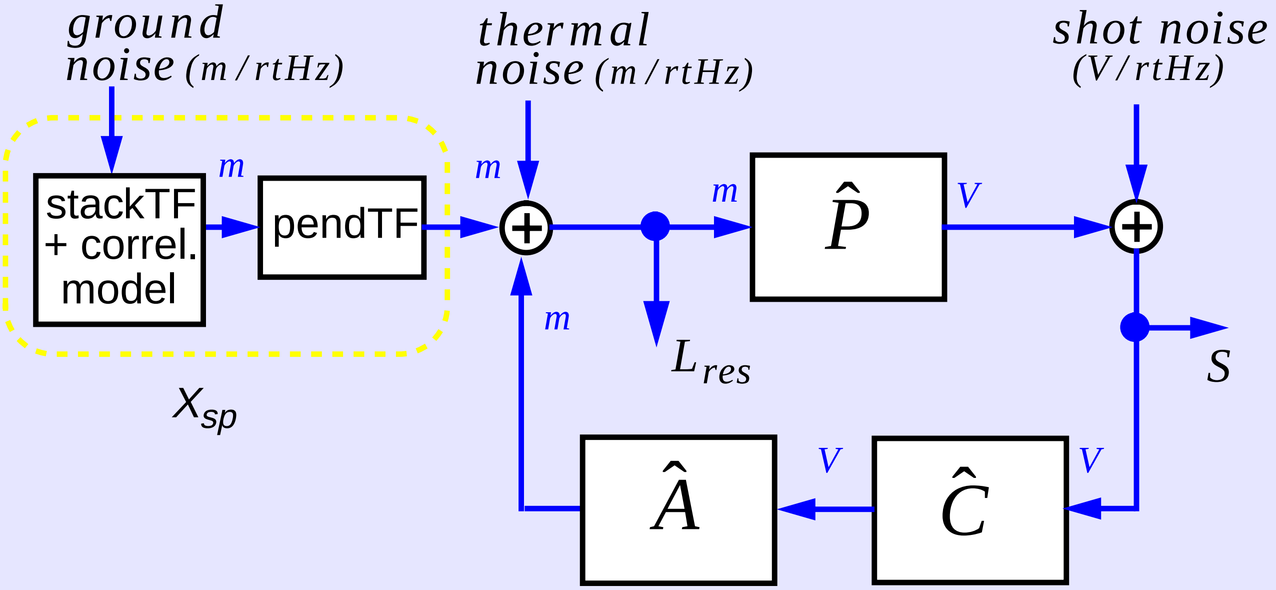
GAIN CONSTRAINTS: MISCELLANEOUS

- ❑ Phase Margins: designed to be at least 50 degrees for stability of response around unity gain frequency
  - ❍ effect of delays included in predicted phase margin (though not explicitly in Matlab models)
- ❑ Electronics noise
  - ❍ Three significant noise sources, sum must be ‘10x below’ SRD sensitivity:
    - photodetector electronics noise
    - ADC input voltage noise
    - DAC output voltage noise
- ❑ Actuator dynamic ranges
  - ❍ ranges of frequency actuators important for designing crossovers
  - ❍ suspensions: supposed to have sufficient range by design (BS & RM ranges have recently been increased so that this is so)

SUMMARY OF GAIN REQUIREMENTS

Degree(s) of freedom	Frequency range	Gain	Reason
All	DC	50 — 200 dB	achieve required residual deviation
Lm, Lp, lp (lm)	9.5 kHz (5.6 kHz)	< –135 dB	internal resonance of RM, TMs (BS)
Lm, lm, lp	13 Hz	> 20 dB resonant gain	vertical mode of suspensions
lm	> 20 Hz	< 0 dB	sensing noise at pick-off couples to GW signal via off-diagonal plant element
Lp	> 40 Hz	> –60 dB relative to overall loop gain	achieve required frequency noise suppression above 40 Hz
Lp	> 40 Hz	> –80 dB relative to overall loop gain	Lp couples to GW signal via TM suspensions imbalance
lp	DC	> 100 dB	residual lp couples to GW signal via demodulator phase error

MULTI-INPUT MULTI-OUTPUT CONTROL SYSTEM MODEL

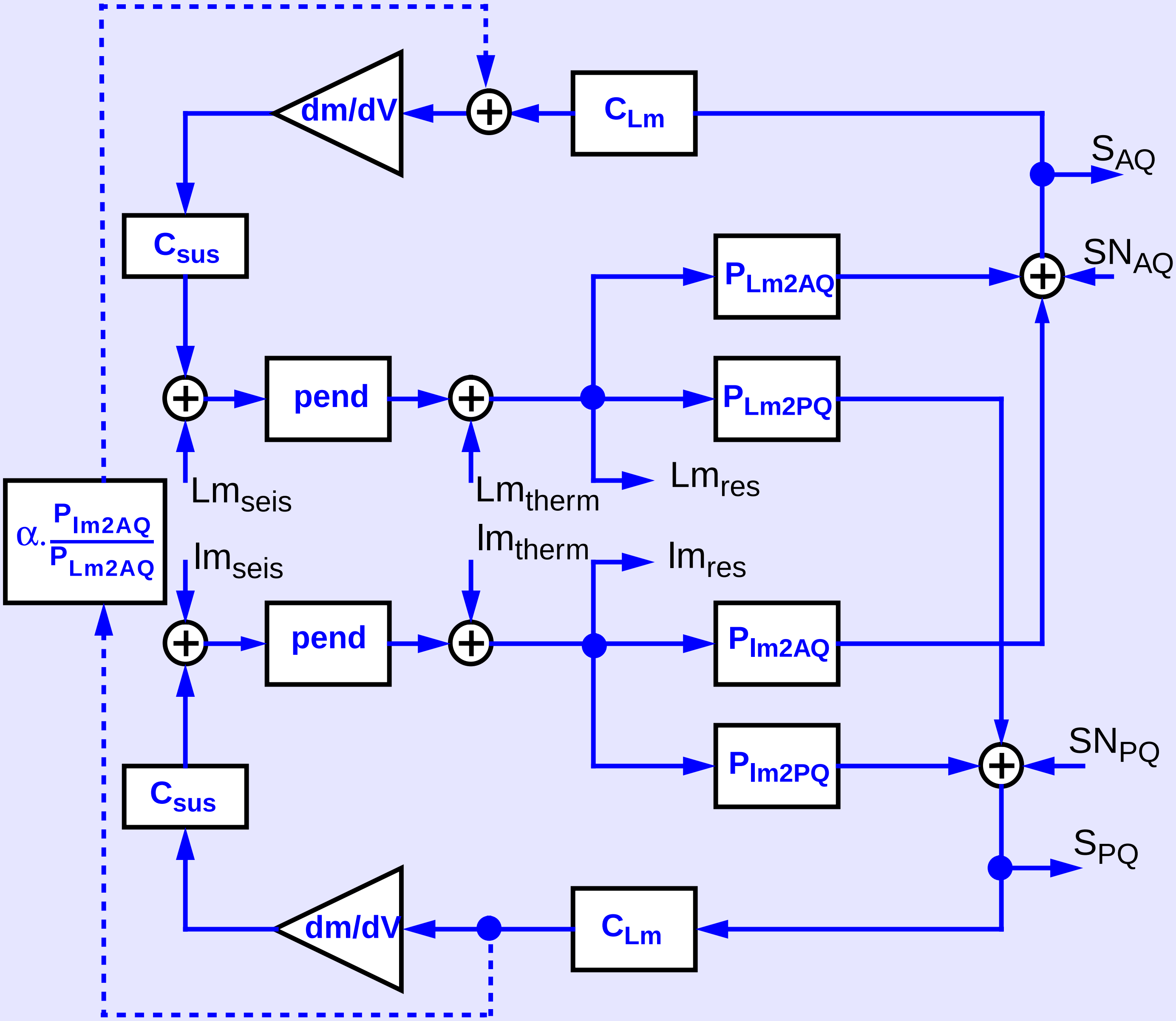


$$\vec{L}_{res} = \hat{M}^{-1}(X_{sp}\vec{L}_{gnd} + \vec{L}_{therm} + \hat{A}\hat{C}\vec{S}_{shot})$$

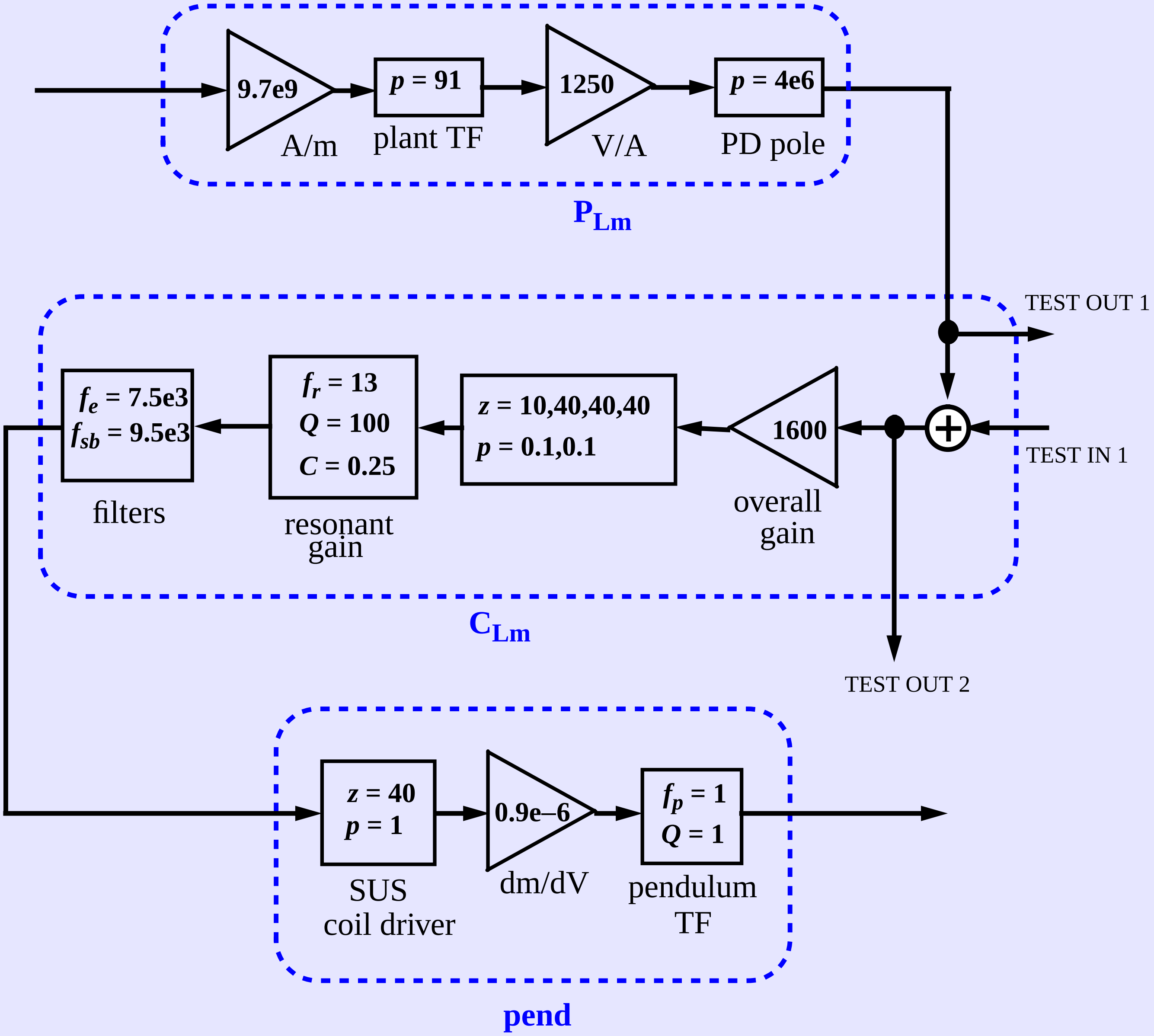
$$\hat{M} = \hat{1} - \hat{A}\hat{C}\hat{P}$$

- **plant matrix block-diagonal ⇒ treat common-mode and differential-mode d.o.f. independently**

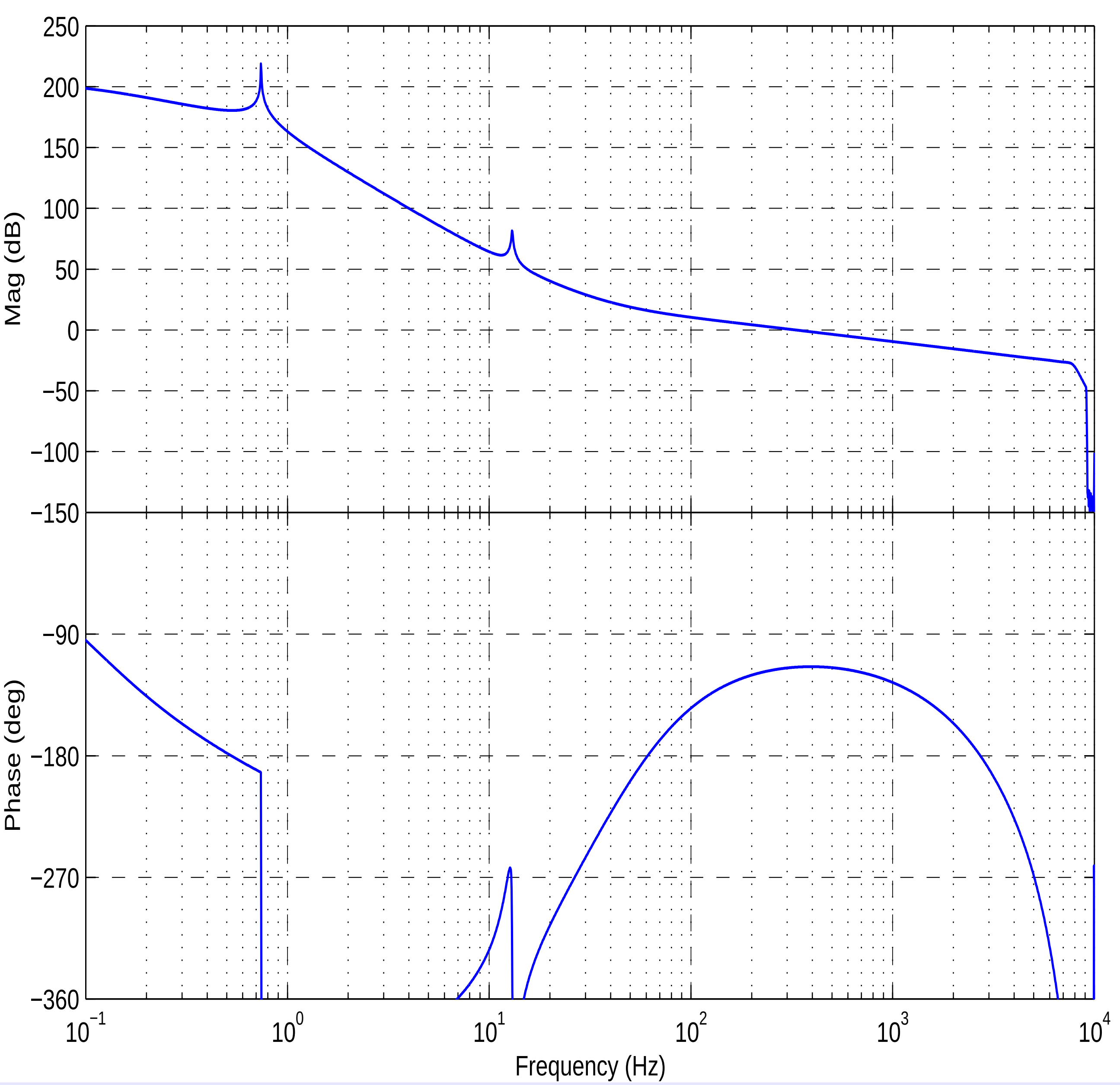
DIFFERENTIAL-MODE SYSTEM BLOCK DIAGRAM



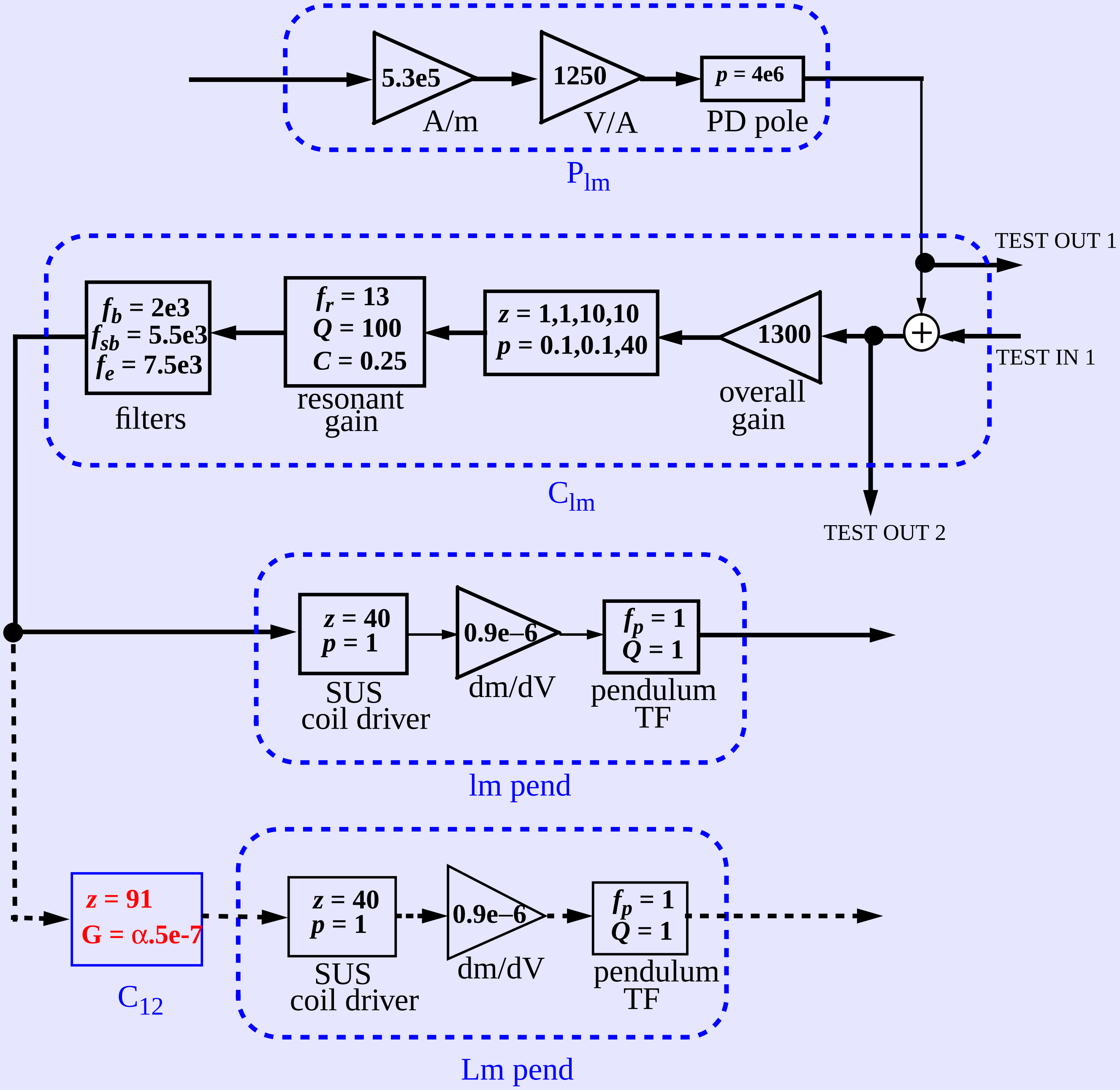
Lm SYSTEM BLOCKS



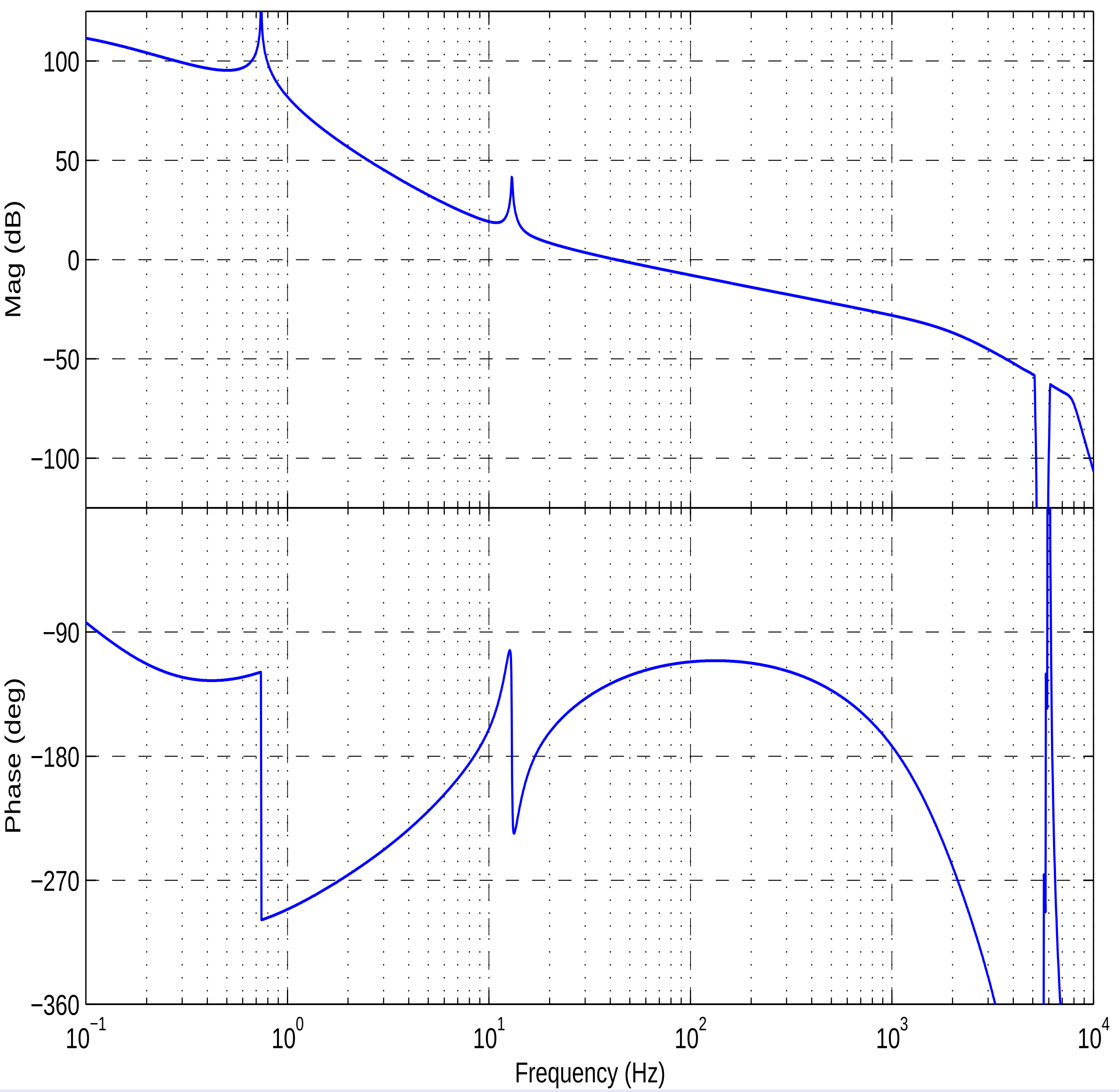
Lm OPEN-LOOP GAIN



lm SYSTEM BLOCKS

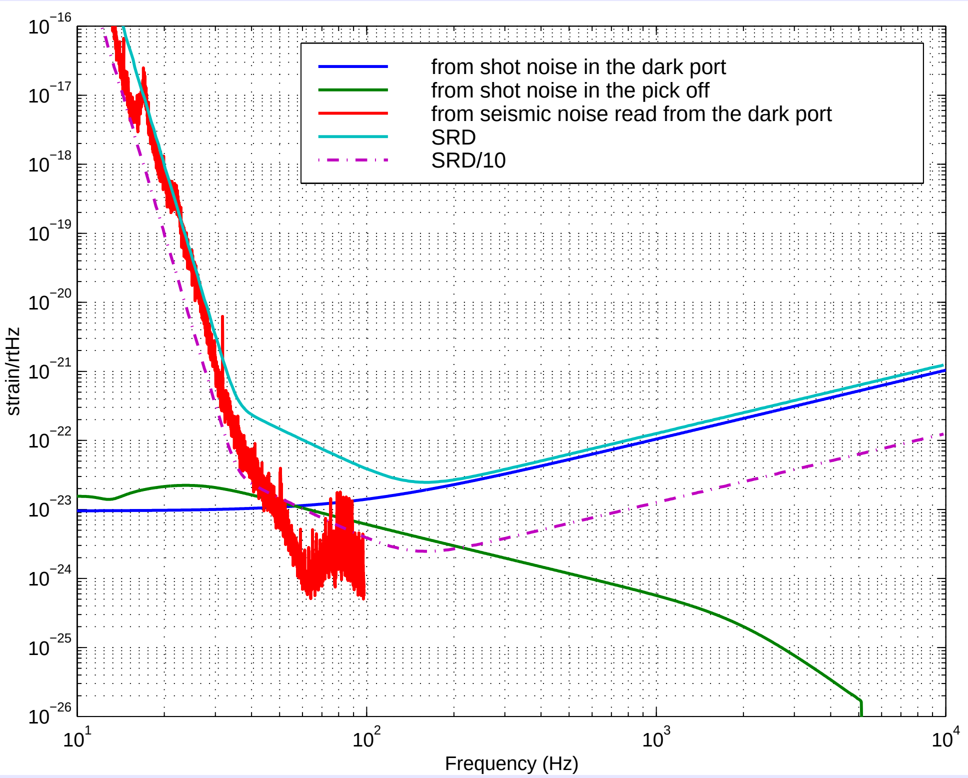


lm OPEN-LOOP GAIN



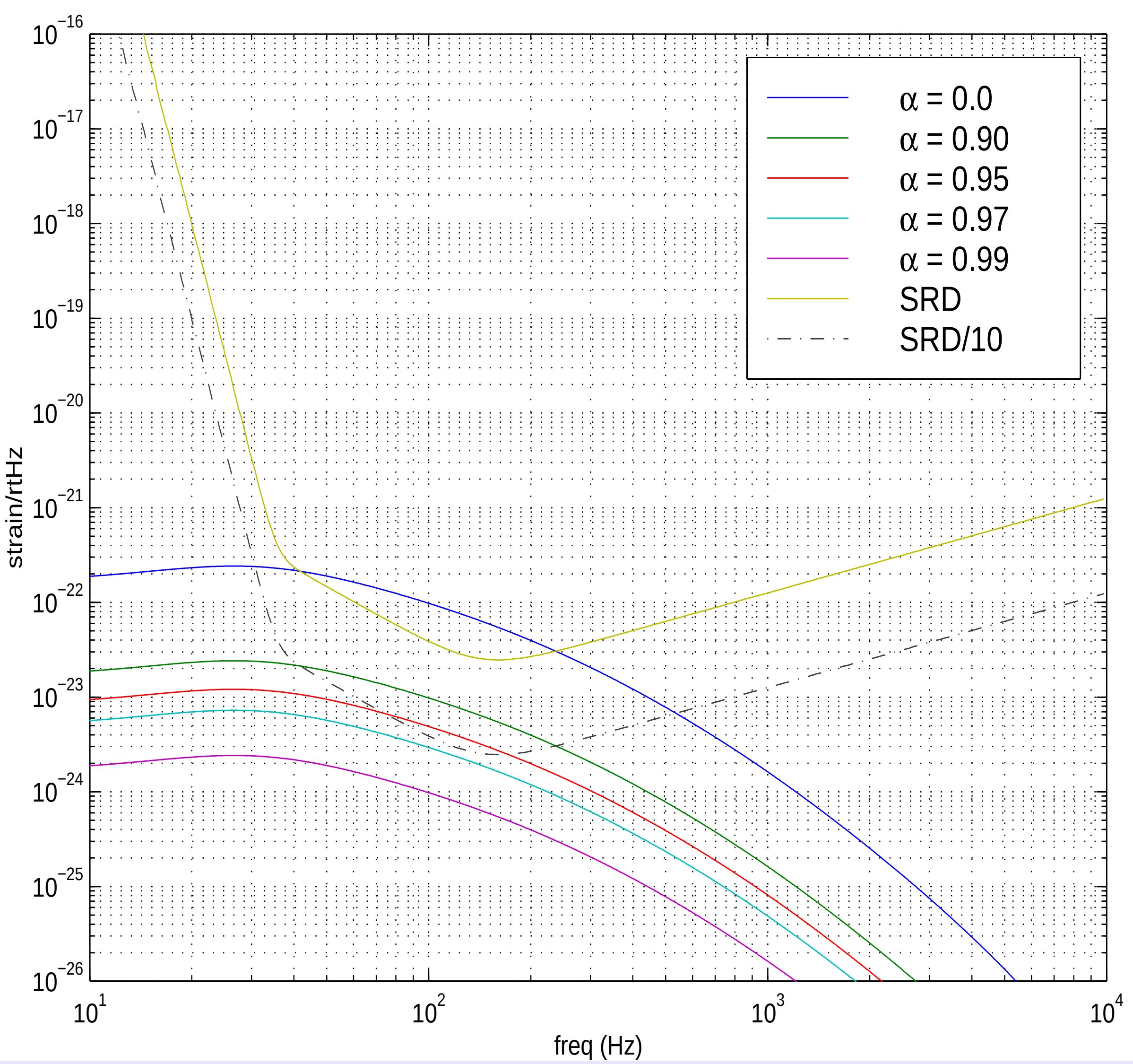
PERFORMANCE OF DIFFERENTIAL-MODE LOOPS

Performance Data	Lm	lm	Units
Gain at DC	205	110	dB
Unity gain bandwidth	330	43	Hz
Phase margin	66	55	degrees
Gain at 9.48 kHz (5.58 kHz)	–140	(–141)	dB
Residual length deviation	10^{-14}	5×10^{-12}	m_{rms}
Control signal at coil driver	3.1	0.13	μm_{rms}

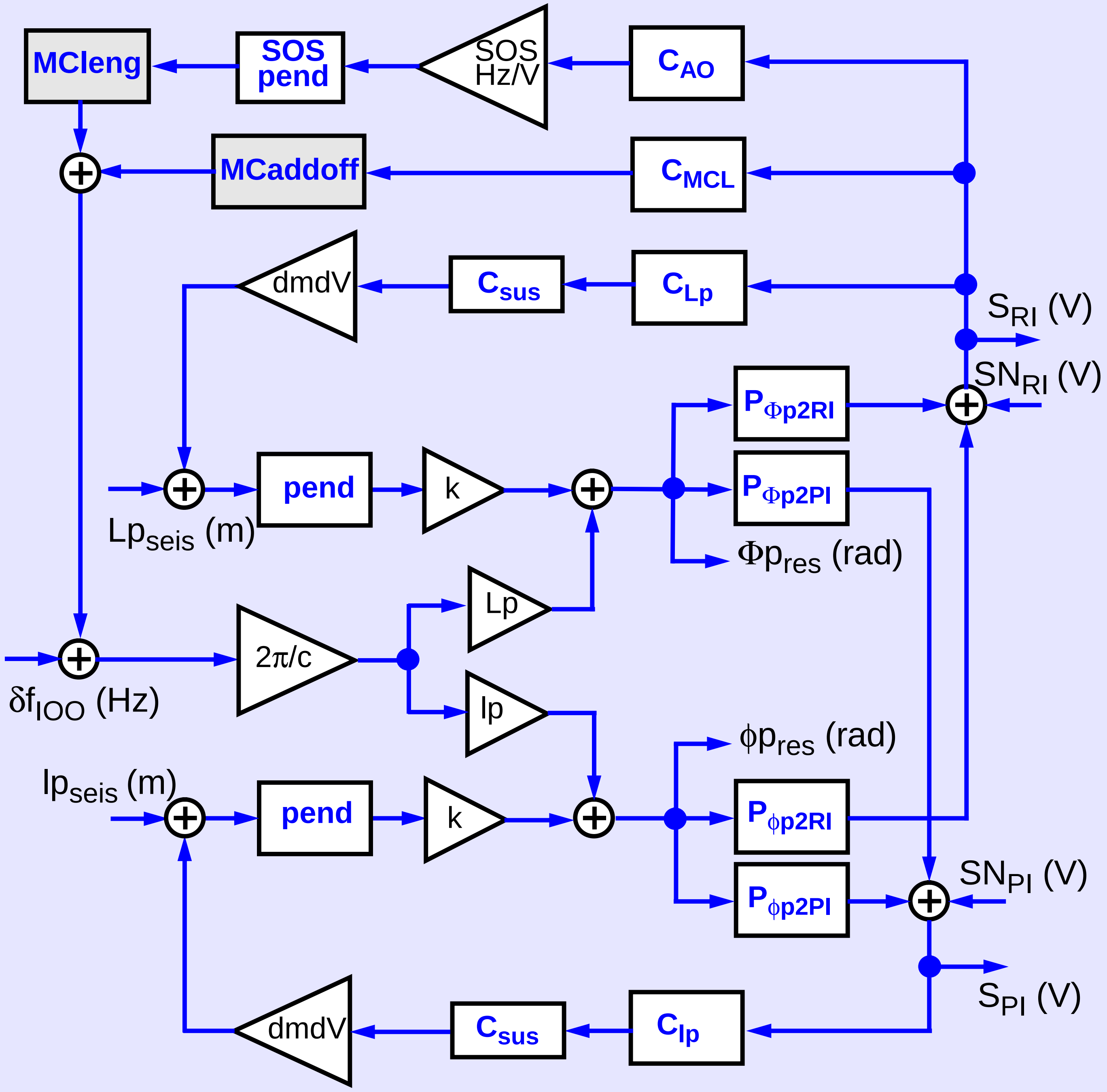


NON-DIAGONAL CONTROL MATRIX AT WORK

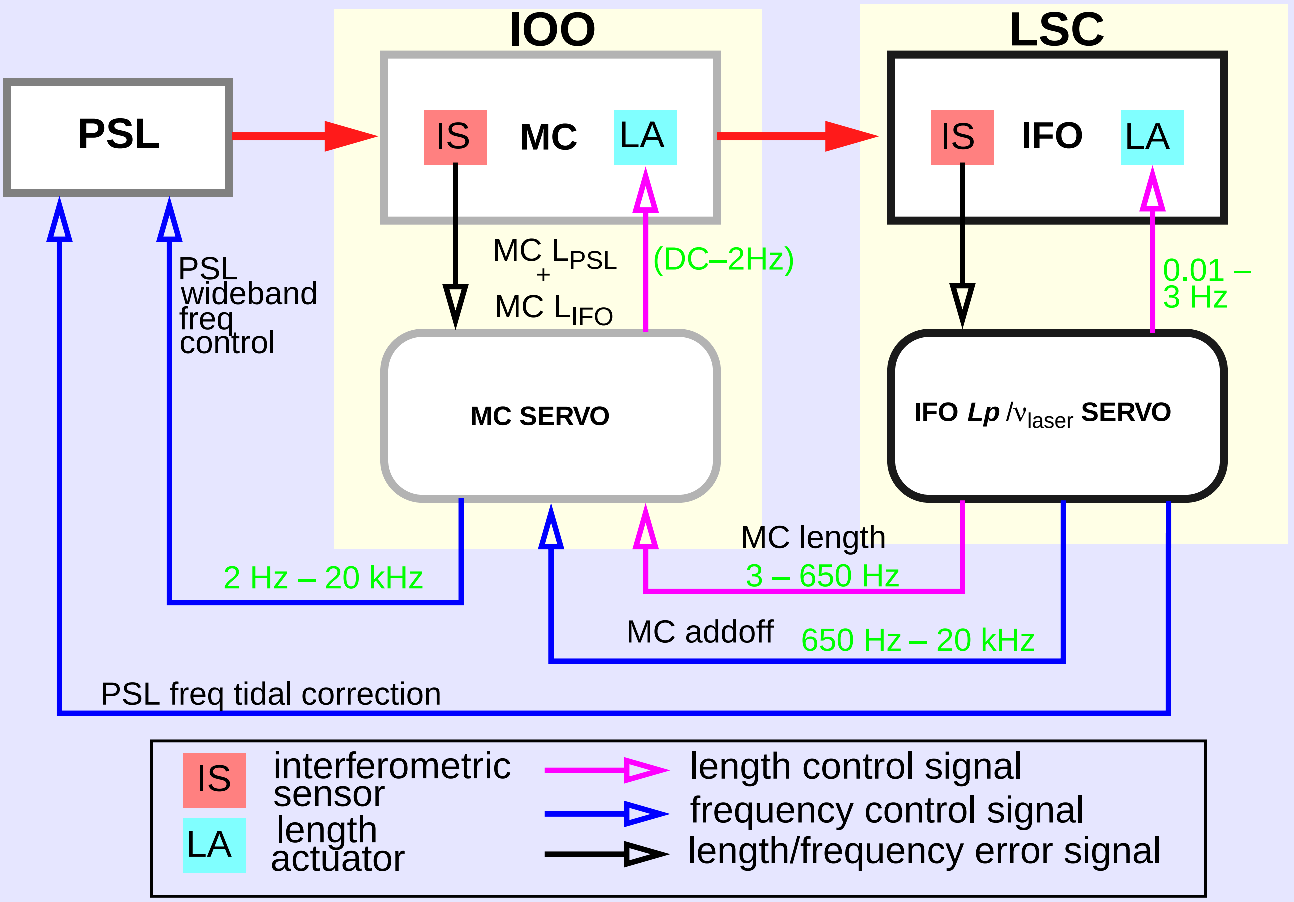
- **add *lm* control signal to *Lm* to cancel feedthrough of shot noise at S_{Pl} to GW signal**



COMMON-MODE SYSTEM BLOCK DIAGRAM



FREQUENCY CONTROL TOPOLOGY



IOO

LSC

PSL

IS MC LA

IS IFO LA

MC L_{PSL} + MC L_{IFO}

(DC–2Hz)

0.01 – 3 Hz

PSL wideband freq control

MC SERVO

IFO Lp/ν_{laser} SERVO

MC length

3 – 650 Hz

2 Hz – 20 kHz

MC addoff

650 Hz – 20 kHz

PSL freq tidal correction

IS interferometric sensor

LA length actuator

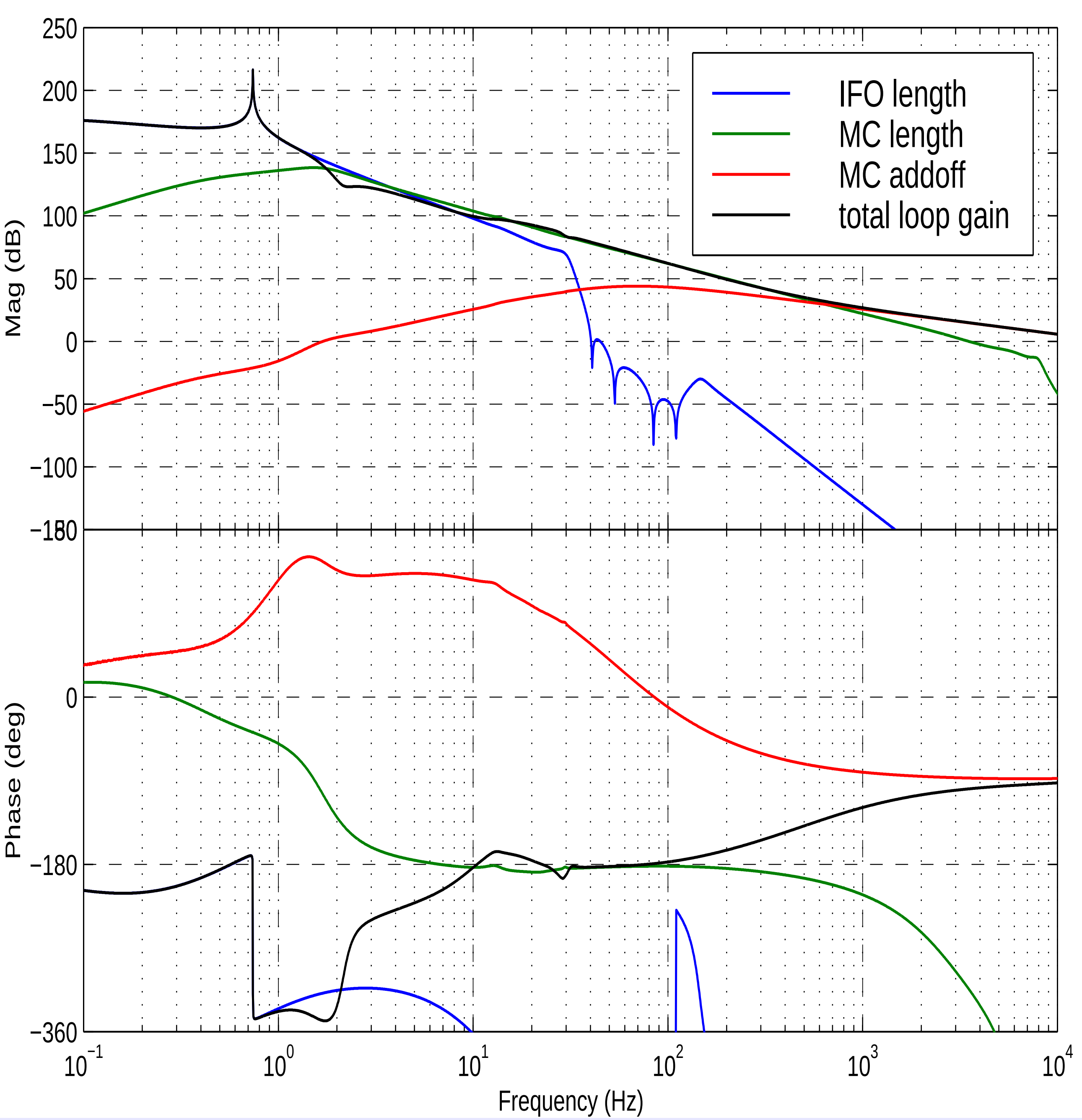
length control signal

frequency control signal

length/frequency error signal

- **Lp/frequency servo has three actuation paths**
  - *Lp* ➡ ETMs
  - δν ➡ mode cleaner length ➡ PSL wideband actuator
  - δν ➡ additive offset of mode cleaner servo error point

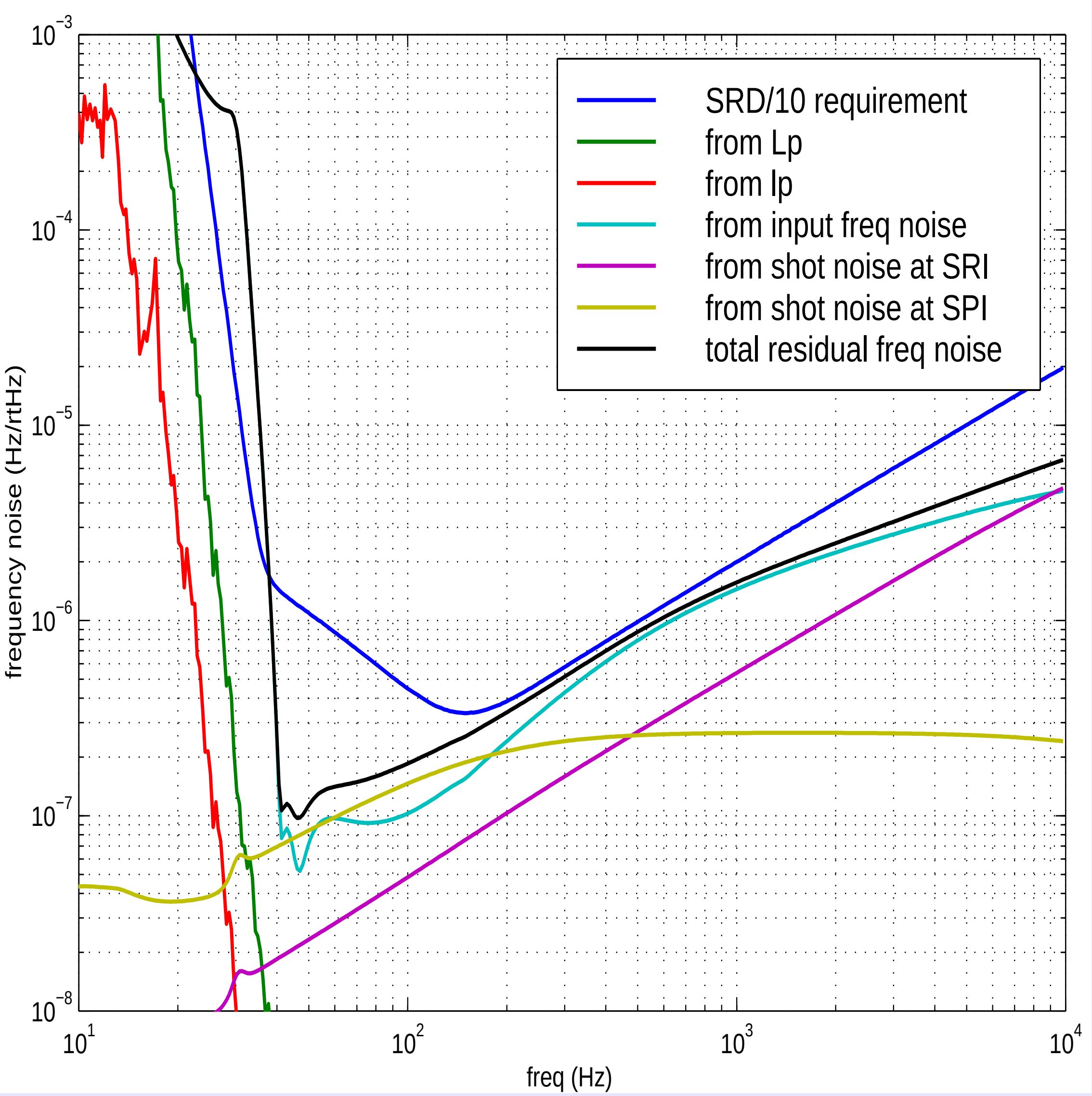
Lp/MCleng/MCaddoff
OPEN-LOOP GAINS



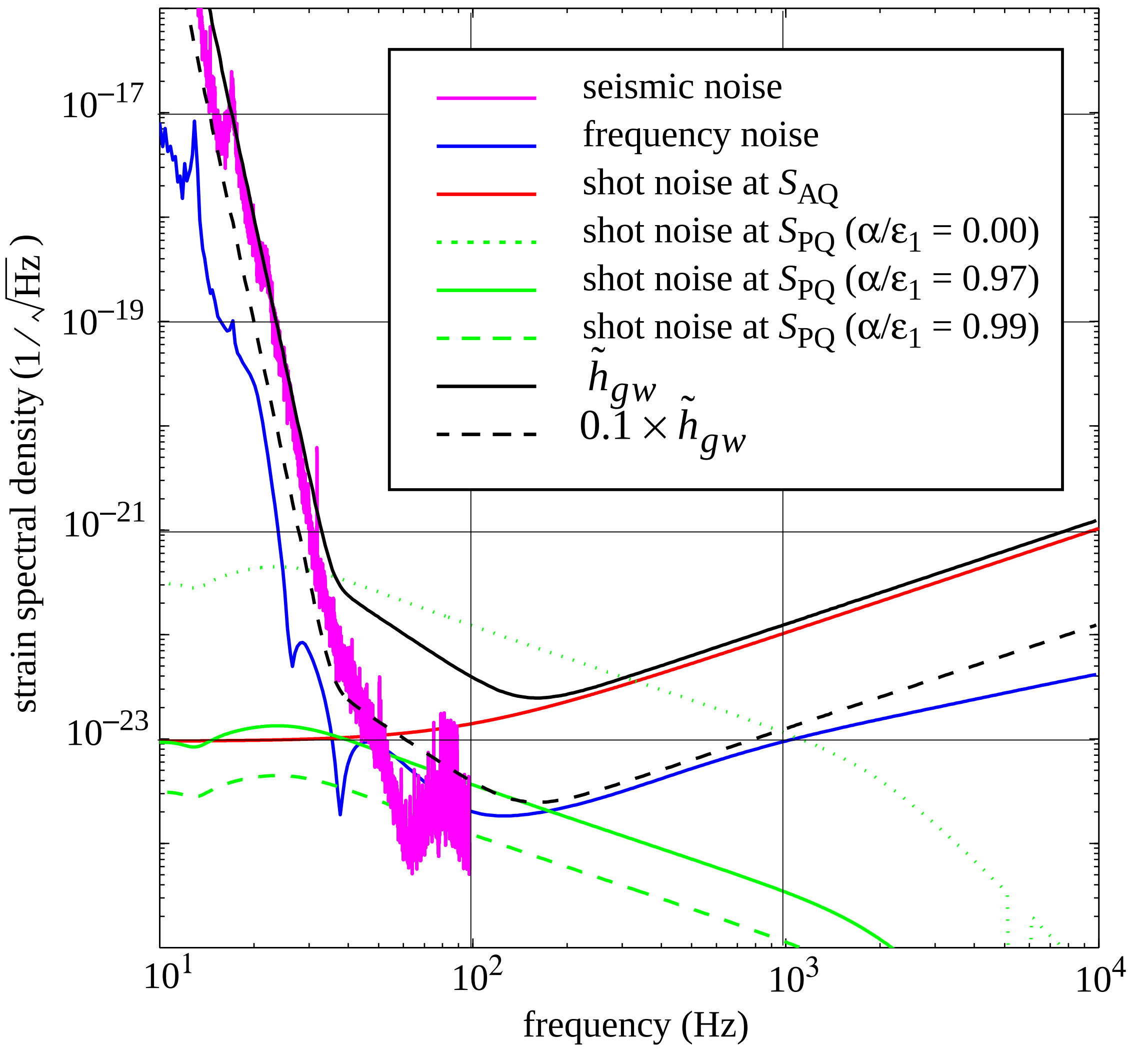
PERFORMANCE SUMMARY FOR COMMON-MODE LOOPS

Performance Data	***Lp*/df**	***lp***	**Units**
Gain at DC	180	120	dB
Unity gain bandwidth	21000	130	Hz
Phase margin	94	70	degrees
Lp/MC length crossover frequency	3.7		Hz
MC length/additive offset crossover freq.	650		Hz
Gain at 9.48 kHz	–200	–160	dB
Residual phase deviations	6×10^{-7}	4×10^{-5}	radian$_{rms}$
Error signals	0.2	0.03	V$_{rms}$
Drive signals	Lp: 3.1 MC leng: 1200 MC addoff: 2.3	lp: 1.1	µm$_{rms}$ Hz$_{rms}$ mHz$_{rms}$

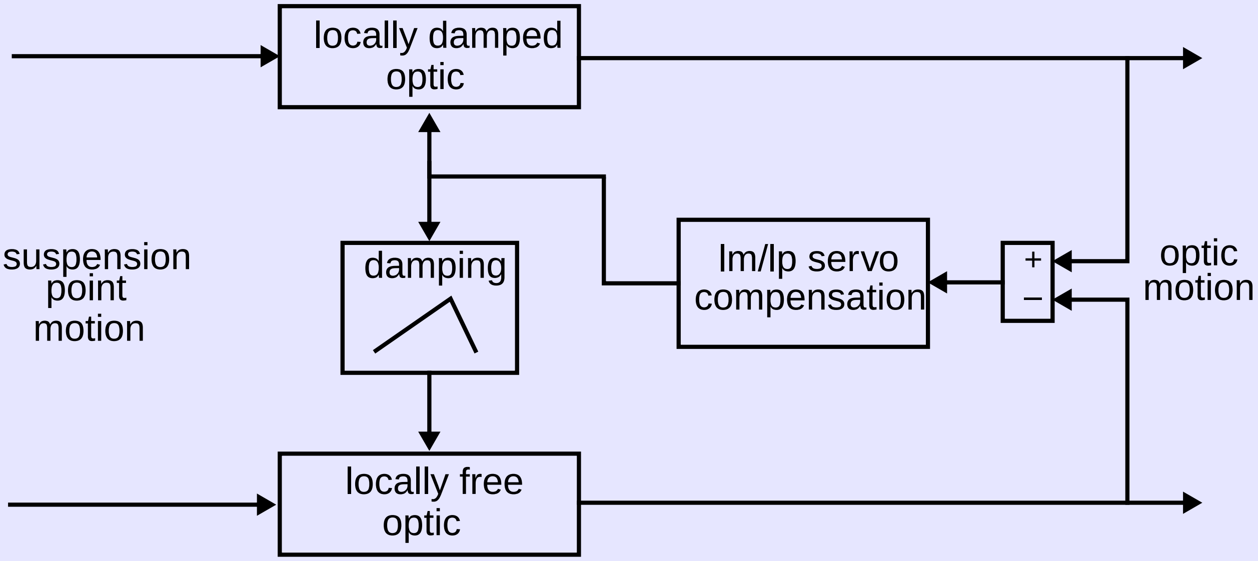
RESIDUAL FREQUENCY NOISE



GW STRAIN SENSITIVITY



TEST MASS DAMPING PATHS



locally damped optic

suspension point motion

damping

lm/lp servo compensation

+

–

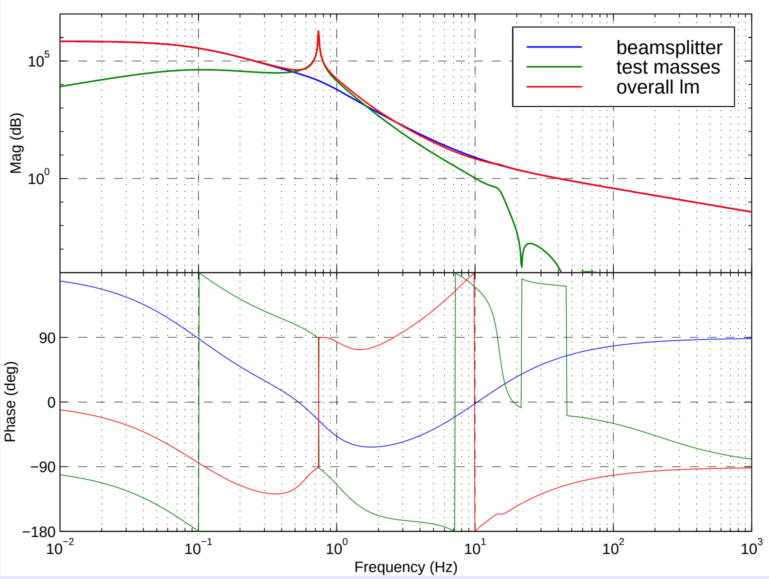
optic motion

locally free optic

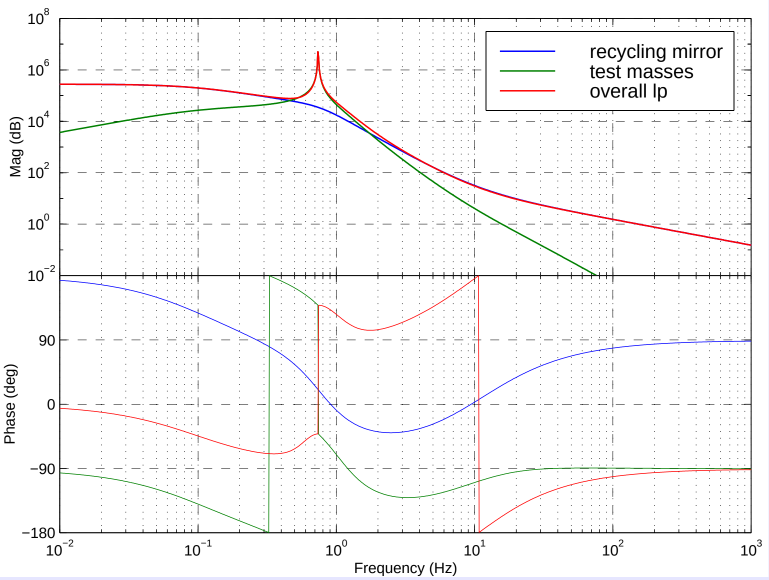
Loop	Sensitivity @ 40 Hz (m/√Hz)	GW channel coupling	Damping path gain req. @ 40 Hz	Damping servo
Recycling cavity	6×10^{-17}	5%	–35 dB	Zero @ DC 2nd order Chebyshev @1 Hz (+ pole@20 Hz for stability)
Michelson	1.5×10^{-16}	100%	–65 dB	Zero @DC 2nd order Chebyshev @1 Hz 4th order elliptic @ 20 Hz, 30 dB stop

lm AND *lp* LOOPS WITH TM DAMPING

- ❍ ***lm* open-loop gain**



- ❍ ***lp* open-loop gain**



LOOKING AHEAD...

- ❑ Detection mode controls
  - ❍ Noise couplings ⇒ estimates?
  - ❍ Optical plant ⇒ as-built? stability? variations?
  - ❍ Controllers ⇒ nearly optimum? optimization?
  - ❍ Actuators ⇒ cross-couplings? variations?
- ❑ Robust control ⇐ uncertainties
  - ❍ Model errors, parameter drifts ⇒ system identification (sysID)
  - ❍ Identify control law, control variables
  - ❍ Characterize control system elements
  - ❍ Adaptive controls
- ❑ Lock Acquisition