For a axis sapphire

$$\frac{\Delta n}{n} = \frac{0.08}{1.76}$$

$$\Gamma(300mm) = \frac{1.4mm}{\lambda} \rightarrow many\ waves$$

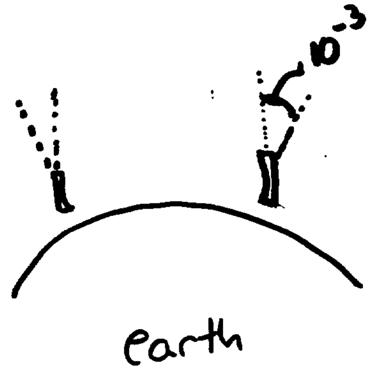
Worst case scenario

$$\Gamma = \frac{\pi}{2} \bmod \pi.$$

Can A-axis sapphire be used in an interferometer?

Peter Beyersdorf

7-21-99

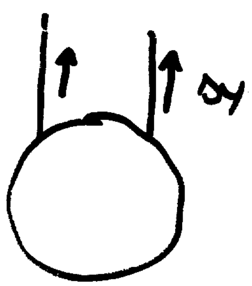


For a Vertical-longitudinal coupling of 10^{-3}

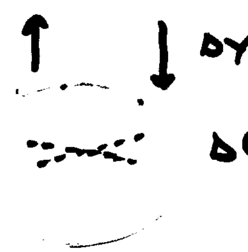
$$\Delta\tilde{y} < 10^{-16}\ \mathrm{m}/\sqrt{\mathrm{Hz}}\ @\ 100\ \mathrm{Hz}$$

to reach a reasonable sensitivity $\sim 5\cdot 10^{-23}\ /\sqrt{\mathrm{Hz}}$ @ 100 Hz

Since bounce mode **MUST** be controlled $< 10^{-16}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$



It seems reasonable that the bounce mode has a similar amplitude



$$\Delta\theta \approx \frac{\Delta Y}{d}$$

This Antisymmetric bounce mode is the Roll mode for this optic for the azimuthal alignment, while [5] and [7] require a control of the roll mode of

$$\tilde{\theta}(\omega) \leq \frac{\tilde{\phi}_{\min}(\omega)}{16\,\theta} \qquad [11]$$

for shot-noise limited performance. Relation [10] requires the azimuthal angle of birefringent optics for the power recycled Fabry-Perot Michelson interferometer be aligned to

$$\theta < 5.6\,10^{-3}\ rad \qquad [12]$$

and [11] requires the roll mode be stabilized to

$$\tilde{\theta}(100\ Hz) \leq 6.9\,10^{-11}\ rad / \sqrt{Hz} \qquad [13]$$

Fabry-Perot Mirror roll

The roll mode of the optics is degenerate for beams reflecting off the centered or passing through isotropic optics. For optics with birefringence, however, roll of the optics alters the polarization state of the light. Assuming the birefringent optics have their principle axes aligned with the polarization state of the beam, as has been proposed for Fabry-Perot mirror made of crystallin optics such as sapphire,

$$\vec{E}_{out} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\gamma} \end{bmatrix} \begin{bmatrix} E_{in} \\ 0 \end{bmatrix}. \qquad [1]$$

If the azimuthal orientation of the optic is misaligned by θ then

$$\vec{E}_{out} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\gamma} \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} E_{in} \qquad [2]$$

To second order in θ this gives

$$\vec{E}_{out} \approx \begin{bmatrix} 1-\theta^2\left(1+e^{i\gamma}\right) \\ \theta\left(1-e^{i\gamma}\right) \end{bmatrix} E_{in} \qquad [3]$$

which represents a rotation of the polarization angle by

$$\theta\left|1-e^{i\gamma}\right| \qquad [4]$$

giving an expected angle between polarization states of the two interfering beams of

$$\beta = \sqrt{2}\theta\left|1-e^{i\gamma}\right| \qquad [5]$$

The excess power on the dark port due to polarization misalignment is

$$P_{out} = \sin^2(\beta)\, P_0 \qquad [6]$$

and the power spectral density of the power due to polarization misalignment is

$$\tilde{P}_{out}(\omega) = 2\sin(\beta)\tilde{\beta}(\omega)\, P_0 \qquad [7]$$

where β is the angle between the polarization state of the interfering beams. To keep the dark port power below a threshold power P_{max} the total differential birefringence seen must be small

$$\beta < \sqrt{\frac{P_{max}}{P_0}} \qquad [8]$$

For sapphire optics with beam propagating along the a axis, the indices of refraction differ by 0.008 around a mean value of 1.76. Over the an optic of length L this gives a retardation

$$\gamma = \frac{2\pi}{\lambda}\frac{\Delta n}{\bar{n}} L \qquad [9]$$

of many radians. It is realistic to assume that there could be as much as π radians of retardation, modulo 2π for such an optic. Thus [5] and [8] give

$$\theta < \frac{1}{2\sqrt{2}}\sqrt{\frac{P_{max}}{P_0}} \qquad [10]$$

Input polarization state (i.e. Power Recycling mirror)

The power recycled Fabry-Perot Michelson is not sensitive to the alignment of the input polarization angle but can see noise if the input polarization angle fluctuates, as the path length difference of the interfering beams couples input polarization fluctuations into differential polarization fluctuations at the detector. The power that does not destructively interference at the detector is

$$P_{out} = \beta^2 P_0, \qquad [14]$$

where β is the angle between the polarization of the interfering beams. So output power fluctuations of

$$\tilde{P}_{out}(\omega) = 2\beta P_0 \tilde{\beta}(\omega) \qquad [15]$$

are present if the polarization angle fluctuates at the output. The asymmetry in path length of the power recycled Fabry-Perot Michelson interferometer converts input fluctuations in polarization angle to output power fluctuations. Consider the main interfering beams with a path length difference of $\Delta L/2$ between the beamsplitter and the Fabry-Perot cavities of the arms. A fluctuation in the input polarization $\theta(t)$ will produce fields at the output of the Fabry-Perot cavities with a fluctuating polarization that is different between the two arms by the amount the input polarization fluctuated as one beam was travelling the additional length ΔL. The Jones vector representation of the output field can be represented by

$$\begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix}_{out} = \frac{E_0}{2\sqrt{2}} \begin{bmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \end{bmatrix} - \frac{E_0}{2\sqrt{2}} \begin{bmatrix} \cos(\theta(t-\Delta L/c)) \\ \sin(\theta(t-\Delta L/c)) \end{bmatrix} \qquad [16]$$

If the Jones polarization axes x and y are chosen so that the nominal input polarization is an equal superposition of x and y polarizations the output filed can be expanded for small fluctuations around the angle $\pi/2$. This gives

$$\begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix}_{out} \approx \frac{E_0}{2} \begin{bmatrix} 1-\Delta\theta(t) \\ 1+\Delta\theta(t) \end{bmatrix} - \frac{E_0}{2} \begin{bmatrix} 1-\Delta\theta(t-\Delta L/c) \\ 1+\Delta\theta(t+\Delta L/c) \end{bmatrix} \qquad [17]$$

to first order in $\Delta\theta$. This can be simplified to

$$\begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix}_{out} \approx \frac{E_0}{2} \begin{bmatrix} -\Delta\theta(t)+\Delta\theta(t-\Delta L/c) \\ \Delta\theta(t)-\Delta\theta(t+\Delta L/c) \end{bmatrix} \qquad [18]$$

which gives a field spectral density of

$$\begin{bmatrix} E_x(\omega) \\ E_y(\omega) \end{bmatrix}_{out} \approx \frac{E_0}{2} \begin{bmatrix} -\Delta\theta(\omega)+\Delta\theta(\omega)e^{i\omega\Delta L/c} \\ \Delta\theta(\omega)-\Delta\theta(\omega)e^{i\omega\Delta L/c} \end{bmatrix} \qquad [19]$$

This can be written as

$$\begin{bmatrix} \tilde{E}_x(\omega) \\ \tilde{E}_y(\omega) \end{bmatrix}_{out} \approx iE_0 \begin{bmatrix} e^{-i\omega\Delta L/2c}\tilde{\theta}(\omega)\sin(\omega\Delta L/2c) \\ -e^{-i\omega\Delta L/2c}\tilde{\theta}(\omega)\sin(\omega\Delta L/2c) \end{bmatrix}, \qquad [20]$$

which corresponds to an output field with magnitude

$$\left|\tilde{E}_{out}(\omega)\right| \approx E_0 \sin(\omega\Delta L / c)\, \tilde{\theta}(\omega). \qquad [21]$$

The mixing of the output field with a local oscillator, used for homodyne or heterodyne detection, gives a power spectral density of the light on the detector of

$$\tilde{P}_{out}(\omega) \approx \sqrt{P_{LO} P_0}\, \frac{\omega\Delta L}{c}\, \tilde{\theta}(\omega). \qquad [22]$$

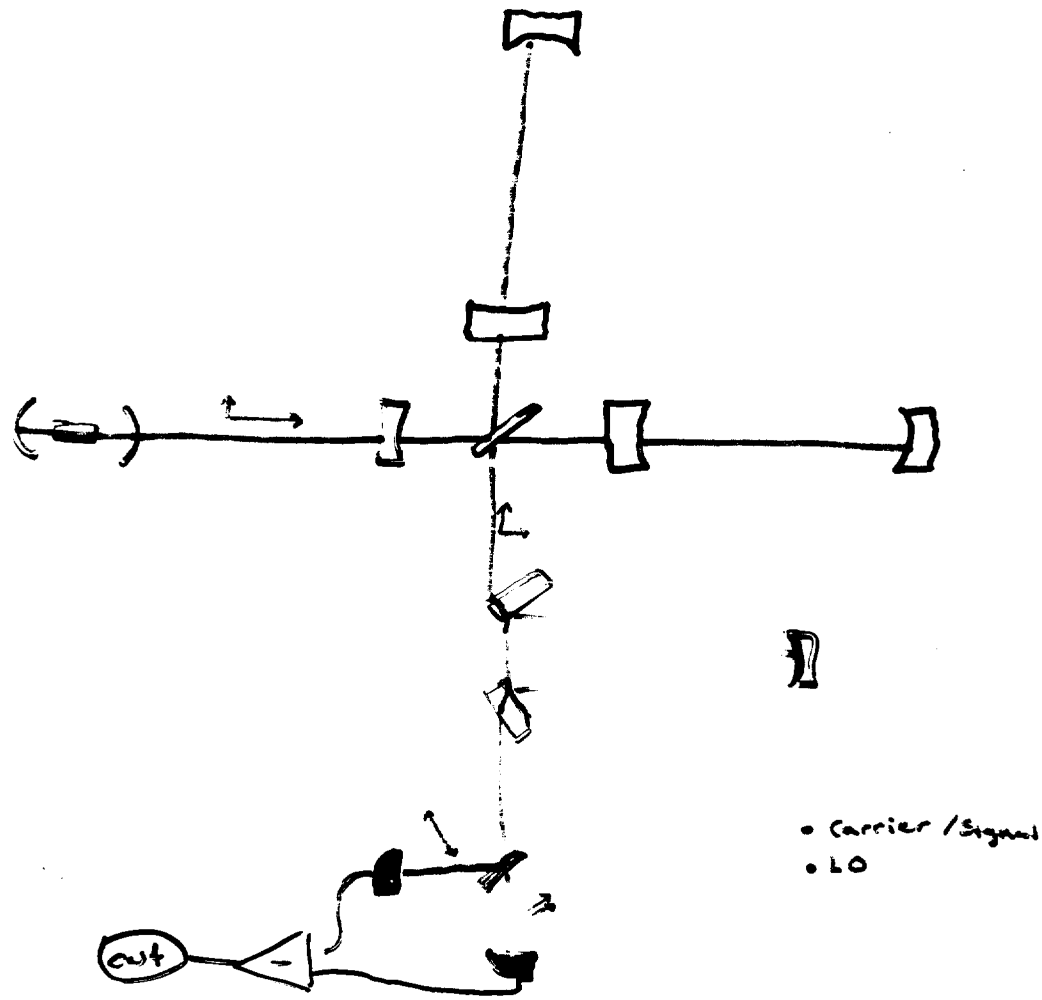
The requirement on input polarization fluctuations for the power recycled Fabry-Perot Michelson interferometer from [22] is

$$\tilde{\theta}(\omega) \leq \frac{c\tilde{\phi}_{\min}(\omega)}{\omega\Delta L}\sqrt{\frac{P_0}{P_{LO}}} \qquad [23]$$

for shot-noise limited detection which gives

$$\tilde{\theta}(100\ Hz) \leq 6.5\, 10^{-6}\ rad / \sqrt{Hz} \qquad [24]$$

Polarization based Signal extraction



- Only carrier frequency must pass through output mode cleaner
- Balanced detection removes sensitivity to amplitude noise
- LO and Signal are passively aligned and locked to each other (const. phase relationship)

Note 1, Linda Turner, 08/17/99 07:56:10 PM
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