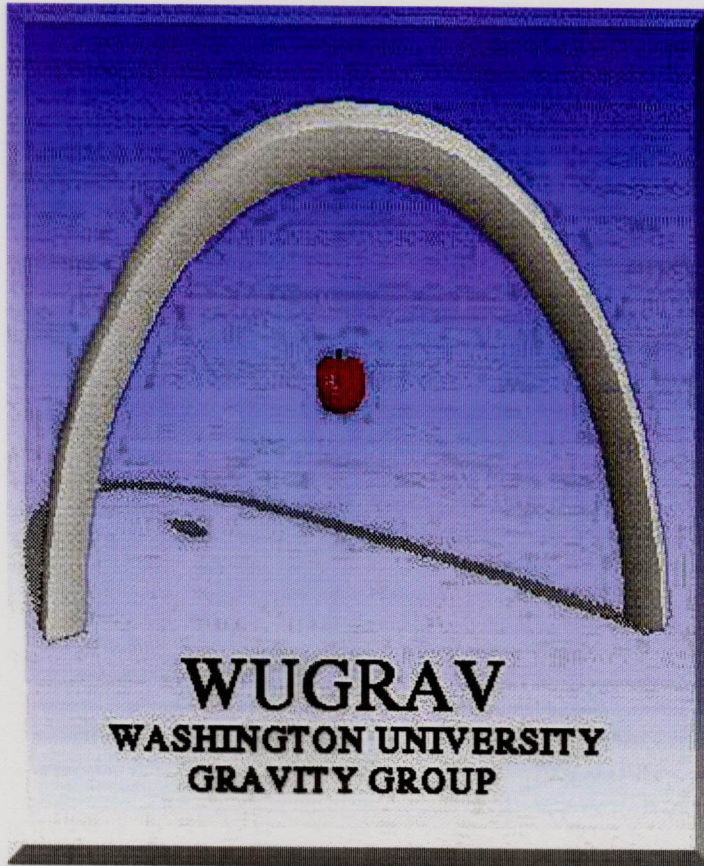


Gravitational Waves and the Validity of General Relativity



- INTRODUCTION
- TESTING GENERAL RELATIVITY: A MILLENNIAL UPDATE
- GRAVITATIONAL-WAVE TESTS OF GR
 - Polarization of Gravitational Waves
 - Speed of Gravitational Waves
 - Tests of Gravitational-wave Damping

THE EINSTEIN EQUIVALENCE PRINCIPLE

- Test bodies fall with the same acceleration
Weak Equivalence Principle (WEP)
- In a local freely falling frame, non-gravitational physics is independent of the frame's velocity
Local Lorentz Invariance (LLI)
- In a local freely falling frame, non-gravitational physics is independent of the frame's location
Local Position Invariance (LPI)

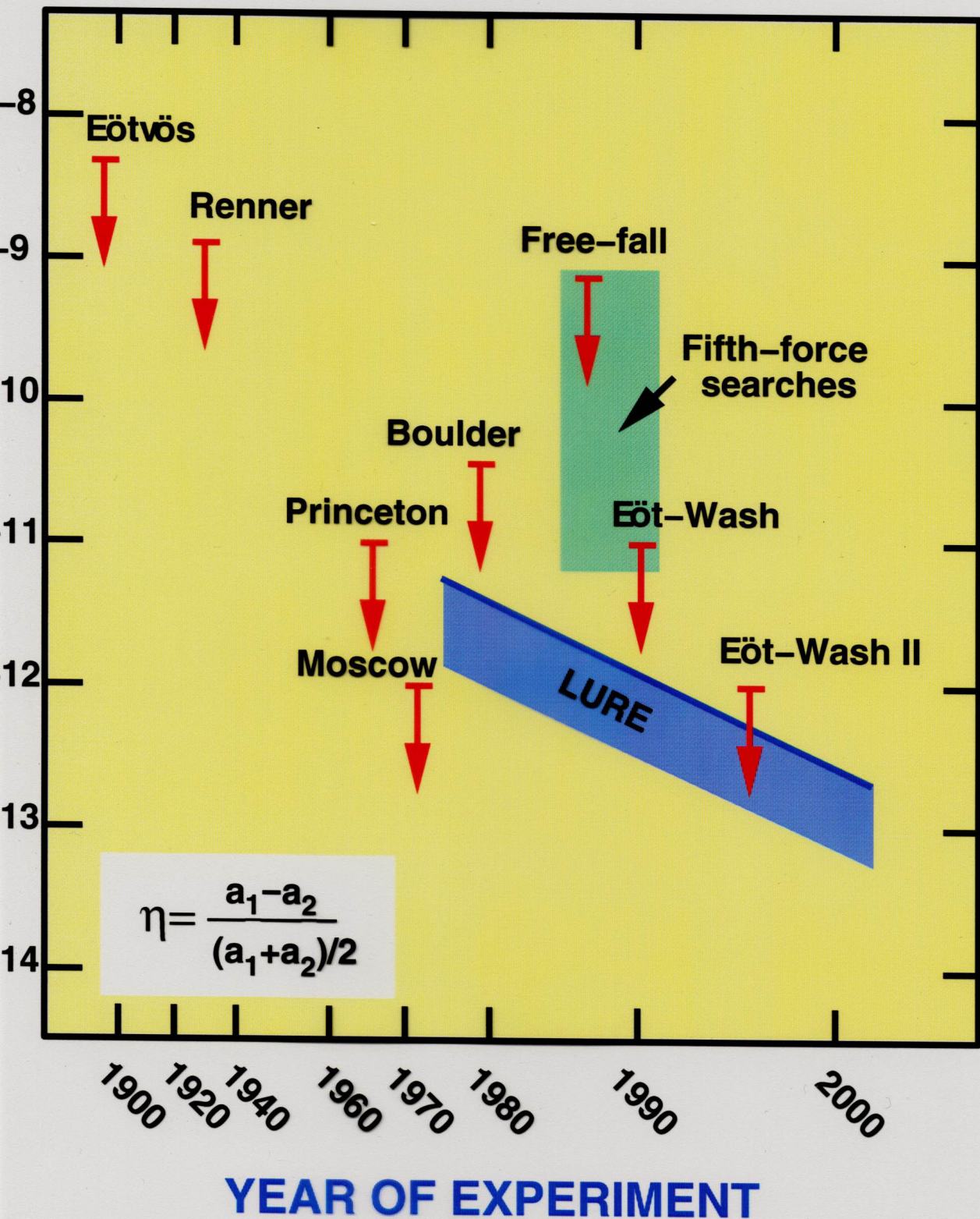
EEP \longrightarrow Metric Theory of Gravity

- symmetric $g_{\mu\nu} \longleftrightarrow \eta_{\mu\nu}$ locally
- 'semicolon' \longleftrightarrow 'comma'

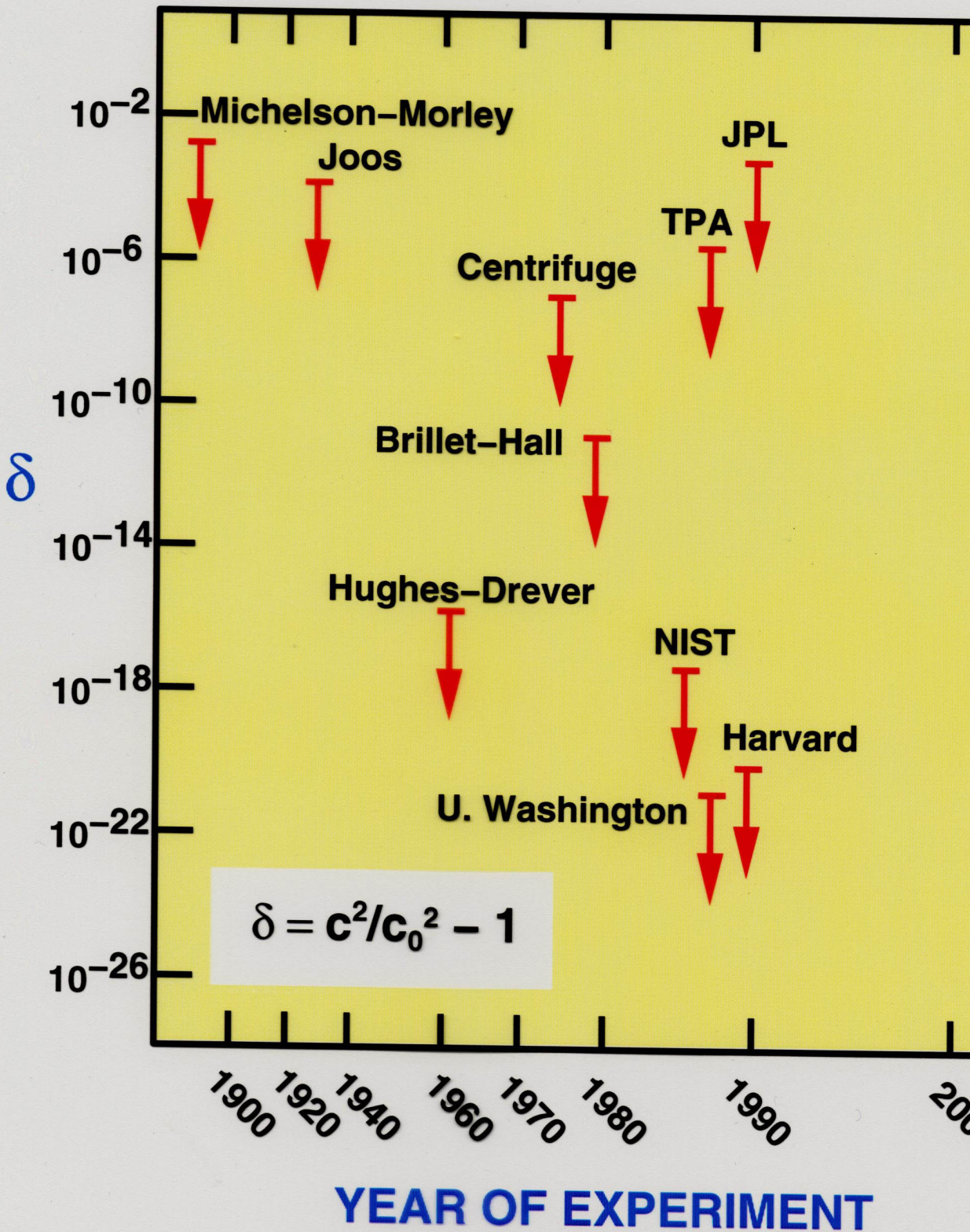
Metric Theories: *GR, Brans-Dicke, Rosen, ...*

Nonmetric Theories: *Moffat's NGT, String Theory ...*

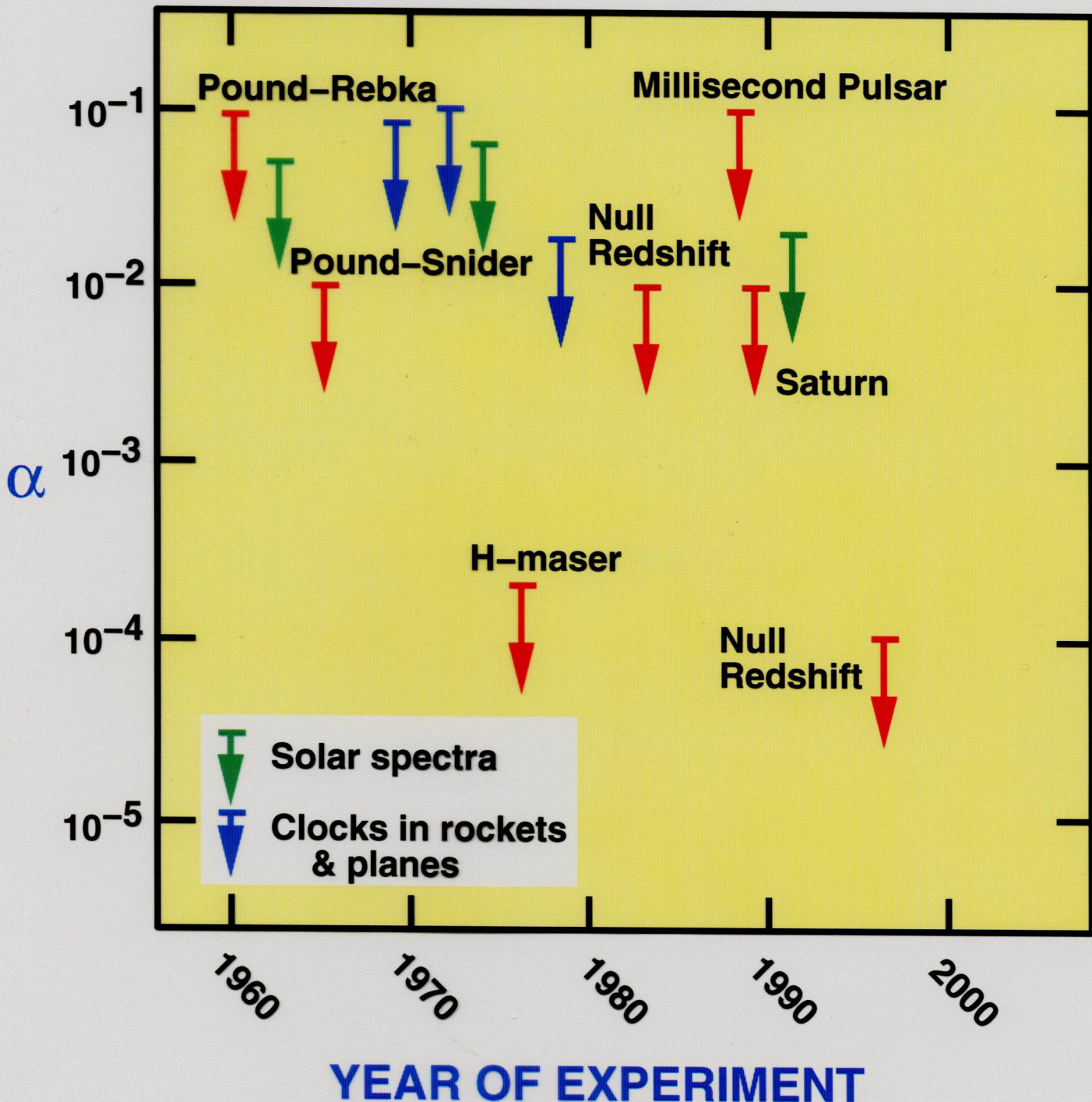
TESTS OF THE WEAK EQUIVALENCE PRINCIPLE



TESTS OF LOCAL LORENTZ INVARIANCE



TESTS OF LOCAL POSITION INVARIANCE



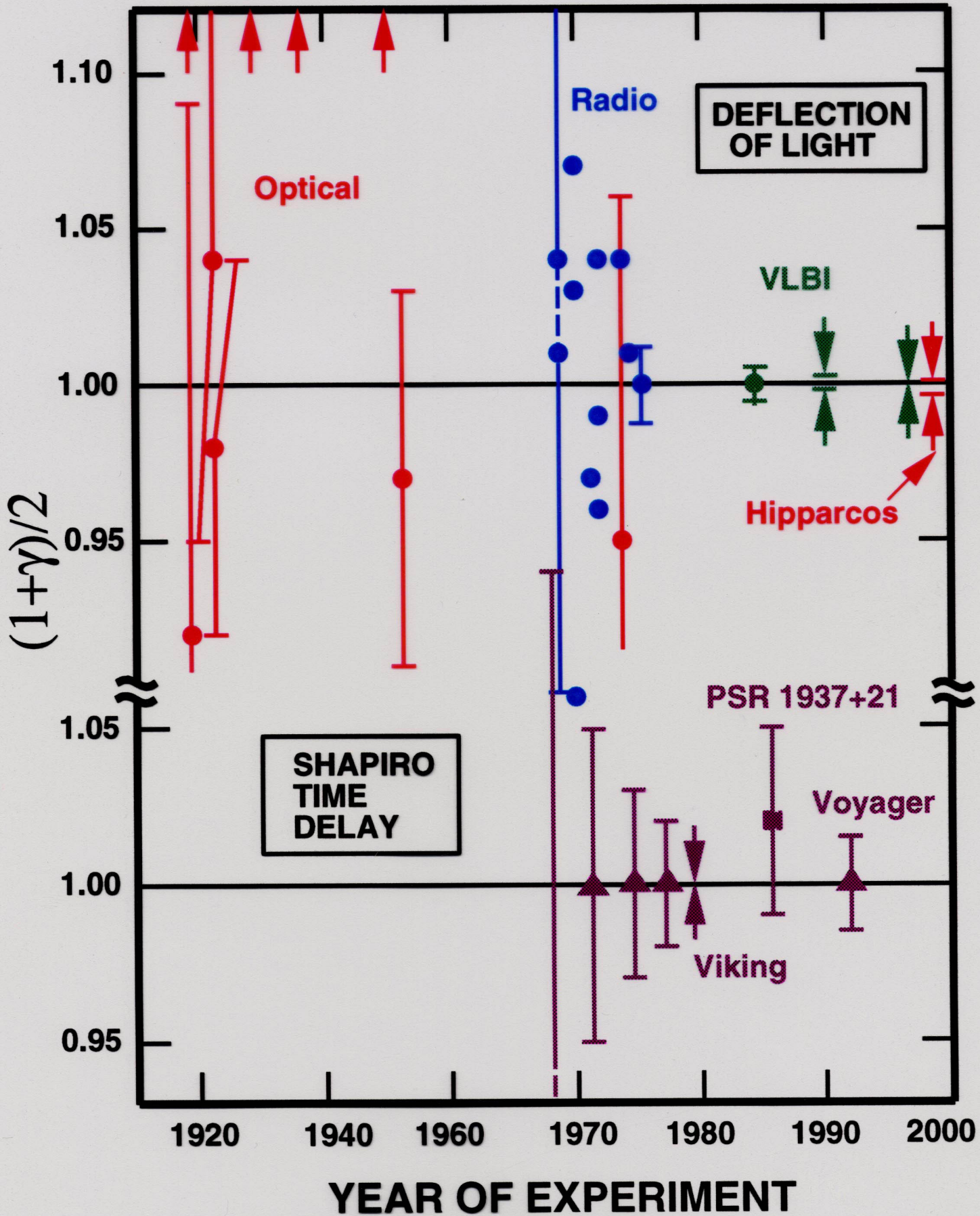
$$\Delta v/v = (1+\alpha)\Delta U/c^2$$

Table 1. The PPN Parameters and Their Significance*

Parameter	What it measures relative to general relativity	Value in general relativity	Value in semi-conservative theories	Value in fully-conservative theories
γ	How much space-curvature is produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" is there in the superposition law for gravity?	1	β	β
ξ	Are there preferred location effects?	0	ξ	ξ
α_1	Are there preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
ζ_1	Is there violation of conservation of total momentum?	0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

*For a compendium of PPN parameter values in alternative theories together with derivations, see TEGP, Chapter 5.

THE PARAMETER $(1+\gamma)/2$



Mercury's Perihelion: Triumph or Trouble?

- ☐ In 1915: triumph
- ☐ 1966–80: trouble
- ☐ 1980 – : resolution

$$\omega = 42''.98 (\lambda_{\text{REL}} + \lambda_{\text{QUAD}})$$

$$\lambda_{\text{REL}} = (2+2\gamma-\beta)/3 = 1 \text{ in GR}$$

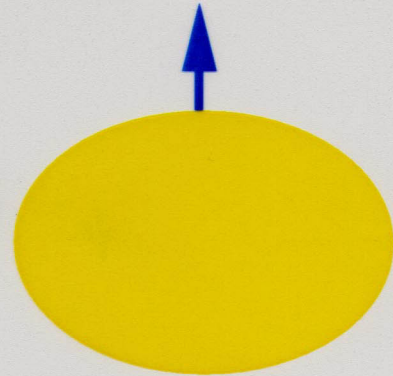
* Triumph

$$\lambda_{\text{OBS}} = 1.000 \pm 0.001 \text{ (Mercury radar)}$$

* Trouble and resolution

$$\lambda_{\text{QUAD}} = 3 \times 10^{-4} (J_2 / 10^{-7})$$

Helioseismology $\rightarrow J_2 \cong 2 \times 10^{-7}$



CURRENT LIMITS ON THE PPN PARAMETERS

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2×10^{-3}	Viking ranging
	light deflection	3×10^{-4}	VLBI
$\beta - 1$	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	6×10^{-4}	$\eta = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging PSR J2317+1439
α_2	solar spin precession	4×10^{-7}	solar alignment with ecliptic
α_3	pulsar acceleration	2×10^{-20}	pulsar \dot{P} statistics
η	Nordtvedt effect ¹	10^{-3}	Lunar laser ranging
ζ_1	-	2×10^{-2}	combined PPN bounds
ζ_2	binary self-boost	4×10^{-5}	\ddot{P} for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	Lunar acceleration
ζ_4	-	-	not independent

¹ Here $\eta = 4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 - 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3$

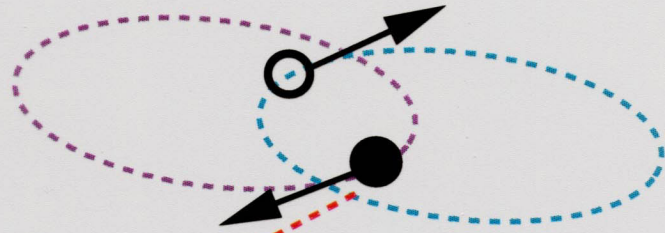
Bound on Scalar-tensor theory:

$$\omega > 3000$$

1979: Waves of Gravity

The Binary Pulsar

Discovery: 1974
Pulse Period: 59 ms (16 cps)
Orbit Period: 8 hours



*1993 Nobel Prize
to Joe Taylor &
Russell Hulse*

Parameter

Value

Keplerian

Pulse period (ms)

59.029997929613(7)

Orbit period (day)

0.322997462736(7)

Eccentricity

0.6171308(4)

Post-Keplerian

Periastron shift ($d\omega/dt$ $^\circ/\text{yr}$)

4.226621(11)

Pulsar clock shifts (γ ms)

4.295(2)

Orbit decay (dP_b/dt 10^{-12})

-2.422(6)

MASS OF COMPANION (solar masses).

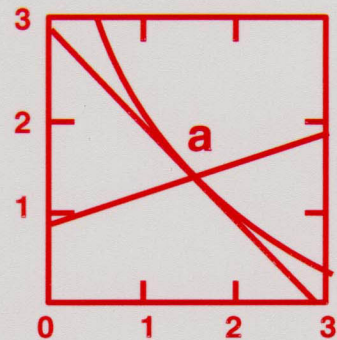
1.41
1.40
1.39
1.38
1.37

1.42 1.43 1.44 1.45 1.46

MASS OF PULSAR (solar masses).

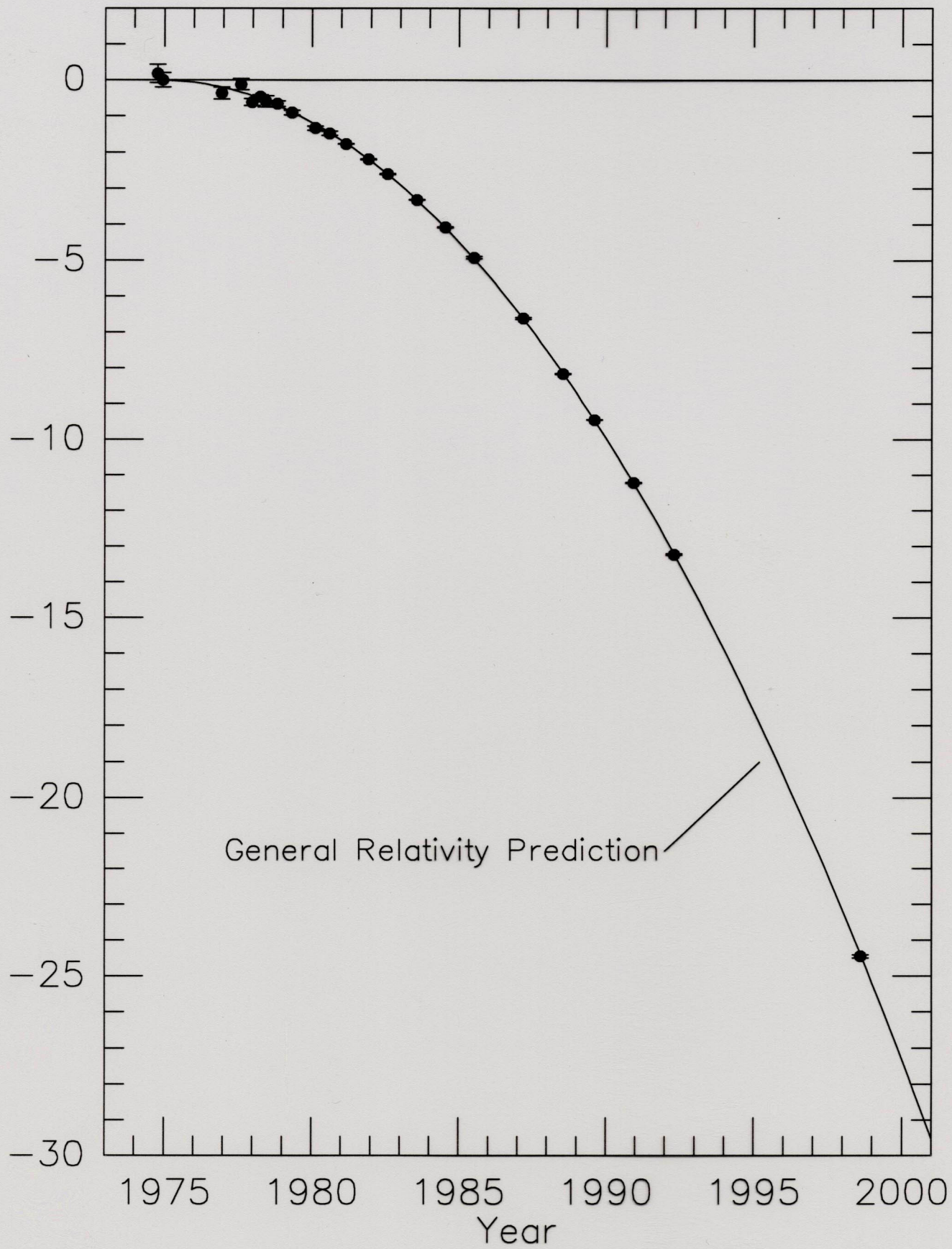
$m_p = 1.4411(7)$ solar masses

$m_c = 1.3874(7)$ solar masses



dC/dt (0.0004%)
 dP_b/dt (0.4%)
 γ' (0.07%)

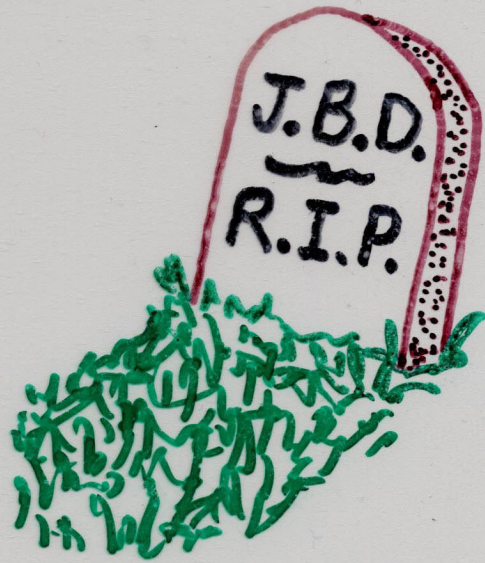
Cumulative shift of periastron time (s)



THE DEMISE OF

JORDAN-BRANS-DICKE GRAVITY

- GR coupled to scalar field
- Coupling constant ω (strength $\propto \omega^{-1}$)
- $G(\phi) \rightarrow G(t)$
- $\frac{GR-BD}{GR} \sim \omega^{-1}$
- PPN bounds $\rightarrow \omega > 3000$ $\left\{ \gamma = \frac{1+\epsilon}{2+\epsilon} \right\}$

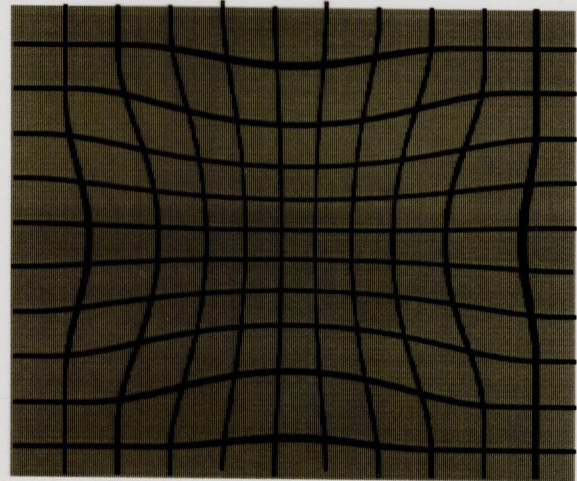
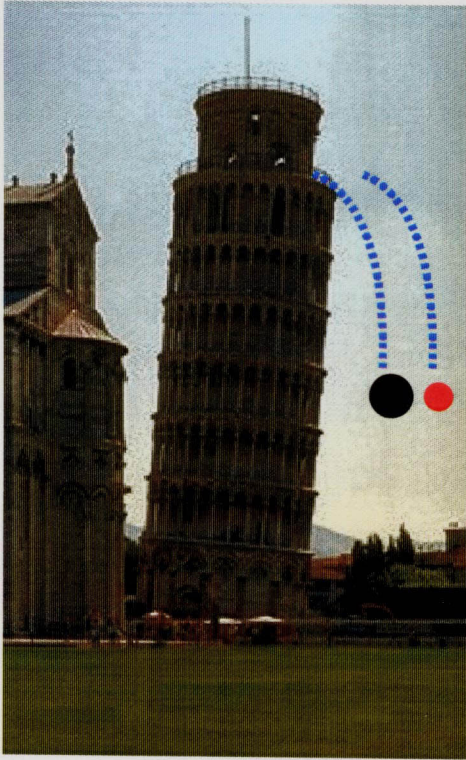


IT'S BAAAACCCKKKK !!!

- ★ Scalar fields in string-inspired theories
 - dilatons, moduli
 - weak violations of WEP
- ★ Extended inflation in cosmology
- ★ Biscalar and multi-scalar theories (Damour & Esposito-Farèse 1992)
- ★ Oscillating- G theories (Steinhardt, Accetta, Will, 1995)
- ★ GR as an “attractor” (Damour & Nordtvedt 1993)
 - $(1/\omega)_{\text{today}} \rightarrow 10^{-4} - 10^{-7}$

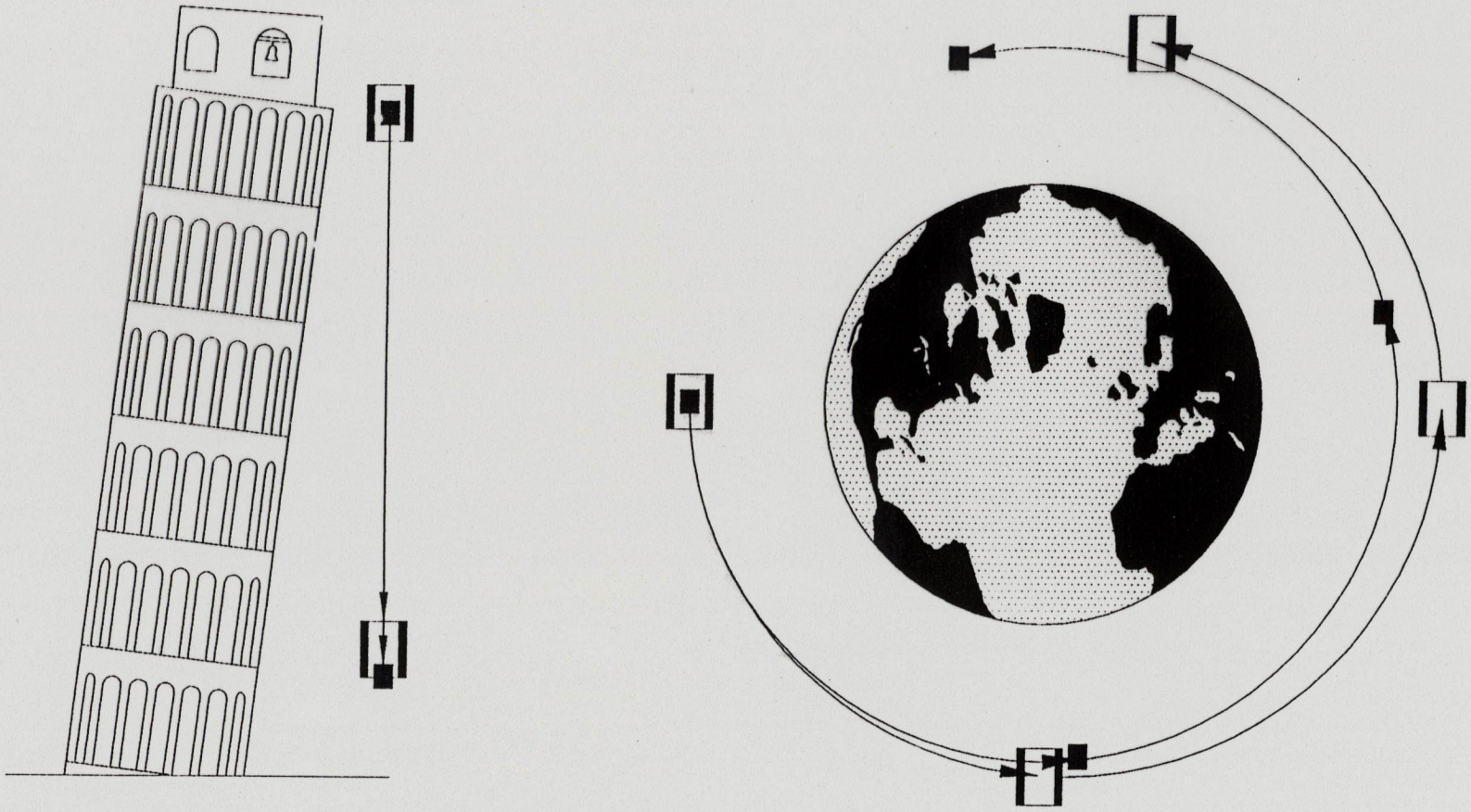


The Equivalence Principle and General Relativity



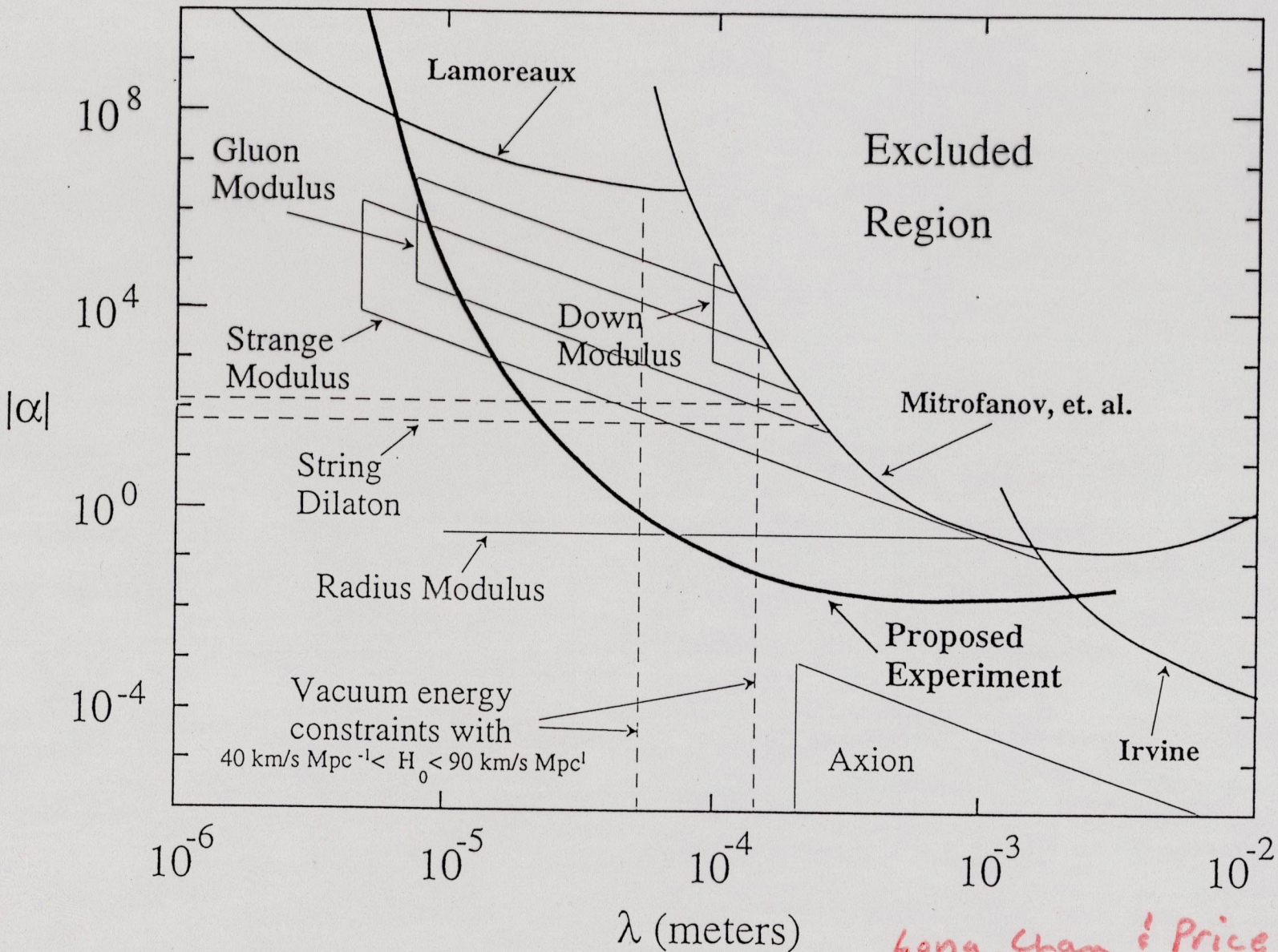
... but how good is it?

- ❑ Current experiments good to 10^{-12}
 - Lab tests
 - Lunar ranging
- ❑ Violations imply new physics
 - String theory predicts violation
 - New elementary particles
- ❑ Space experiment to reach 10^{-18}



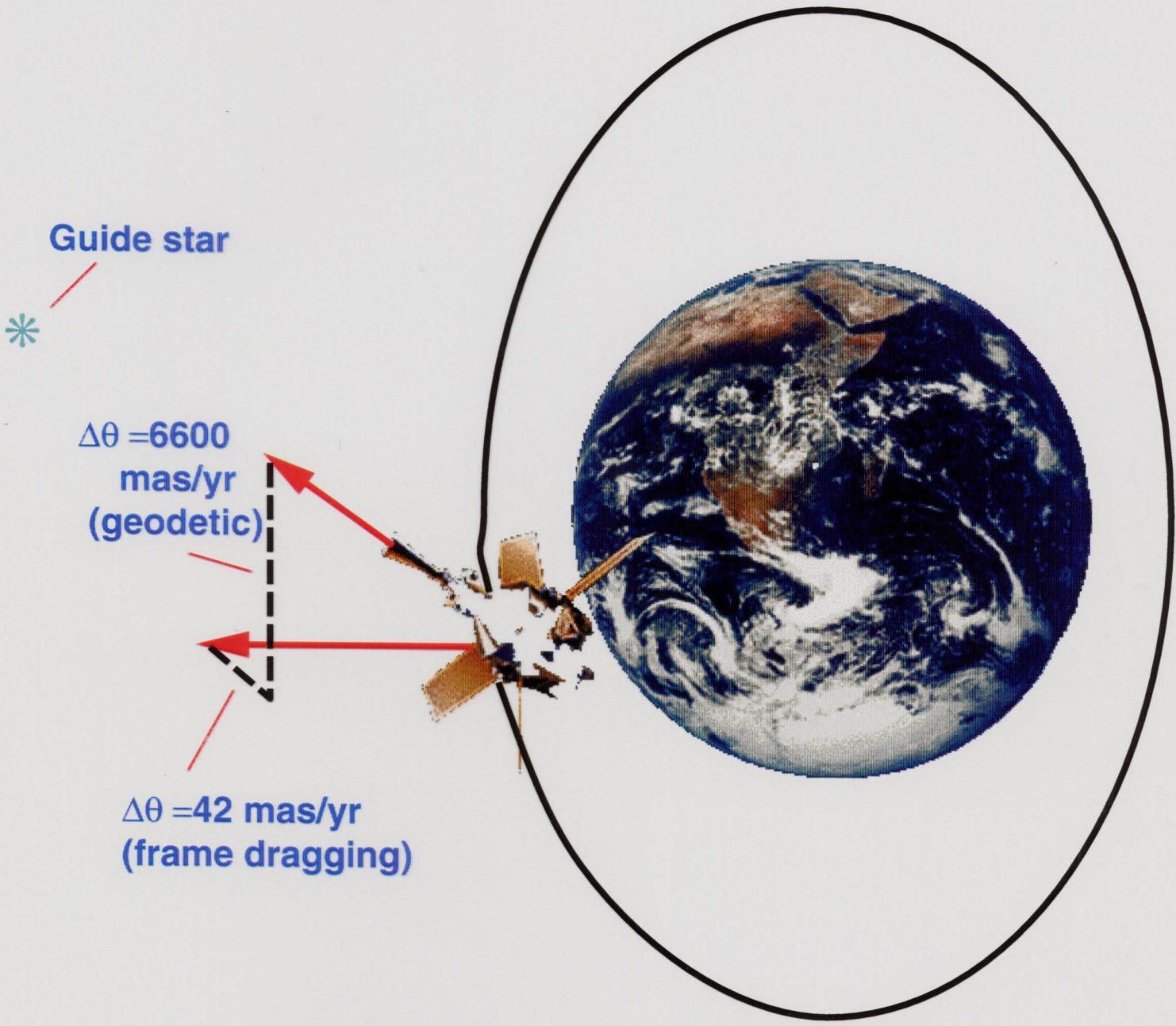
EFFECT OF EQUIVALENCE PRINCIPLE VIOLATION

Search for Forces at Sub-mm Scales



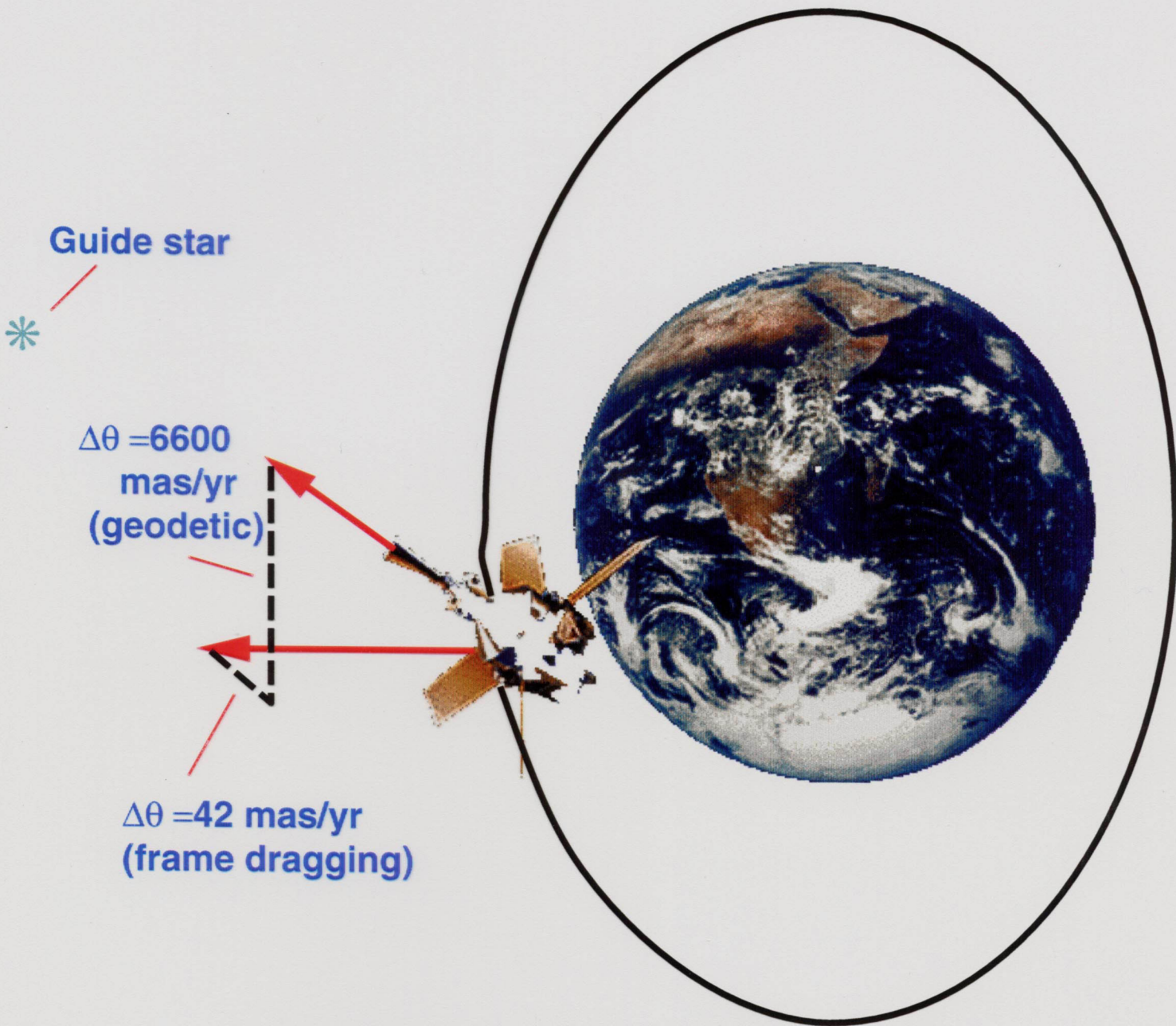
Long, Chan & Price (1998)

GRAVITY PROBE B



GOAL: 0.4 mas/yr
LAUNCH: Early 2001

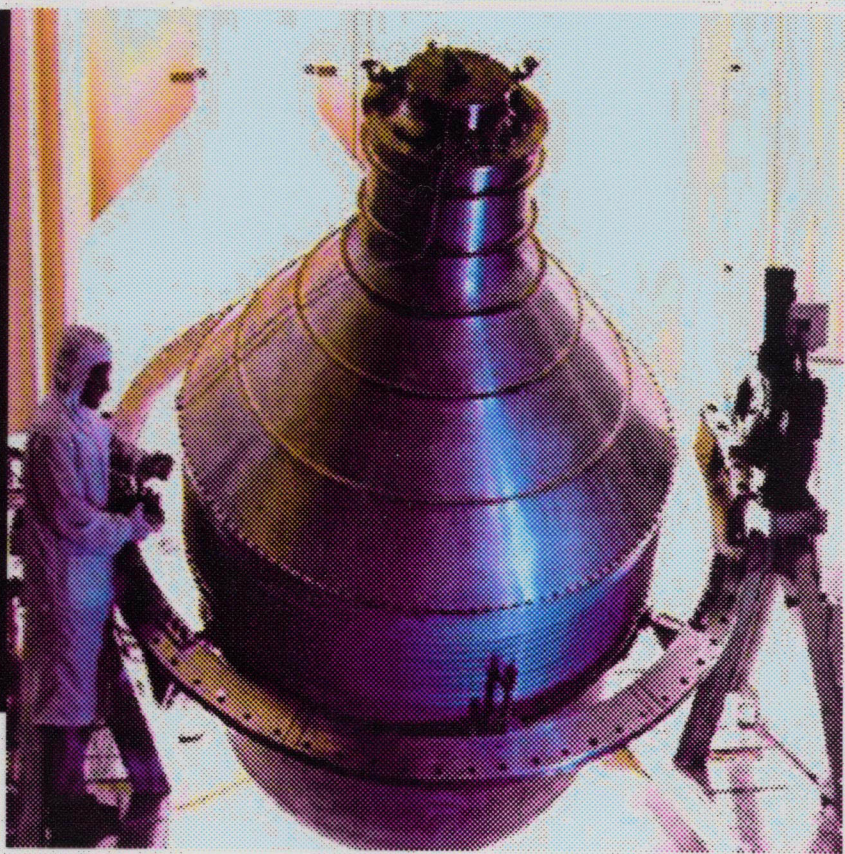
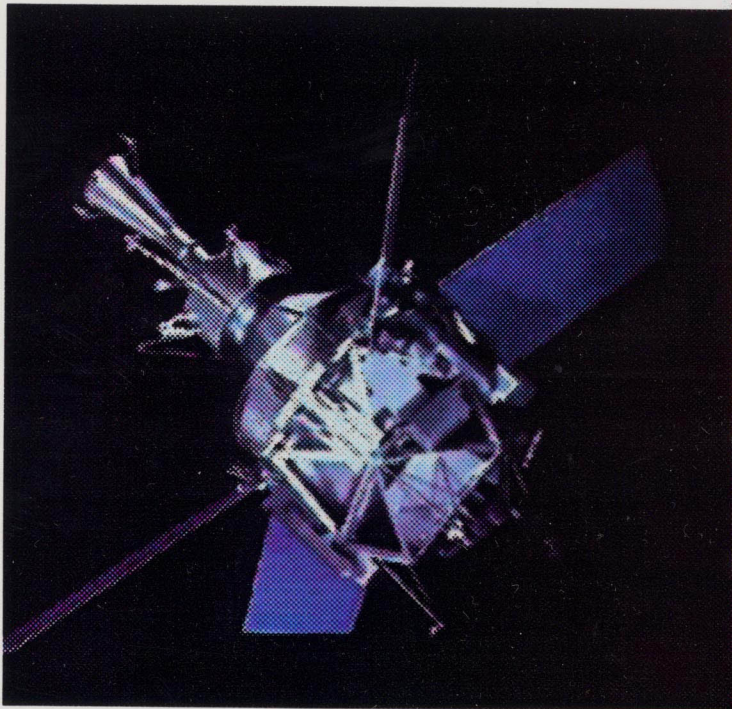
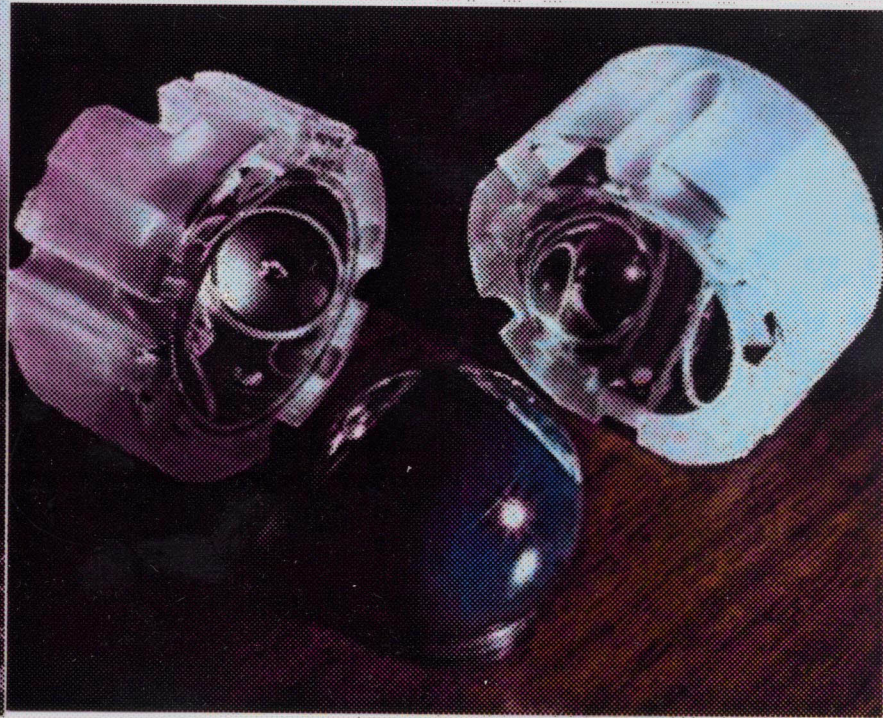
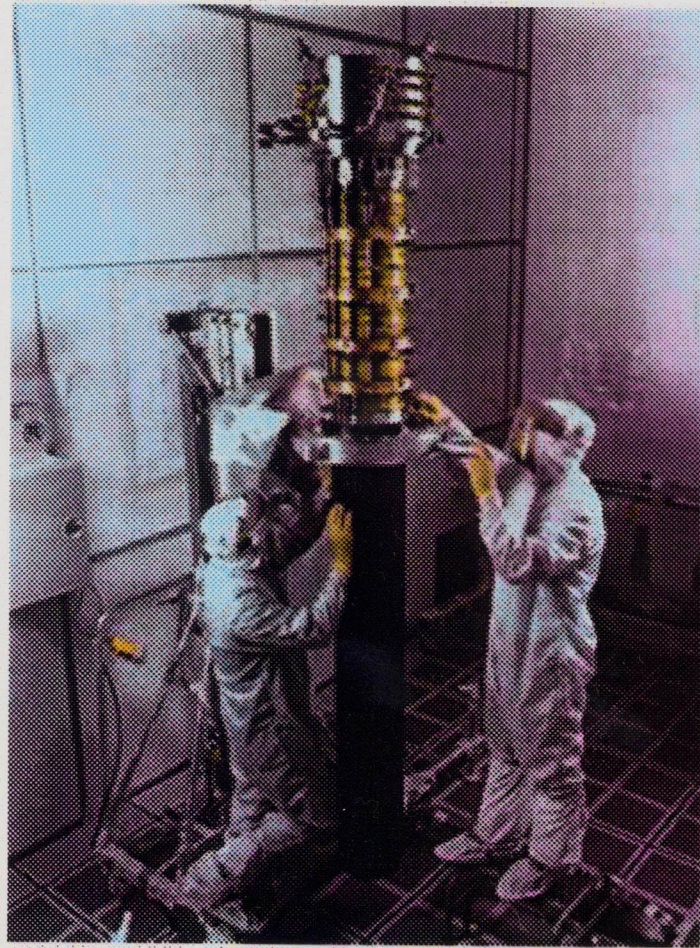
GRAVITY PROBE B



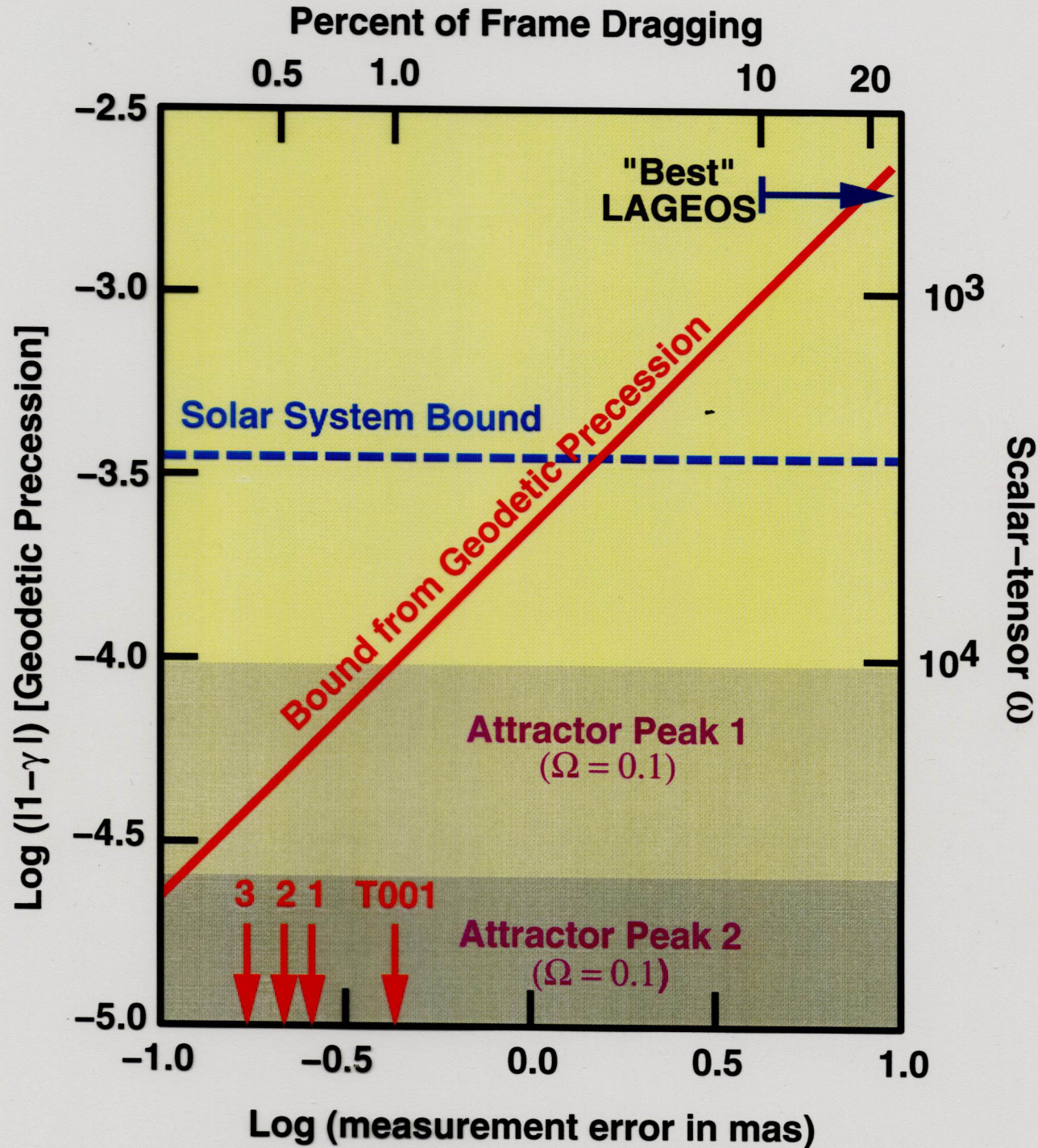
GOAL: 0.4 mas/yr

LAUNCH: Early 2001

GRAVITY PROBE-B



Implications of Gravity Probe B



WHAT CAN LIGO/VIRGO DO?

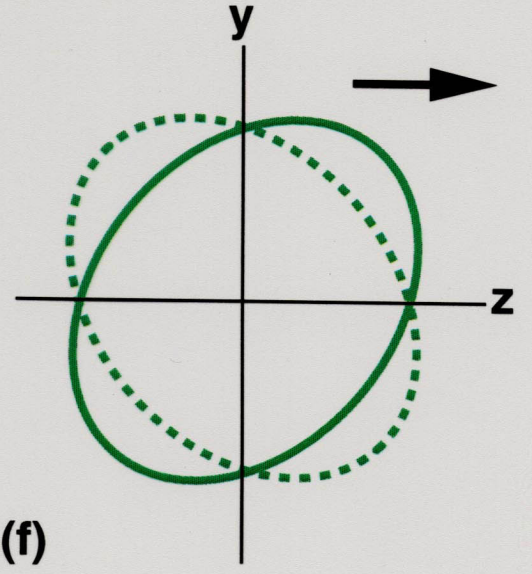
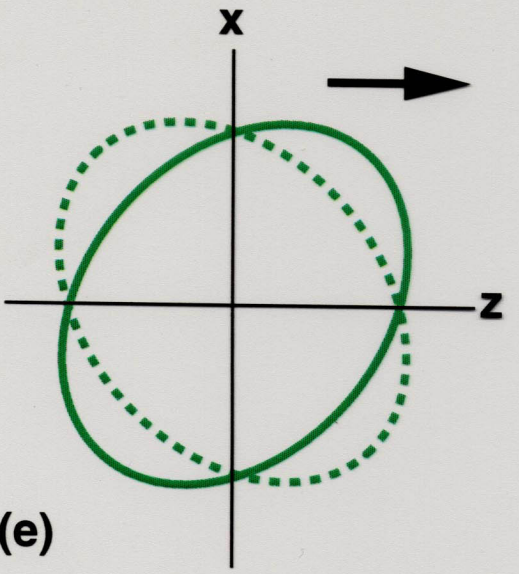
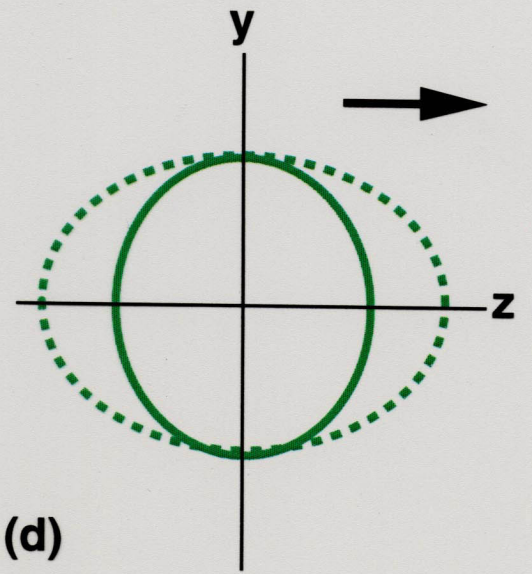
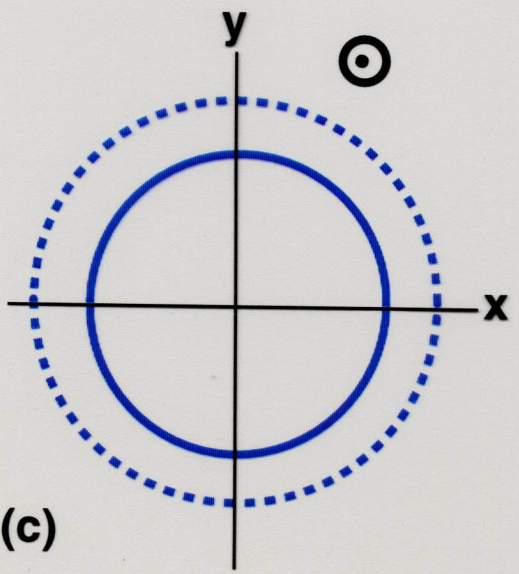
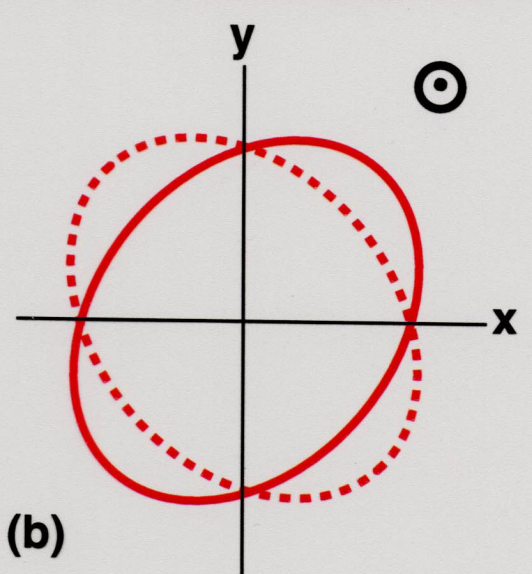
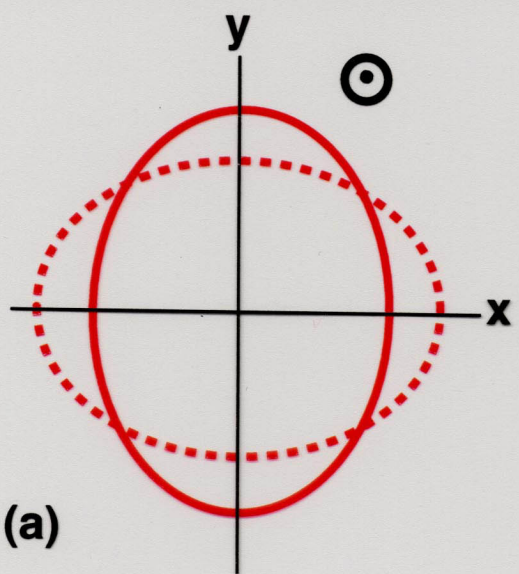
◆ Tests of General Relativity

- Speed of gravitational waves
 - Polarization
 - GR in strong field regime
-

◆ Observe and study sources of GW

- Supernovae
- Inspiralling binaries
 - NS–NS, BH–BH, NS–BH
 - Gamma ray bursts?
- New hot neutron stars
- Pulsars
- Cosmological background
- The unexpected

Gravitational-Wave Polarization



GR =  ST =  + 

SPEED OF GRAVITATIONAL WAVES

WHY SPEED COULD BE DIFFERENT FROM LIGHT SPEED

- Massive graviton: $v_g^2 = 1 - \left(\frac{m_g}{E_g}\right)^2$
- Gravity coupling to “background” fields: $v_g = F(\phi, \vec{K}, \mathbf{H}, \dots)$

EXAMPLES

- *General Relativity.* For $\lambda \ll \mathcal{R}$, GW move on null geodesics of background spacetime, as do photons. $v_g \equiv 1$.
- *Scalar Tensor Gravity.* Tensor waves have $v_g \equiv 1$, even if scalar is massive.
- *Theories with background flat metric.* Eg. Rosen’s Bimetric Theory (1970s). GW follow null cones of η , light follows cones of g .

POSSIBLE LIMITS

$$1 - v_g \approx 5 \times 10^{-17} \left(\frac{200\text{Mpc}}{D}\right) [\Delta t_a - (1 + Z)\Delta t_e]$$

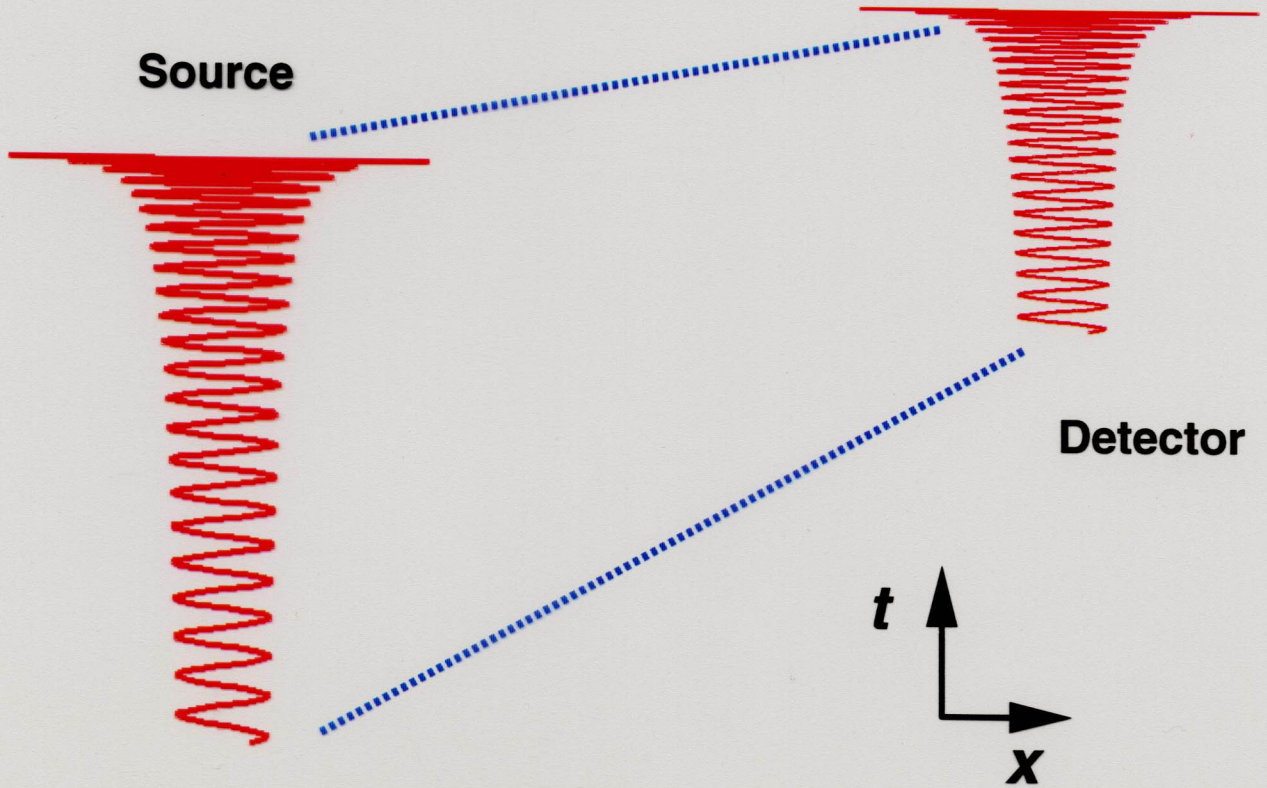
D = distance of source Z = redshift of source

Δt_a (Δt_e) = time difference, in seconds

$$\lambda_g > 3 \times 10^{12} \text{ km} \left(\frac{D}{200\text{Mpc}}\right)^{1/2} \left(\frac{100\text{Hz}}{f}\right)^{1/2} \left(\frac{1}{f\Delta t}\right)^{1/2}$$

$$3 \times 10^{12} \text{ km} = (4 \times 10^{-22} \text{ eV})^{-1}$$

Bounding the Graviton Mass Using Inspiralling Compact Binaries



m_1	m_2	Distance (Mpc)	Bound on λ_g (km)
Ground Based (LIGO/VIRGO)			
1.4	1.4	300	4.6×10^{12}
10	10	1500	6.0×10^{12}
Space Based (LISA)			
10^6	10^6	3000	5.4×10^{16}
10^4	10^4	3000	0.7×10^{16}

Other Limits on the Graviton Mass

- $\lambda_g > 3 \times 10^{12}$ km (solar system $1/r^2$ law)
- $\lambda_g > 6 \times 10^{19}$ km (galaxy & cluster dynamics)

Testing Gravitational Radiation Reaction

$$\frac{df}{dt} = \frac{96}{5\pi\mathcal{M}^2} (\pi\mathcal{M}f)^{11/3} \left[1 \right. \quad \text{Measure chirp Mass } \mathcal{M}$$

N

$$- \left(\frac{743}{336} + \frac{11}{4}\eta \right) (\pi\mathcal{M}f)^{2/3} \quad \text{Measure } m_1 \text{ \& } m_2$$

1PN

"Tail" term: Test GR

$$+ (4\pi - \beta)(\pi\mathcal{M}f)$$

1.5PN

Measure spins

$$+ \left(\frac{34103}{18144} + \frac{13661}{2016}\eta + \frac{59}{18}\eta^2 + \sigma \right) (\pi\mathcal{M}f)^{4/3}$$

2PN

Test GR

$$+ O[(\pi\mathcal{M}f)^{5/3}]$$

$$M = m_1 + m_2; \quad \eta = m_1 m_2 / M^2; \quad \mathcal{M} = \eta^{3/5} M$$

$$\beta = \text{Spin - Orbit Effect}; \quad \sigma = \text{Spin - Spin Effect}$$

Testing Scalar Tensor Gravity with Advanced LIGO

- Requirement: NS-BH binary inspiral

