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Fringe structure of LHO 2k FP a modeler's view

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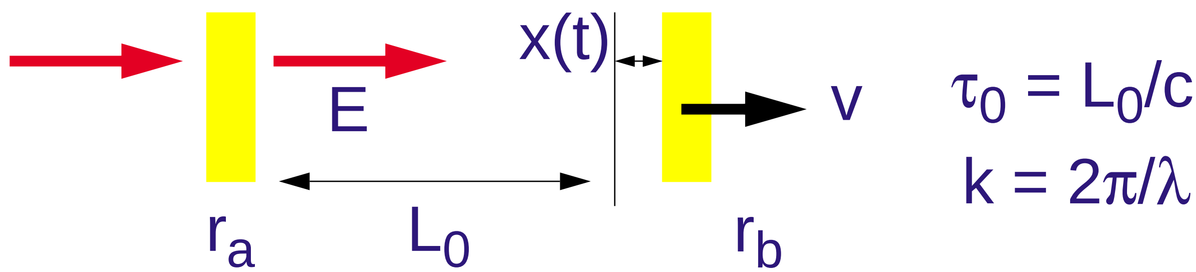
Fringe structure of LHO 2k FP

a modeler's view

- Presented by Hiro Yamamoto of CIT/LIGO Lab at the lunch time meeting at LHO on December 9, 1999
- The analysis method was developed in 1997 when the 40m one arm FP measurement was done and analyzed by Matt Evans, Malik Rakhmanov and Hiro Yamamoto. This analysis was presented in the 3rd PAC Meeting at LHO in November 6-7, 1997 (LIGO Note L970507).
- The analytic form of the wiggling part was first derived by M.Rakhmanov during the course of the above research, and the derivation of the expression can be found in the thesis of M.Rakhmanov. The calculation was improved by M.Evans and H.Yamamoto.
- The fringe structure analysis tool for this 2k FP data has been developed using matlab, and will be installed at LHO.

Fabry-Perot cavity field

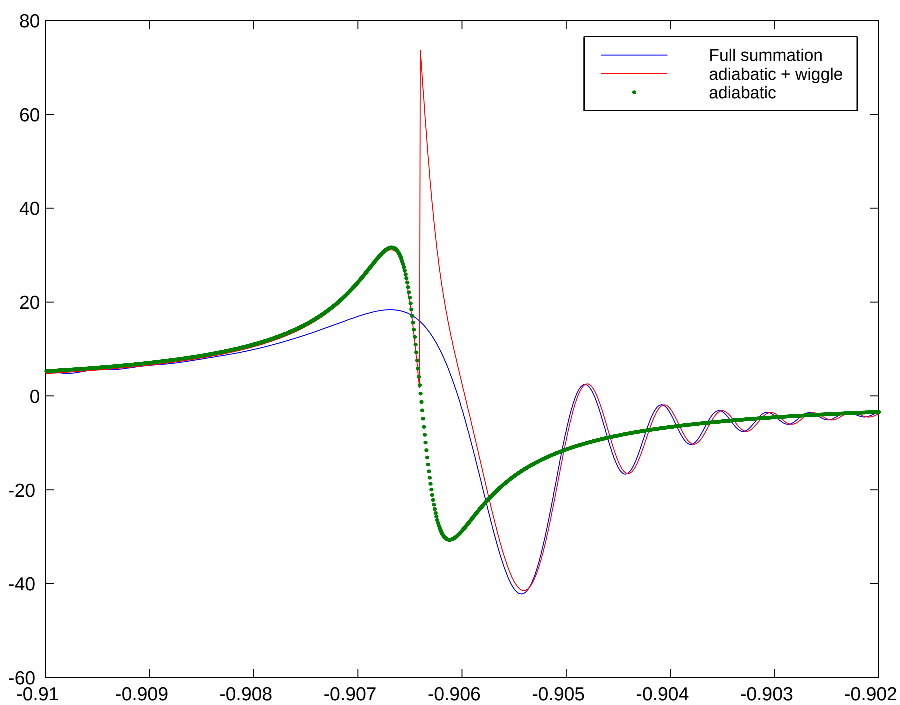
simple expression



- $$E(t) = \sum_{m=0}^{N = \frac{t}{2\tau_0}} \exp(-(B + iC) \cdot m + i(A \cdot m^2))$$

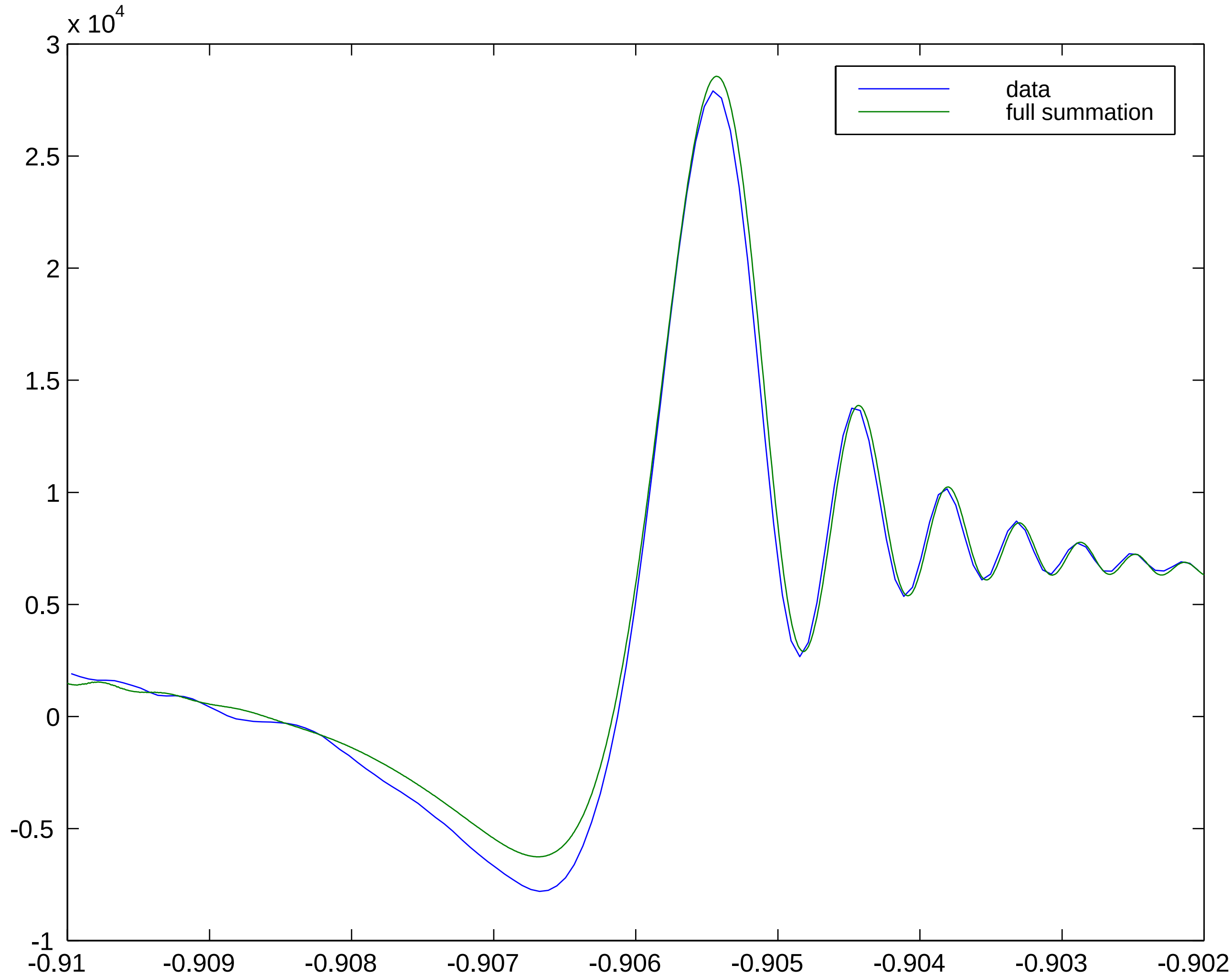
  $$A = 2k\tau_0 v \qquad B = \ln\left(\frac{1}{r_a r_b}\right) \qquad C = 2kx(t)$$

- $$E(t) \approx \frac{1}{1 - r_a r_b \exp(-2kx)} + \sqrt{\frac{i \cdot \pi}{A}} \exp\left(-\frac{\ln\left(\frac{1}{r_a r_b}\right)}{2\tau_0}(t - t_0)\right) \exp\left(i\left(\frac{B^2}{4A} - \frac{kv}{2\tau_0}(t - t_0)^2\right)\right)$$



Data vs calculation

preview of results

- 

- Calculation is done using the parameters determined by the procedures described blow, except for the DC offset and mixing angle (assumbed to be ideal).

Velocity and resonant point

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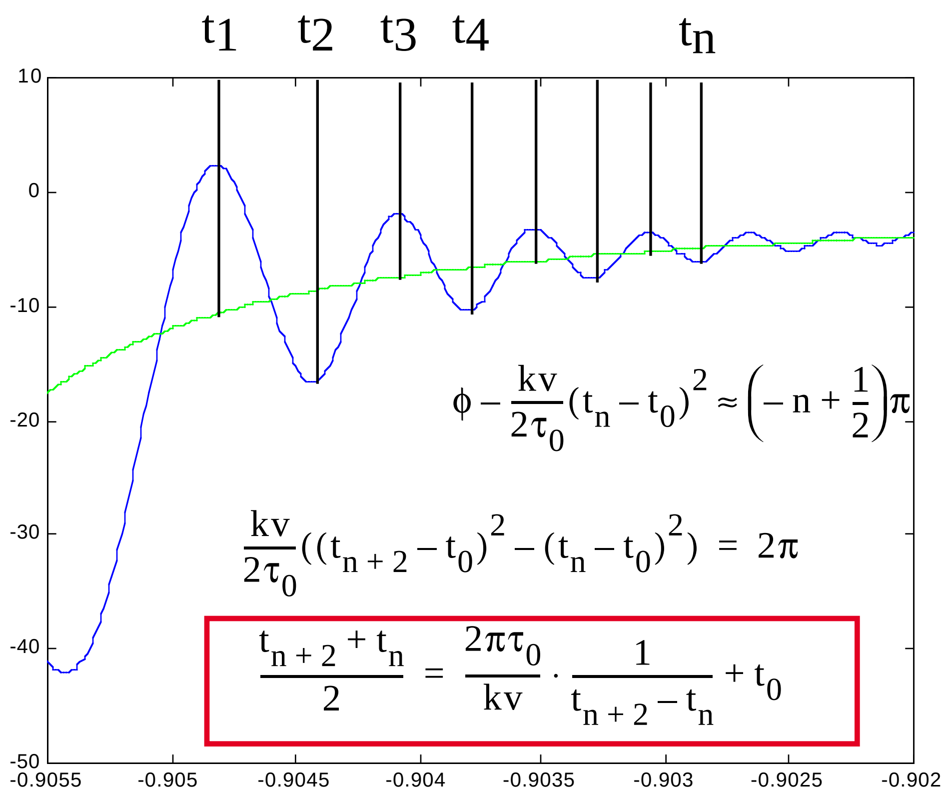

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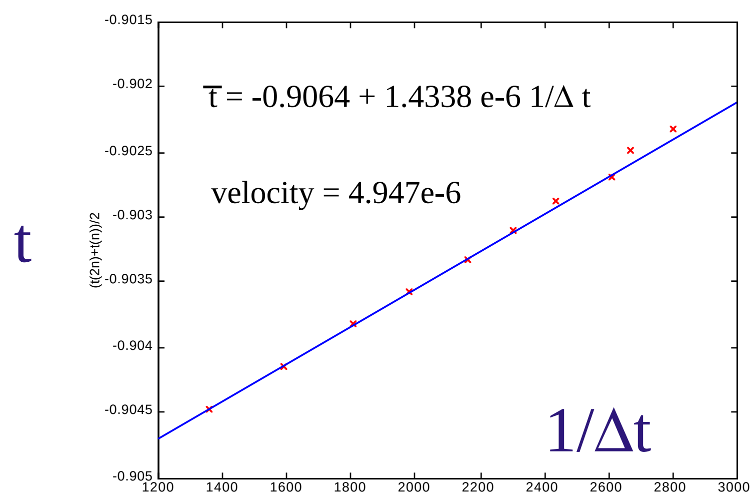

Finesse

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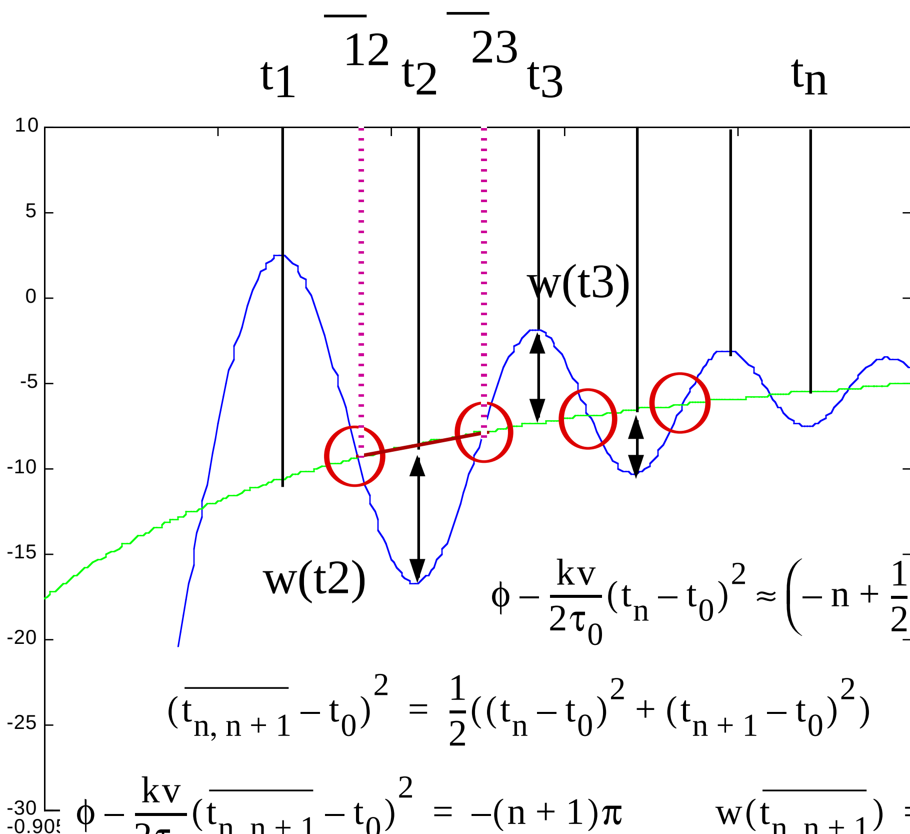

$$\phi - \frac{kv}{2\tau_0}(t_n - t_0)^2 \approx \left(-n + \frac{1}{2}\right)\pi$$

$$(\overline{t_{n,\,n+1}} - t_0)^2 = \frac{1}{2}((t_n - t_0)^2 + (t_{n+1} - t_0)^2)$$

$$\phi - \frac{kv}{2\tau_0}(\overline{t_{n,\,n+1}} - t_0)^2 = -(n+1)\pi \qquad w(\overline{t_{n,\,n+1}}) = 0$$

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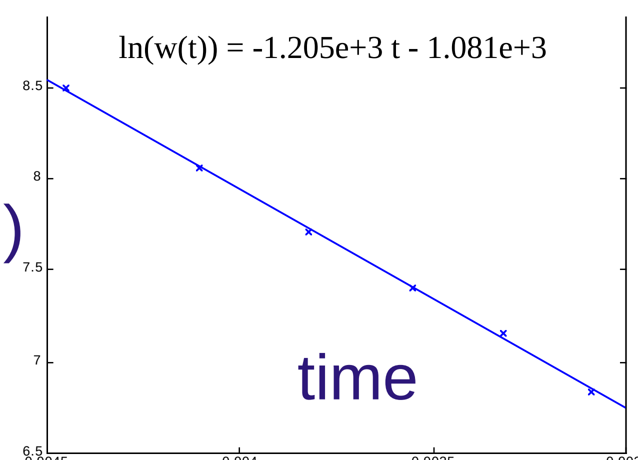
ln(w(t))



$$\text{slope} = \frac{\ln \frac{1}{ra \cdot rb}}{2 \cdot \tau_0}$$

$$RaRb = 0.9684$$

$$T_a J_0 J_1 Pwr = 575$$

Summary of velocity and finess
preliminary

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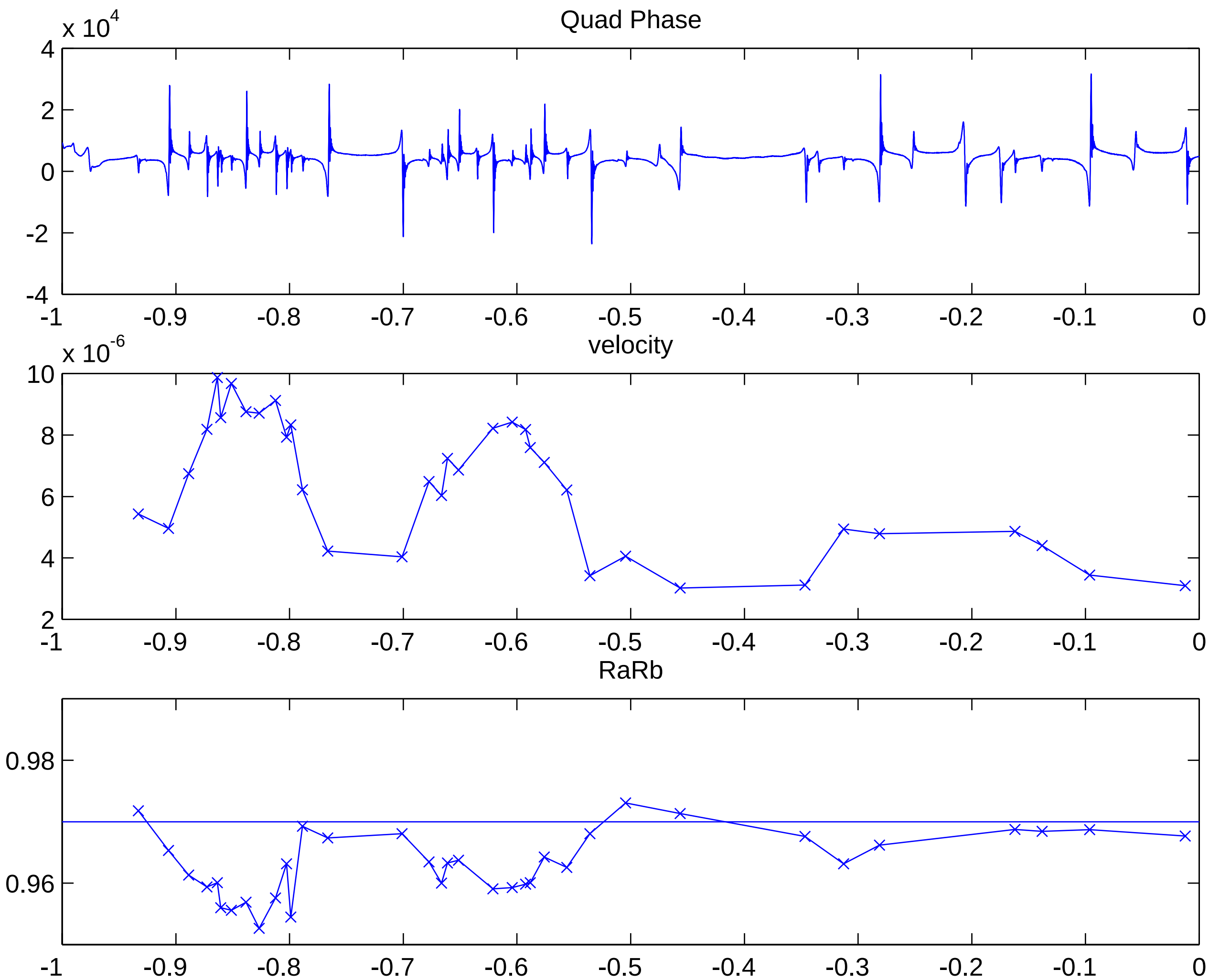

Signal normalization

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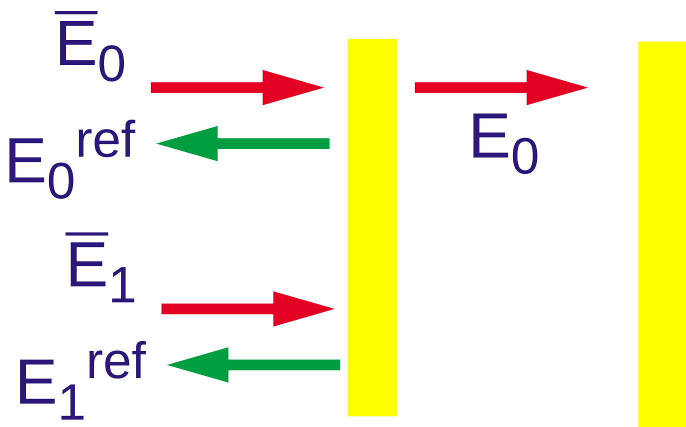

$$E_1^{ref} = r_a \overline{E_1}$$

$$E_0^{ref} = r_a \overline{E_0} + t_a E_0$$

$$E_0 = \frac{1}{1 - r_a r_b e^{-i2kl}} + \text{wiggle}$$

$$\text{demod} \sim \overline{E_1} \cdot E_0$$

- Error signal and slope - static limit

$$\text{error} = 2 \cdot \text{Pwr} \cdot J_0(\Gamma) \cdot J_1(\Gamma) \cdot T_a \cdot r_a \cdot \text{Imag}\left(\frac{1}{1 - r_a r_b \exp(-2ikl)}\right)$$

$$= 575 \cdot \text{Imag}\left(\frac{1}{1 - r_a r_b \exp(-2ikl)}\right)$$

$$\text{slope} = \frac{d}{dl}\text{error} = 575 \cdot \frac{4\pi r_a r_b \frac{1}{\lambda}}{(1 - r_a r_b)^2} = 575 \times 5.1e10 = 2.7e13$$