LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

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# Reduction of degenerate modes excitation in an imperfect FP cavity for LG33 beam

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## 1. Introduction

The excitation of degenerate modes in a FP cavity with imperfect mirrors can contaminate the signal when LG33 beam is used and degradation of the contrast defect affects the SN ratio. In this note, artificial irregularity is placed on the mirror surface at the node radius of the LG33 field, and the reduction of the degenerate mode excitation is studied.
It is shown that these methods do suppress the degenerate mode excitation, but it is accomplished at the high cost of the signal loss.

## 2. Characteristics of reflected field

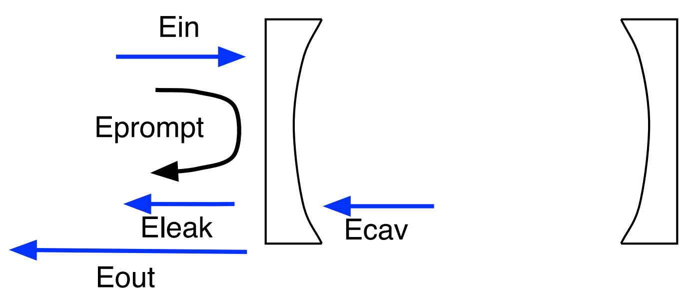


**Figure 1 Reflected field components**

When TEM33 field, Ein, is injected to a cavity, the reflected field, Eout, consisted of two components, Eprompt and Eleak. Eprompt is the field that Ein is reflected by the front mirror, and Eleak is the field that the cavity field Ecav transmits through the front mirror.

For a high finesse cavity, the following relationship exists.

$$E_{prompt} \approx E_{in}$$
$$E_{leak} = -2((1-\varepsilon)E_{in} + E_{om})$$
$$E_{out} = -(1-2\varepsilon)E_{in} - 2E_{om} = -E_{in} + dE$$
$$dE \equiv 2\varepsilon \cdot E_{in} - 2E_{om}$$

ε is the net loss of the base mode in the cavity, and Eom is the component of the field in the cavity other than the base mode. The total power in the cavity is Pmax ( (1-2ε) + η ), where Pmax is the power in the arm when the cavity is perfect and there is no loss and no mode mismatch. η is the impurity in the cavity, or the fraction of the power of the field with modes other than the base mode:

$$\eta \approx (1+2\varepsilon)\frac{E_{om}}{E_{in}^2}^2 \approx \frac{E_{om}}{E_{in}^2}^2$$

Among the total reflected field, ε and Eom depend on the specifics of the arm and can contribute the contrast defect. But, simply calculating the power of dE may be overestimation.

$$dP_{out} / 4P_{in} = \varepsilon^2 + \eta$$

When the aberration of optics in two arms are different and the degenerate mode excitation is random, then Eom's from two arms will be different and their powers can be added to estimate the contrast defect. But the loss of the base mode in the cavity, ε, is always positive and the amplitude different due to different ε's, ε1 and ε2, is (ε1- ε2)Ein.

$$E_{dark} = E_1 - E_2 = 2(\varepsilon_1 - \varepsilon_2)E_{in} - 2(E_{om}^1 - E_{om}^2)$$
$$P_{dark} / 4P_{in} = (\varepsilon_1 - \varepsilon_2)^2 + \eta_1 + \eta_2$$

The loss of the base mode degrade the SN ratio, but the inclusion of that effect in the estimation of the contrast defect can give overestimation. Based on this argument, the measure of the contrast defect in this note is calculated by using η, i.e., the impurity of the field in the arm.

## 3. FP Cavity simulation

The ROCs of two mirrors are both 2857m, the cavity length is 3994.5m, test mass apertures are both 34cm, T(ITM)=1.4% and T(ETM)=5ppm. The beam size on the test mass is 3.84cm and the diffractive loss is 1ppm when both mirrors have no aberration.

The cavity was locked using the PDH technique based on the reflected field.

## 4. Test masses

In order to study the problems of the excitation of the degenerate modes, four aLIGO test masses are used. At the time of this analysis, there is no test masses with acceptable coating on it, and all aberrations are due to polishing. The following figure shows PSDs of these test masses.

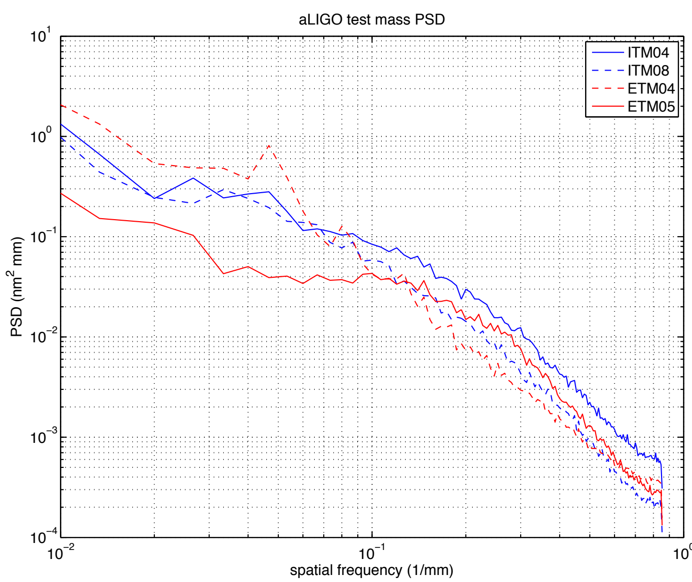


**Figure 2 Advanced LIGO test mass PSD**

RMS of these optics calculated in the central 16cm aperture is 0.230nm (ITM04), 0.209nm (ITM08), 0.263nm (ETM04) and 0.125nm (ETM05).

## 5. Degenerate modes in a FP cavity

Table.1 shows the fraction of modes in FP cavities with imperfect mirrors. The last two rows, (f) and (g), show fractions with artificial imperfections to suppress degenerate mode excitation, which are discussed in the following sections.

| | | LG(4,±1) | LG(3,-3) | LG(2, ±5) | LG(1, ±7) | LG(0, ±9) | Impurity |
|---|---|---|---|---|---|---|---|
| (a) | ITM4 + ETM5 | 1.79 | 0.10 | 1.66 | 0.02 | 0.05 | 3.62 |
| (b) | ETM5 + ETM5 | 1.07 | 0.02 | 0.76 | 0.01 | 0.01 | 1.87 |
| (c) | ITM4(90° rotated) + ETM5 | 1.06 | 0.01 | 1.05 | 0.06 | 0.40 | 2.19 |
| (d) | ITM8 + ETM4 | 3.13 | 0.08 | 3.36 | 0.19 | 0.01 | 6.77 |
| (e) | ITM4 + ETM5 astigmatism removed | 0.45 | 0.08 | 0.41 | 0.05 | 0.05 | 1.04 |
| (f) | ITM4 + ETM5 2mm null ring | 0.37 | 0.04 | 0.05 | 0 | 0 | 0.45 |
| (g) | ITM4 + ETM5 6mm cusp ring | 0.64 | 0.05 | 0.08 | 0 | 0 | 0.77 |

**Table 1 Mode fractions in a FP cavity units in %**

Zernike terms up to power are removed from the surface maps in all cases, except (e) for which Zernike terms up to astigmatism are removed.

For case (a), the phase map of ITM04 is used for the front mirror and that of ETM05 is used for the back mirror. For case (b), the phase map of ETM05, which is the best among the four test masses used in this analysis, is used for both the front and back mirror. For case (c), the phase map of ITM4 is used for the front mirror, same as in case (a), but the map is rotated by 90°. For case (d), the phase map of ITM08 and ETM05 are used for front and back mirrors.

Comparison of these four cases shows that the purity varies by factor of several depending on what kind of mirrors are used. Another observation is that the dominant degenerate modes are LG(4,1) and LG(2,5). The fraction of LG(4,-1) and LG(2,-5) are around the same size as LG(3,-3). In the rest of the analysis, the pair of ITM04 and ETM05 are used.

For case (e), astigmatic terms are subtracted from the ITM04 and ETM05 phase maps. The impurity drops by a factor of 3, and still larger than 1%.

## 6. Suppression of degenerate modes by artificial irregurarity

Various combinations of aLIGO optics show variations of the impurity in the cavity, but it is still of the order of 1%, much higher than a target of the purity of the order of 100ppm.

Several possibilities have been suggested to reduce the impurity by placing artificial irregularities on the mirror surface at the nodes of the LG33 field. For the cavity analyzed in this note, the node radius' are at r1 = 3.97cm, r2 = 6.26cm and r3 = 8.82cm.

Two methods are studied in this note about the reduction of the impurity and the loss of the signal. One is to make the reflectivity of mirrors to be 0. In this case, the reflectivity on both mirrors are changed to 0 around the node radius using a smooth function of r, s(r, r0, dr) (see footnote about the this function[1]). The reflectance is calculated as nominal value of the reflectance x (1 - s( r, r1, dr))*(1 - s(r, r2, dr))*(1 - s(r, r3, dr)).

The other is to add cusp on the rings by adding height x ( s(r, r1, dr) + s(r, r2, dr) + s(r, r3, dr) ) on top of the mirror phase map.

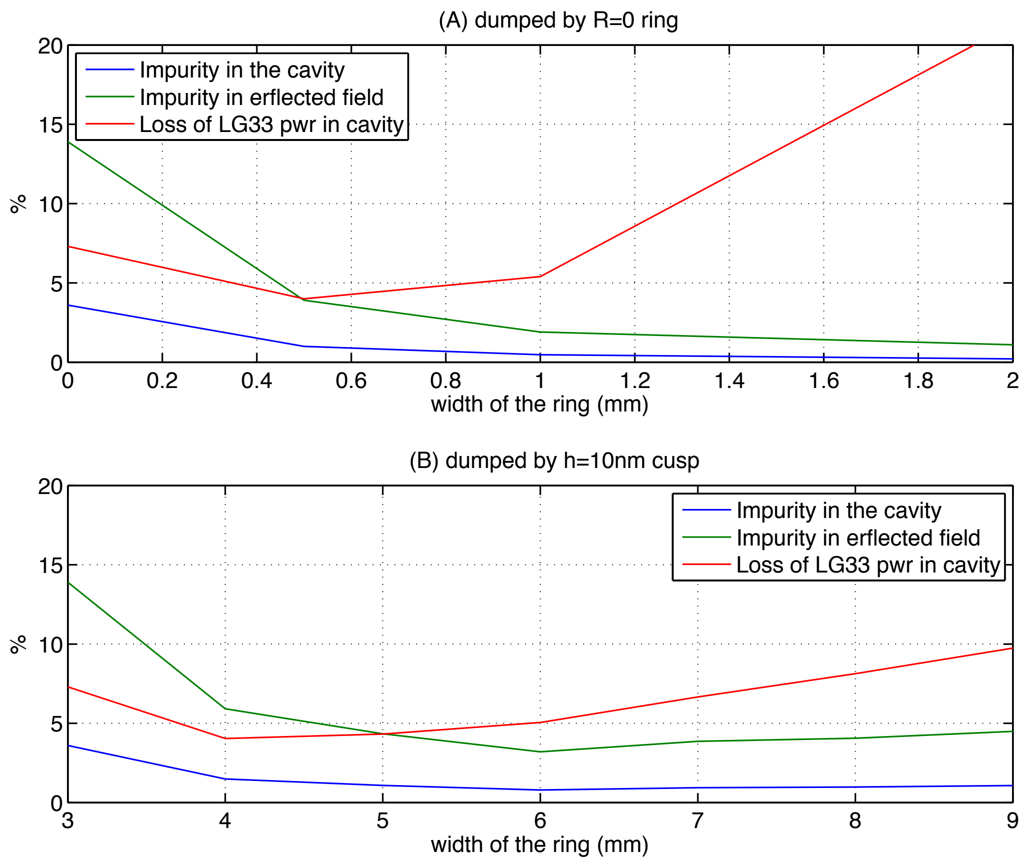


**Figure 3 Impurity and loss**

Fig.3(A) shows the result when R=0 ring is placed and Fig.3(B) when cusp ring with height of 10nm is placed, both as a function of the ring width dr. The blue line is the impurity in the cavity.

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[1] The smooth function s(r,r0,dr) is characterized by the center (r0) and the width (dr). s(r<r-dr,r0,dr) = 0, s(r=r0,d0,dr) = 1, s(r>r-dr,r0,dr) = 0, derivative of s is 0 at r=r0±dr and at r=r0, and 3rd order polynomial is used to connect between r=r0-dr and r0+dr.

The red line is the loss of the LG33 mode power defined as 1 – (Power of LG33 with aberrations and ring irregularities) / (Power of LG33 when there is no aberration). This includes the mode matching effect between the arm cavity and the outside system[2] and the diffractive loss, so this is a good quantity to estimate the signal loss. The blue and red lines are to be used as the figure of merit.

When R=0 rings are placed, the impurity becomes smaller as the ring width becomes larger. But, with larger ring width, the loss of the signal becomes large. When h=10nm rign cusp is placed, the impurity has the minimum at around 6~8mm, with the loss over 5%. There is no acceptable solution for either case.

This does not cover all possible irregularities. For example, when the height of the cusp is increased to 20nm, the purity does improve at some width, but no order of magnitude change was found when several alternative parameters were used.

Case (f) and (g) in Table.1 show the content of modes in the cavity. As is seen from this table, LG(4,1) is the largest mode remaining in the cavity as the dominant impurity source.

[2] In this note, standalone FP cavity is studied, but, in the real situation, the arm is a part of an interferometer and the mode matching between the arm and the rest of the system is to be taken into account. The power loss defined in this note takes into account this mode matching as the coupling of the injected field and the arm mode.