Actuator Sizing of a Quad Pendulum for Gravitational Wave Detectors

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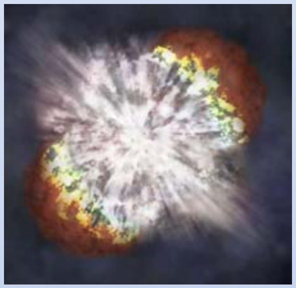
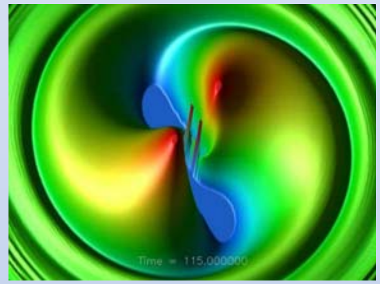
June 29

Gravitational Waves



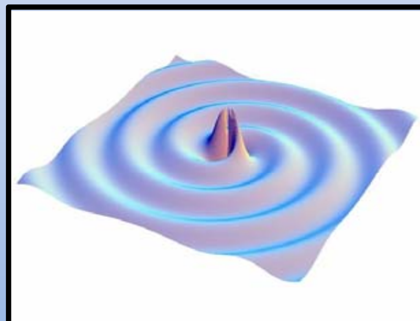
Supernova

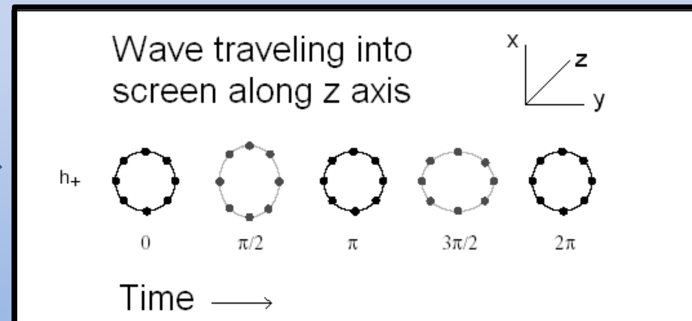


Merging Black Holes

Pulsar

Wave of strain amplitude h

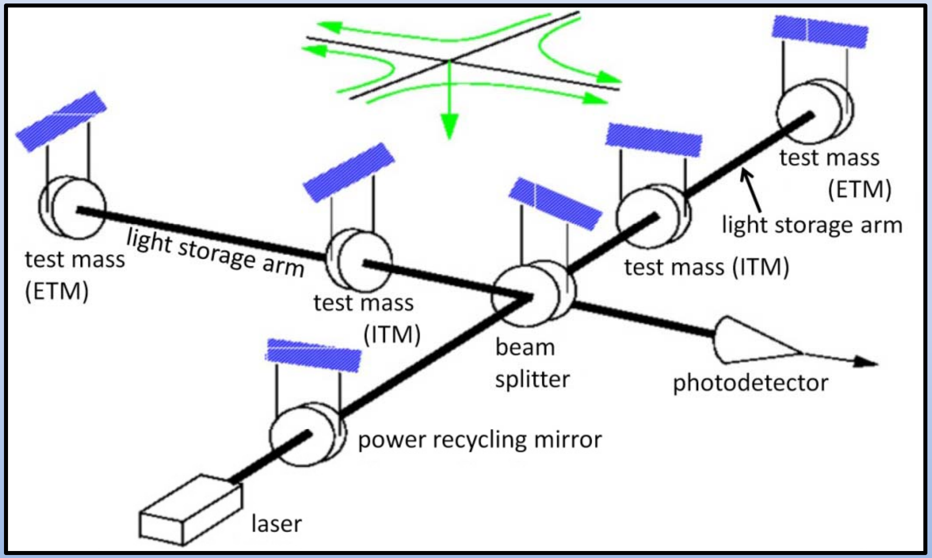




- Supernovae
  - Asymmetry required
- Coalescing Binaries
  - Black Holes or Neutron Stars Mergers
- Pulsars
  - Asymmetry required
- Stochastic Background (Big bang, etc.)

The Laser Interferometer Gravitational-wave Observatory (LIGO)

Funded by



Hanford, WA

Livingston, LA

- ***3, 4 km and long interferometers at 2 sites in the US***
- ***Michelson interferometers with Fabry-Pérot arms***
- ***Optical path enclosed in vacuum***
- ***Sensitive to displacements around $10^{-19} m_{rms}$***

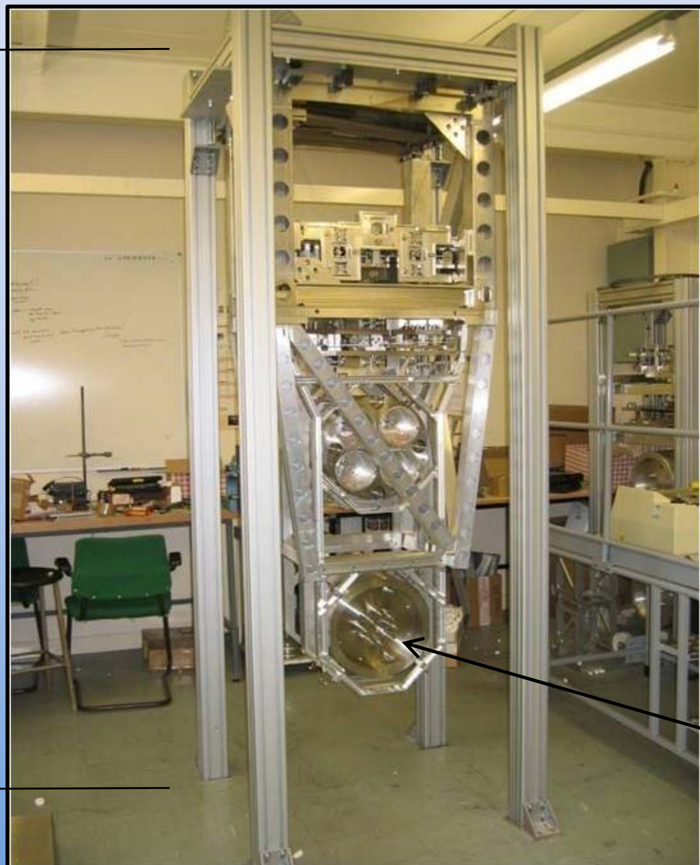
LIGO Scientific Collaboration

- Australian Consortium for Interferometric Gravitational Astronomy
- The Univ. of Adelaide
- Andrews University
- The Australian National Univ.
- The University of Birmingham
- California Inst. of Technology
- Cardiff University
- Carleton College
- Charles Sturt Univ.
- Columbia University
- CSU Fullerton
- Embry Riddle Aeronautical Univ.
- Eötvös Loránd University
- University of Florida
- German/British Collaboration for the Detection of Gravitational Waves
- University of Glasgow
- Goddard Space Flight Center
- Leibniz Universität Hannover
- Hobart & William Smith Colleges
- Inst. of Applied Physics of the Russian Academy of Sciences
- Polish Academy of Sciences
- India Inter-University Centre for Astronomy and Astrophysics
- Louisiana State University
- Louisiana Tech University
- Loyola University New Orleans
- University of Maryland
- Max Planck Institute for Gravitational Physics
- University of Michigan
- University of Minnesota
- The University of Mississippi
- Massachusetts Inst. of Technology
- Monash University
- Montana State University
- Moscow State University
- National Astronomical Observatory of Japan
- Northwestern University
- University of Oregon
- Pennsylvania State University
- Rochester Inst. of Technology
- Rutherford Appleton Lab
- University of Rochester
- San Jose State University
- Univ. of Sannio at Benevento, and Univ. of Salerno
- University of Sheffield
- University of Southampton
- Southeastern Louisiana Univ.
- Southern Univ. and A&M College
- Stanford University
- University of Strathclyde
- Syracuse University
- Univ. of Texas at Austin
- Univ. of Texas at Brownsville
- Trinity University
- Tsinghua University
- Universitat de les Illes Balears
- Univ. of Massachusetts Amherst
- University of Western Australia
- Univ. of Wisconsin-Milwaukee
- Washington State University
- University of Washington

Quadruple Pendulum

Prototype quad pendulum

≈2 m



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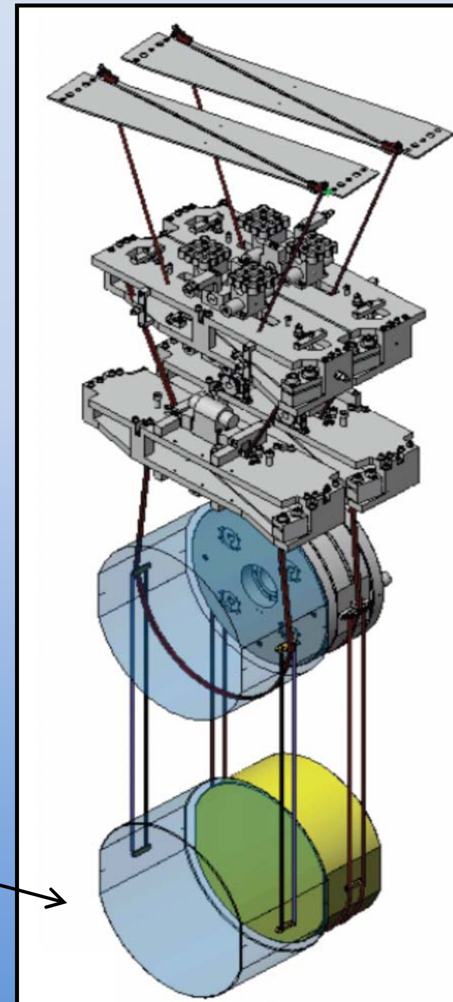
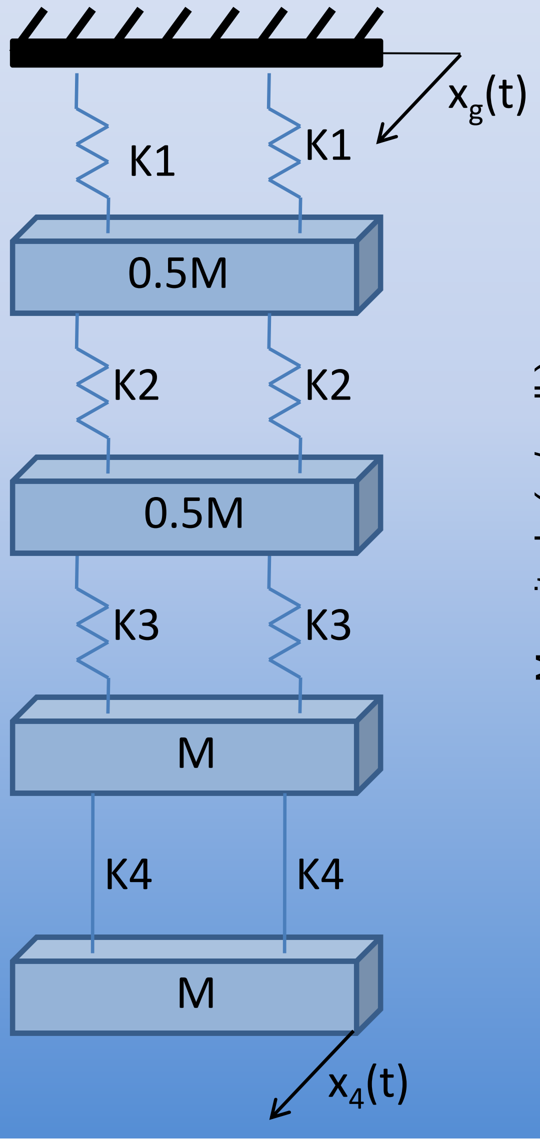
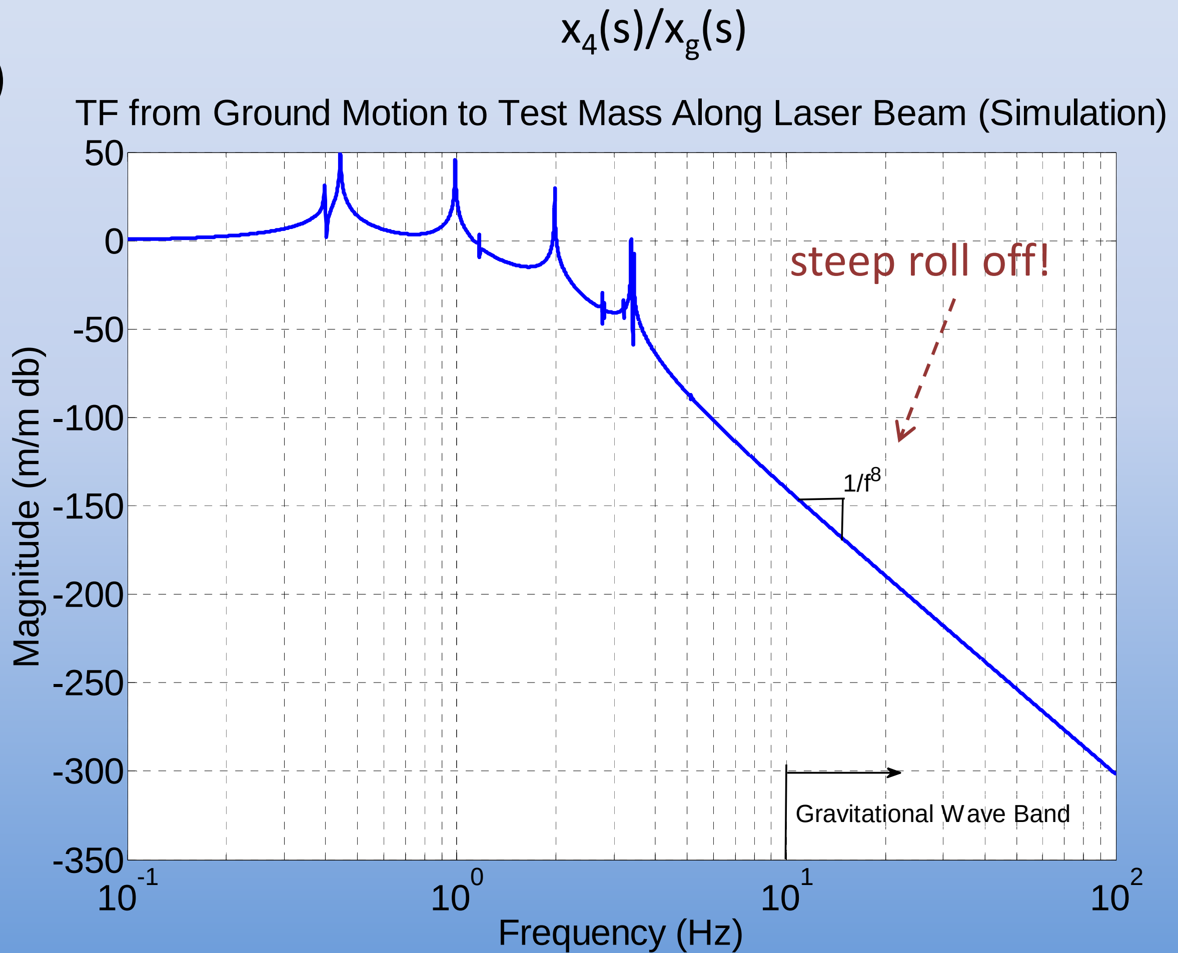
Test Mass
40 Kg



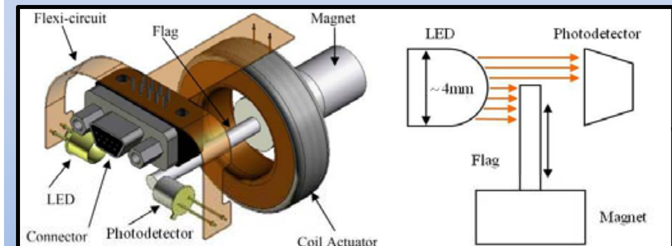
Quadruple Pendulum (Quad) Isolation



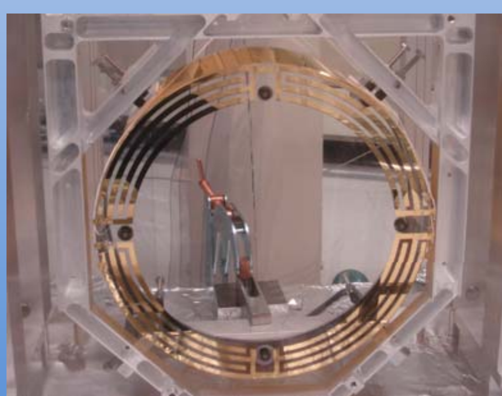


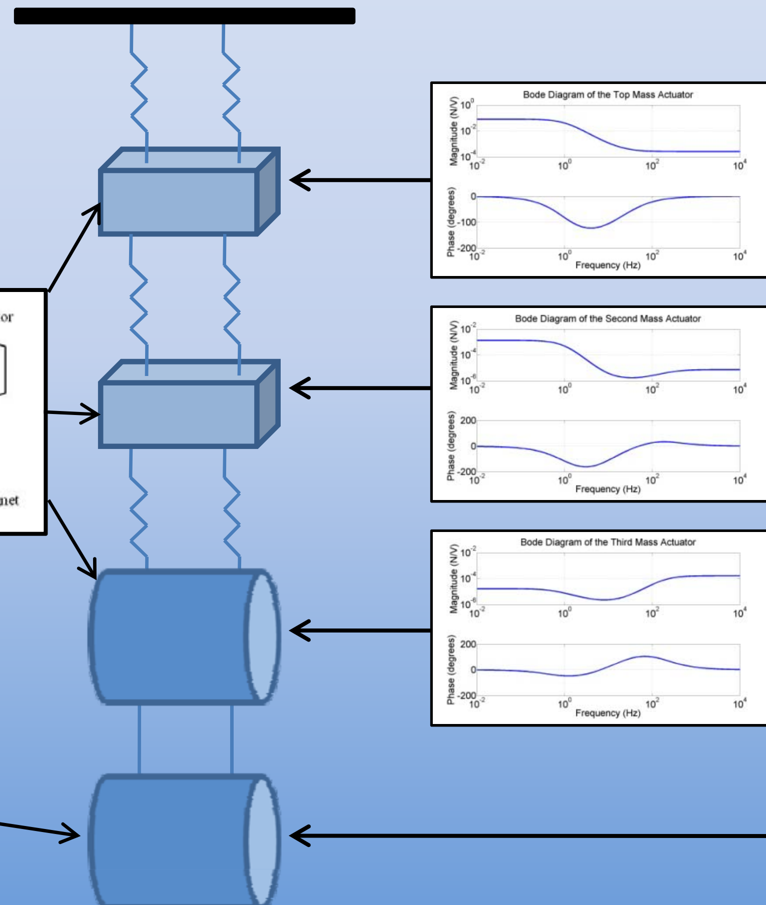
Actuator Frequency Response

Electromagnetic actuator



Electrostatic actuator





u_1 205 mN DC

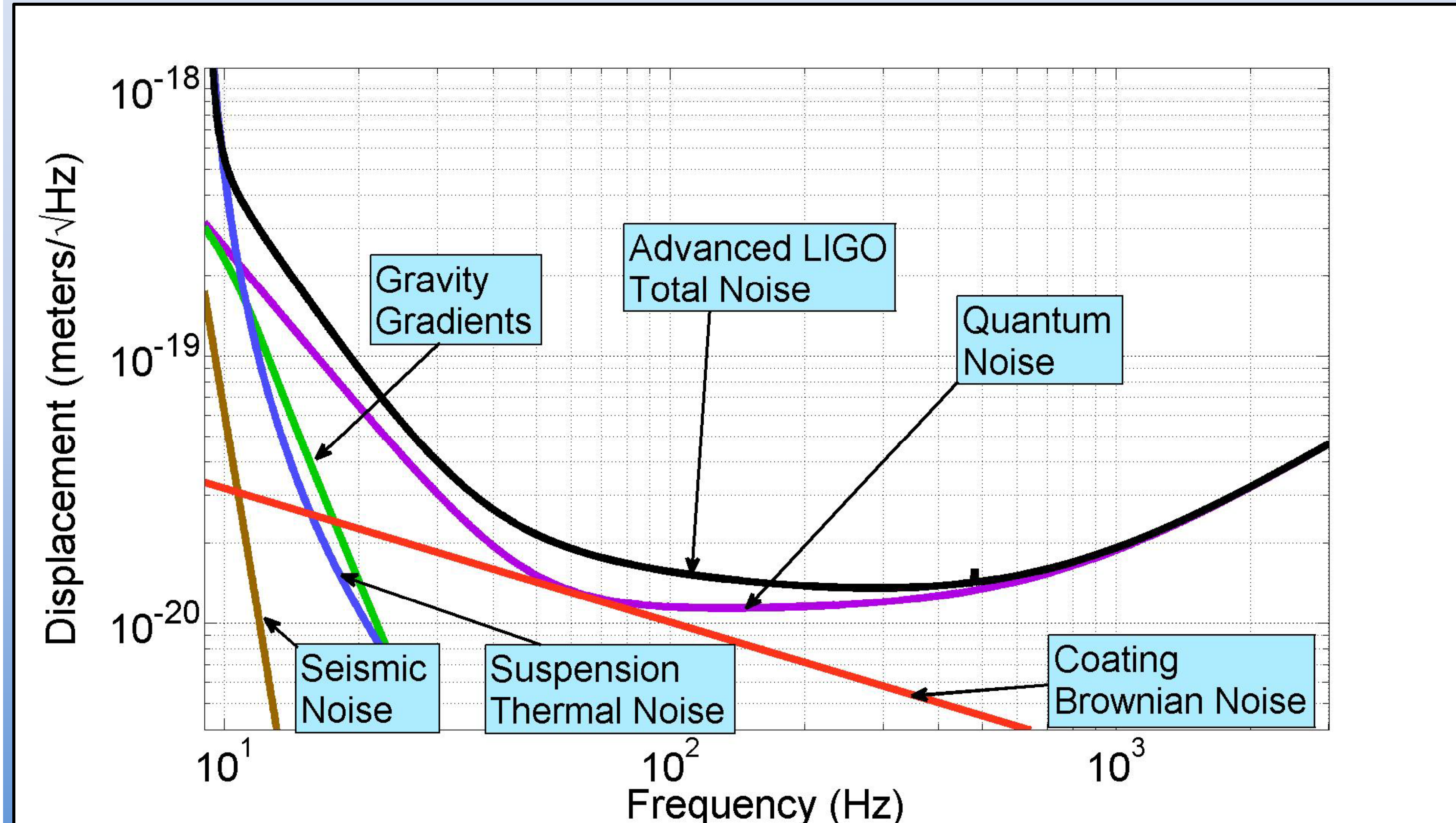
u_2 205 mN DC

u_3 3.4 mN DC

u_4 0.095 mN

Location of Quadruple Pendulums

Advanced LIGO Sensitivity



Outline

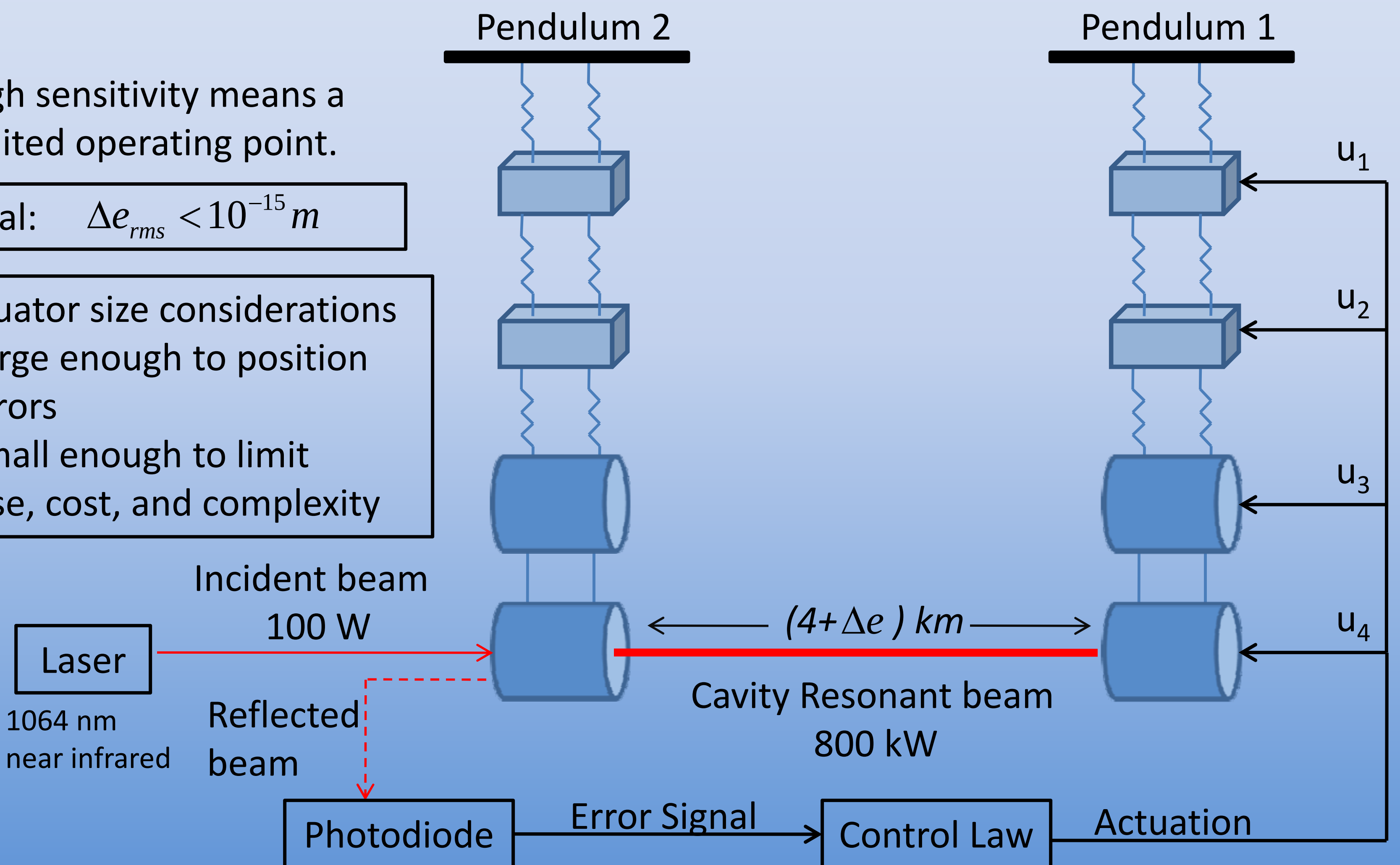
- Gravitational Waves and LIGO
- Quadruple Pendulum
- Quad Pendulum Actuator Specifications
- Actuator Sizing
- Conclusion

Optical Arm Cavity Control

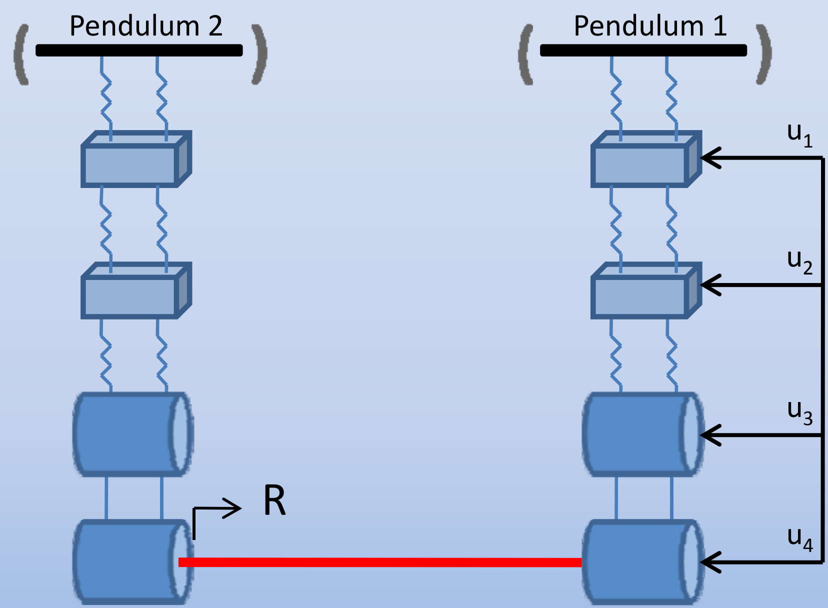
High sensitivity means a limited operating point.

Goal: $\Delta e_{rms} < 10^{-15} m$

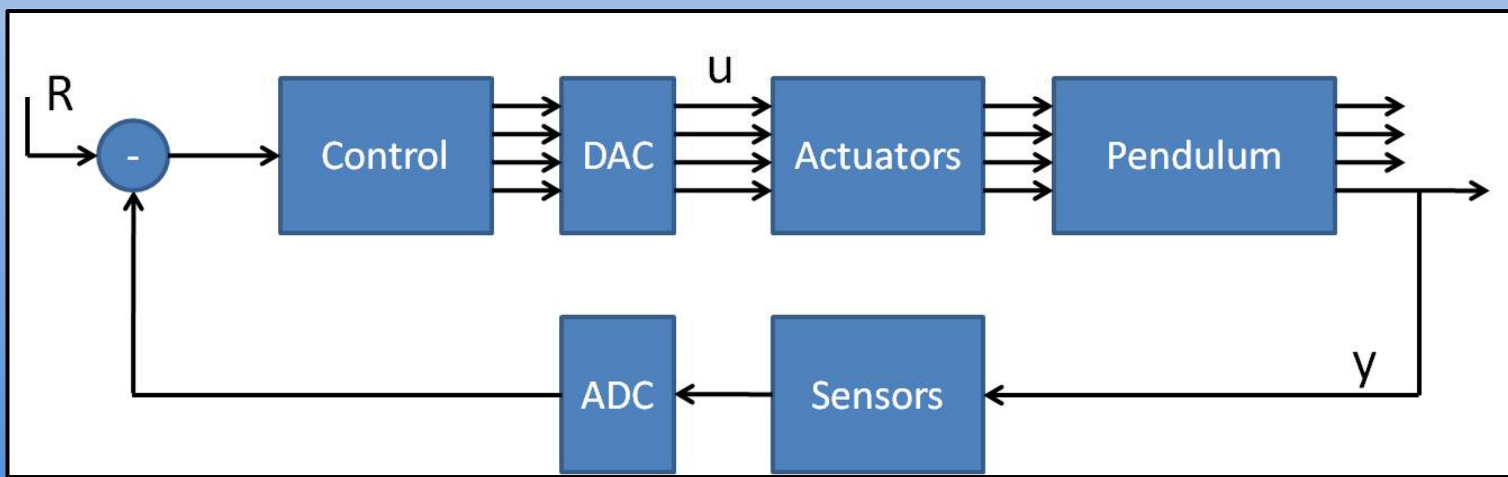
Actuator size considerations
- Large enough to position mirrors
- Small enough to limit noise, cost, and complexity



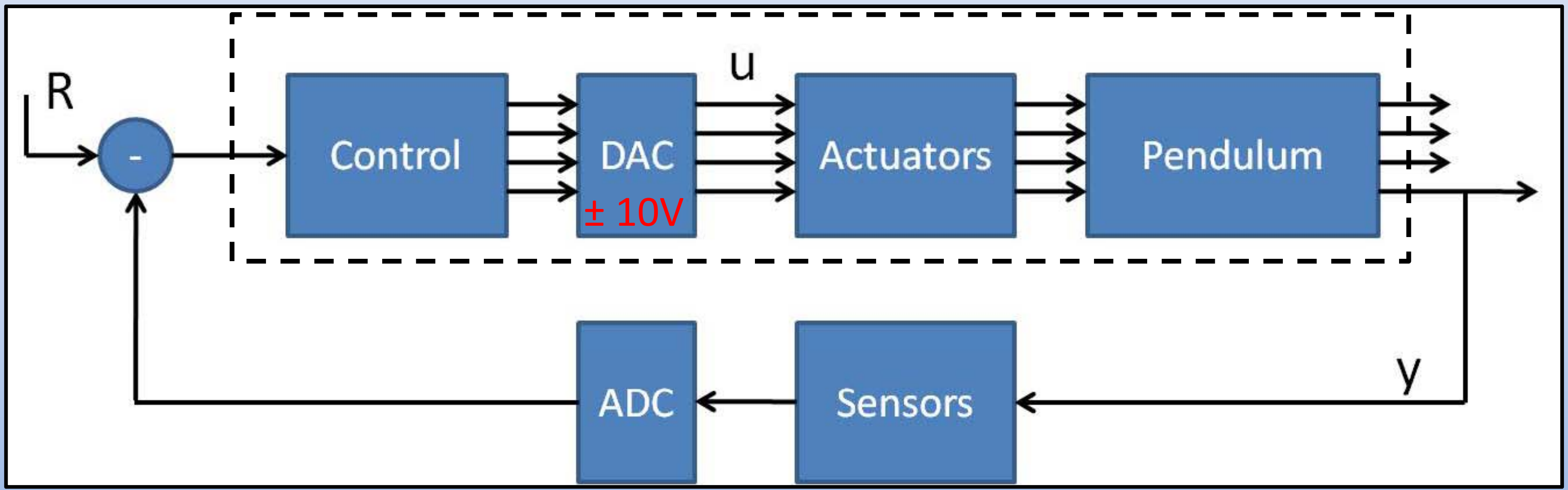
Optical Arm Cavity Control



Standard tracking diagram

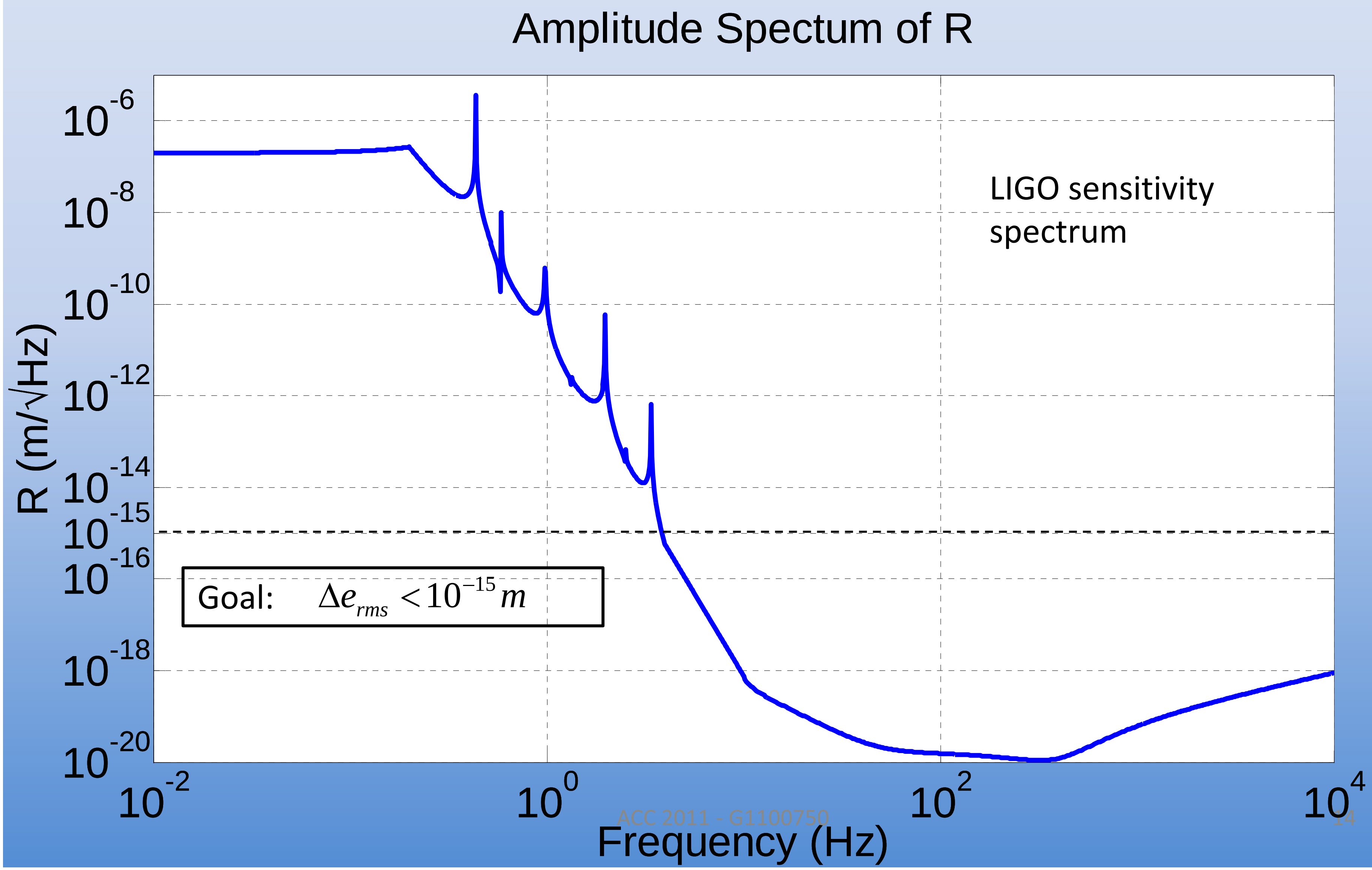


Arm Control Block Diagram



* What is the minimum baseline target size for each actuator, without considering an infinite set of possible controller designs?

Disturbance Spectrum



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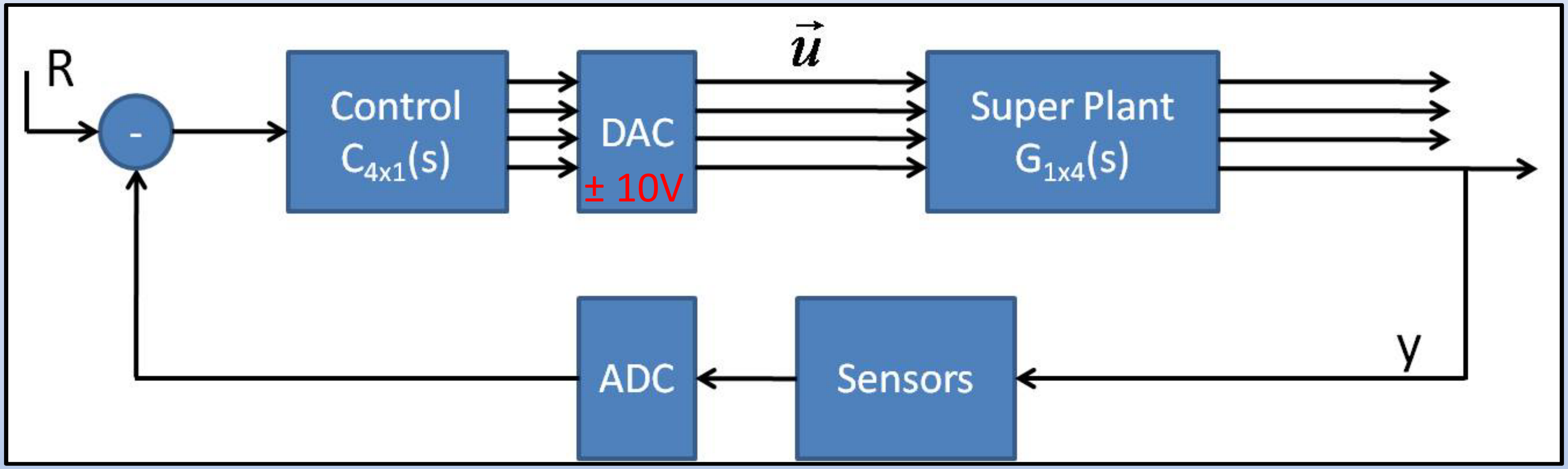
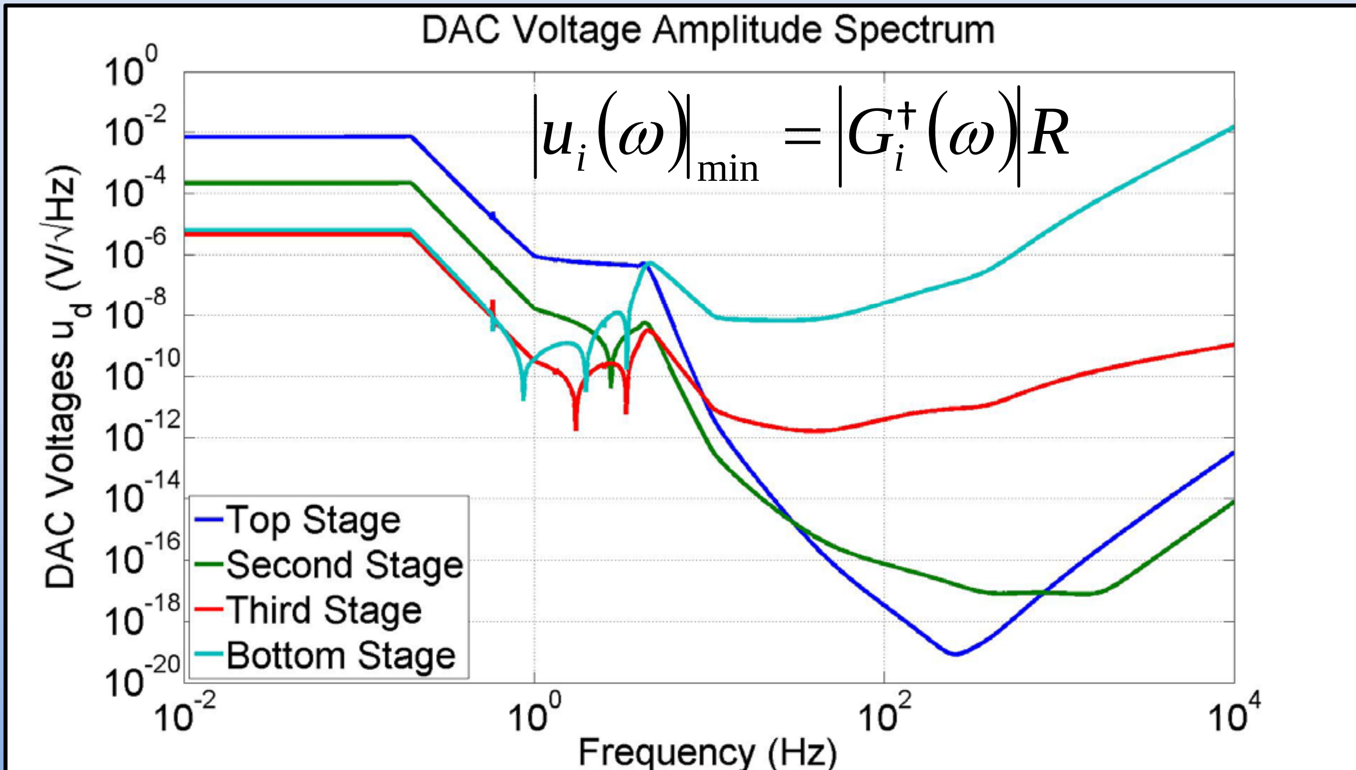
Arm Control Block Diagram



- $y = G\vec{u} = R \quad \Longrightarrow \quad \vec{u}_{\min} = G^{\dagger} R$
- $y = \dfrac{GC}{1+GC} R \approx R \qquad GC >> 1$
- $\vec{u} = \dfrac{C}{1+GC} R \qquad GC >> 1 \quad \Longrightarrow \quad \vec{u} \approx G^{\dagger} R = \vec{u}_{\min}$

† -> Moore-Penrose pseudoinverse

Minimum Least Squares Actuation



The relative magnitudes between curves quantifies the relative 'effectiveness' of an actuator.

	RMS (V)	p
Top Stage	3.3×10^{-3}	0
Second Stage	1.0×10^{-4}	0
Third Stage	2.1×10^{-6}	0
Mirror	0.57	3.1×10^{-69}

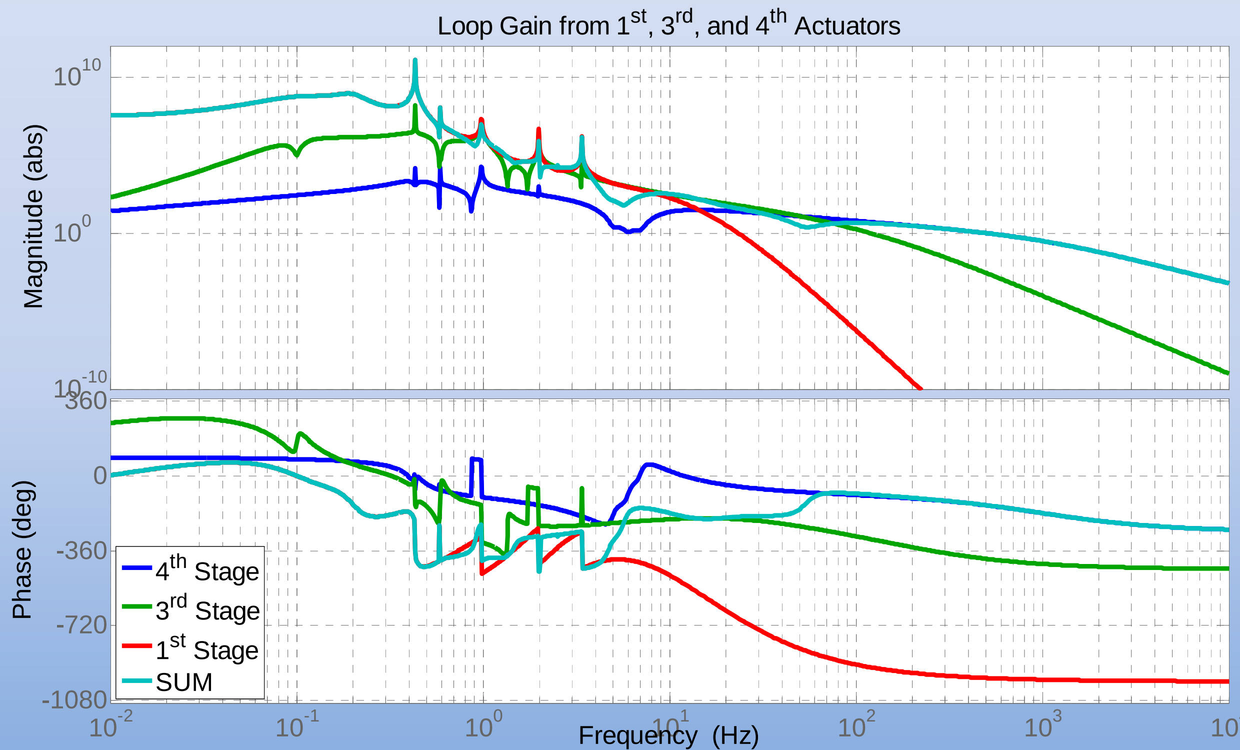
$$p_i = 1 - erf\left(\frac{10}{\sqrt{2}u_{i,RMS}}\right)$$

Conclusions

- The pseudoinverse of the combined plant TF shows the actuators have enough range with reasonable margin.
- Also shows which actuator is most effective at each frequency.
- No feedback design is required.
- The pseudoinverse can guide feedback design.

Back Ups

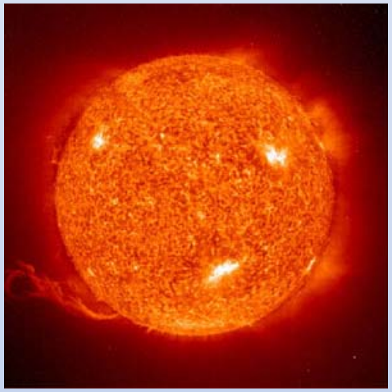
Example Feedback Loop Gains



	RMS (V)	p	Feedback RMS (V)
Top Stage	3.3×10^{-3}	0	3.7*10^-3
Second Stage	1.0×10^{-4}	0	0
Third Stage	2.1×10^{-6}	0	2*10^-2
Mirror	0.57	3.1×10^{-69}	8*10^-5

Seismic Noise

- Tides - ≈ 10^{-5} Hz

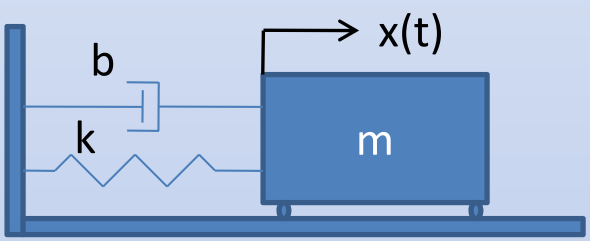


- Microseismic peak - ≈ 0.1 to 0.3 Hz

- Anthropogenic Noise - ≈ 1 to 10 Hz

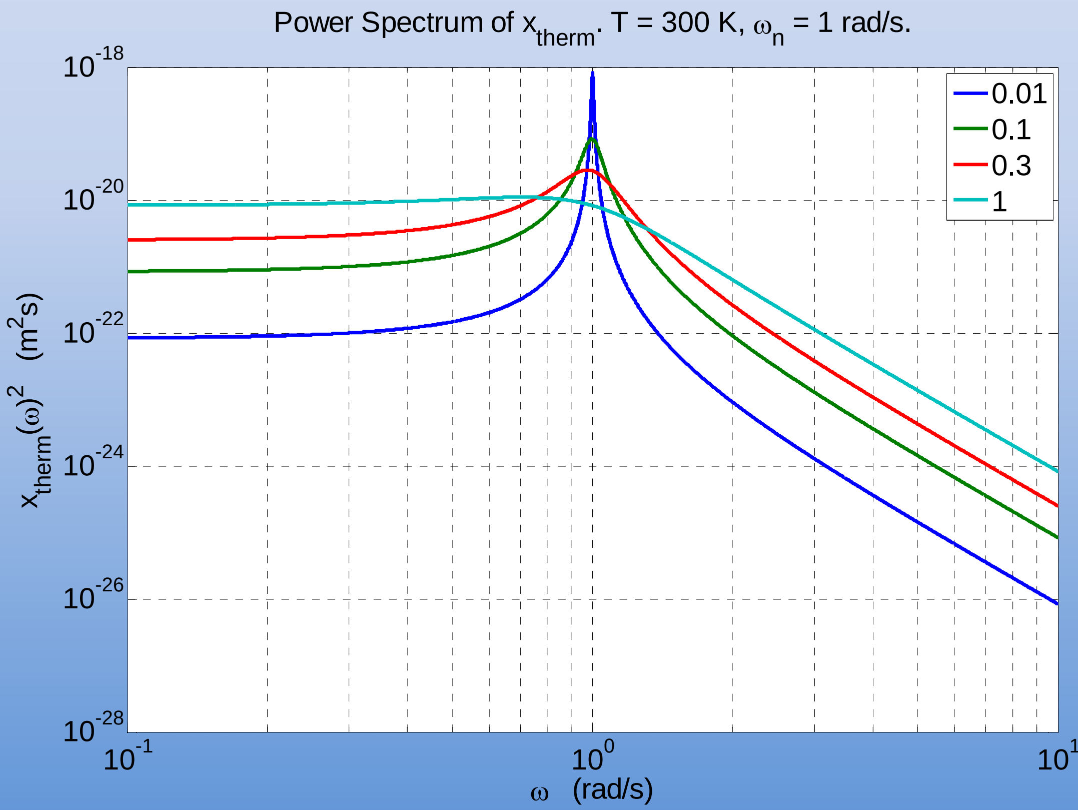
Thermal Noise

System in *thermal equilibrium*



b = dissipation factor (damping)
T = Temperature (Kelvin)
k_b = Boltzmann's Constant

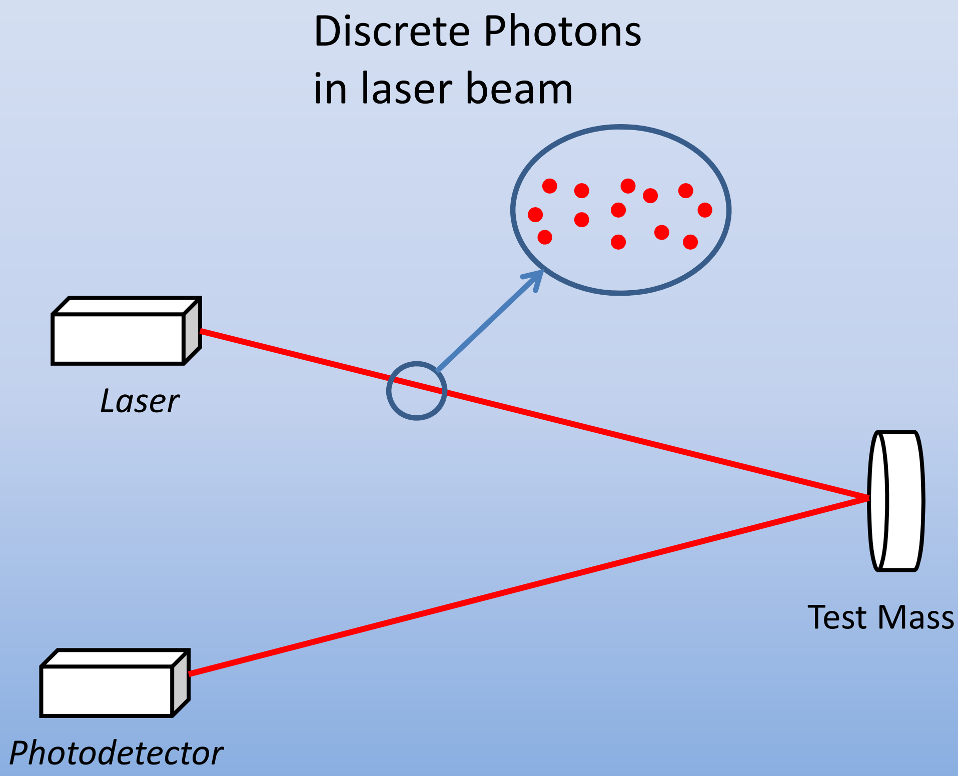
$$x^2_{therm}(\omega) = \frac{2k_B T b}{\omega^2[b^2 + m(\omega - \frac{\omega_n^2}{\omega})^2]}$$



Mean energy: (equipartition function)

$$\overline{x^2} = \frac{k_B T}{k}$$

Quantized Optical Noise

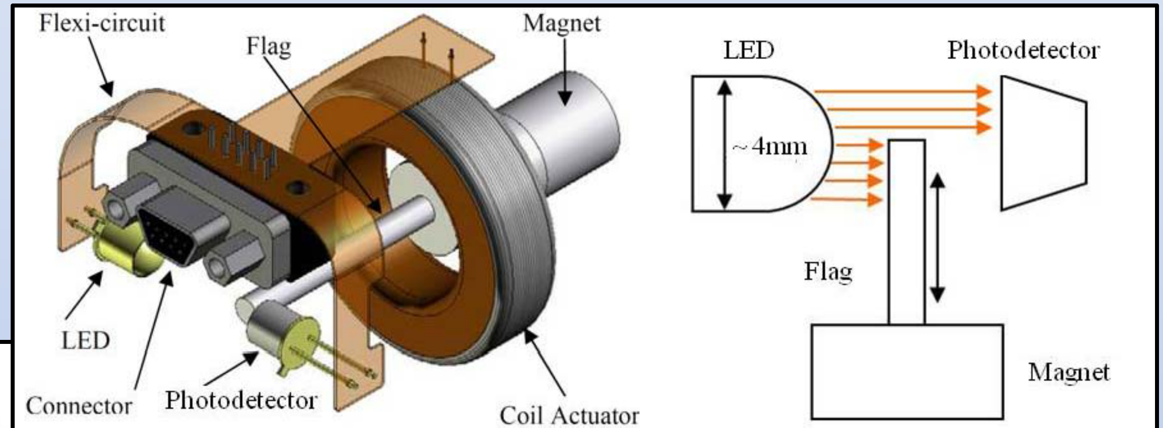


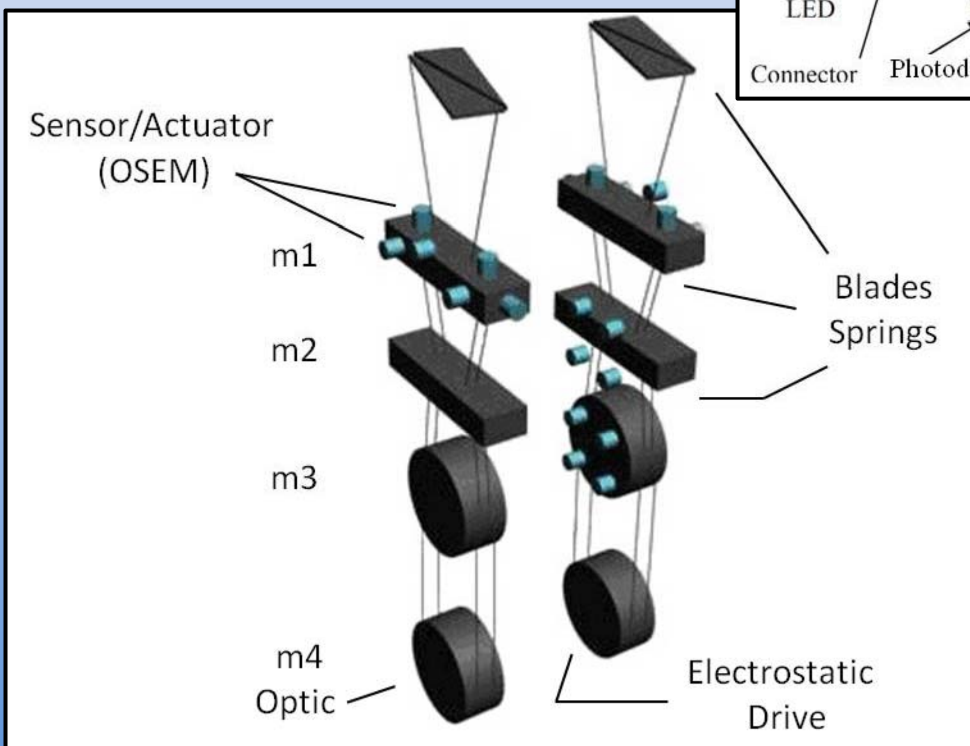
Laser light consists of a finite (and uncertain) numbers of photons.

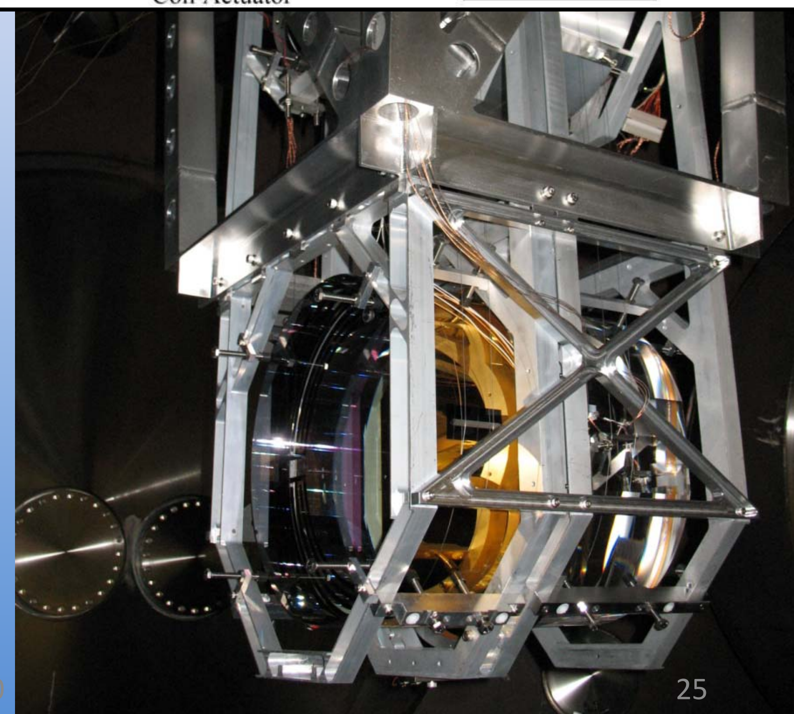
The momentum transfer onto the test mass from a random distribution of photons produces ***radiation pressure noise***.

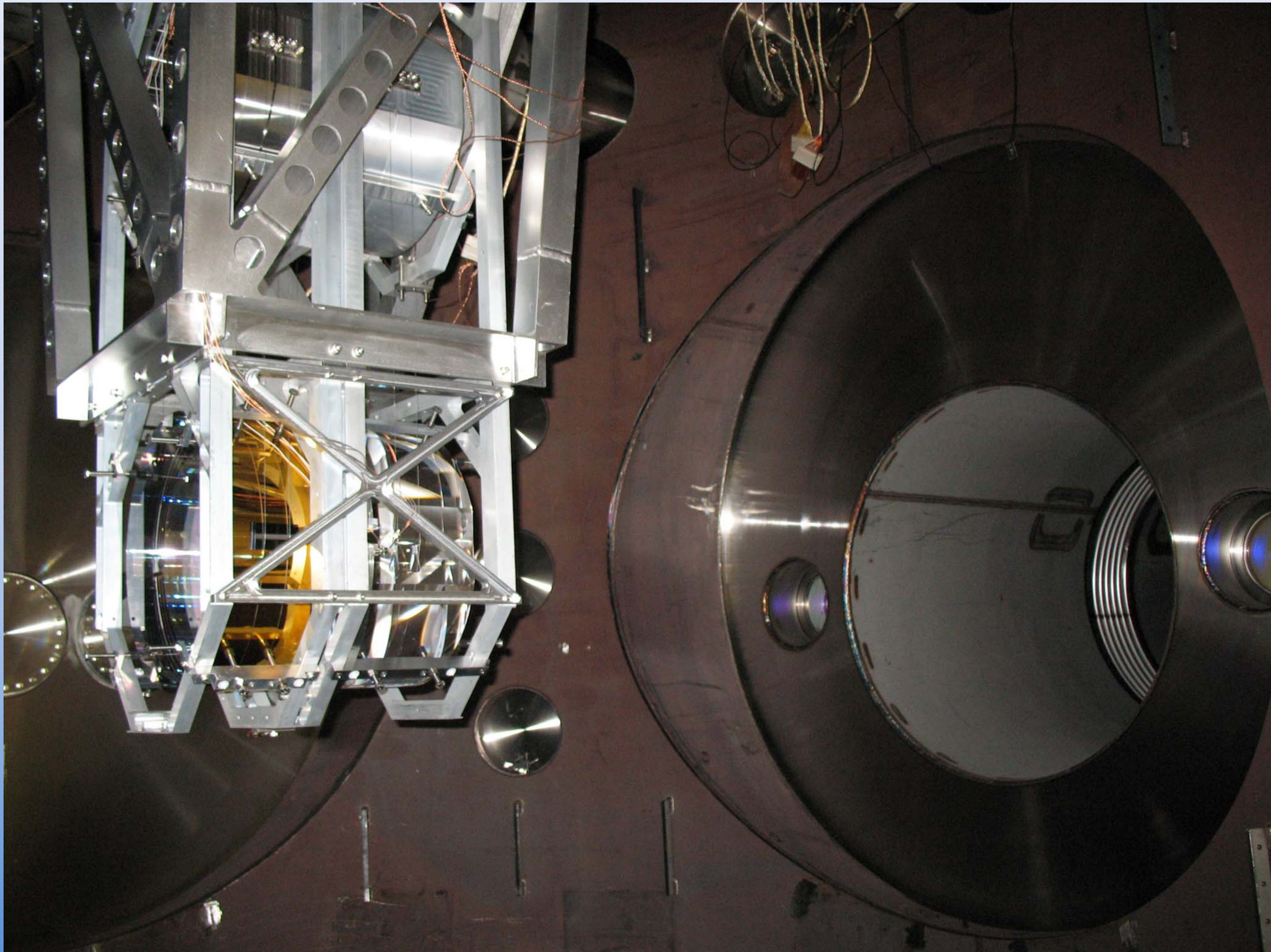
The random distribution of photons arriving at the photodetector produces ***shot noise***.

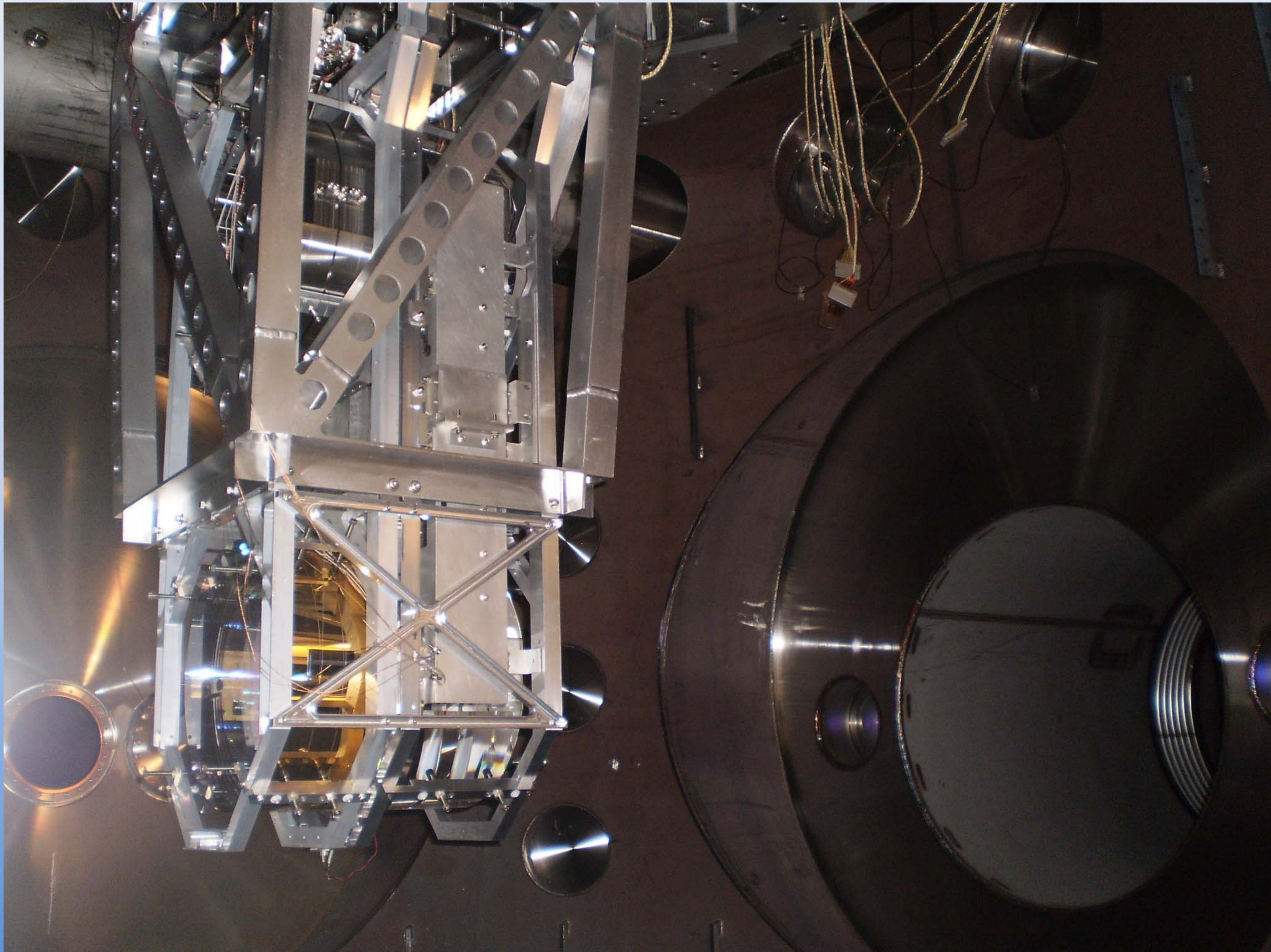
Actuators


Flexi-circuit
Flag
Magnet
LED
Connector
Photodetector
Coil Actuator
LED
Photodetector
~4mm
Flag
Magnet

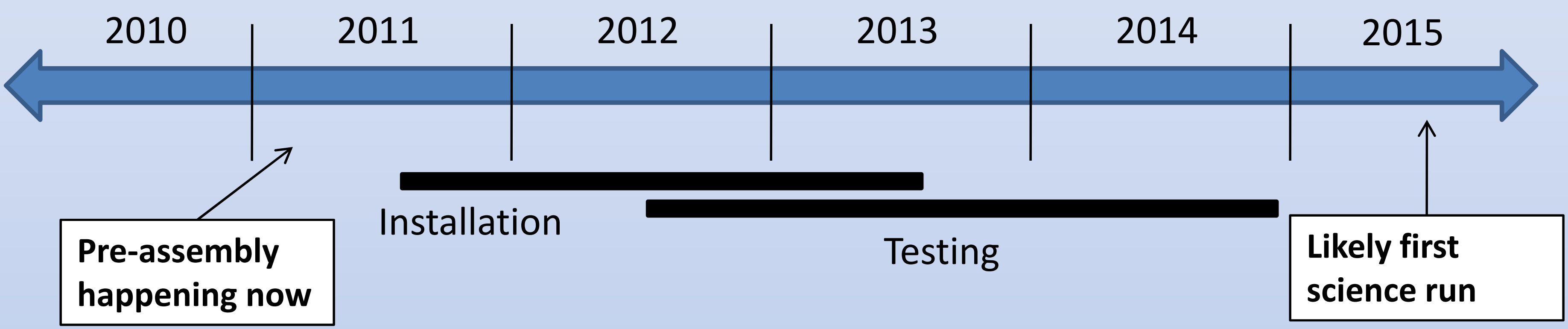

Sensor/Actuator (OSEM)
m1
m2
m3
m4 Optic
Blades Springs
Electrostatic Drive



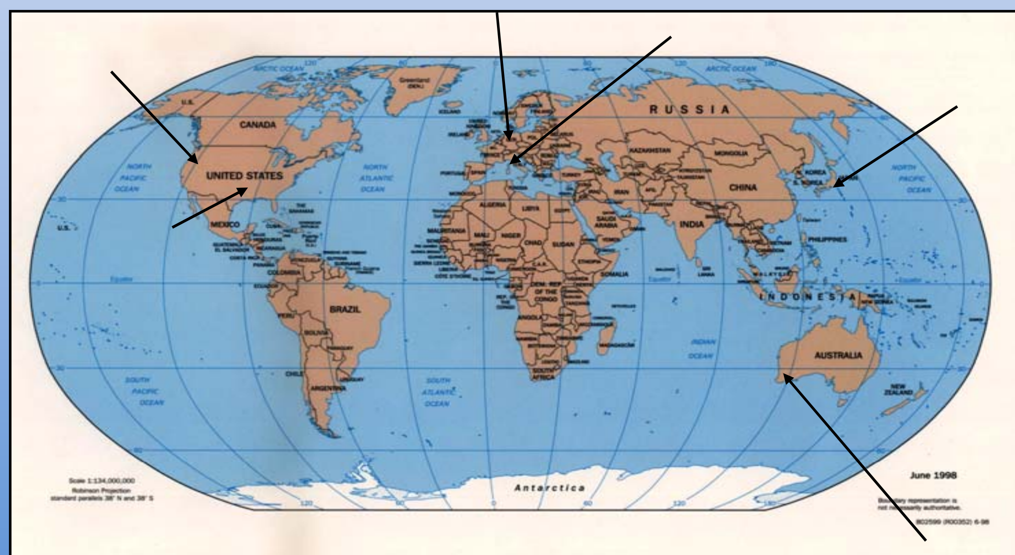




Advanced LIGO Schedule

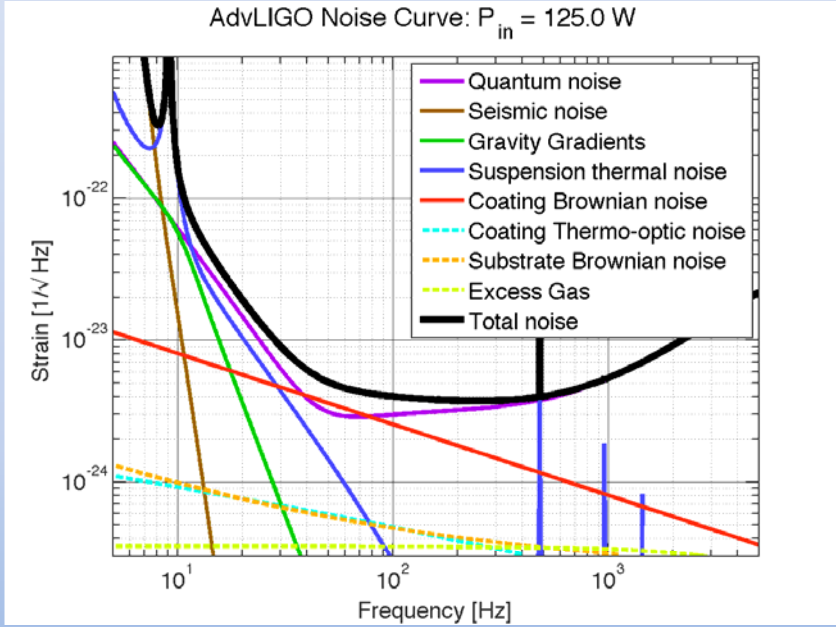


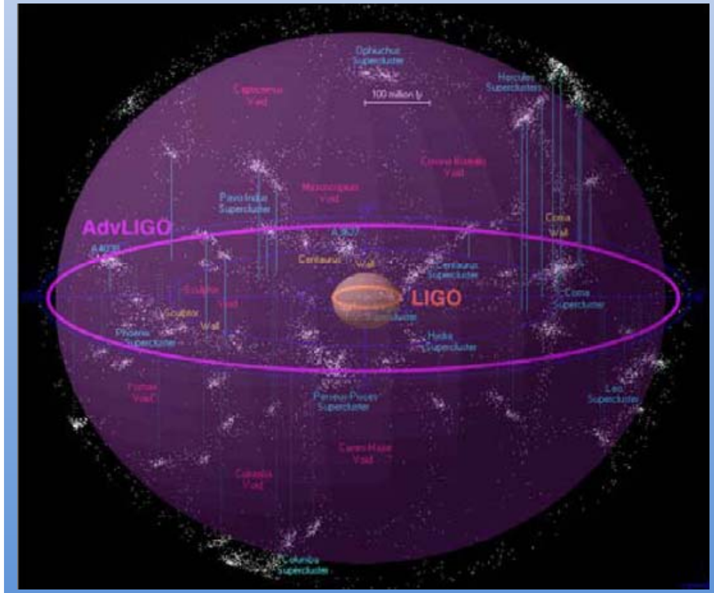
• Other observatories around the world collecting data during this time.



Advanced LIGO

- **Quantum noise limited in much of band**
- **Thermal noise in most sensitive region**
- **About factor of 10 better sensitivity**
- **Expected sensitivity**
  - Neutron star inspirals to about 200 Mpc, ~**40/yr**
  - 10 M_O black hole inspirals to 775 Mpc, ~**30/y**





Advanced LIGO Astronomical Reach

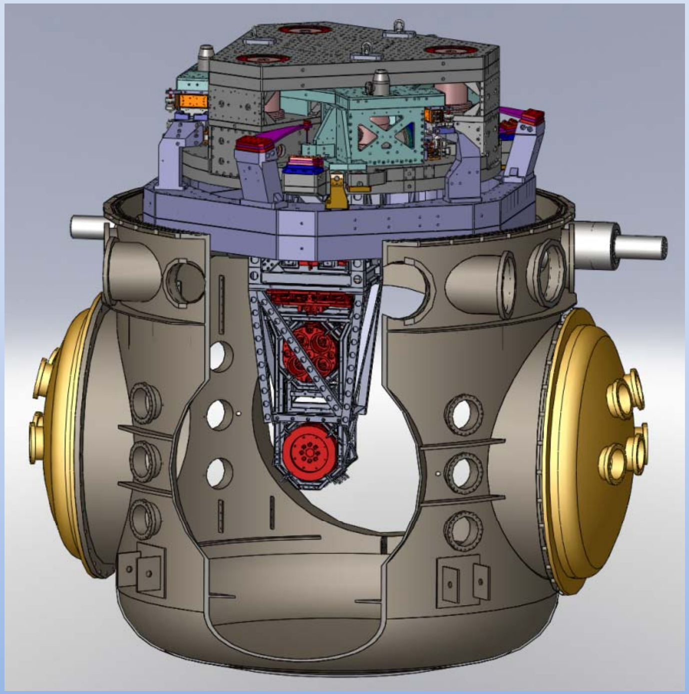
LIGO infrastructure designed for a progression of instruments

- Nominal 30 year lifetime

All subsystems to be replaced and upgraded

- More powerful laser – from 10W to 180 W
- Larger test masses – from 10 kg to 40 kg
- More aggressive seismic isolation
- Lower thermal noise coatings

Seismic Isolation



ISI and Quad in LIGO Vacuum Chamber

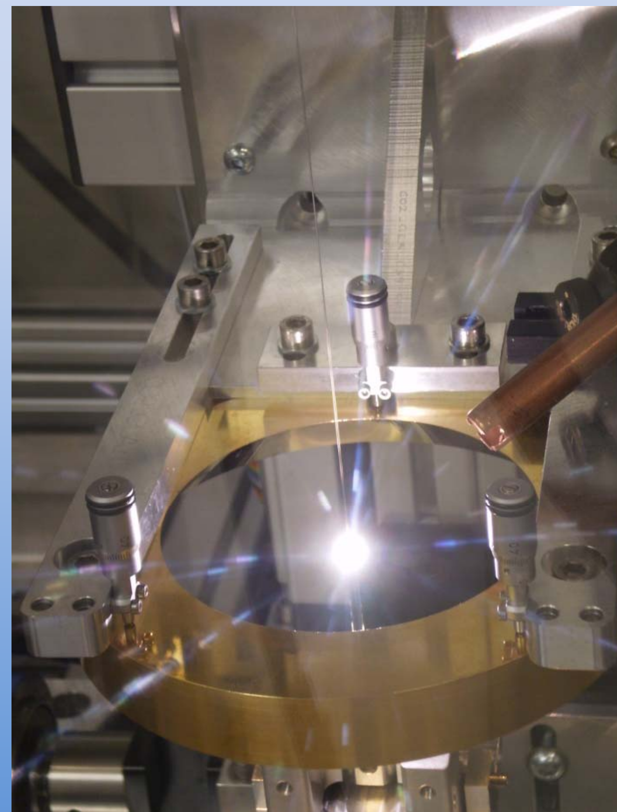
Prototype ISI and Quad at MIT

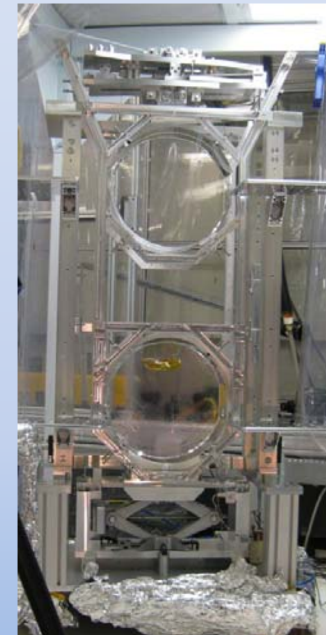
Overall Isolation 9 to 10 orders of magnitude at 10 Hz.

- 7 cascaded stages of seismic isolation
  - External Hydraulic Pre-Isolator (HEPI). Active isolation up to 10 Hz.
  - 2 stage Internal Seismic Isolation (ISI). Active isolation up to 30 Hz, passive above.
  - 4 stage Quadruple Pendulum (Quad). Mirror is the final stage. Passive isolation above 1 Hz.

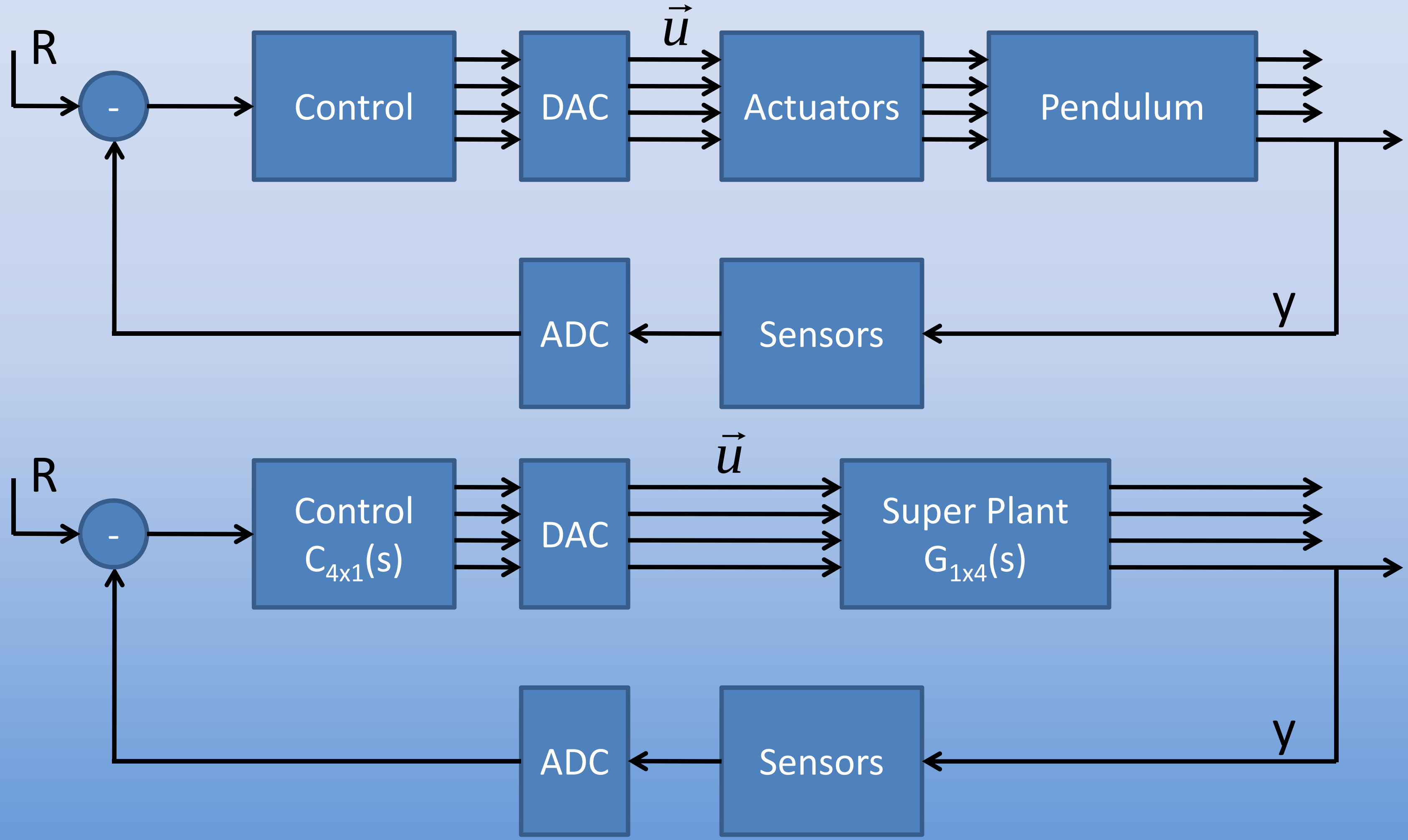
Mirrors Suspend from Glass Fibers

- Developed by the University of Glasgow
- Suspension thermal noise suppressed by suspending low loss fused silica test masses from fused silica fibers.





- 0.6m long, 400 μm diameter silica fibers pulled from 3 mm diameter stock and laser welded between the two silica lower stages of the quadruple pendulum.


R
-
Control
DAC
$\vec{u}$
Actuators
Pendulum
y
ADC
Sensors
R
-
Control
$C_{4x1}(s)$
DAC
$\vec{u}$
Super Plant
$G_{1x4}(s)$
y
ADC
Sensors