

**CALIFORNIA INSTITUTE OF TECHNOLOGY**  
**Laser Interferometer Gravitational Wave Observatory (LIGO) Project**

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Subject: Internal Modes of Testmasses

MODE FREQUENCY (Hz)	EFFECTIVE MASS (End / Vertex) <sup>a</sup>	N_diametric	N_parity	N_order
6788.472592		2	1	1
9421.511818	0.576803 / 0.497243	0	1	1
11176.171312		2	0	1
12395.991604		3	1	1
12505.690841		1	0	1
13810.572752		1	0	2
14301.593214	8.240901 / 7.868536	0	0	1
14987.907139		1	1	1
17077.955709		3	0	1
17821.830937		4	1	1
18740.201201	22.775114 / 5.209551	0	1	2
19191.655179		2	0	2
19951.897227		2	1	2
21503.625145		1	1	2
22126.719668		4	0	1
22204.901804	0.295987 / 0.217764	0	1	3
23081.515718		1	1	3
23740.979462		2	1	3
24406.321839		3	1	2
24816.979527		1	0	3
25139.459161	0.613834 / 0.581486	0	0	2
25216.624536		3	0	2
25939.236387		1	0	4
26222.806734		2	0	3
27176.523617		2	1	4
27694.232989		2	1	5
27811.992017		4	0	2
28109.490933		3	0	3
28618.512673		4	1	2
28983.253389		2	0	4
29100.823543	65.952693 / 390.108796	0	0	3
29431.411109		1	0	5
29587.672147	3.659772 / 1.224146	0	1	4
30762.304995		1	1	4
30782.353924	0.114216 / 0.086612	0	0	4
31512.783158		3	0	4
32396.862306		1	0	6
32516.415991		3	0	5
32718.423224		3	1	3

a. Beamsports used were 4.565cm for end mirror and 3.634cm for vertex mirror.

The internal modes for the LIGO test masses of dimensions 10cm height and 12.5cm radius are given in the table above. The columns in this table give the mode resonance frequency, effective mass coefficient, the diametric (or circumferential) mode number, the parity mode number and the mode order number for given diametric mode. These modes are calculated using Kent Blackburn's corrected code based on the original codes of Gillespie and Hutchinson. The code uses the theory of elasticity to approximate the modes found in the testmasses. For the axisymmetric modes ( $N_{\text{diametric}} = 0$ ), the code will also calculate the effective mass used in the determination of the thermal noise contribution from a particular mode. No model exists within the project to calculate the thermal noise contribution from the higher diametric modes given in the table. The thermal noise from the non-axisymmetric modes will approach zero as the beam on the mirror face approaches center. The results of this code are not exact. Infinite series expansions of the mode representations are approximated by finite series having 40 terms in the radial and azimuthal directions. The code has shown good agreement with measurements taking by both LIGO and VIRGO in the past.

The table can be used to assist in determining the size of the aliased signal from mode resonances outside the Nyquist frequency associated with the desired sampling rate. Current plans for LIGO call for rates of 16,384 samples per second, with a Nyquist frequency of 8,192 Hz. This may prove too low a sample rate, making the anti-aliasing filters difficult or impossible to design with adequate roll-off to attenuate these resonances and still preserve unity gain of the gravitational wave signal of scientific interest. Mode frequencies in the table include all resonances below 32,768 Hz having diametric mode numbers less than or equal to 4, thereby allowing a characterization of the influence of the testmass modes in a data acquisition system having sampling rates 4 times higher than the current design.

Of the modes shown in the table the three at 18,740Hz, 22,205Hz and 25139Hz pose the most potential for aliasing at a sampling rate of 32,768 samples per second. As a result of the thermal noise power having a dependency as the inverse of the effective mass, the mode at 22,205Hz could come into the band at higher amplitude than the mode at 18,740Hz depending on the filter characteristics.

The gravitational bandwidth needed to carry out binary inspiral detection and parameter extraction requires unity gain of the signal out to about 25 times the frequency of greatest sensitivity. The frequency of greatest sensitivity will be approximately 140Hz in LIGO requiring a bandwidth of 3500Hz. Pulsar searches will require unity gain to frequencies on the order of 1000Hz. Quasi-normal mode ring-down of newly formed black holes will probably only be possible to frequencies of order 3000Hz. Millisecond burst events from supernovae will require a few kilohertz bandwidth to detect. These number are approxiamate and should only be used as guidelines.

Large non-Guassian events above the Nyquist frequency will probably pose the greatest challenge as they will be aliased into the gravitational wave bandwidth in a non predictable fashion.

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Chronological File

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