A Modelled Cross-Correlation Search for Gravitational Waves from Scorpius X-1

John T. Whelan

Center for Computational Relativity & Gravitation Rochester Institute of Technology, Rochester, NY, USA;

john.whelan@astro.rit.edu

Abstract

The low-mass X-ray binary (LMXB) Scorpius X-1 (Sco X-1) is a promising source of gravitational waves in the advanced detector era. A variety of methods have been used or proposed to perform the directed search for gravitational waves from a binary source in a known sky location with unknown frequency and residual uncertainty in binary orbital parameters. One of these is modification of the cross-correlation method used in the stochastic search which takes into account the signal model of a rotating neutron star to allow cross-correlation of data from different times. By varying the maximum allowed time lag between cross-correlated segments, one can tune this semicoherent search and strike a balance between sensitivity and computing cost.

Gravitational Waves from LMXBs

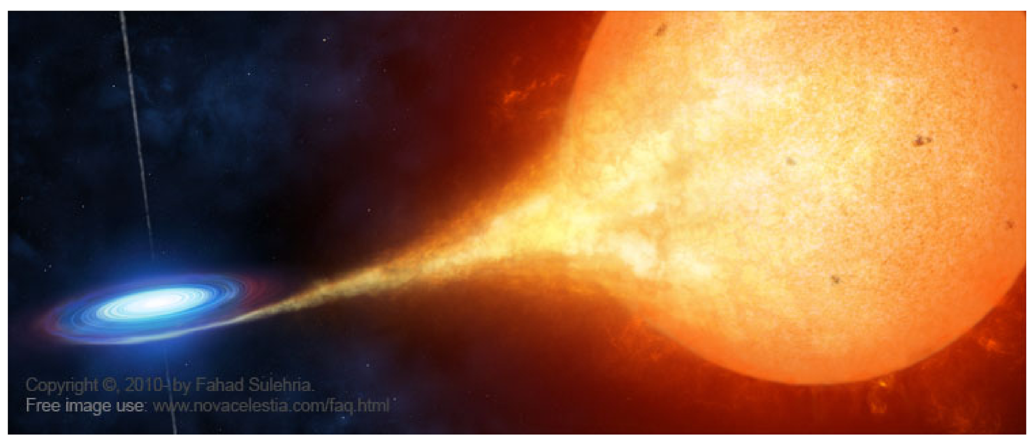

Figure 1: *Artist's impression of a low-mass X-ray binary. From Astronomical Illustrations and Space Art, by Fahad Sulehria, http://www.novacelestia.com/*

A low-mass X-ray binary is a binary of a compact object (neutron star or black hole) & a companion star. If the CO is a NS, accretion from the companion can produce a hot spot & power GW emission from the non-axisymmetric NS. If GW spin-down balances accretion spinup, GW strength can be estimated from X-ray flux, and GW freq $\approx$ constant [1]. Sco X-1, the brightest LMXB, is thought to be a $1.4M_\odot$ NS + $0.42M_\odot$ companion[2]. Proposed & applied search methods include a fully coherent search over a small amount of data [3], an unmodelled search for a monochromatic stochastic signal [4], a search for a pattern of sidebands arising from the Doppler modulation of the signal by the binary orbit [5], and the modelled cross-correlation search described here[6].

Cross-Correlation Method

- Divide data into segments of length T_{sft} & take "short Fourier transform" (SFT) $\tilde{x}_I(f)$.
- Label segments by $I, J, \dots$ (I & J can be same or different times or detectors)
- Label pairs by $\alpha, \beta, \dots$.
- Use CW signal model ($\mathcal{A}_+ = \frac{1+\cos^2\iota}{2}$; $\mathcal{A}_\times = \cos\iota$)

$$h(t) = h_0 \left[\mathcal{A}_+ \cos\Phi(\tau(t)) F_+ + \mathcal{A}_\times \sin\Phi(\tau(t)) F_\times\right]$$

- expected cross-correlation btwn SFTs I & J

$$E\left[\tilde{x}_I^*(f_{k_I})\,\tilde{x}_J(f_{k_J})\right] = \tilde{h}_I^*(f_{k_I})\,\tilde{h}_J(f_{k_J}) = h_0^2\,\tilde{\mathcal{G}}_{IJ}\,\delta_{T_{\text{sft}}}(f_{k_I}-f_I)\,\delta_{T_{\text{sft}}}(f_{k_J}-f_J)$$

☞ f_I is signal freq @ time T_I Doppler shifted for detector I

☞ $\delta_{T_{\text{sft}}}(f-f') = \int_{-T_{\text{sft}}/2}^{T_{\text{sft}}/2} e^{i2\pi(f-f')t}\,dt$ so $\delta_{T_{\text{sft}}}(0) = T_{\text{sft}}$.

- Construct $\mathcal{Y}_{IJ} = \frac{\tilde{x}_I^*(f_{k_I})\tilde{x}_J(f_{k_J})}{(T_{\text{sft}})^2}$ (where $f_{k_I} \approx f_I$) s.t.

$$E[\mathcal{Y}_\alpha] \approx h_0^2 \tilde{\mathcal{G}}_\alpha \qquad \mathrm{Var}[\mathcal{Y}_{IJ}] \approx \sigma_{IJ}^2 = \frac{S_I(f_0)S_J(f_0)}{4(T_{\text{sft}})^2}$$

- Optimally combine into $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$ w/$u_\alpha \propto \frac{\tilde{\mathcal{G}}_\alpha^*}{\sigma_\alpha^2}$ so $E[\rho] = h_0^2 \sqrt{2\sum_\alpha \frac{|\tilde{\mathcal{G}}_\alpha|^2}{\sigma_\alpha^2}}$ & $\mathrm{Var}[\rho] = 1$

Computational considerations limit coherent integration time. Can make tunable semi-coherent search by restricting which SFT pairs α are included in $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$ E.g., only include pairs where $|T_I - T_J| \equiv |T_\alpha| \le T_{\text{max}}$

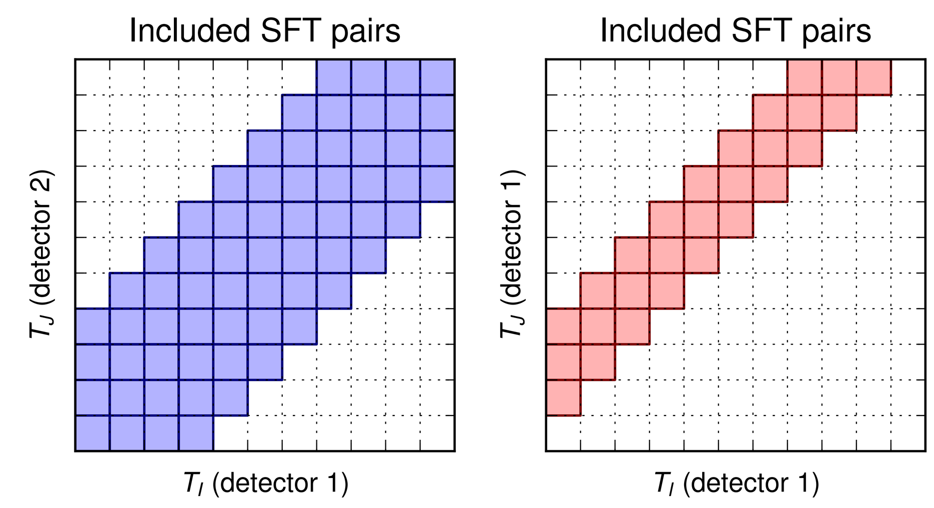

Figure 2: *SFT pairs for inclusion in sliding cross-correlation search. Left: data from different detectors at same or different times. Right: data from same detector at different times. In this illustrative example, $T_{max} = 3T_{sft}$.*

Parameter Space Metric

Consider dependence of ρ on parameters $\boldsymbol{\lambda} \equiv \{\lambda_i\}$. Can define Parameter space metric via

$$\frac{E[\rho] - E[\rho^{\text{true}}]}{E[\rho^{\text{true}}]} = -g_{ij}(\Delta\lambda^i)(\Delta\lambda^j) + \mathcal{O}([\Delta\lambda]^3)$$

$$g_{ij} = -\frac{1}{2}\frac{E[\rho_{,ij}]|_{\boldsymbol{\lambda}=\boldsymbol{\lambda}_{\text{true}}}}{E[\rho^{\text{true}}]}$$

Assume dominant contribution to $E[\rho_{,ij}]$ is from variation of $\Delta\Phi_{IJ} = \Phi_I - \Phi_J$; get phase metric

$$g_{ij} = \frac{1}{2}\frac{\sum_\alpha \Delta\Phi_{\alpha,i}\Delta\Phi_{\alpha,j}|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}{\sum_\beta |\tilde{\mathcal{G}}_\beta|^2/\sigma_\beta^2} \equiv \frac{1}{2}\langle \Delta\Phi_{\alpha,i}\Delta\Phi_{\alpha,j}\rangle_\alpha$$

Note $\langle\,\rangle_\alpha$ is average over pairs weighted by $\frac{|\tilde{\mathcal{G}}_\alpha|^2}{\sigma_\alpha^2}$; ignoring weighting factor gives usual metric [7]

$$\langle \Phi_{I,i}\Phi_{I,j}\rangle_I - \langle \Phi_{I,i}\rangle_I \langle \Phi_{J,j}\rangle_J$$

Define $T_{IJ} = T_I - T_J \equiv T_\alpha$ as time offset btwn SFTs; T_α^{av} is average time. For each detector pair, avg over pairs is avg over T_α & T_α^{av}. If we assume the avg over T_α^{av} evenly samples orbital phase, the metric in $\{f_0, a_p, \tilde{T}\}$ space is

$$\mathbf{g} = \begin{pmatrix} 2\pi^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \langle T_\alpha^2 \rangle_{T_\alpha} + \begin{pmatrix} \pi^2 a_p^2 & \pi^2 f_0 a_p & 0 \\ \pi^2 f_0 a_p & \pi^2 f_0^2 & 0 \\ 0 & 0 & 4\pi^4 f_0^2 a_p^2/P_{\text{orb}}^2 \end{pmatrix} \left\langle \sin^2 \frac{\pi T_\alpha}{P_{\text{orb}}} \right\rangle_{T_\alpha}$$

Since $\langle T_\alpha^2\rangle_{T_\alpha} \gg a_p^2 \left\langle \sin^2 \frac{\pi T_\alpha}{P_{\text{orb}}}\right\rangle_{T_\alpha}$, metric approximately diagonal. Using this metric to place templates gives template count illustrated in Fig. 3.

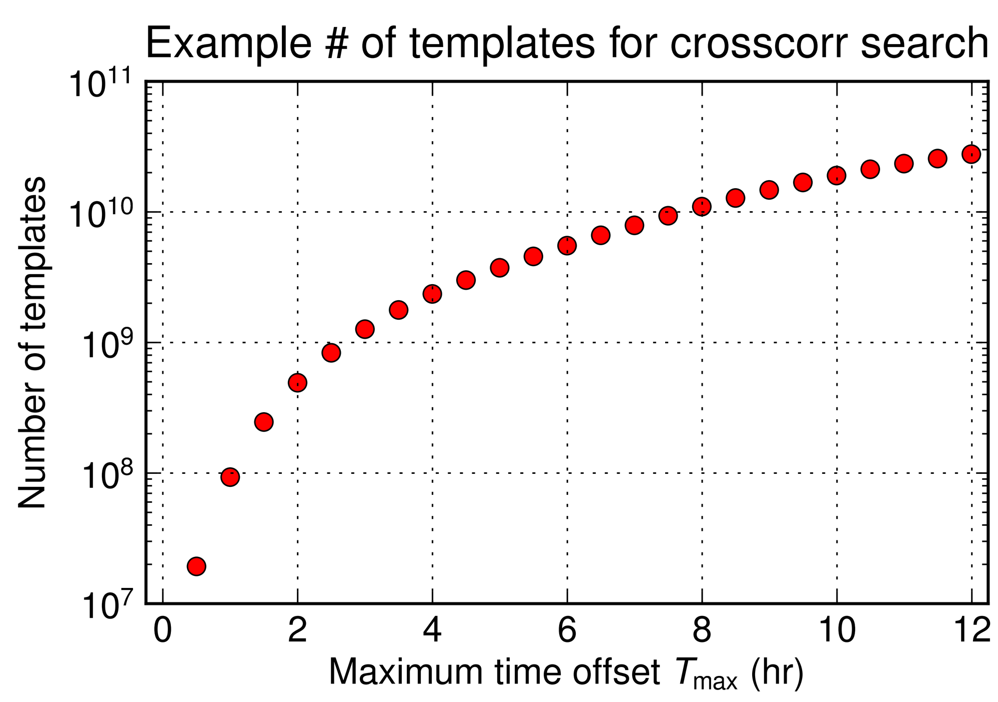

Figure 3: *Illustration of dependence of template count on T_{max}; don't read too much into absolute numbers*

Sensitivity Estimates

Search is sensitive to signal of amplitude

$$h_0 = \left(\frac{\mathcal{S}^2}{\sum_\alpha |\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}\right)^2$$

where $\mathcal{S}$ is a statistical factor. $\tilde{\mathcal{G}}_\alpha$ depends on (unknown) spin orientation angles ι & ψ; standard approach is to average value of $\tilde{\mathcal{G}}_\alpha$ over $\cos\iota$ & ψ. The ψ effect is small after average over sidereal time. The ι effect means actually

$$E[\rho] \approx h_0^2 \frac{\mathcal{A}_+^2 + \mathcal{A}_\times^2}{2} \sqrt{2\sum_\alpha |\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}$$

Net effect is to change statistical factor $\mathcal{S}$; for 10% false-alarm & -dismissal, h_0 sensitivity is a factor of 1.4 worse. In Fig. 4 we illustrate the dependence of search sensitivity on the maximum allowed time offset T_{max}

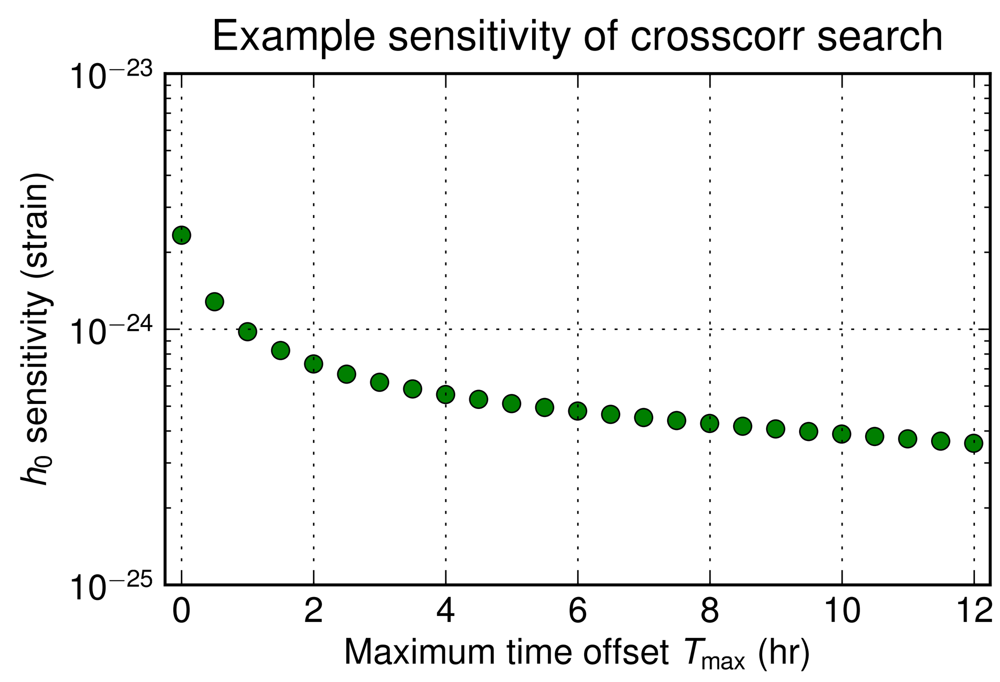

Figure 4: *Illustration of dependence of template count on T_{max}; don't read too much into absolute numbers. note that the $T_{max} = 0$ measurement is effectively the directed stochastic "radiometer" search.*

As an illustration of the sensitivity of a practical search, we show in Fig. 5 the sensitivity with one year of data, using $T_{\text{max}} = 6\,\text{hr}$.

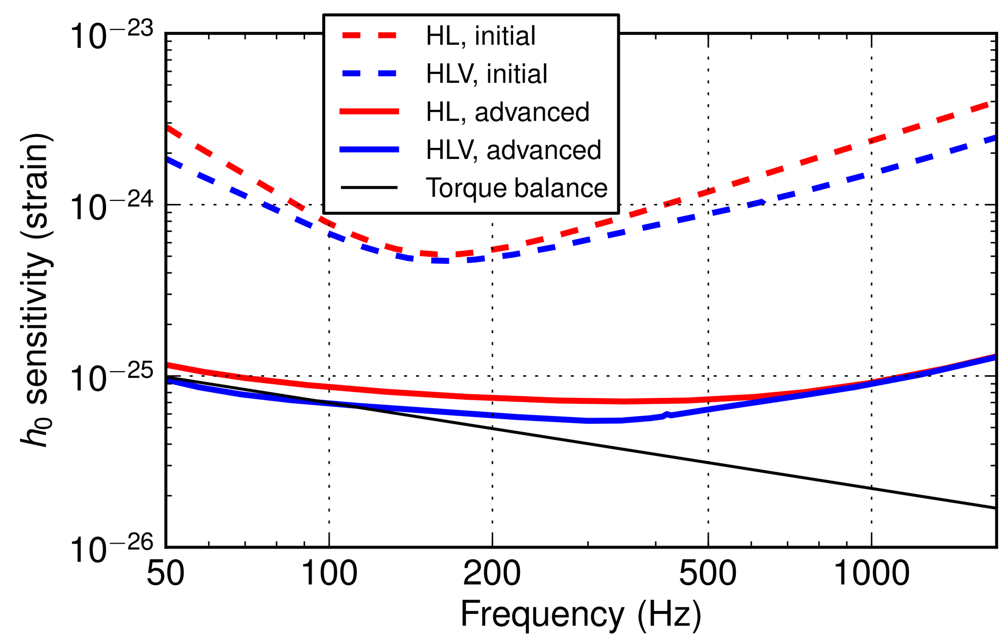

Figure 5: *Sensitivity of a cross-correlation search with one year of LIGO and LIGO/Virgo design-sensitivity data, assuming 10% false-alarm &-dismissal, with $T_{max} = 6\,\text{hr}$.*

References

[1] Bildsten, *ApJL* **501**, L89 (1998)

[2] Steeghs & Casares *ApJ* **568**, 273 (2002)

[3] LSC, *PRD* **76**, 082001 (2007)

[4] Ballmer, *CQG* **23**, S179 (2006)
LSC, *PRD* **76**, 082003 (2007)

[5] Messenger & Woan, *CQG* **24**, S469 (2007)

[6] Dhurandhar et al, *PRD* **77**, 082001 (2008)

[7] Pletsch, *PRD* **82**, 042002 (2010)