LIGO-T1200188-v1 4/26/12

LCGT
ETM Transmission Monitor Telescope Performance

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1 INTRODUCTION

This document presents an analysis of the Guoy phase error of a proposed off-axis spherical mirror telescope for monitoring the transmitted beam through the ETM mirror of bLCGT.

The function of the TMS Telescope is to provide signals for determining the tilt and lateral displacement of the main interferometer arm cavity beam.

1.1 Applicable Documents

2 TRANSMISSION MONITOR (TMS) TELESCOPE

2.1 TMS TELESCOPE DESCRIPTION

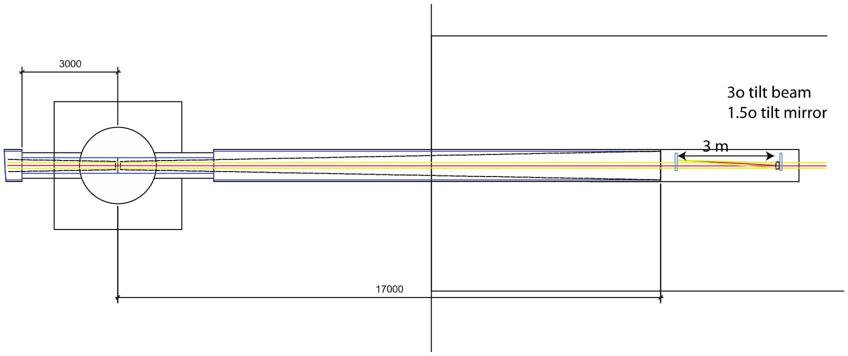
A proposed design for the TMS Telescope is presented in **JGW-T1201013**-v1, which is excerpted here.

KAGRA will need an end telescope. LIGO was forced to make a very short one, and this made it very complex and prohibitively expensive, mostly because of the requirement of parabolic offset mirrors.

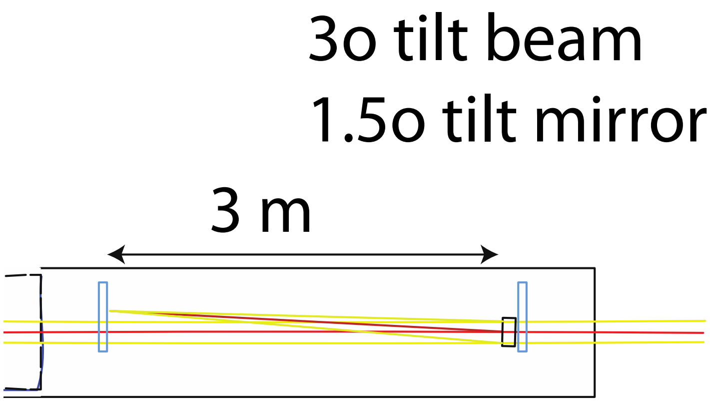
Can we make a cheaper telescope with a smaller angle and spherical mirrors?

The first strawman design is about the end telescope.

It has to sit after the cryogenic baffles and has to be suspended.

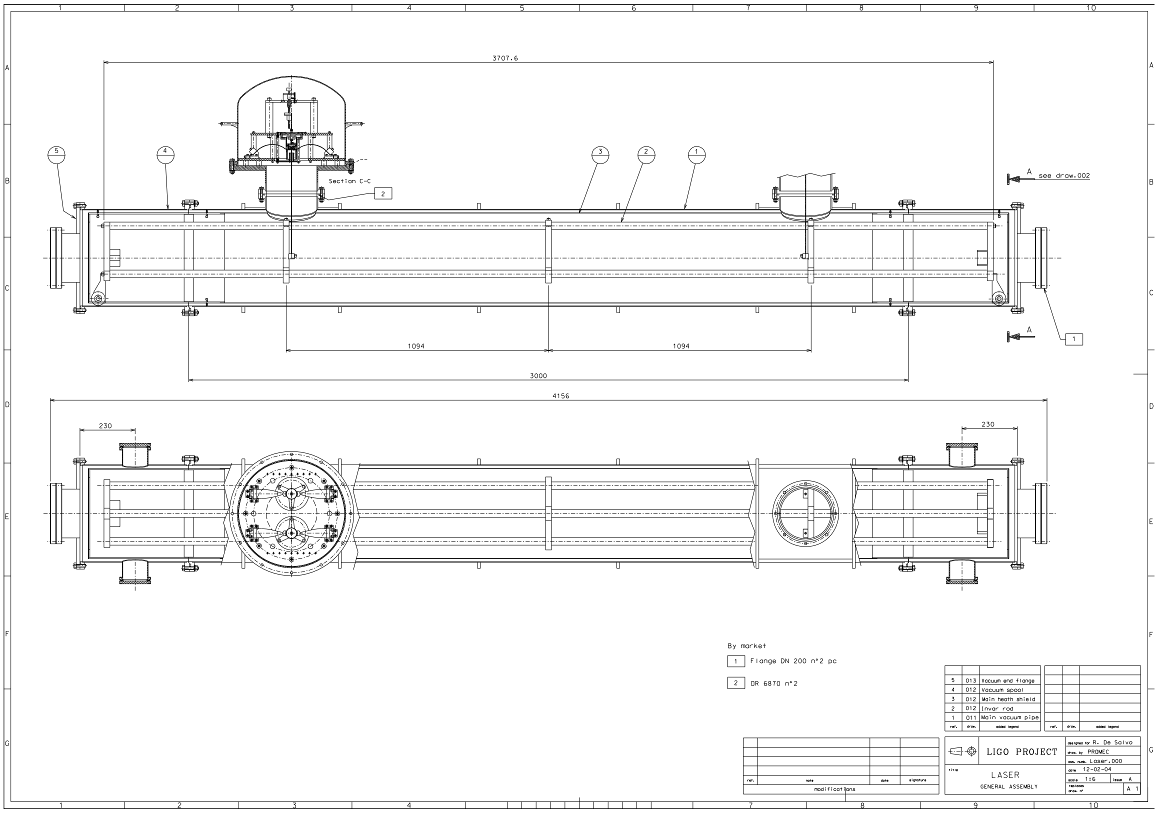


I assume a 220 mm primary beam to catch the entire transmitted beam, the same diameter of the main mirror (a smaller primary beam would make the problem easier). I also assume a 3 m long telescope. A 1.5o mirror tilt, 3o beam inclination is sufficient to shift the reflected, focused beam by more than 1 radius.

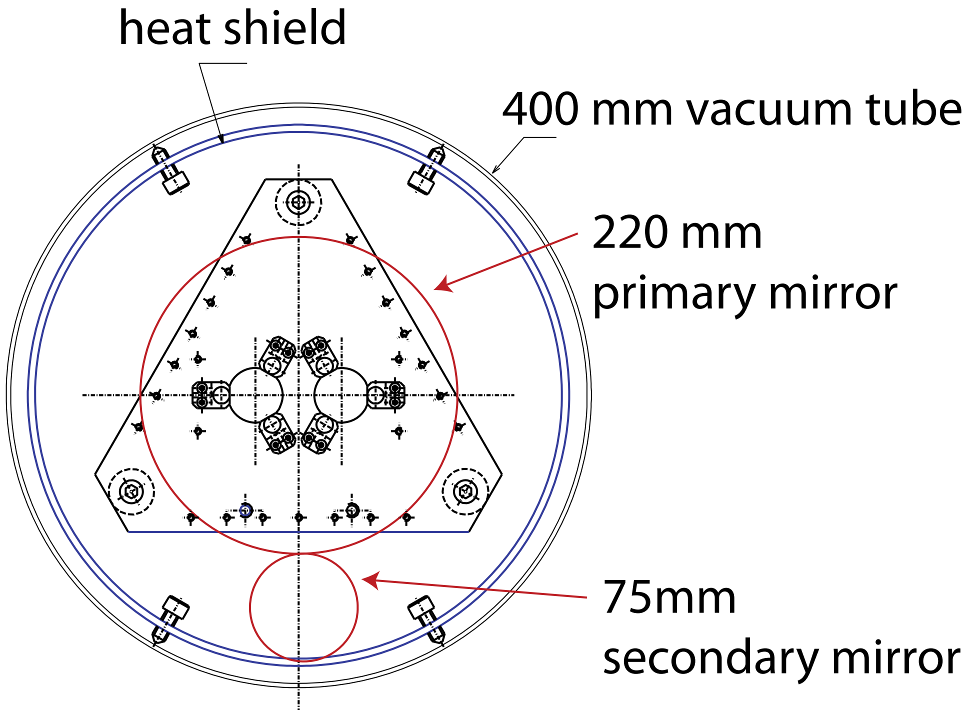


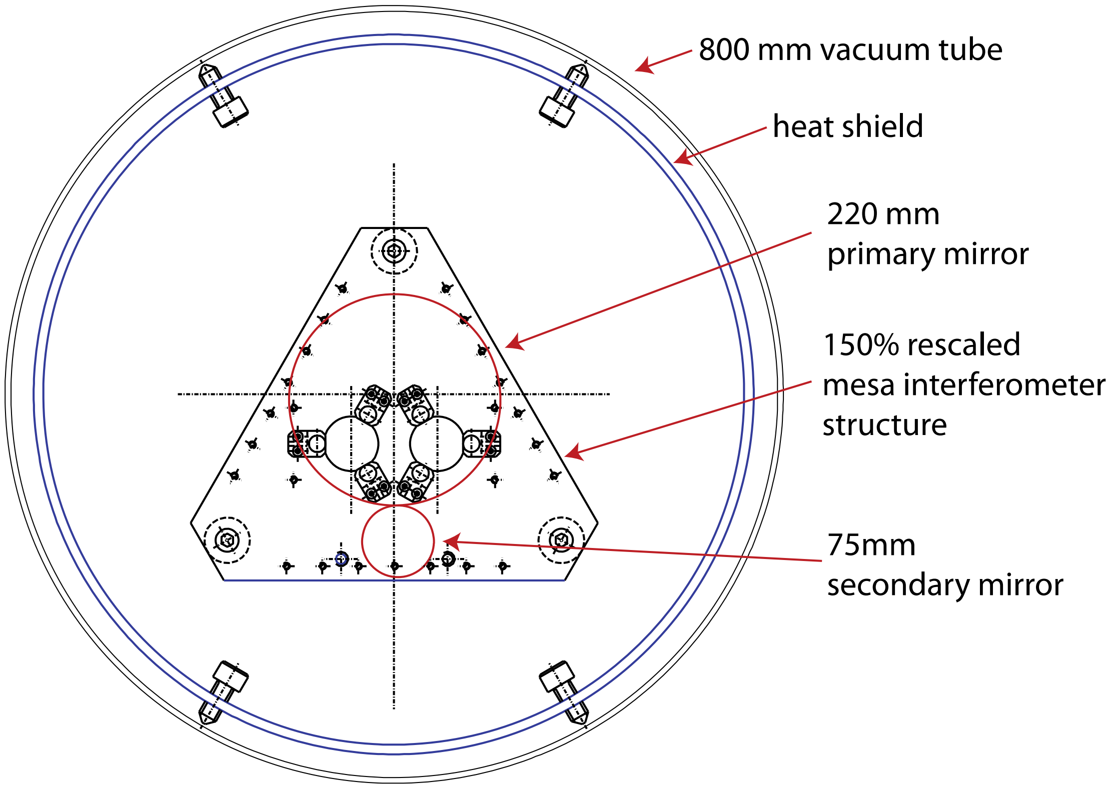
The mesa beam interferometer design is a triangular structure built around three invar rods, each 3 m long, which would allow construction of the 3^{o} tilt telescope described above. The structure of the mesa interferometer fits inside a 400 mm vacuum tube, is suspended by four small GAS filters, mounted in two mushroom tanks above the vacuum tube that also allow pitch and vertical alignment. A pure aluminum shroud inside the pipe guarantees a high level of thermal uniformity in the structure.

The structure of the mesa beam interferometer is too small to contain the KAGRA end telescope with a 220 mm primary mirror, however it only requires ~ 50% rescaling to do the job and would easily fit inside a standard KAGRA 80 mm vacuum tube.



The mesa beam structure was supported by two pairs of GAS blades. The end telescope would be suspended by two scaled GAS filters.





An attenuation of ~1000 above 10 Hz can be achieved.

Horizontal flexures can be added to the design for yaw alignment, if necessary.

2.2 TMS TELESCOPE OPTICAL PARAMETERS

wavelength, mm	$\lambda := 1.064 \cdot 10^{-3}$
index of refraction of fused silica	$n := 1.44963$
off-axis angle, deg	$\theta := 1.5$
primary lens focal length, mm	$f_{10} := -3000$
telescope magnification ratio	$m := 20$
secondary lens focal length, mm	$f_{20}(f_{10}) := \frac{-f_{10}}{m}$
	$f_{20}(f_{10}) = 150$
defocus of TMS telescope, mm	$\Delta_f := 5.043$
input beam waist, mm	$w_{011} := 14.7$
input location, mm	$z_{11} := 1500000$

2.3 TMS TELESCOPE ASTIGMATIC FOCI

An off-axis spherical telescope will exhibit two independent foci for the tangential and sagittal rays. The beam shape and location of the foci are dependent upon the angle of incidence that the principal ray makes with the axis of curvature of the mirror, and the defocus parameter, which is the deviation of the primary to secondary mirror spacing from the perfectly focused condition.

note: Fundamentals of Optics, Jenkins & White
ftan = f*cos(theta); fsag = f/cos(theta); where
theta is the incidence angle

tangential primary focal length, mm

$$f_{1tan}(f_{10}, \theta) := f_{10} \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right)$$

$$f_{1tan}(f_{10}, \theta) = -2.999 \times 10^{3}$$

tangential secondary focal length, mm

$$f_{2tan}(f_{10}, \theta) := f_{20}(f_{10}) \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right)$$

$$f_{2tan}(f_{10}, \theta) = 149.949$$

sagittal primary focal length

$$f_{1sag}(f_{10}, \theta) := \frac{f_{10}}{\cos\left(\theta \cdot \frac{\pi}{180}\right)}$$

$$f_{1sag}(f_{10}, \theta) = -3.001 \times 10^{3}$$

sagittal secondary focal length

$$f_{2sag}(f_{10}, \theta) := \frac{f_{20}(f_{10})}{\cos\left(\theta \cdot \frac{\pi}{180}\right)}$$

$$f_{2sag}(f_{10}, \theta) = 150.051$$

Tangential focus

tangential virtual beam waist
location inside tel measured from primary,
mm

$$z_{12tan}(z_{11}, f_{10}, \theta) := f_{1tan}(f_{10}, \theta) + f_{1tan}(f_{10}, \theta)^2 \cdot \frac{(z_{11} - f_{1tan}(f_{10}, \theta))}{\left[(z_{11} - f_{1tan}(f_{10}, \theta))^2 + \left(\pi \cdot \frac{w_{011}{}^2}{\lambda}\right)^2\right]}$$

$$z_{12tan}(z_{11}, f_{10}, \theta) = -2.994\times 10^3$$

tangential virtual beam waist
size inside tel, mm

$$w_{012tan}(z_{11}, f_{10}, \theta) := \left[\frac{1 \cdot \left(1 - \frac{z_{11}}{f_{1tan}(f_{10}, \theta)}\right)^2}{w_{011}{}^2} + \frac{1 \cdot \left(\pi \cdot \frac{w_{011}}{\lambda}\right)^2}{f_{1tan}(f_{10}, \theta)^2}\right]^{-0.5}$$

$$w_{012tan}(z_{11}, f_{10}, \theta) = 0.027$$

$$w_{021tan}(z_{11}, f_{10}, \theta) := w_{012tan}(z_{11}, f_{10}, \theta)$$

$$z_{21tan}(\Delta_f, z_{11}, f_{10}, \theta) := f_{10} + f_{20}(f_{10}) - z_{12tan}(z_{11}, f_{10}, \theta) + \Delta_f$$

tangential output beam waist
location, measured
from second lens of
tel, mm

$$z_{22tan}(\Delta_f, z_{11}, f_{10}, \theta) := f_{2tan}(f_{10}, \theta) + f_{2tan}(f_{10}, \theta)^2 \cdot \frac{(z_{21tan}(\Delta_f, z_{11}, f_{10}, \theta) - f_{2tan}(f_{10}, \theta))}{\left[(z_{21tan}(\Delta_f, z_{11}, f_{10}, \theta) - f_{2tan}(f_{10}, \theta))^2 + \left(\pi \cdot \frac{w_{021tan}(z_{11}, f_{10}, \theta)^2}{\lambda}\right)^2\right]}$$

$$z_{22tan}(\Delta_f, z_{11}, f_{10}, \theta) = -3.852\times 10^3$$

2.3.1 Non-astigmatic Telescope Beam Waist Location and Size

The size and position of the beam waist for a non-astigmatic telescope is calculated below, as measured from the position of the 2ndary mirror.

NON-ASTIGMATIC BEAM POSITION FROM SECONDARY MIRROR

$$z_{22}(\Delta_f, z_{11}, f_{10}) := z_{22tan}(\Delta_f, z_{11}, f_{10}, 0)$$

tan output beam
waist size,
tel, mm

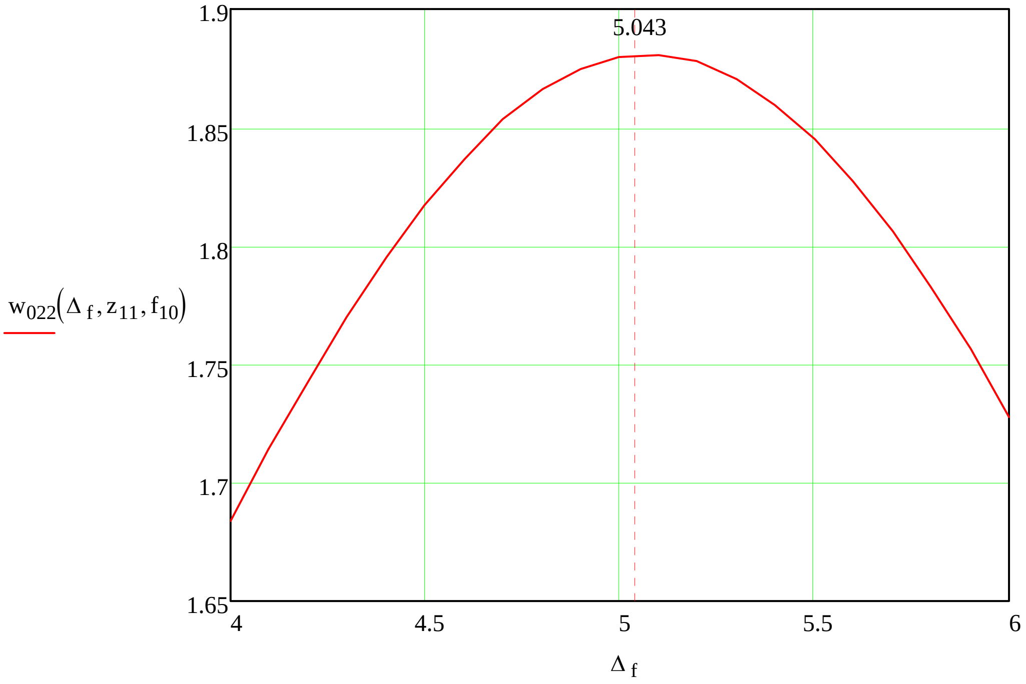
$$w_{022tan}(\Delta_f, z_{11}, f_{10}, \theta) := \left[\frac{1 \cdot \left(1 - \frac{z_{21tan}(\Delta_f, z_{11}, f_{10}, \theta)}{f_{2tan}(f_{10}, \theta)}\right)^2}{w_{021tan}(z_{11}, f_{10}, \theta)^2} + \frac{1 \cdot \left(\pi \cdot \frac{w_{021tan}(z_{11}, f_{10}, \theta)}{\lambda}\right)^2}{f_{2tan}(f_{10}, \theta)^2} \right]^{-0.5}$$

$$w_{022tan}(0, z_{11}, f_{10}, \theta) = 0.631$$

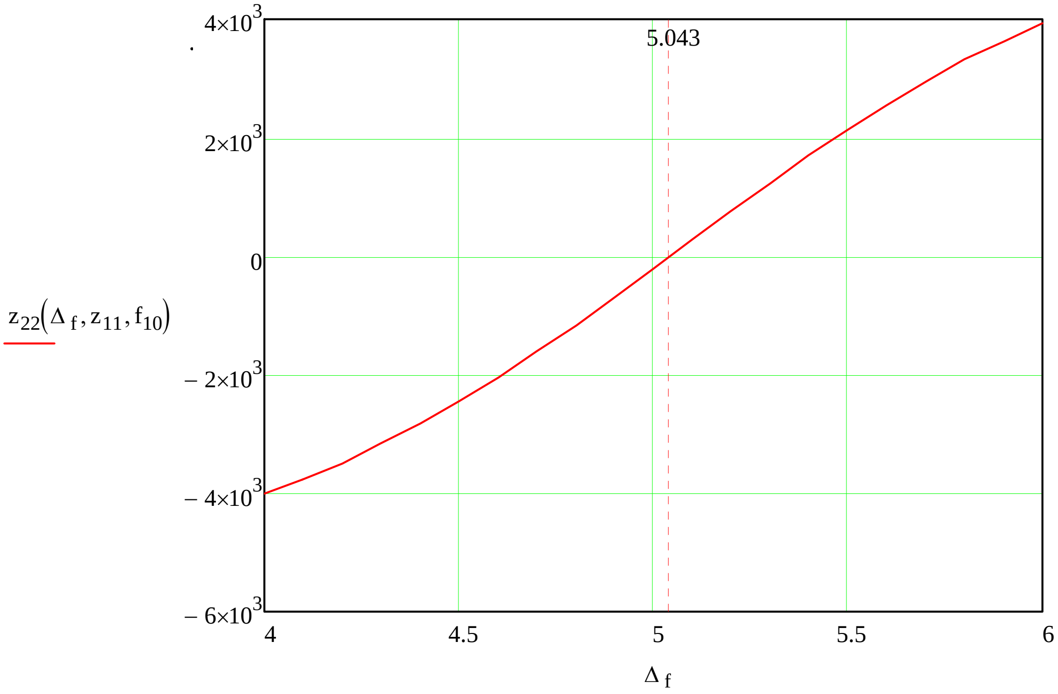
NON-ASTIGMATIC BEAM SIZE

$$w_{022}(\Delta_f, z_{11}, f_{10}) := w_{022tan}(\Delta_f, z_{11}, f_{10}, 0)$$

NON-ASTIGMATIC SPOT SIZE VS DEFOCUS



NON-ASTIGMATIC SPOT DISTANCE FROM 2NDARY MIRROR
VS DEFOCUS (SHORT FOCUS = 5.043 mm)



2.3.2 Astigmatic Telescope Beam Waist Location and Size

Sagittal focus

sagittal virtual beam waist location inside tel measured from primary, mm

$$z_{12sag}(z_{11}, f_{10}, \theta) := f_{1sag}(f_{10}, \theta) + f_{1sag}(f_{10}, \theta)^2 \cdot \frac{\left(z_{11} - f_{1sag}(f_{10}, \theta)\right)}{\left[\left(z_{11} - f_{1sag}(f_{10}, \theta)\right)^2 + \left(\pi \cdot \frac{w_{011}{}^2}{\lambda}\right)^2\right]}$$

$$z_{12sag}(z_{11}, f_{10}, \theta) = -2.996 \times 10^3$$

sagittal virtual beam waist size inside tel, mm

$$w_{012sag}(z_{11}, f_{10}, \theta) := \left[\frac{1 \cdot \left(1 - \frac{z_{11}}{f_{1sag}(f_{10}, \theta)}\right)^2}{w_{011}{}^2} + \frac{1 \cdot \left(\pi \cdot \frac{w_{011}}{\lambda}\right)^2}{f_{1sag}(f_{10}, \theta)^2}\right]^{-0.5}$$

$$w_{012sag}(z_{11}, f_{10}, \theta) = 0.027$$

$$w_{021sag}(z_{11}, f_{10}, \theta) := w_{012sag}(z_{11}, f_{10}, \theta)$$

$$z_{21sag}(\Delta_f, z_{11}, f_{10}, \theta) := f_{10} + f_{20}(f_{10}) - z_{12sag}(z_{11}, f_{10}, \theta) + \Delta_f$$

$$z_{21sag}(\Delta_f, z_{11}, f_{10}, \theta) = 150.994$$

sagittal output beam waist
location, measured
from second lens of
tel, mm

$$z_{22sag}(\Delta_f, z_{11}, f_{10}, \theta) := f_{2sag}(f_{10}, \theta) + f_{2sag}(f_{10}, \theta)^2 \cdot \frac{\left(z_{21sag}(\Delta_f, z_{11}, f_{10}, \theta) - f_{2sag}(f_{10}, \theta)\right)}{\left[\left(z_{21sag}(\Delta_f, z_{11}, f_{10}, \theta) - f_{2sag}(f_{10}, \theta)\right)^2 + \left(\pi \cdot \frac{w_{021sag}(z_{11}, f_{10}, \theta)^2}{\lambda}\right)^2\right]}$$

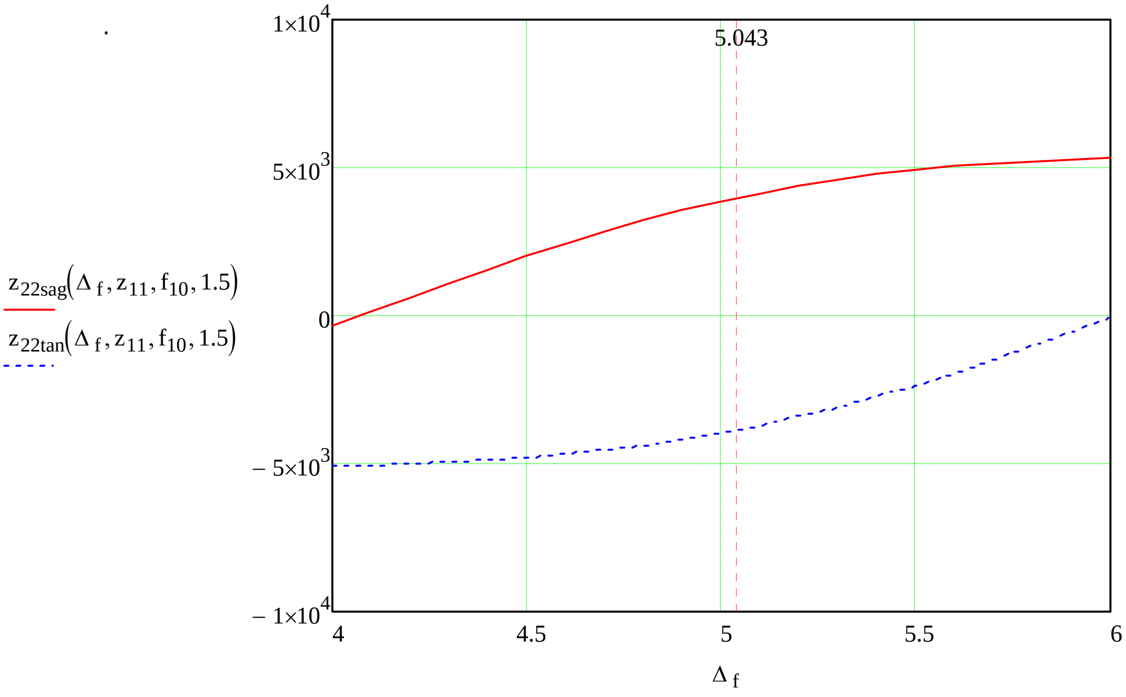
$$z_{22sag}(\Delta_f, z_{11}, f_{10}, \theta) = 3.986 \times 10^3$$

sagittal output beam
waist size,
tel, mm

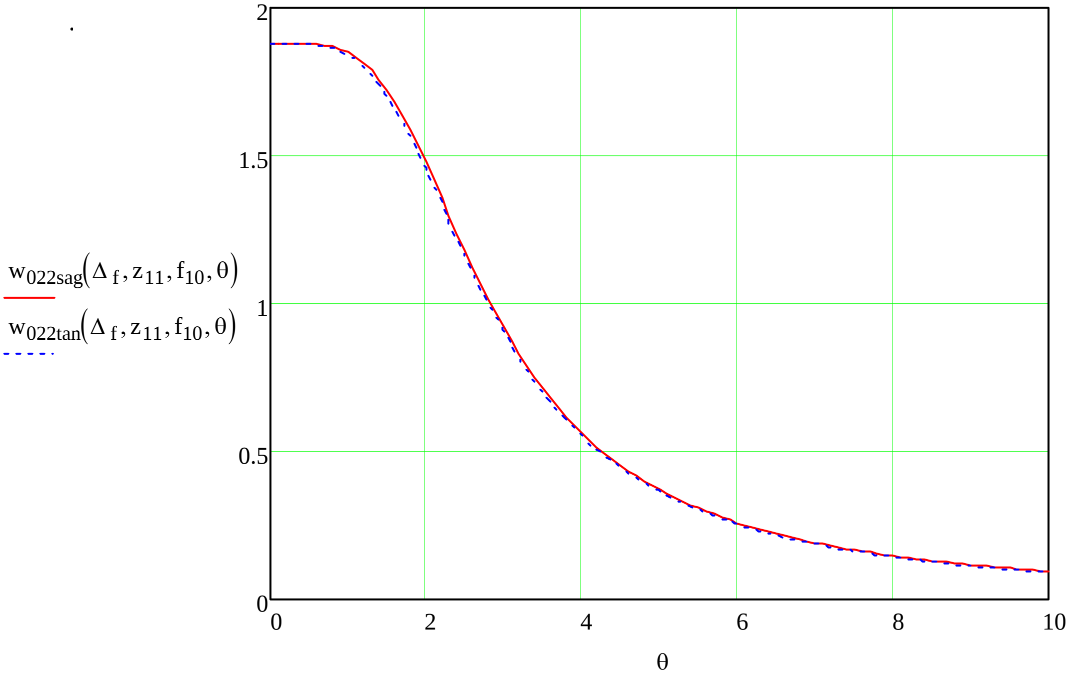
$$w_{022sag}(\Delta_f, z_{11}, f_{10}, \theta) := \left[\frac{1 \cdot \left[1 - \frac{\left(z_{21sag}(\Delta_f, z_{11}, f_{10}, \theta)\right)}{f_{2sag}(f_{10}, \theta)}\right]^2}{w_{021sag}(z_{11}, f_{10}, \theta)^2} + \frac{1 \cdot \left(\pi \cdot \frac{w_{021sag}(z_{11}, f_{10}, \theta)}{\lambda}\right)^2}{f_{2sag}(f_{10}, \theta)^2}\right]^{-0.5}$$

$$w_{022sag}(\Delta_f, z_{11}, f_{10}, \theta) = 1.723$$

ASTIGMATIC SPOT DISTANCE FROM 2NDARY MIRROR VS DEFOCUS, WITH 1.5 DEG OFF-AXIS ANGLE



ASTIGMATIC SPOT SIZE VS OFF-AXIS ANGLE, WITH CLOSE FOCUS



2.4 TMS TELESCOPE GUOY PHASE ERROR

tangential Guoy phase, deg

$$\phi_{gtan}(\Delta_f, z_{11}, f_{10}, \theta) := \frac{180}{\pi} \cdot \operatorname{atan}\left[\frac{z_{22tan}(\Delta_f, z_{11}, f_{10}, \theta) - z_{22}(\Delta_f, z_{11}, f_{10})}{\frac{\left[\pi \cdot \left(w_{022}(\Delta_f, z_{11}, f_{10})\right)^2\right]}{\lambda}}\right]$$

$$\phi_{gtan}(\Delta_f, z_{11}, f_{10}, \theta) = -20.249$$

sagittal Guoy phase, deg

$$\phi_{gsag}(\Delta_f, z_{11}, f_{10}, \theta) := \frac{180}{\pi} \cdot \operatorname{atan}\left[\frac{z_{22sag}(\Delta_f, z_{11}, f_{10}, \theta) - z_{22}(\Delta_f, z_{11}, f_{10})}{\frac{\left[\pi \cdot \left(w_{022}(\Delta_f, z_{11}, f_{10})\right)^2\right]}{\lambda}}\right]$$

$$\phi_{gsag}(\Delta_f, z_{11}, f_{10}, \theta) = 20.882$$

Guoy phase error, deg

$$\Delta\phi_g(\Delta_f, z_{11}, f_{10}, \theta) := \phi_{gsag}(\Delta_f, z_{11}, f_{10}, \theta) - \phi_{gtan}(\Delta_f, z_{11}, f_{10}, \theta)$$

$$\Delta\phi_g(\Delta_f, z_{11}, f_{10}, \theta) = 41.131$$

GUOY PHASE ERROR VS OFF-AXIS ANGLE, FOR VARIOUS DEFOCUS