# Searching for Gravitational Waves from Periodic Sources

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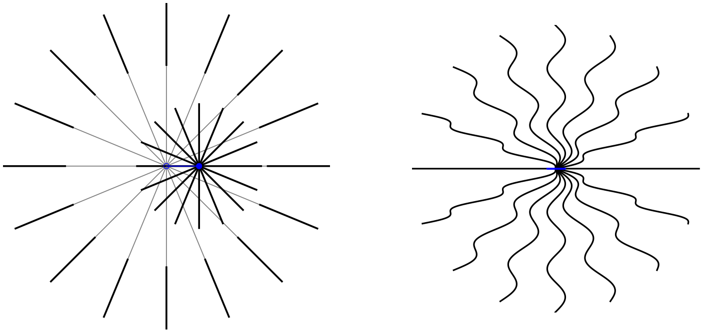
# Outline

1. Periodic Gravitational Waves
   - Physical Picture
   - Mathematical Description
2. Signals and Signal Processing
   - Signal Model & Parameters
   - Coherent Search Methods
   - Semicoherent Methods
3. Astrophysical Searches w/LIGO & Virgo
   - Targeted Searches for GWs from Known Pulsars
   - All-Sky Searches for Unknown Neutron Stars
   - Directed Searches for GWs from Known Sky Positions

# Outline

# Gravity + Causality = Gravitational Waves



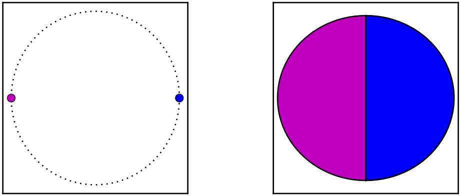
- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light
  $\longrightarrow$ time-dep grav fields must propagate like light waves

# Generation of Gravitational Waves

- EM waves generated by moving/oscillating charges
- GW generated by moving/oscillating masses
- Lowest multipole is quadrupole
- Different types of signals:
  - Burst (transient, unmodelled)
  - Stochastic (long-lived, unmodelled)
  - Binary coalescence (transient, modelled)
  - Periodic (long-lived, modelled)
- Periodic sources have simpler waveforms,
  but interaction w/detector complicated by signal modulation

# Sources of Periodic Gravitational Waves

- System w/quadrupole moment oscillating at frequency $\Omega$ emits periodic GWs w/frequency $f_{\text{gw}} = 2\frac{\Omega}{2\pi}$
- Hulse-Taylor binary pulsar 1913+16 (slowly inspiralling) $P_{\text{orb}} \approx 7.75\,\text{hr} \longrightarrow f_{\text{gw}} \approx 72\,\mu\text{Hz}$ (too low)
- White-dwarf binary $f_{\text{gw}} \sim 1 - 10\,\text{mHz}$ (LISA/NGO source) e.g., AM CVn $P_{\text{orb}} \approx 10^3\,\text{s} \longrightarrow f_{\text{gw}} \approx 2\,\text{mHz}$
- Triaxial neutron star (pulsar or LMXB) $f_{\text{gw}} \sim 1 - 10^3\,\text{Hz}$ (LIGO/Virgo source) e.g., Crab $f_{\text{rot}} \approx 30\,\text{Hz} \longrightarrow f_{\text{gw}} \approx 60\,\text{Hz}$



# Gravity as Geometry

- Minkowski Spacetime:

$$ds^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$
$$= \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}^{\text{tr}} \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\text{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu$$

# Gravitational Wave as Metric Perturbation

- For GW propagation & detection, work to 1st order in $h_{\mu\nu} \equiv$ difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

($h_{\mu\nu}$ "small" in weak-field regime, e.g. for GW detection)

- Convenient choice of gauge is transverse-traceless:

$$h_{0\mu} = h_{\mu 0} = 0 \qquad \eta^{\nu\lambda} \frac{\partial h_{\mu\nu}}{\partial x^\lambda} = 0 \qquad \eta^{\mu\nu} h_{\mu\nu} = \delta^{ij} h_{ij} = 0$$

In this gauge:

- Test particles w/constant coörds are freely falling
- Vacuum Einstein eqns $\Longrightarrow$ wave equation for $\{h_{ij}\}$:

$$\left( -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2 \right) h_{ij} = 0$$

# Gravitational Wave Polarization States

- Far from source, GW looks like plane wave prop along $\vec{k}$
  TT conditions mean, in convenient basis,

$$\{k_i\} \equiv \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \{h_{ij}\} \equiv \mathbf{h} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

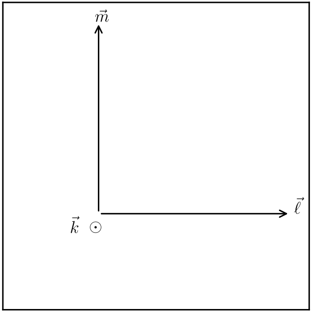
  where $h_+\left(t - \frac{x^3}{c}\right)$ and $h_\times\left(t - \frac{x^3}{c}\right)$ are components in "plus" and "cross" polarization states
- More generally

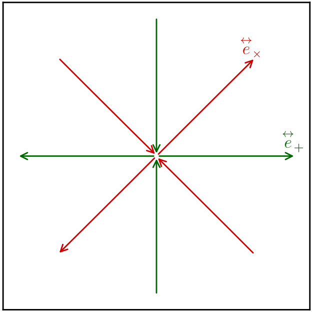
$$\overleftrightarrow{h} = h_+\left(t - \frac{\vec{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_+ + h_\times\left(t - \frac{\vec{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_\times$$

# The Polarization Basis

- wave propagating along $\vec{k}$;
  construct $\overleftrightarrow{e}_{+,\times}$ from $\perp$ unit vectors $\vec{\ell}$ & $\vec{m}$:

$$\overleftrightarrow{e}_{+} = \vec{\ell} \otimes \vec{\ell} - \vec{m} \otimes \vec{m} \qquad \overleftrightarrow{e}_{\times} = \vec{\ell} \otimes \vec{m} + \vec{m} \otimes \vec{\ell}$$

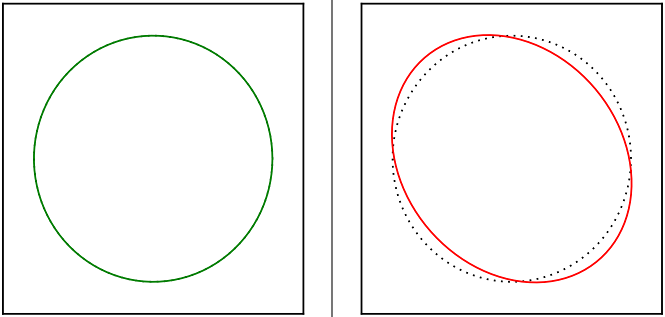


# Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

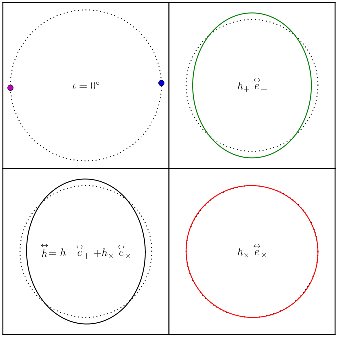
| Plus (+) Polarization | Cross (×) Polarization |
|---|---|

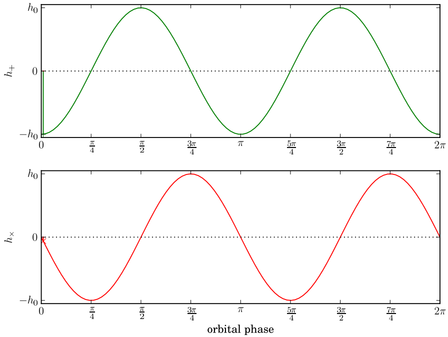


# Example: Circular polarization

"Face-on"; inclination $\iota = 0°$

$$h_+ = A \cos \Phi(t) \qquad h_\times = A \sin \Phi(t)$$

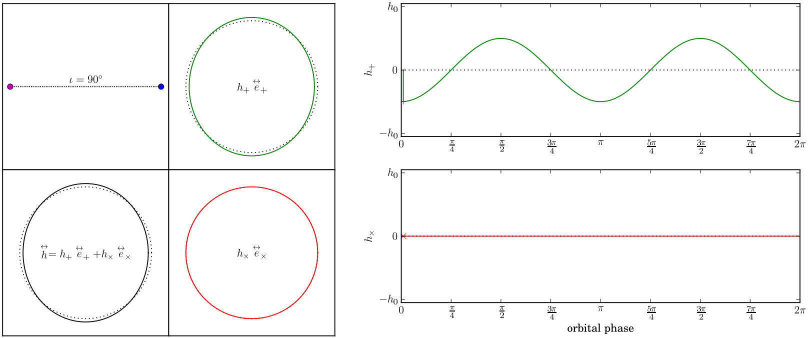


# Example: Linear polarization

"Edge-on"; inclination $\iota = 90°$

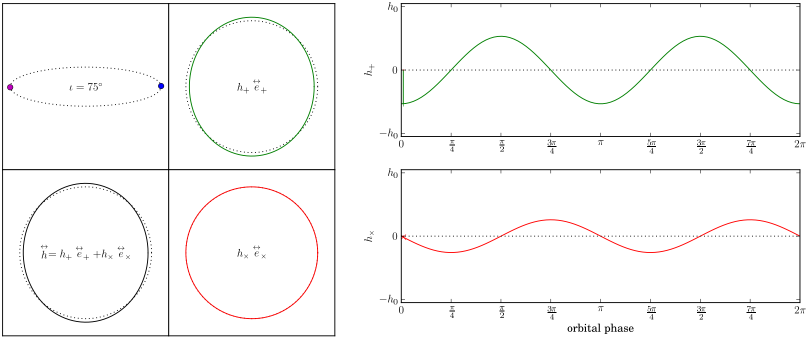
$$h_+ = A\cos\Phi(t) \qquad h_\times = 0$$



# Example: Elliptical polarization

General situation w/inclination $\iota$: $A_+ \propto \frac{1+\cos^2\iota}{2}$; $A_\times \propto \cos\iota$

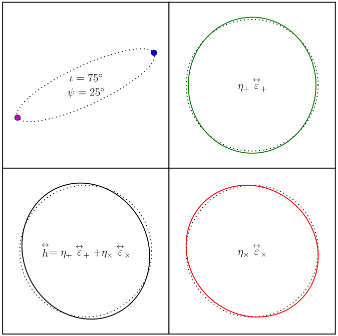
$$h_+ = A_+ \cos\Phi(t) \qquad h_\times = A_\times \sin\Phi(t)$$

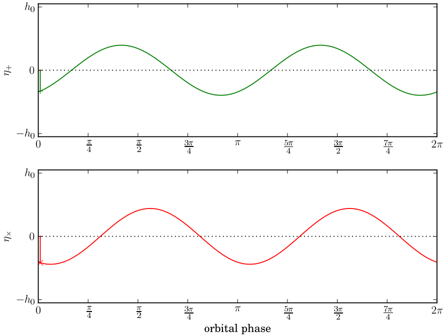


# Elliptical Polarization Resolved in Arbitrary Basis

If $+$ & $\times$ basis tensors chosen arbitrarily, not 90° out of phase

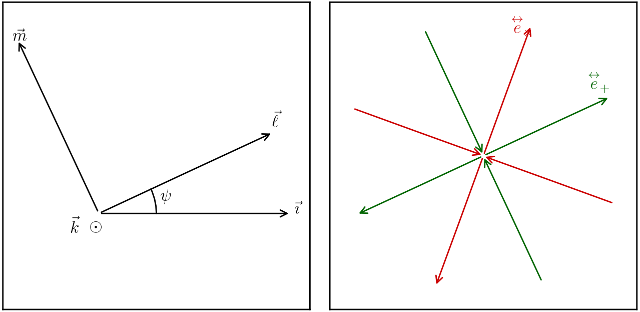
$$\overleftrightarrow{h} = \eta_+ \overleftrightarrow{\varepsilon}_+ + \eta_\times \overleftrightarrow{\varepsilon}_\times = h_+ \overleftrightarrow{e}_+ + h_\times \overleftrightarrow{e}_\times$$



# Natural Polarization Basis

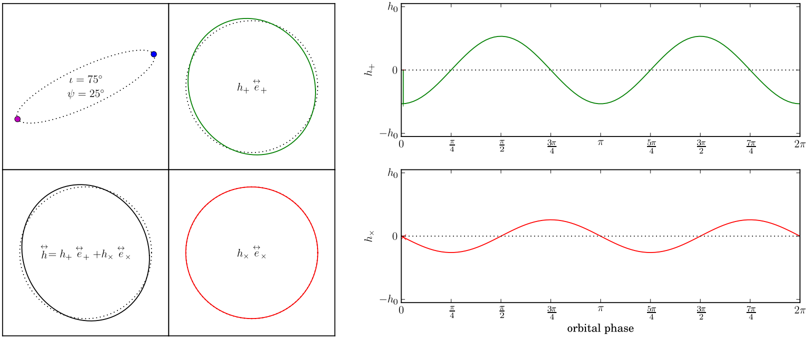
- Free to choose $\vec{\ell}$ within plane $\perp$ $\vec{k}$ (fixes $\vec{m} = \vec{k} \times \vec{\ell}$)
- Choose it in orbital plane (binary) or equatorial plane (NS) $\longrightarrow$ $h_+$ & $h_\times$ are $90^\circ$ out of phase
- Pol angle $\psi$ relates $\vec{\ell}$ to some reference direction $\vec{\imath}$ (e.g., "West on the sky")



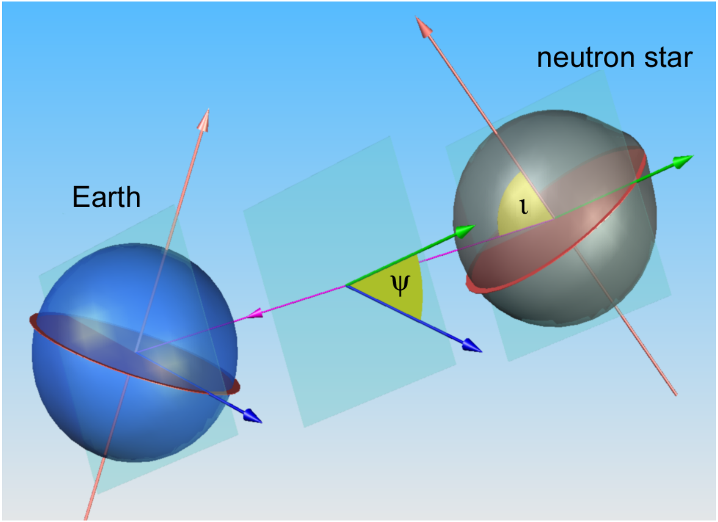
# Elliptical Polarization Resolved in Preferred Basis

$h_+$ & $h_\times$ are 90° out of phase ($\iota$ & $\psi$ give alignment of system)

$$h_+ = A_+ \cos\Phi(t) \qquad h_\times = A_\times \sin\Phi(t)$$



# Inclination & Polarization Angles for Neutron Star



# Outline

# GW Signal from Periodic Source

GW signal arriving time $\tau$ at Solar System Barycenter

$$\overleftrightarrow{h}(\tau) = h_0 \left[ \frac{1+\cos^2\iota}{2} \cos\Phi(\tau) \overleftrightarrow{e}_+ + \cos\iota \sin\Phi(\tau) \overleftrightarrow{e}_\times \right]$$

- Amplitude $h_0$ depends on distance, frequency, ellipticity
- Pol basis $\{\overleftrightarrow{e}_+, \overleftrightarrow{e}_\times\}$ depends on sky position $\{\alpha, \delta\}$ and polarization angle $\psi$
- Phase evolution e.g., $\Phi(\tau) = \phi_0 + 2\pi \left( f_0\tau + \frac{f_1\tau^2}{2} + \cdots \right)$ (+Doppler mod if NS in binary; note constant Doppler shift OK)
- Signal $h(t) = \overleftrightarrow{h}(\tau(t)) : \overleftrightarrow{d}$ received in detector has $\{\alpha, \delta\}$-dep Doppler shift $\tau(t)$ due to daily & yearly motion of detector
- Divide signal parameters into
    - amplitude params: $\{h_0, \iota, \psi, \phi_0\}$
    - phase params: $\{\alpha, \delta, f_0, f_1, \ldots\}$ + orbital params for LMXB

# Coherent Maximum-Likelihood Search ($\mathcal{F}$-statistic)

- Divide signal parameters into
  - amplitude params: $\{h_0, \iota, \psi, \phi_0\}$
  - phase params: $\lambda \equiv \{\alpha, \delta, f_0, f_1, \ldots\}$ + orb params for LMXB
- Jaranowski, Królak, Schutz *PRD* **58**, 063001 (1998) showed signal linear in $\{\mathcal{A}^\mu\}$, fcns of amplitude params

$$h(t) = \mathcal{A}^\mu h_\mu(t) \qquad \text{(assume } \textstyle\sum_{\mu=1}^{4}\text{)}$$

  template waveforms $h_\mu(t)$ depend on phase params $\lambda$
- Mismatch of obs data w/signal model quadratic in $\{\mathcal{A}^\mu\}$:

$$\chi^2(\mathcal{A}, \lambda) = \mathcal{A}^\mu \mathcal{M}_{\mu\nu}(\lambda)\mathcal{A}^\nu - 2\mathcal{A}^\mu x_\mu(\lambda) + \chi^2(0, \lambda)$$

- $\mathcal{F}$-stat method uses best-fit amp params $\widehat{\mathcal{A}}^\mu = \mathcal{M}^{\mu\nu}(\lambda)x_\nu(\lambda)$ ($\mathcal{M}^{\mu\nu}$ is inv of $\mathcal{M}_{\mu\nu}$); detection statistic is max log-likelihood

$$\mathcal{F} = -\frac{\chi^2(\widehat{\mathcal{A}}, \lambda) - \chi^2(0, \lambda)}{2} = \frac{1}{2} x_\mu(\lambda)\mathcal{M}^{\mu\nu}(\lambda)x_\nu(\lambda)$$

# Bayesian Interpretation ($\mathcal{B}$-statistic)

- Assume $\lambda$ known; likelihood $P(x|\mathcal{A}) \propto e^{-\chi^2(\mathcal{A})/2}$
- Bayes's theorem says $P(\mathcal{H}|x) = \frac{P(x|\mathcal{H})P(\mathcal{H})}{P(x)}$
- Odds ratio $\frac{P(\mathcal{H}_1|x)}{P(\mathcal{H}_0|x)} = \frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)} \frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)}$; Bayes Factor $\mathcal{B}_{10} = \frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)}$
- $\mathcal{H}_1 \equiv$ noise + signal w/some $\mathcal{A}$; $\mathcal{H}_0 \equiv$ noise only
- $\mathcal{F}$-stat is maximized log-likelihood: $\max_{\mathcal{A}} \frac{P(x|\mathcal{A})}{P(x|0)} = e^{\mathcal{F}}$
- But $\mathcal{H}_1$ is composite hypoth. $P(x|\mathcal{H}_1) = \int P(x|\mathcal{A})P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A}$
- Don't maximize; marginalize! $\mathcal{B}$-statistic (Prix): $\mathcal{B} = \int \frac{P(x|\mathcal{A})}{P(x|0)} P(\mathcal{A}|\mathcal{H}_1) d^4\mathcal{A} = \int e^{-\frac{1}{2}\mathcal{A}^\mu \mathcal{M}_{\mu\nu}\mathcal{A}^\nu + \mathcal{A}^\mu x_\mu} P(\mathcal{A}|\mathcal{H}_1) d^4\mathcal{A}$
- Prix & Krishnan *CQG* **26**, 204013 (2009): If $P(\mathcal{A}|\mathcal{H}_1)$ uniform in $\{\mathcal{A}^\mu\}$, $\mathcal{B} = e^{\mathcal{F}}$
  Unphysical; implies $P(h_0, \cos\iota, \psi, \phi_0|\mathcal{H}_1) \propto h_0^3(1-\cos^2\iota)^3$
- Prix & JTW working on approximations for evaluating $\mathcal{B}$-stat integral w/physical priors

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- $\mathcal{F}$-stat is maximized log-likelihood: $\max_{\mathcal{A}} \frac{P(x|\mathcal{A})}{P(x|0)} = e^{\mathcal{F}}$
- But $\mathcal{H}_1$ is composite hypoth. $P(x|\mathcal{H}_1) = \int P(x|\mathcal{A})P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A}$
- Don't maximize; marginalize! $\mathcal{B}$-statistic (Prix): $\mathcal{B} = \int \frac{P(x|\mathcal{A})}{P(x|0)} P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A} = \int e^{-\frac{1}{2}\mathcal{A}^\mu \mathcal{M}_{\mu\nu}\mathcal{A}^\nu + \mathcal{A}^\mu x_\mu} P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A}$
- Prix & Krishnan *CQG* **26**, 204013 (2009): If $P(\mathcal{A}|\mathcal{H}_1)$ uniform in $\{\mathcal{A}^\mu\}$, $\mathcal{B} = e^{\mathcal{F}}$
  Unphysical; implies $P(h_0, \cos\iota, \psi, \phi_0|\mathcal{H}_1) \propto h_0^3(1-\cos^2\iota)^3$
- Prix & JTW working on approximations for evaluating $\mathcal{B}$-stat integral w/physical priors

# Bayesian Interpretation ($\mathcal{B}$-statistic)

- Assume $\lambda$ known; likelihood $P(x|\mathcal{A}) \propto e^{-\chi^2(\mathcal{A})/2}$
- Bayes's theorem says $P(\mathcal{H}|x) = \frac{P(x|\mathcal{H})P(\mathcal{H})}{P(x)}$
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# Computational Costs & Phase Parameter Resolution

- If $\lambda \equiv$ {freq, sky pos etc} known, can do most sensitive fully coherent search (correlate all data)
- If some params unknown, have to search over them
- Long coherent observation $\rightarrow$ fine resolution in freq etc $\rightarrow$ need too many templates $\rightarrow$ computationally impossible

$$\text{e.g.} \qquad N_{\text{tmplts}} \sim \frac{1}{\Delta f} \frac{1}{\Delta \dot{f}} \frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2$$

- Most CW searches semi-coherent: deliberately limit coherent integration time & param space resolution to keep number of templates manageable

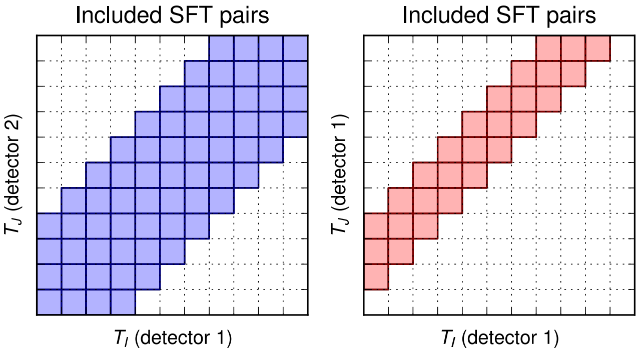
# One Semicoherent Method: Cross-Correlation

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)
Chung, Melatos, Krishnan & JTW *MNRAS* **414**, 2650 (2011)
(Currently being applied by JTW, Peiris, Krishnan, et al)

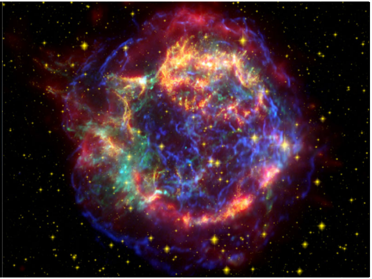
- Divide data into segments of length $T_{\text{sft}}$ & take "short Fourier transform" (SFT) $\tilde{x}_I(f)$
- Label SFTs by $I$, $J$, … and pairs by $\alpha$, $\beta$, …
  - ☞ $I$ & $J$ can be same or different times or detectors
- Construct cross-correlation $\mathcal{Y}_{IJ} = \frac{\tilde{x}_I^*(f_{\tilde{k}_I})\tilde{x}_J(f_{\tilde{k}_J})}{(T_{\text{sft}})^2}$
  - ☞ $f_{\tilde{k}_I} \approx$ signal freq @ time $T_I$ Doppler shifted for detector $I$
- Use CW signal model to determine expected cross-correlation btwn SFTs & combine pairs into optimal statistic $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$

# Tuning the Cross-Correlation Search

- Computational considerations limit coherent integration time
- Can make tunable semi-coherent search by restricting which SFT pairs $\alpha$ are included in $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$
- E.g., only include pairs where $|T_I - T_J| \equiv |T_\alpha| \leq T_{\max}$



# Outline

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# Computing Cost Motivates Search Strategies

All-sky coherent search of full phase param space infeasible:
# of templates skyrockets w/increasing integration time
E.g, for all-sky search with one spindown,

$$N_{\text{tmplts}} \sim \frac{1}{\Delta f} \frac{1}{\Delta \dot{f}} \frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2 \propto T^5$$

Different strategies depending on knowledge of object:

- Known pulsars: all phase parameters known, can do fully coherent **Targeted Search**
  Note $f_{\text{gw}} = 2 f_{\text{rot}}$ for triaxial ellipsoid rotating about principal axis
- Unknown objects: need to use semi-coherent methods for **All-Sky Search**
- Known objects not seen as pulsars (e.g., SN remnants, LMXBs): can do **Directed Search** but need to cope w/uncertain remaining phase parameters

# Searching for Known Pulsars

- Phase params (rotation, sky pos [& binary params]) known
  Pulsar ephemerides (timing) detail phase evolution
- Can search over amplitude params ($h_0$, $\iota$, $\psi$, $\phi_0$);
  search cost NOT driven by observing time
- Different options for amplitude parameters:
  - Maximize likelihood analytically ($\mathcal{F}$-statistic)
  - Marginalize likelihood numerically ($\mathcal{B}$-statistic)
  - Get posterior prob distribution w/Markov-Chain Monte Carlo
  - Use astro observations to constrain spin orientation ($\iota$ & $\psi$)
- Spindown produces indirect upper limit
  - GW emission above limit $\longrightarrow$ more spindown than seen
  - Pulsars w/rapid spindown have “more room” for GW
  - LIGO/Virgo have surpassed spindown limit for Crab & Vela

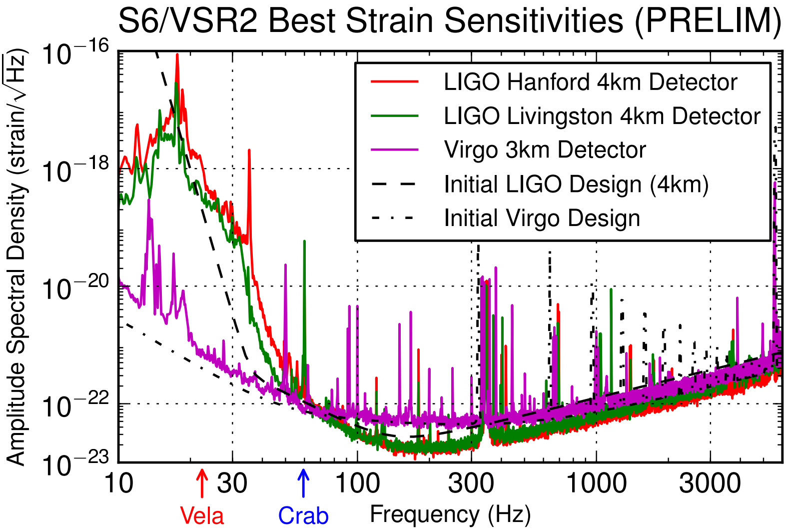
## LSC/Virgo Crab Pulsar Upper Limit

- Pulsar in Crab Nebula
- Created by SN 1054
- $\sim 2\,\mathrm{kpc}$ away
- $f_{\mathrm{rot}} = 29.7\,\mathrm{Hz}$
- $f_{\mathrm{gw}} = 59.4\,\mathrm{Hz}$

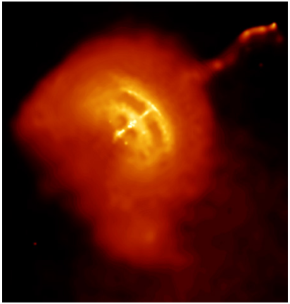
Image credit: Hubble/Chandra

- Initial LIGO (S5) upper limit beats spindown limit
- Abbott et al (LSC) *ApJL* **683**, L45 (2008)
- Abbott et al (LSC & Virgo) + Bégin et al *ApJ* **713**, 671 (2010)
- No more than 2% of spindown energy loss can be in GW

# Initial Virgo Targets the Vela Pulsar



# LSC/Virgo Vela Pulsar Upper Limit



- Pulsar in Vela SN remnant
- Created $\sim$ 12,000 years ago
- $\sim$ 300 pc away
- $f_{\text{rot}} = 11.2\,\text{Hz}$
- $f_{\text{gw}} = 22.4\,\text{Hz}$

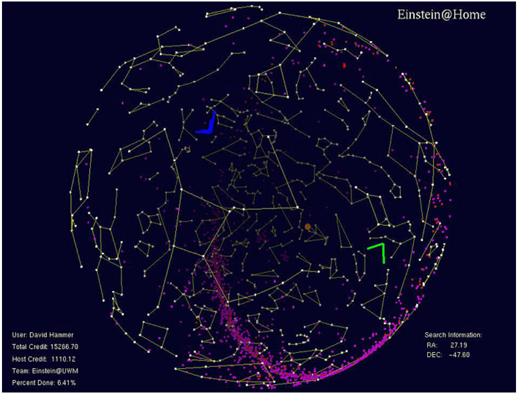
Image credit: Chandra

- GW frequency below initial LIGO "seismic wall"
- Virgo has better low-frequency sensitivity
- VSR2 upper limit beats spindown limit
- No more than 10% of spindown energy loss can be in GW

Abadie et al (LSC & Virgo) + Buchner et al *ApJ* **737**, 93 (2011)

# Einstein@Home

Semicoherent methods needed to handle phase param space;
Increase computing resources by enlisting volunteers
Distributed using BOINC & run as screensaver

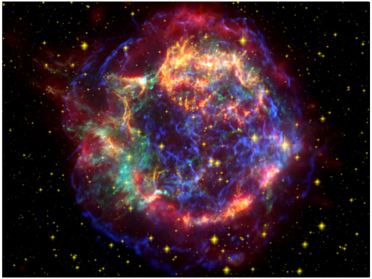



http://www.einsteinathome.org/

# Directed Searches for NS at Known Sky Positions

- Known or suspected neutron stars not seen as pulsars
- Knowledge of sky position reduces parameter space
- Can do fully coherent search on short stretch of data using $\mathcal{F}$-statistic method (Jaranowski, Królak, Schutz *PRD* **58**, 063001 (1998)):
  - Search over remaining phase params (freq & orbit)
  - Analytically maximize likelihood ratio over amp params
  - Use maximized likelihood as detection statistic
- To use all available data instead, need to combine coherent sub-searches incoherently

# LSC/Virgo Cassiopeia A Upper Limit



- Cas A SN remnant
- $\sim$ 2 kpc away
- $\sim$ 300 yr old
- central compact object seen in x-rays; spin period unknown

Image: Spitzer/Hubble/Chandra

- Indirect limit on GW emission from age of neutron star
- Sky position known, can search over $f$, $\dot{f}$, $\ddot{f}$ param space using $\mathcal{F}$-stat on 12 days of LIGO S5 Data upper limit surpasses indirect limit below 300 Hz

Abadie et al (LSC & Virgo) *ApJ* **722**, 1504 (2010)

# Gravitational Waves from Low-Mass X-Ray Binaries


- LMXB: compact object (neutron star or black hole) in binary orbit w/companion star
- If NS, accretion from companion provides "hot spot"; rotating non-axisymmetric NS emits gravitational waves
- Bildsten *ApJL* **501**, L89 (1998) suggested GW spindown may balance accretion spinup; GW strength can be estimated from X-ray flux
- Torque balance would give ≈ constant GW freq
- Signal at solar system modulated by binary orbit

# Brightest LMXB: Scorpius X-1

- Scorpius X-1
  - $1.4M_\odot$ NS w/$0.4M_\odot$ companion
  - unknown params are $f_0$, $a \sin i$, orbital phase
- LSC/Virgo searches for Sco X-1:
  - Coherent $\mathcal{F}$-stat search w/6 hr of S2 data
    Abbott et al (LSC) *PRD* **76**, 082001 (2007)
  - Directed stochastic ("radiometer") search (unmodelled)
    Abbott et al (LSC) *PRD* **76**, 082003 (2007)
    Abbott et al (LSC) arXiv:1109.1809
- Proposed directed search methods:
  - Look for comb of lines produced by orbital modulation
    Messenger & Woan, *CQG* **24**, 469 (2007)
  - Cross-correlation specialized to periodic signal
    Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)
    Prabath Peiris working w/JTW on implementing this search
- Promising source for Advanced Detectors

# Summary

- Periodic signals generated by orbiting binaries or spinning neutron stars targeted by space- and ground-based detectors, respectively
- Signal depends on amplitude (extrinsic) & phase (intrinsic) parameters
- Search methods can maximize or marginalize over unknown parameters
- Coherent searches possible when phase params known (targeted); semicoherent methods used for directed (sky position known) or all-sky searches