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HAM3, HAM4 Offset Spool Flange Scattering

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Michael Smith

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of the LIGO Laboratory.

**California Institute of Technology**  
**LIGO Project – MS 18-34**  
**1200 E. California Blvd.**  
**Pasadena, CA 91125**  
Phone (626) 395-2129  
Fax (626) 304-9834  
E-mail: [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)

**Massachusetts Institute of Technology**  
**LIGO Project – NW22-295**  
**185 Albany St**  
**Cambridge, MA 02139**  
Phone (617) 253-4824  
Fax (617) 253-7014  
E-mail: [info@ligo.mit.edu](mailto:info@ligo.mit.edu)

**LIGO Hanford Observatory**  
**P.O. Box 159**  
**Richland WA 99352**  
Phone 509-372-8106  
Fax 509-372-8137

**LIGO Livingston Observatory**  
**P.O. Box 940**  
**Livingston, LA 70754**  
Phone 225-686-3100  
Fax 225-686-7189

<http://www.ligo.caltech.edu/>



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## 1 INTRODUCTION

This technical document presents a calculation comparing the scattered light displacement noise caused by the exposed offset spool flanges between the BSC2 chamber and the HAM3 and HAM4 chambers versus hiding the offset flange with a baffle.

The only significant sources of stray light that hit the offset flanges are scattering from the BS HR, BS AR, and the ITM HR surfaces due to point defects in the coatings. All of the stray light from the arms is blocked by the arm cavity baffles and the ITM elliptical baffles.

The calculations indicate that the scattered light displacement noise caused by the exposed offset spool flanges between the BSC2 and HAM 3 and HAM 4 chambers is acceptable, as shown in Table 2; therefore baffles are not needed to mitigate the scattered light.

### 1.1 Scope

These calculations are based on the assumption that the HR surface of the ITM and the HR and AR surfaces of the BS exhibit Lambertian scattering due to point defects in the coatings equal to 10ppm of the incident light power hitting the surface (this assumption has not been verified for the BS coatings).

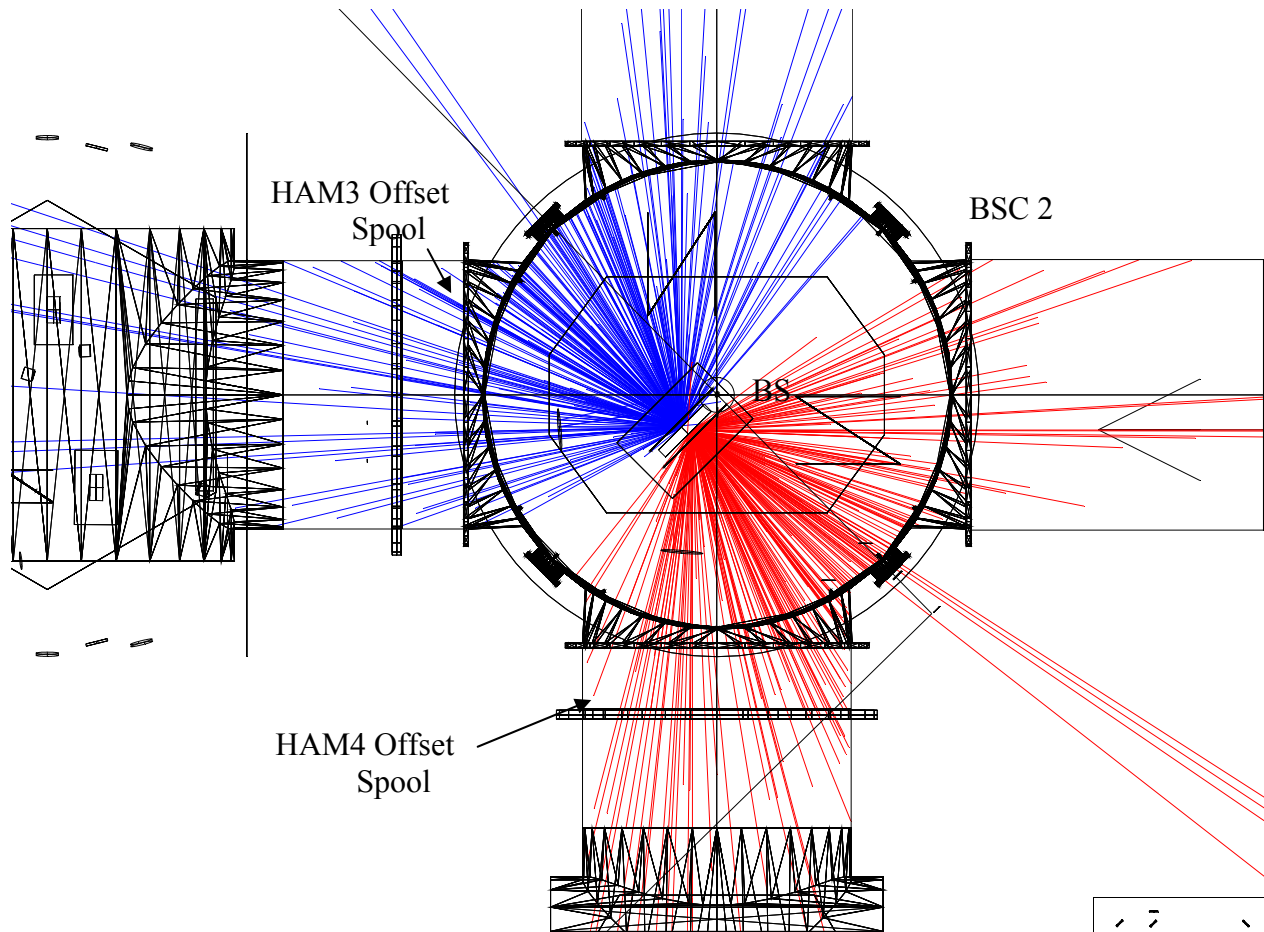
A simplifying assumption was also made that 100% of the scattering from the BS HR is distributed normal to the mirror surface towards HAM3, and 100% of the scattering from the BS AR is distributed normal to the mirror surface towards HAM4; this should account for the total scattered light. The ITM forward scatter was reduced by the Transmissivity of the ITM HR surface.

### 1.2 Applicable Documents

## 2 ZEMAX DATA

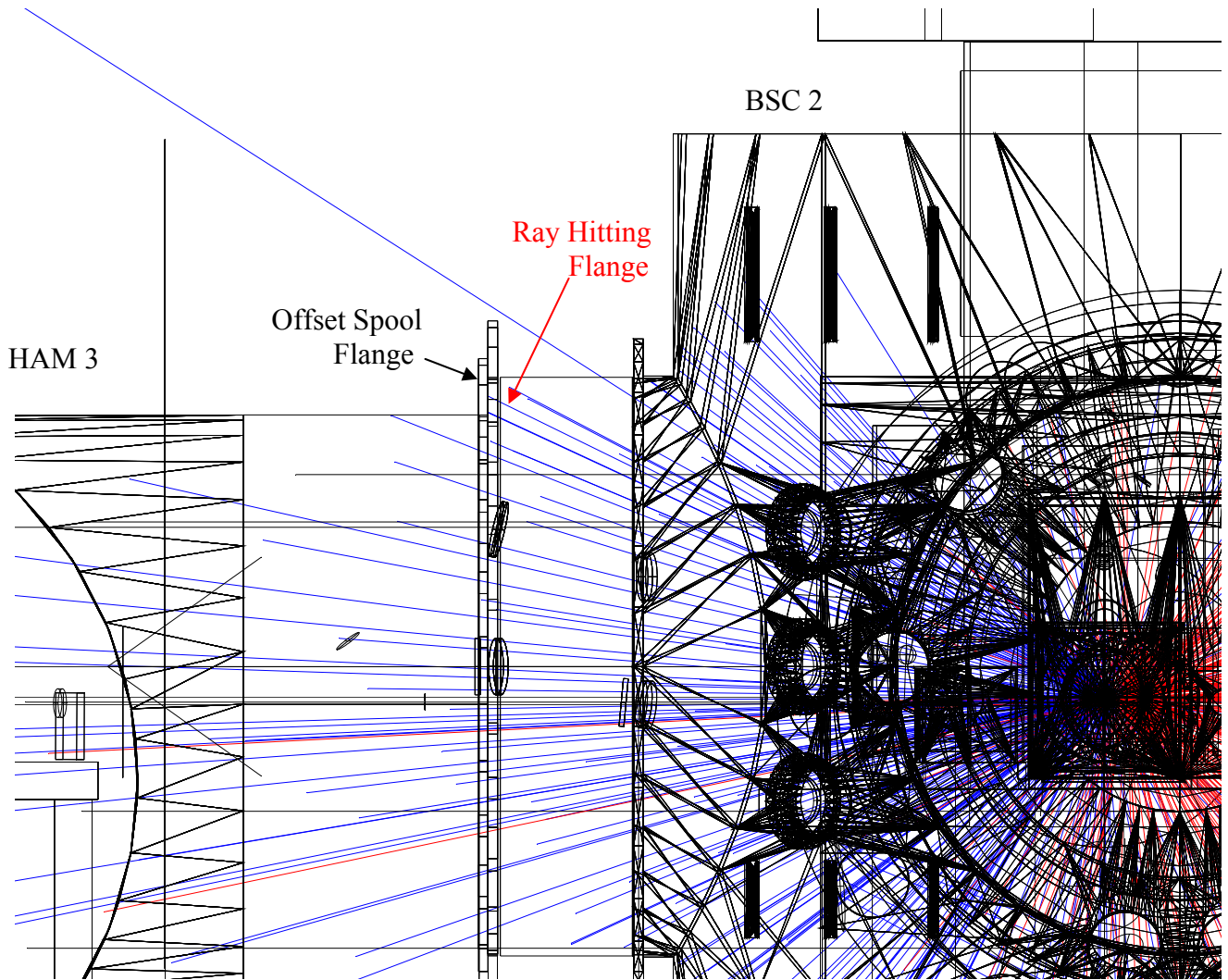
### 2.1 Scattering Geometry

ZEMAX non-sequential ray tracing was used to determine the fraction of wide angle scattered light hitting the offset spool flanges.

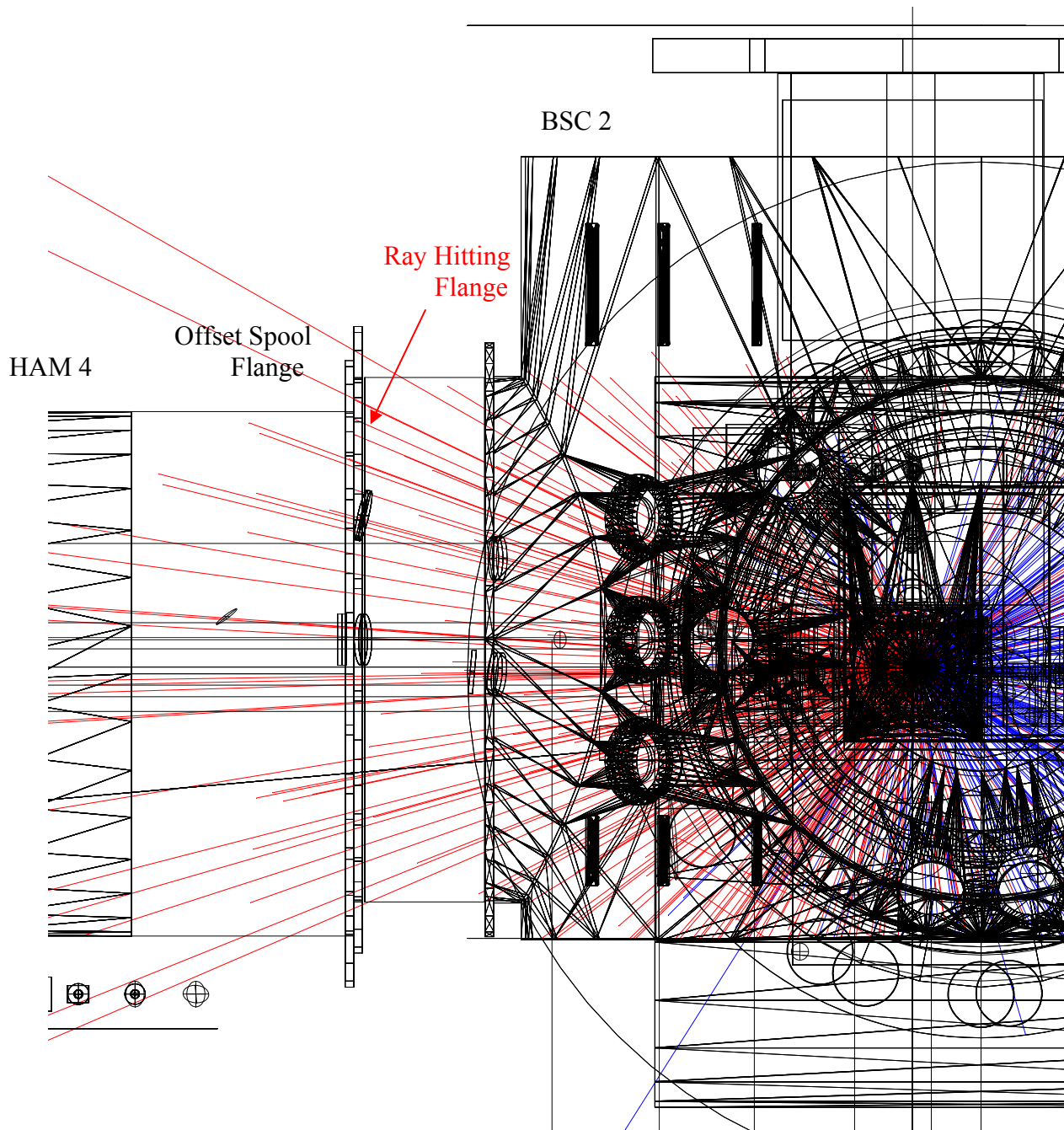


**Figure 1: Scatter from BS Coating**

For the scattering calculations, it was assumed that 100% of the incident scattered rays from the BS that hit the frontal surfaces of the offset spool flange will reflect onto the inner wall of the spool tube and retro-reflect to the BS surface; typical incident rays are shown in Figure 2 and Figure 3.



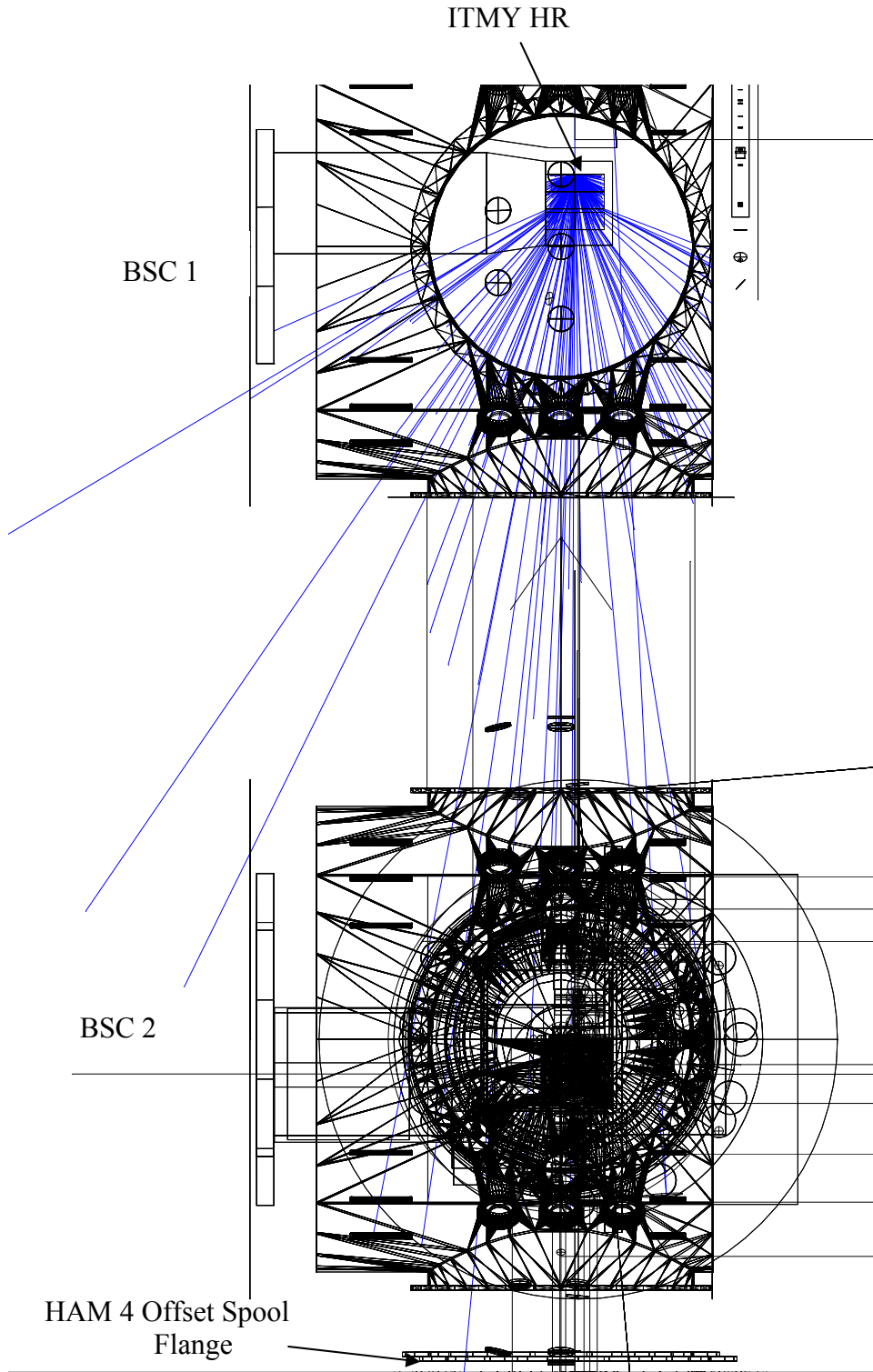
**Figure 2: Scatter from BS HR Surface Hits the HAM3 Offset Spool Flange**



**Figure 3: Scatter from BS AR Surface Hits the HAM4 Offset Spool Flange**

Scattered light from the ITM HR surface will pass through the ITM toward the BS; however, most of that light will be blocked by the BS suspension structure and will be precluded from hitting the offset spool flange, as shown in Figure 4.





**Figure 4: No ITMY HR Scatter Hits HAM 4 Offset Spool Flange**

## 2.2 Scattered Light Fraction

The average incident angles and distance to the scattering surface of the rays hitting the offset spool flange were estimated from the ZEMAX ray traces and are shown below. The fractional scatter from the BS hitting a particular surface was computed by the ZEMAX detector viewer file program.

**Table 1: Scattered Light Fraction Hitting Surface**

Surface	Fractional scatter	scattering length	incident angle on scatter surface, deg	incident angle on BS/ITM, deg
<b>BS HR surface scatter</b>				
HAM_3 optics table	<b>8.30E-03</b>	<b>2732</b>	<b>12</b>	<b>46</b>
Offset Spool HAM3 flange	<b>1.04E-02</b>	<b>1813</b>	<b>26</b>	<b>51</b>
<b>BS AR surface scatter</b>				
HAM_4 optics table	<b>6.63E-03</b>	<b>2710</b>	<b>12</b>	<b>-44</b>
Offset Spool HAM4 flange	<b>8.62E-03</b>	<b>1792</b>	<b>26</b>	<b>-39</b>
<b>ITM HR surface scatter</b>				
Offset Spool HAM4 flange	<b>0</b>	<b>6878</b>	<b>7</b>	<b>7</b>

## 3 SCATTERED LIGHT DISPLACEMENT NOISE

### 3.1 Scattered Light Requirement

A DARM signal is obtained when the differential arm length is modulated as a result of a gravity wave strain. The DARM signal was calculated in reference, T060073-00 Transfer Functions of Injected Noise, and is defined by the following expression:

$$V_{\text{signal}} := \text{DARM} \cdot L \cdot h_{\text{SRD}} \cdot \sqrt{P_0}$$

Where  $L$  is the arm length,  $h_{\text{SRD}}$  is the minimum SRD gravity wave strain spectral density requirement,  $P_0$  is the input laser power into the IFO, and DARM is the signal transfer function.

In a similar manner, an apparent signal (scattered light noise) occurs when a scattered light field with a phase shift is injected into the IFO at some particular location, e.g. through the back of the ETM mirror. The scattered light noise is defined by the following expression:

$$V_{\text{noise}} := \text{SNXXX} \cdot \delta_{\text{SN}} \cdot \sqrt{P_{\text{SN}i}}$$

$P_{\text{SN}i}$  is the scattered light power injected into the IFO mode,  $\delta_{\text{SN}}$  is the phase shift of the injected field, and SNXXX is the noise transfer function for that particular injection location.

The phase shift spectral density of the injected field due to the motion of the scattering surface is given by

$$\delta_{\text{SN}i} := \frac{4 \cdot \pi \cdot x_s}{\lambda}$$

where  $x_s$  is the spectral density of the longitudinal motion of the scattering surface.

In general, the different scattering sources are not coherent and must be added in quadrature. The requirement for total scattered light displacement noise can be stated with the following inequality:

$$\sqrt{\sum_{i=1}^n \left( \frac{\text{SNXXX}}{\text{DARM}} \cdot \frac{4 \cdot \pi \cdot x_s}{\lambda} \cdot \sqrt{\frac{P_{\text{SN}i}}{P_0}} \right)^2} < \frac{1}{10} \cdot L \cdot h_{\text{SRD}}$$

The SNXXX/DARM scattered light noise transfer function ratios for various injection locations within the IFO are shown in Figure 5: Scattered Light Noise Transfer Functions.

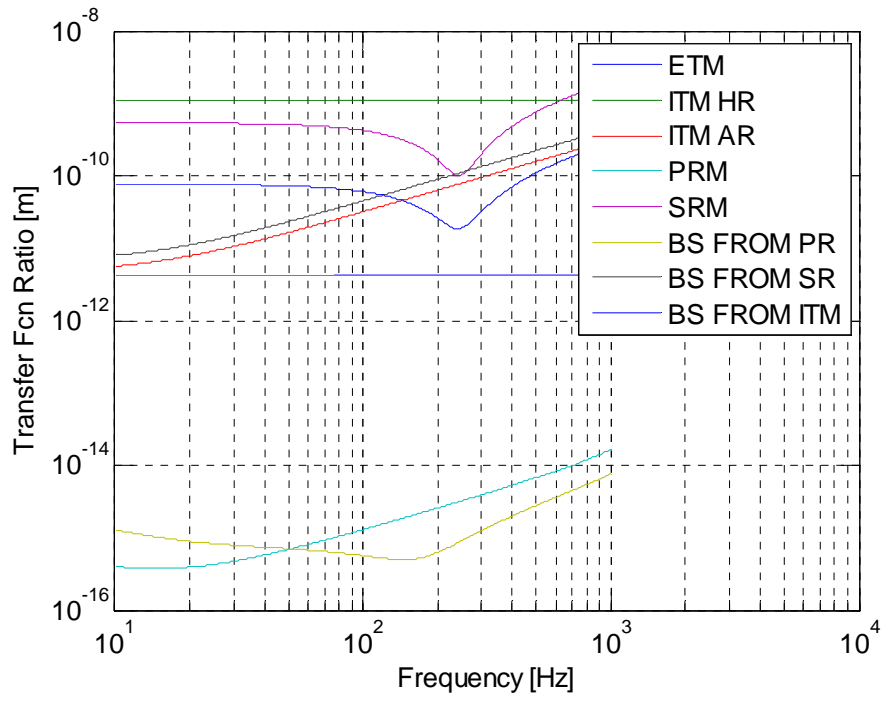


Figure 5: Scattered Light Noise Transfer Functions

### 3.2 Scattered Light Parameters

Displacement noise requirement @ 100 Hz, m/rt Hz	$D_{\text{req}} := 1 \cdot 10^{-21}$
Motion of manifold @ 100 Hz, m/rt Hz	$x_{\text{manifold}} := 8 \cdot 10^{-11}$
Motion of HAM table @ 100 Hz, m/rt Hz	$x_{\text{hamtable}} := 5 \cdot 10^{-12}$
Transfer function @ 100 Hz, BS from SR	$TF_{\text{srbs}} := 4.46 \cdot 10^{-11}$
Transfer function @ 100 Hz, BS from PR	$TF_{\text{prbs}} := 5.66 \cdot 10^{-16}$
Transfer function @ 100 Hz, ITM HR	$TF_{\text{itmhr}} := 1.1 \cdot 10^{-9}$
BRDF of chamber wall, sr <sup>-1</sup>	$BRDF_{\text{wall}} := 0.1$
BRDF of BD, sr <sup>-1</sup>	$BRDF_{\text{bd}} := 0.03$
BRDF of oxidized polished steel, sr <sup>-1</sup>	$BRDF_{\text{oxipolish}} := 0.03$
laser wavelength, m	$\lambda := 1.064 \cdot 10^{-6}$
wave number, m <sup>-1</sup>	$k := 2 \cdot \frac{\pi}{\lambda}$
hemispherical scattering loss fraction TM wide angle(ref: T070089)	$\alpha_{\text{TM}} := 10 \times 10^{-6}$
see T070303 with arm cavity gain = 13000	
Transmissivity of ITM HR	$T_{\text{itmhr}} := 0.01$

recycling cavity power, W

$$P_a := 8.125 \times 10^5$$

power in power recycling cavity each arms, W

$$P_{rc} := \frac{P_a \cdot T_{itmhr}}{4}$$

$$P_{rc} = 2.844 \times 10^3$$

input laser power, W

$$P_{psl} := 125$$

The following data comes from ZEMAX sensor data; see /ALIGO/SLC/BS Elliptical Baffle/wide angle scatter\_BS\_L1\_power.xls

### SCATTER PATH LENGTH

distance from BSHR to HAM3 optics table, m

$$L_{ham3table} := 2.73$$

distance from BSHR to HAM3 offset spool flange, m

$$L_{ham3offsetflange} := 1.81$$

distance from BSAR to HAM4 optics table, m

$$L_{ham4table} := 2.71$$

distance from BS to HAM4 offset spool flange, m

$$L_{ham4offsetflange} := 1.79$$

distance from ITMy to HAM4 offset spool flange, m

$$L_{itmyham4offsetflange} := 6.88$$

### COG INCIDENT ANGLE

incident angle from BS to HAM3 optics table, rad

$$\theta_{ham3table} := 0.81$$

incident angle from BS to HAM3 offset spool flange, rad

$$\theta_{ham3offsetflange} := 0.88$$

incident angle from BS to HAM4 optics table, rad

$$\theta_{ham4table} := -0.76$$

incident angle from BS to HAM4 offset spool flange, rad  $\theta_{\text{ham4offsetflange}} := -0.69$

incident angle from ITMy to HAM4 offset spool flange, rad  $\theta_{\text{itmyham4offsetflange}} := 0.12$

### FRACTION of LAMBERTIAN SCATTER FROM COC HITTING SURFACE

fractional power from BS to HAM3 optics table  $\text{PF}_{\text{ham3table}} := 8.3 \cdot 10^{-3}$

fractional power from BS to HAM3 offset spool flange  $\text{PF}_{\text{ham3offsetflange}} := 1.04 \cdot 10^{-2}$

fractional power from BS to HAM4 optics table  $\text{PF}_{\text{ham4table}} := 6.6 \cdot 10^{-3}$

fractional power from BS to HAM4 offset spool flange  $\text{PF}_{\text{ham4offsetflange}} := 8.6 \cdot 10^{-3}$

fractional power from ITMy to HAM4 offset spool flange  $\text{PF}_{\text{itmyham4offsetflange}} := 0$

### INCIDENT POWER

BS to HAM3 optics table, W

$$P_{\text{ham3table}} := P_{\text{rc}} \cdot \text{PF}_{\text{ham3table}} \cdot \alpha_{\text{TM}}$$

$$P_{\text{ham3table}} = 2.36 \times 10^{-4}$$

BS to HAM3 offset spool flange, W

$$P_{\text{ham3offsetflange}} := P_{\text{rc}} \cdot \text{PF}_{\text{ham3offsetflange}} \cdot \alpha_{\text{TM}}$$

$$P_{\text{ham3offsetflange}} = 2.957 \times 10^{-4}$$

BS to HAM4 optics table, W

$$P_{\text{ham4table}} := P_{\text{rc}} \cdot \text{PF}_{\text{ham4table}} \cdot \alpha_{\text{TM}}$$

$$P_{\text{ham4table}} = 1.877 \times 10^{-4}$$

BS to HAM4 offset spool flange, W

$$P_{\text{ham4offsetflange}} := P_{\text{rc}} \cdot \text{PF}_{\text{ham4offsetflange}} \cdot \alpha_{\text{TM}}$$

$$P_{\text{ham4offsetflange}} = 2.446 \times 10^{-4}$$

ITMy to HAM4 offset spool flange, W

$$P_{\text{itmyham4offsetflange}} := P_{\text{a}} \cdot \text{PF}_{\text{itmyham4offsetflange}} \cdot \alpha_{\text{TM}}$$

$$P_{\text{itmyham4offsetflange}} = 0$$

### 3.2.1 Scattered Light Calculations

### 3.2.2 Scattered Light from Offset Baffle and HAM ISI table

In this section, scattered light from an assumed offset baffle will be calculated.

power scattered from surface back to TM

$$P_{\text{sTM}} := P_{\text{inc}} \cdot \text{BRDF}_{\text{s}} \cdot \frac{\pi \cdot w_{\text{ifo}}^2}{L^2}$$

power scattered from surface & re-scattered from TM into IFO

$$P_{\text{sTMifo}} := P_{\text{sTM}} \cdot \frac{\alpha_{\text{TM}}}{\pi} \cdot \cos(\theta_{\text{inc}}) \cdot \frac{\lambda^2}{(\pi \cdot w_{\text{ifo}}^2)}$$

combining the two equations,

$$P_{\text{sTMifo}} := P_{\text{inc}} \cdot \text{BRDF}_{\text{s}} \cdot \frac{\alpha_{\text{TM}}}{\pi \cdot L^2} \cdot \lambda^2 \cdot \cos(\theta_{\text{inc}})$$



**BS to HAM3 optics table scatter, W**

$$P_{\text{ham3tableifo}} := P_{\text{ham3table}} \cdot \left( \alpha_{\text{TM}} \cdot \frac{\lambda^2}{\pi \cdot L_{\text{ham3table}}^2} \cdot \text{BRDF}_{\text{wall}} \cdot \cos(\theta_{\text{ham3table}}) \right)$$

$$P_{\text{ham3tableifo}} = 7.869 \times 10^{-24}$$

**BS to HAM3 offset spool baffle scatter, W**

$$P_{\text{sham3offsetbaffifo}} := P_{\text{ham3offsetflange}} \cdot \left( \alpha_{\text{TM}} \cdot \frac{\lambda^2}{\pi \cdot L_{\text{ham3offsetflange}}^2} \cdot \text{BRDF}_{\text{oxipolish}} \cdot \cos(\theta_{\text{ham3offsetflange}}) \right)$$

$$P_{\text{sham3offsetbaffifo}} = 6.218 \times 10^{-24}$$

**BS to HAM3 offset spool flange reflection, W**

$$P_{\text{rham3offsetflangeifo}} := P_{\text{ham3offsetflange}} \cdot \left( \frac{\alpha_{\text{TM}}}{\pi} \cdot \frac{\lambda^2}{\pi \cdot w_{\text{ifo}}^2} \cdot \cos(\theta_{\text{ham3offsetflange}}) \right)$$

$$P_{\text{rham3offsetflangeifo}} = 2.161 \times 10^{-22}$$

**BS to HAM4 optics table scatter, W**

$$P_{\text{ham4tableifo}} := P_{\text{ham4table}} \cdot \left( \alpha_{\text{TM}} \cdot \frac{\lambda^2}{\pi \cdot L_{\text{ham4table}}^2} \cdot \text{BRDF}_{\text{wall}} \cdot \cos(\theta_{\text{ham4table}}) \right)$$

$$P_{\text{ham4tableifo}} = 6.675 \times 10^{-24}$$

**BS to HAM4 offset spool baffle scatter, W**

$$P_{\text{sham4offsetbaffifo}} := P_{\text{ham4offsetflange}} \cdot \left( \alpha_{\text{TM}} \cdot \frac{\lambda^2}{\pi \cdot L_{\text{ham4offsetflange}}^2} \cdot \text{BRDF}_{\text{oxipolish}} \cdot \cos(\theta_{\text{ham4offsetflange}}) \right)$$

$$P_{\text{sham4offsetbaffifo}} = 6.364 \times 10^{-24}$$

**ITMy to HAM4 offset spool baffle scatter, W**

$$P_{\text{sitmyham4offsetbaffifo}} := P_{\text{itmyham4offsetflange}} \cdot \left( \alpha_{\text{TM}} \cdot \frac{\lambda^2}{\pi \cdot L_{\text{itmyham4offsetflange}}^2} \cdot \text{BRDF}_{\text{oxipolish}} \cdot \cos(\theta_{\text{itmyham4offsetflange}}) \right)$$

$$P_{\text{sitmyham4offsetbaffifo}} = 0$$

**3.2.3 Retro-reflected Scattered Light from Offset Flange without Baffle**

power reflected from the surface back to TM

$$P_{\text{rTM}} := P_{\text{inc}}$$

combining the two equations,

$$P_{\text{rTMifo}} := P_{\text{inc}} \cdot \frac{\alpha_{\text{TM}}}{\pi} \cdot \cos(\theta_{\text{inc}}) \cdot \frac{\lambda^2}{(\pi \cdot w_{\text{ifo}})^2}$$

**BS to HAM4 offset spool flange reflection, W**

$$P_{\text{rham4offsetflangeifo}} := P_{\text{ham4offsetflange}} \cdot \left( \frac{\alpha_{\text{TM}}}{\pi} \cdot \frac{\lambda^2}{\pi \cdot w_{\text{ifo}}^2} \cdot \cos(\theta_{\text{ham4offsetflange}}) \right)$$

$$P_{\text{rham4offsetflangeifo}} = 2.164 \times 10^{-22}$$

**ITMy to HAM4 offset spool flange reflection, W**

$$P_{\text{ritmyham4offsetflangeifo}} := P_{\text{itmyham4offsetflange}} \cdot \left( \frac{\alpha_{\text{TM}}}{\pi} \cdot \frac{\lambda^2}{\pi \cdot w_{\text{ifo}}^2} \cdot \cos(\theta_{\text{itmyham4offsetflange}}) \right)$$

$$P_{\text{ritmyham4offsetflangeifo}} = 0$$

**3.2.4 Displacement Noise by Light Scattered from Offset Flange with Baffle, and HAM ISI table****BS to HAM3 optics table**

$$\text{DN}_{\text{ham3tableifo}} := \text{TF}_{\text{prbs}} \cdot \left( \frac{P_{\text{ham3tableifo}}}{P_{\text{psl}}} \right)^{0.5} \cdot x_{\text{hamtable}} \cdot 2 \cdot k$$

$$\text{DN}_{\text{ham3tableifo}} = 8.386 \times 10^{-33}$$

**BS to HAM3 offset spool baffle scatter**

$$\text{DN}_{\text{sham3offsetbaff}} := \text{TF}_{\text{prbs}} \cdot \left( \frac{P_{\text{sham3offsetbaffifo}}}{P_{\text{psl}}} \right)^{0.5} \cdot x_{\text{manifold}} \cdot 2 \cdot k$$

$$\text{DN}_{\text{sham3offsetbaff}} = 1.193 \times 10^{-31}$$

**BS to HAM4 optics table**

$$\text{DN}_{\text{ham4tableifo}} := \text{TF}_{\text{srbs}} \cdot \left( \frac{P_{\text{ham4tableifo}}}{P_{\text{psl}}} \right)^{0.5} \cdot x_{\text{hamtable}} \cdot 2 \cdot k$$

$$\text{DN}_{\text{ham4tableifo}} = 6.086 \times 10^{-28}$$

**BS to HAM4 offset spool baffle scatter**

$$DN_{\text{sham4offsetbaff}} := TF_{\text{srbs}} \cdot \left( \frac{P_{\text{sham4offsetbaffifo}}}{P_{\text{psl}}} \right)^{0.5} \cdot x_{\text{manifold}} \cdot 2 \cdot k$$

$$DN_{\text{sham4offsetbaff}} = 9.508 \times 10^{-27}$$

**3.2.5 Displacement Noise by Light Retro-reflected from Offset Flange without Baffle****BS to HAM3 offset spool flange reflection**

$$DN_{\text{rham3offsetflange}} := TF_{\text{prbs}} \cdot \left( \frac{P_{\text{rham3offsetflangeifo}}}{P_{\text{psl}}} \right)^{0.5} \cdot x_{\text{manifold}} \cdot 2 \cdot k$$

$$DN_{\text{rham3offsetflange}} = 7.032 \times 10^{-31}$$

**BS to HAM4 offset spool flange reflection**

$$DN_{\text{rham4offsetflange}} := TF_{\text{srbs}} \cdot \left( \frac{P_{\text{rham4offsetflangeifo}}}{P_{\text{psl}}} \right)^{0.5} \cdot x_{\text{manifold}} \cdot 2 \cdot k$$

$$DN_{\text{rham4offsetflange}} = 5.544 \times 10^{-26}$$

**ITMy to HAM4 offset spool flange reflection**

$$DN_{\text{ritmyham4offsetflange}} := TF_{\text{srbs}} \cdot \left( \frac{P_{\text{ritmyham4offsetflangeifo}}}{P_{\text{psl}}} \right)^{0.5} \cdot x_{\text{manifold}} \cdot 2 \cdot k$$

$$DN_{\text{ritmyham4offsetflange}} = 0$$

## 4 Summary of Displacement Noise

The scattered light displacement noise caused by the exposed offset spool flange between the BSC2 and HAM 3 and HAM 4 chambers is acceptable, as shown in Table 2; therefore baffles are not needed to mitigate the retro-reflected scattered light from the offset spool flange, even though baffles would reduce the displacement noise by approximately a factor 6.

**Table 2: Scattered Light Displacement Noise, m/rtHz @ 100 Hz**

Surface	Displacement Noise, m/rtHz @ 100 Hz
<b>AOS Requirement</b>	<b>&lt;1E-21 @ 100 Hz</b>
<b>Scatter from Surface with Offset Spool Baffle</b>	
HAM_3 optics table	<b>8.39E-33</b>
Offset Spool HAM3 Baffle	<b>1.19E-31</b>
HAM_4 optics table	<b>6.09E-28</b>
Offset Spool HAM4 Baffle	<b>9.51E-27</b>
<b>Retro-reflection from Surface without Offset Spool Baffle</b>	
Offset Spool HAM3 Flange	<b>7.03E-31</b>
Offset Spool HAM4 Flange	<b>5.54E-26</b>