Accurate measurement of quantum efficiency

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Quantum efficiency measurement

Background

♦Squeezing level will be limited by the quantum efficiency(QE) of a photodiode(PD).
-> Not sufficient accuracy for PDs with a high QE (close to 99%)
♦The accuracy of QE measurement is limited by the accuracy of the incident laser power.

Objectives

♦Measure the quantum efficiency of PD within 1% uncertainty
♦It correspond to make power meter with high accuracy
=> contribute to estimate an accurate squeezing level

Method

♦Michelson interferometer with a tiny mirror
=> Tiny mirror is sensitive for changing input power
(Application of the tiny mirror in RPN measurement)
=> Accurate measurement of the laser power (i.e. number of photon)
=> We can get an accurate quantum efficiency of a PD

Theory

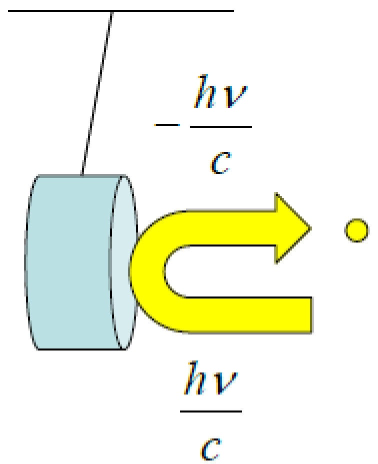
$$QE = \frac{N_e}{N_p}$$

Fine measurement of N_p lead to a high accuracy of QE.

<< Notation >>

h: Planck constant
N_P: Number of photons in input light
m: Mass of small mirror
c: Speed of light
ν: Laser frequency
ω: Frequency of the intensity modulation
ϕ: Angle of the incident laser to mirror
QE = Quantum Efficiency
(Efficiency to convert light power to current)
N_e: Number of electrons in output current of PD

One photon



Equation of motion

(Mechanical response)

$$m\ddot{x} = \frac{2h\nu}{c}$$

$$\rightarrow d\tilde{X} = -\frac{2h\nu N_P}{m\omega^2 c} \cdots (1)$$

$$N_p = \frac{P}{h*\nu}$$

Response of Michelson IFO

$$d\tilde{X} = \frac{\lambda}{2\pi V_0} dV_{PD} \cdots (2)$$

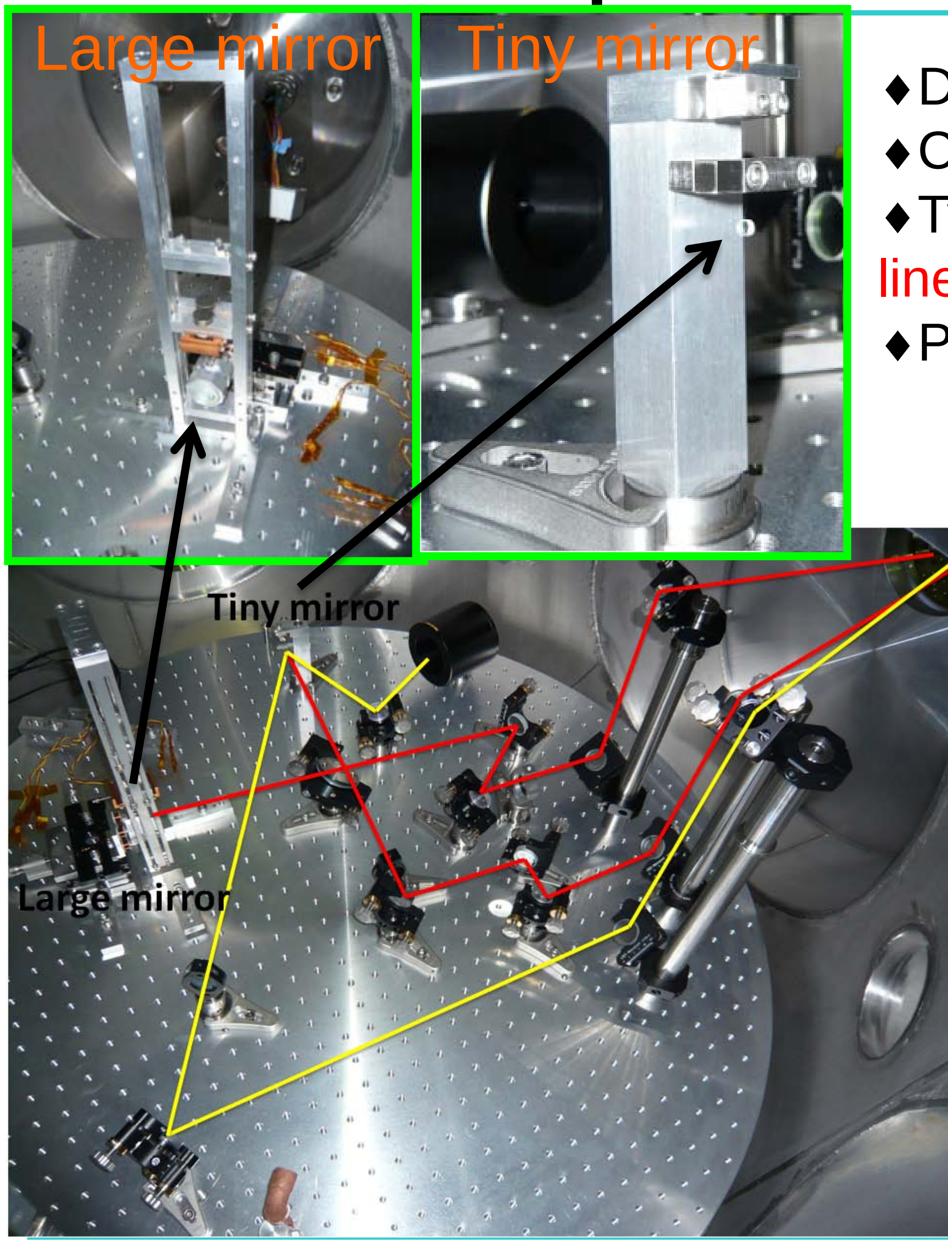
(1) + (2) = opto-mechanical response through radiation pressure

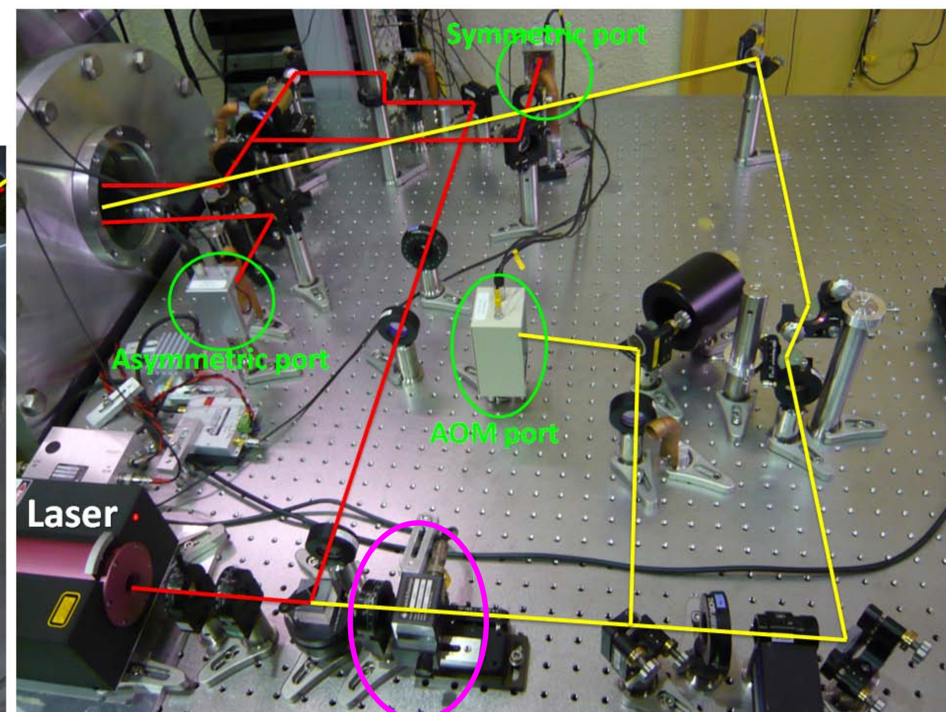
$$dP = \frac{m*c*\omega^2*\lambda}{4*\pi*V_o} * dV_{PD}$$

An opto-mechanical response makes a new kind of power meter

Experimental setup

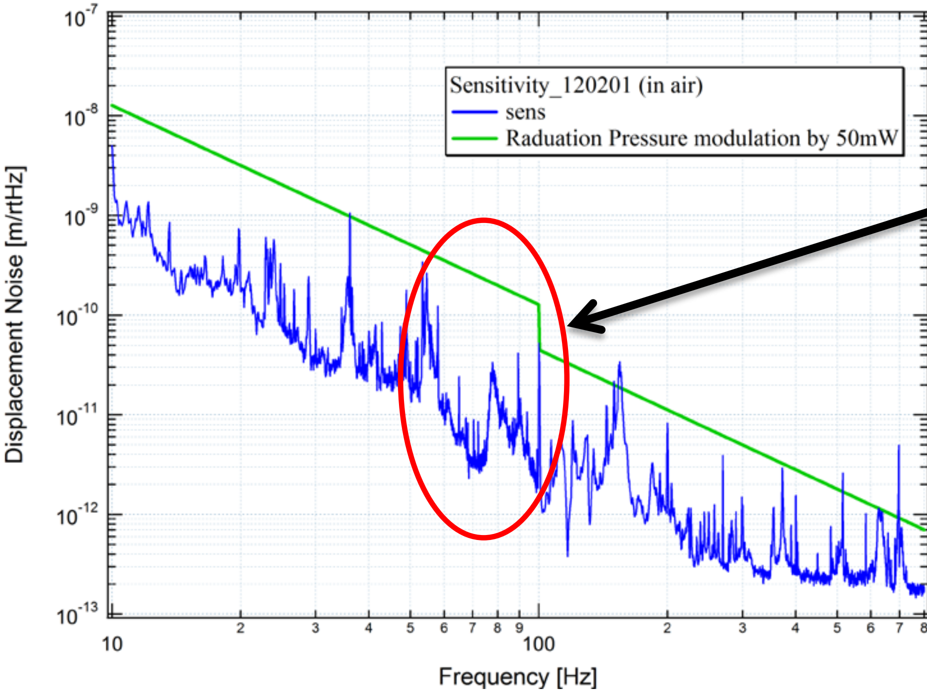
- Displacement sensor: Michelson IFO
- Control: Mid-fringe lock by coil actuator
- Two path: for shaking mirror (Yellow line) and for MI (Red line)
- Power modulation: AOM





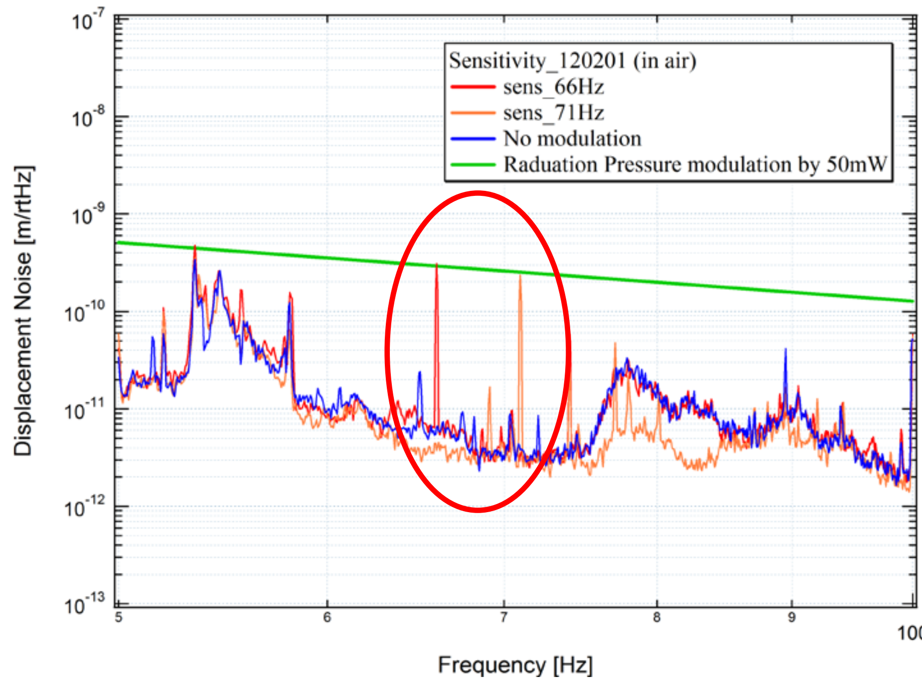
AOM

Displacement by shaken radiation pressure



Suitable region for measurement

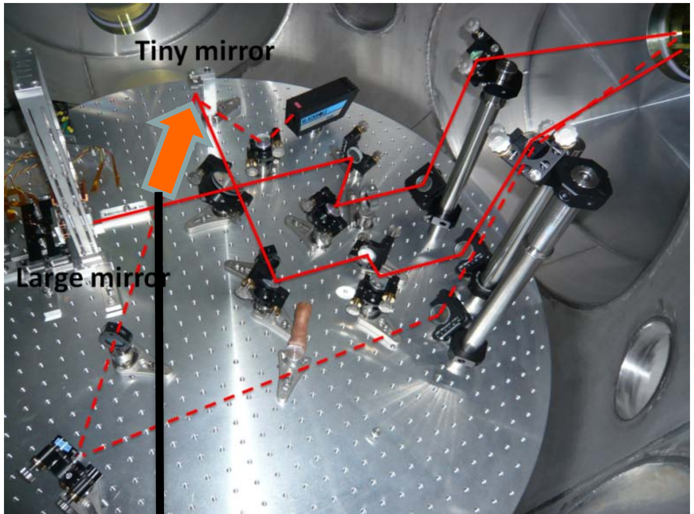
When I shake the laser power using AOM at 66Hz and 71Hz

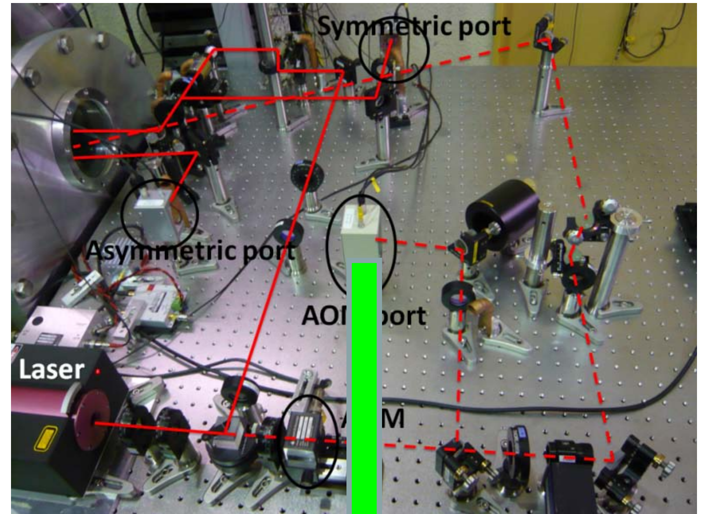


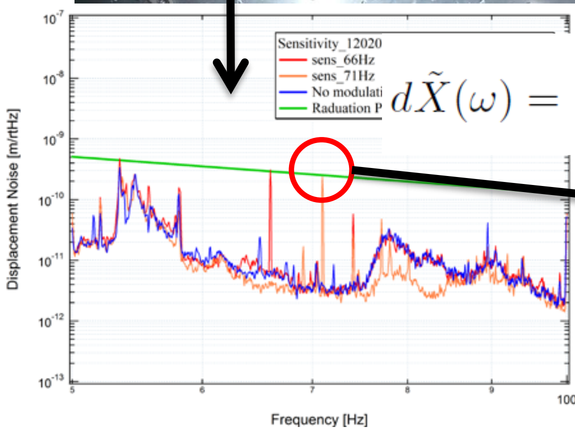
Large amplitude of displacement comparing with floor noise is observed

=> It corresponds to 50mW shaking

Calibration as a power meter





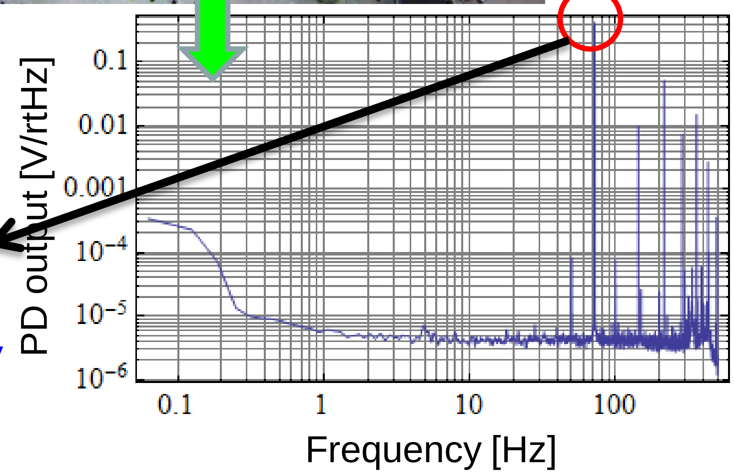


$$d\tilde{X}(\omega) = \frac{2P_{\rm m}(\omega)\cdot\cos\phi}{c\,m\omega^2}$$

W/rtHz

V/rtHz

$\Rightarrow$ 0.301 W/V

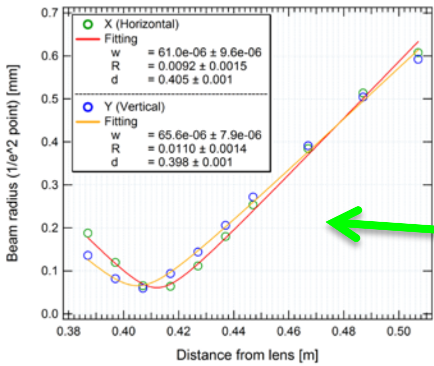


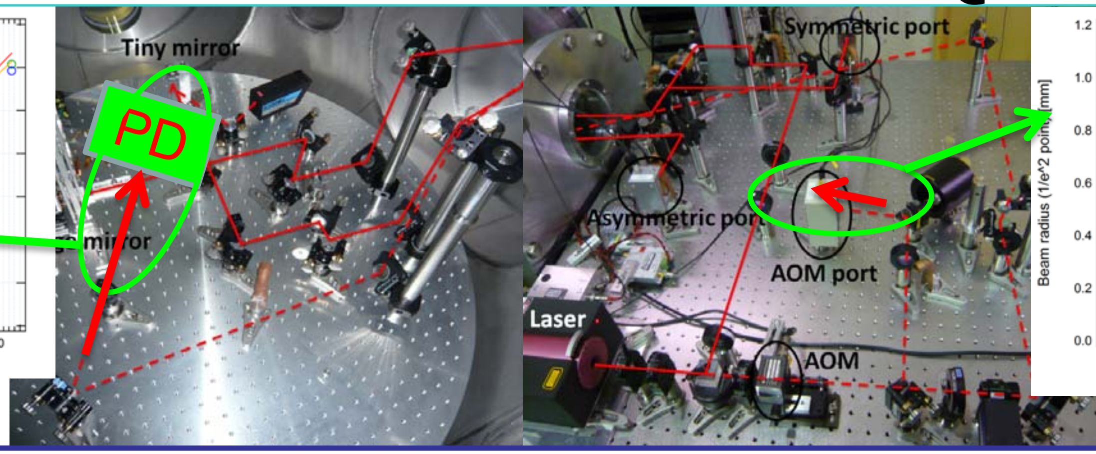
PD on the AOM port is calibrated as a power meter

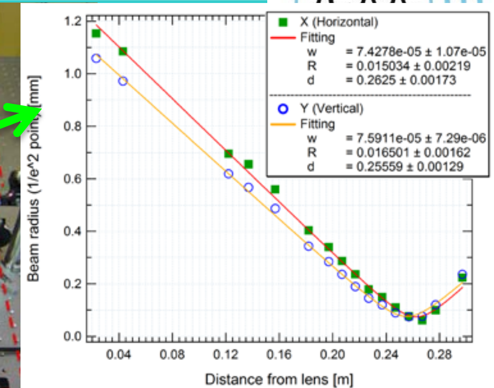
- Commercial power meter (± 2.5%)： 250.0 ± 6.3 mW
- Our experimental result： 244 mW

Consistent within uncertainty

Measurement of QE







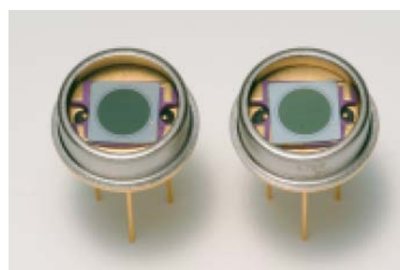
- Ratio between two PDs was measured by exchanging the position of two PDs
  Efficiency ratio (0.939 : 1), Laser power ratio (102 : 1)
- At almost same size of beam radius by measuring beam profiles (*w* = 0.2 mm)

Result of QE

⇒ PD inside tank:
0.371 → 0.319 A/W

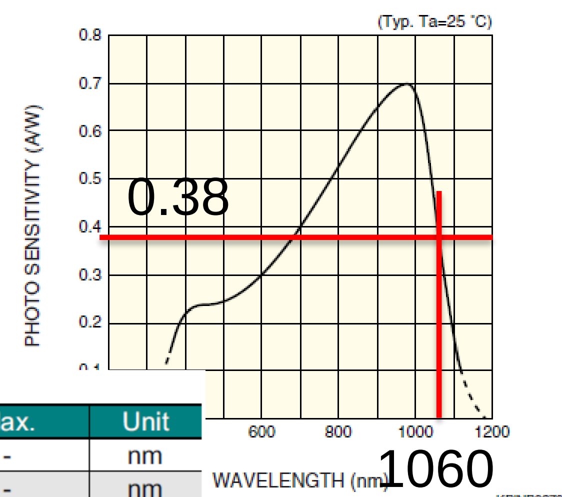
⇒ PD on AOM port:
0.396 → 0.340 A/W

Reasonable value!



Si PIN photodiode
S3759 by HAMAMTSU

■ Spectral response



■ Electrical and optical characteristics (Ta=25 °C)

Parameter	Symbol	Condition	Min.	Typ.	Max.	Unit
Spectral response range	λ		-	360 to 1120	-	nm
Peak sensitivity wavelength	λp		-	980	-	nm
Photo sensitivity	S	λ=1060 nm	0.3	0.38	-	A/W

Contribution to uncertainty

Detailed formula of the laser power is $P = \frac{c\lambda}{4\pi H_m V_{pp} \alpha_{\mathrm{m}} \cos\phi} G_{\mathrm{CL}} \cdot T_{AH} \cdot V_f$

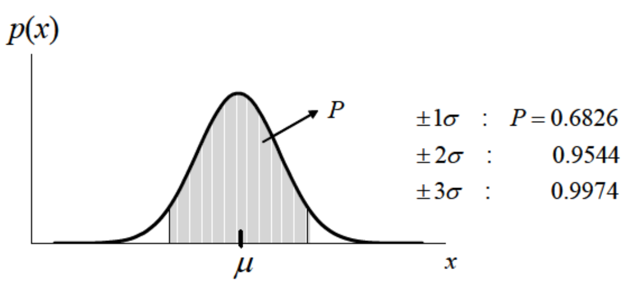
The propagation low of uncertainty (total standard deviation) is expressed as $\sigma_F = \sqrt{\sum_{j=1}^{s} \left(\frac{\partial F}{\partial X_j} \sigma_j \right)^2}$

$$\frac{\sigma_P}{P} = \sqrt{\left(\frac{\sigma_G}{G_{\mathrm{CL}}}\right)^2 + \left(\frac{\sigma_T}{T_{AH}}\right)^2 + \left(\frac{\sigma_{V_f}}{V_f}\right)^2 + \left(\frac{\sigma_H}{H_m}\right)^2 + \left(\frac{\sigma_{V_{pp}}}{V_{pp}}\right)^2 + \left(\frac{\sigma_\alpha}{\alpha_{\mathrm{m}}}\right)^2 + (\sigma_\phi \cdot \tan\phi)^2}$$

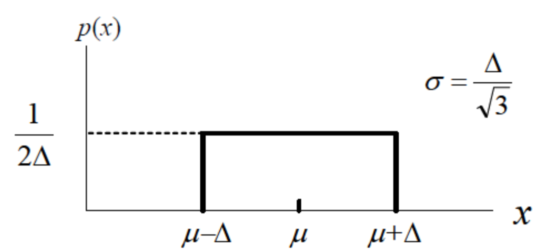
≦1%

Evaluation method: Standard uncertainty
(ISO, *Guide to the Expression of Uncertainty in Measurement*；GUM)

- ♦ Type-A
  (for statistical error)
  Gaussian probability density
  => Standard deviation



- ♦ Type-B
  (for not statistical uncertainty)
  Uniform probability density
  => Corresponding value to standard deviation

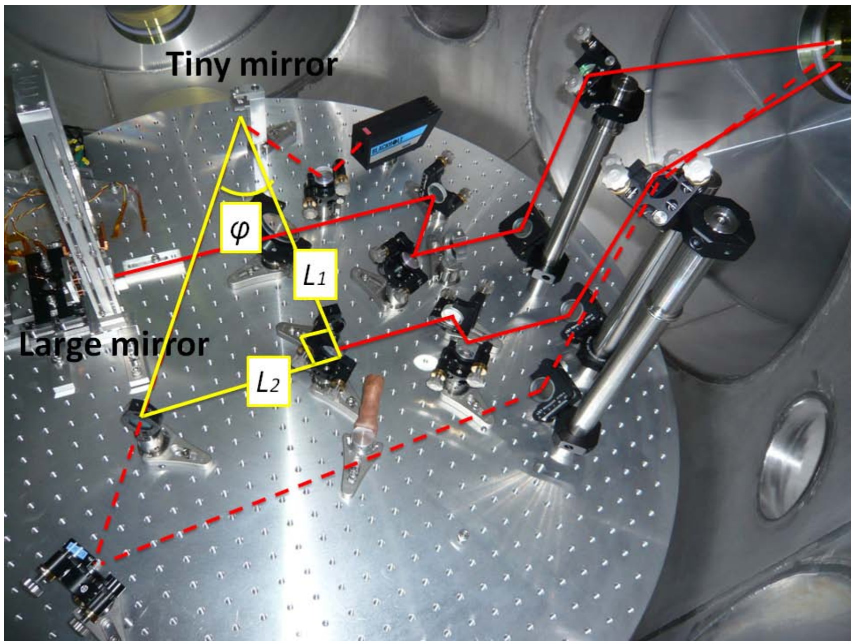


Incident angle to mirror (Type-B)

$$P = \frac{c\lambda}{4\pi H_m V_{pp} \alpha_{\rm m} \cos\phi} G_{\rm CL} \cdot T_{AH} \cdot V_f$$

Φ: Incident angle to small mirror

Horizontal direction



L1 and *L2* is measured to decide incident angle

L1 = 38.8 cm ± 0.1 cm

L2 = 20.2 cm ± 0.2 cm

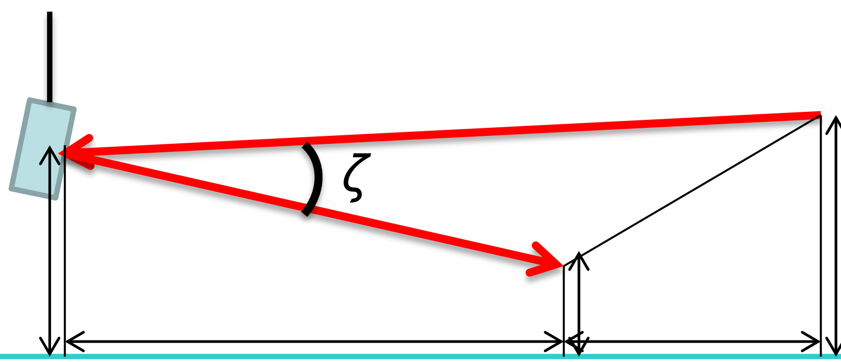
Φ = 27.5° ± 0.3°

$\sigma_\phi \cdot \tan\phi$

=Tan[0.48]*0.00511/sqrt(3)

= 0.15%

Vertical direction



Incident angle is

ζ / 2 = 2.63° ± 0.19°

$\sigma_\phi \cdot \tan\phi$ = 0.009%

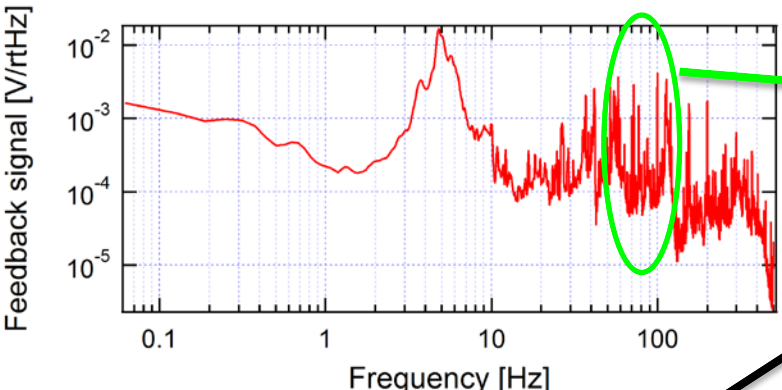
Because tan(*ζ / 2)* is quite small

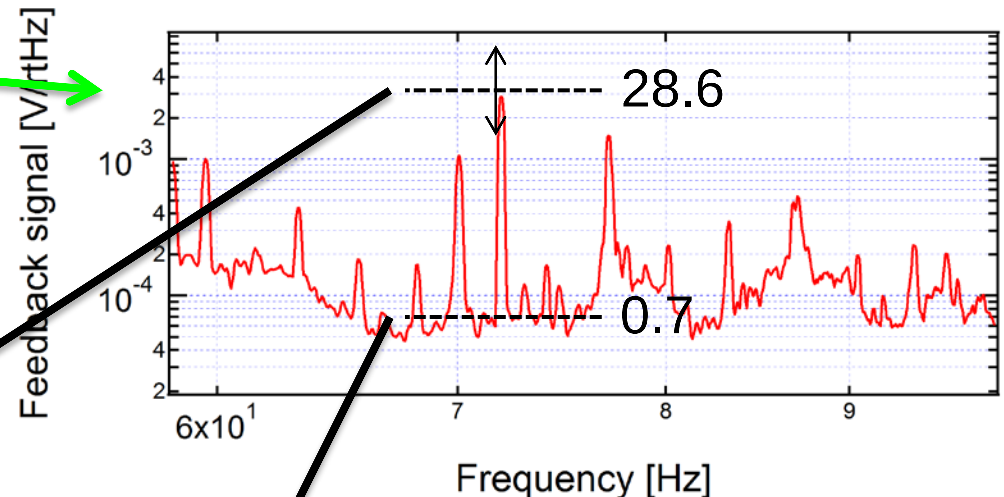
Feedback signal (Type-A)

$$P = \frac{c\lambda}{4\pi H_m V_{pp} \alpha_{\mathrm{m}} \cos\phi} G_{\mathrm{CL}} \cdot T_{AH} \cdot V_f$$

V_f: Feedback signal for control

1 kHz sampling with AAF(500Hz), 5 x 10^5 points, Time series → FFT → Flat-top window, Ave. num. 31 → Standard Deviation





$$\frac{\sigma_{V_f}}{V_f} = 0.30\%$$

★Including Intensity noise, thermal drift and so on.

Floor noise is 40 times lower than signal
(1/40)^2 = 0.000625 ≒ 0.06%

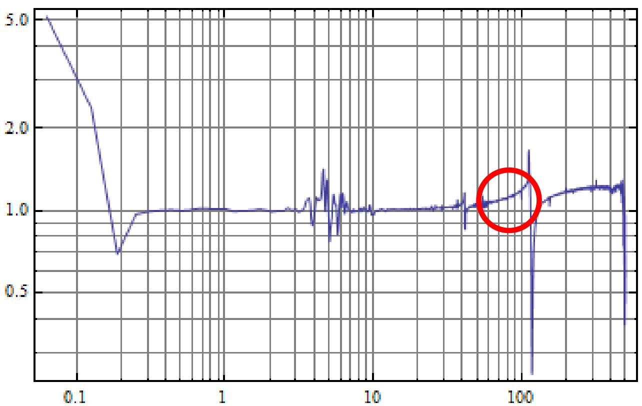
★Including frequency noise, seismic noise, and other disturbances

Calibration factors (Type-A)

$$P = \frac{c\lambda}{4\pi H_m V_{pp} \alpha_{\rm m} \cos\phi} G_{\rm CL} \cdot T_{AH} V_f$$

Feedback signal [V/rtHz] is calibrated to displacement [m/rtHz] using these parameters.

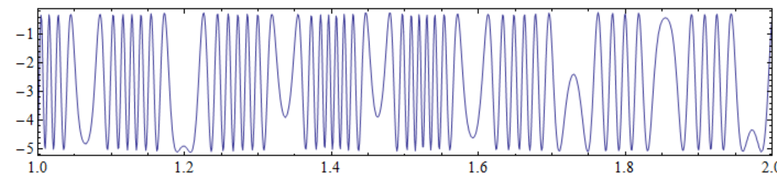
G_{CL}: (1+G)/G



1.094@ 72Hz

$$\frac{\sigma_G}{G_{\rm CL}} = 0.44\%$$

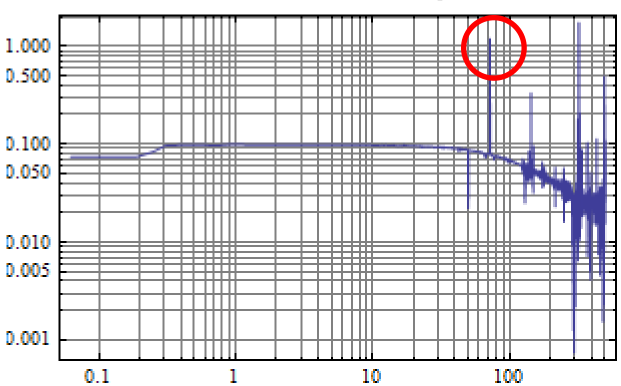
V_{pp}: Peak to Peak of error signal of Michelson IFO



V_{pp} = 3.437V ± 0.011V

$$\frac{\sigma_{V_{pp}}}{V_{pp}} = 0.33\%$$

T_{AH}: Actuator response



Shaken by single frequency to improve S/N

$$\frac{\sigma_T}{T_{AH}} = 0.485\%$$

(Deviation from Mid-fringe: V_f, G_{CL}, T_{AH})

Drift of the offset in electrical circuit

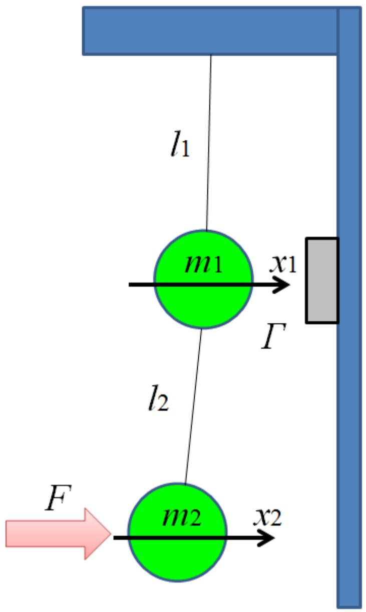
Cos[0.03] = 0.045 (Drift was 3%)

⇒ Drift of 3% cause uncertainty of 0.045% (Type-B)

Deviation from free mass (Type-B)

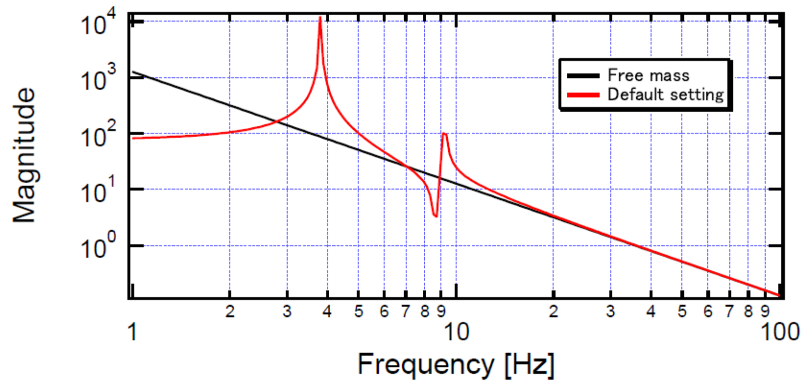
$$P=\frac{c\lambda}{4\pi H_m V_{pp}\alpha_{\rm m}\cos\phi}G_{\rm CL}\cdot T_{AH}\cdot V_f$$

Hm: response function of pendulum

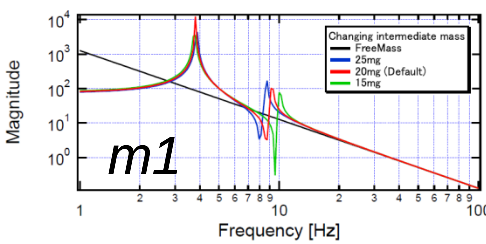


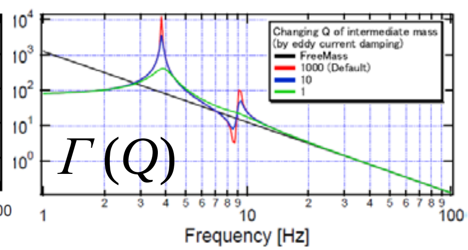
$$\begin{cases} m_1\ddot{x}_1=-\dfrac{(m_1+m_2)g}{l_1}x_1-\dfrac{m_2\,g}{l_2}(x_1-x_2)-\Gamma\dot{x}_1 \\ m_2\ddot{x}_2=-\dfrac{m_2\,g}{l_2}(x_2-x_1)+F \end{cases}$$

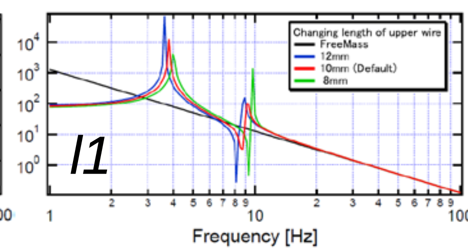
Default value
m1: 20 mg
m2: 20 mg
l1: 1 cm
l2: 1 cm

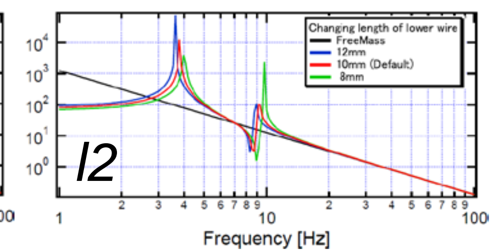


Free mass and Default is different by 0.3% at 93Hz
⇒ Default value is better to use for analysis









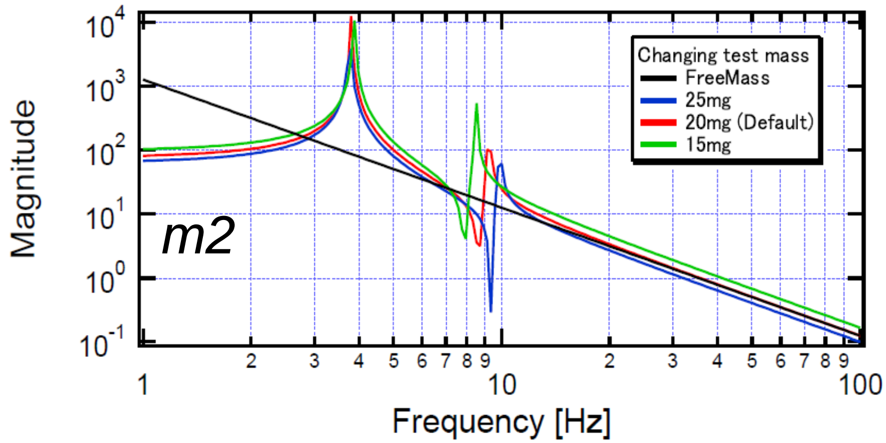
±5 mg, bellow 0.01%	Q = 1, bellow 0.01%	±2 mm, bellow 0.01%	±2 mm, 0.12 %

Possible deviation from the default value at 72Hz

Measurement of Tiny Mirror

$$P = \frac{c\lambda}{4\pi H_m V_{pp} \alpha_m \cos\phi} G_{CL} \cdot T_{AH} \cdot V_f$$

*H*m: response function of pendulum



Uncertainty of *m2* affect directly

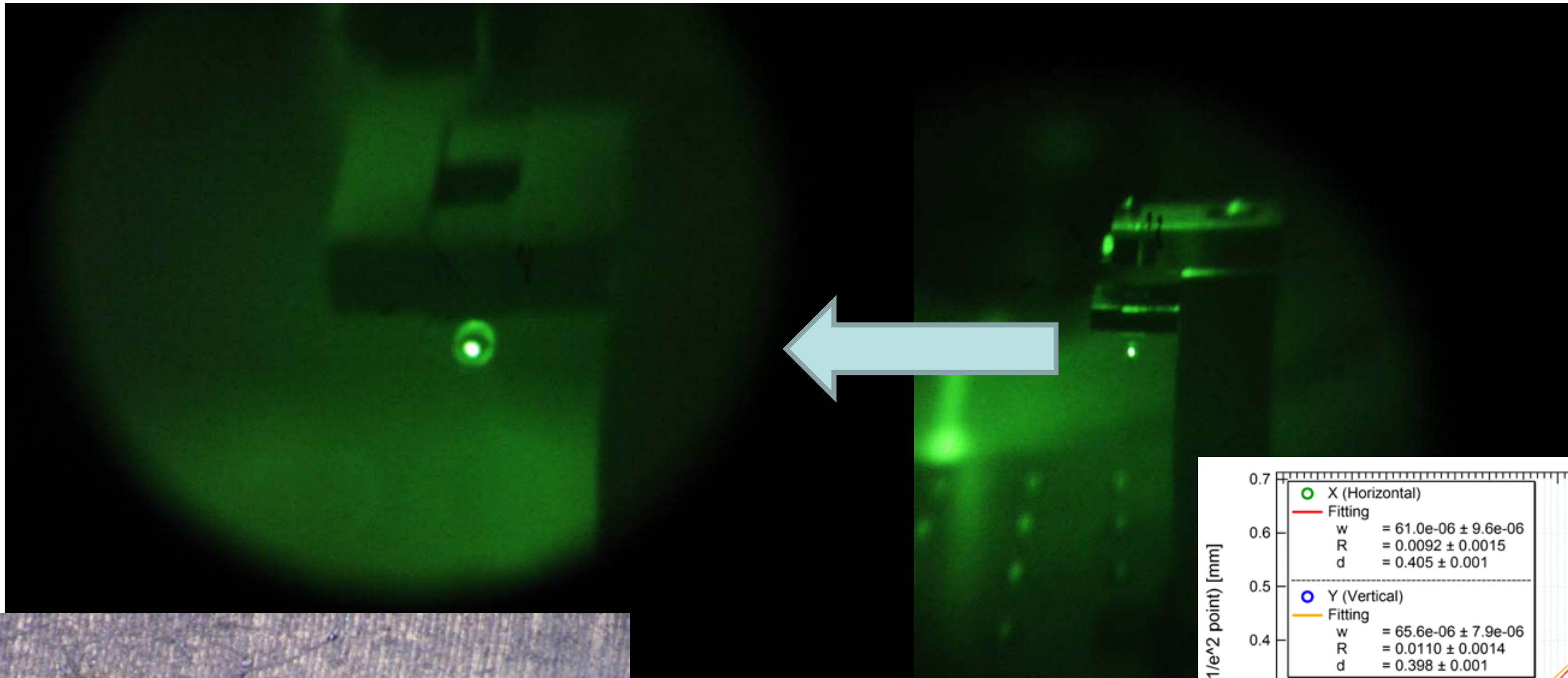
Micro Analytical Balances (A&D) can measure small-mirror mass within 0.01% ~ 0.15% uncertainty.

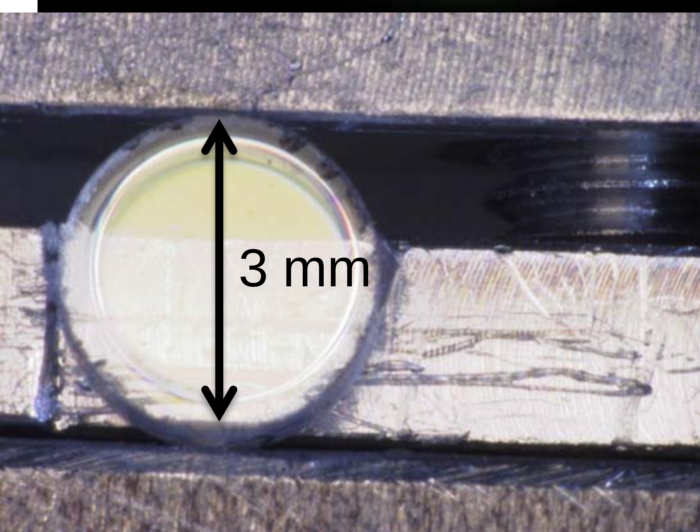
(Type-A)

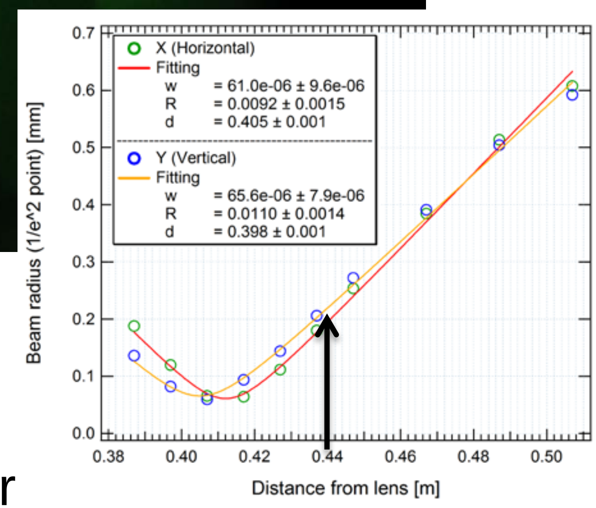
Specifications

	BM-20	BM-22	BM-252	BM-200	BM-300	BM-500
Weighing capacity	22 g	5.1 g / 22 g*	250 g	220 g	320 g	520 g
Minimum weighing value	0.001 mg	0.001 mg / 0.01 mg	0.01 mg	0.1 mg		
Repeatability (standard deviation)	0.0025 mg (for 1 g)	0.004 mg (for 1 g) / 0.01 mg	0.03 mg (for 100 g)	0.1 mg	0.2 mg	

Spot size (Type-B)







Beam spot size:
w = 0.2 mm
⇒ w ≧ 0.4 mm area
include 99.97% of power

80% of mirror surface is covered by reflection coating

Scattering and Absorption of mirror

$$P = \frac{c\lambda}{4\pi H_m V_{pp} \alpha_m \cos\phi} G_{CL} \cdot T_{AH} \cdot V_f$$

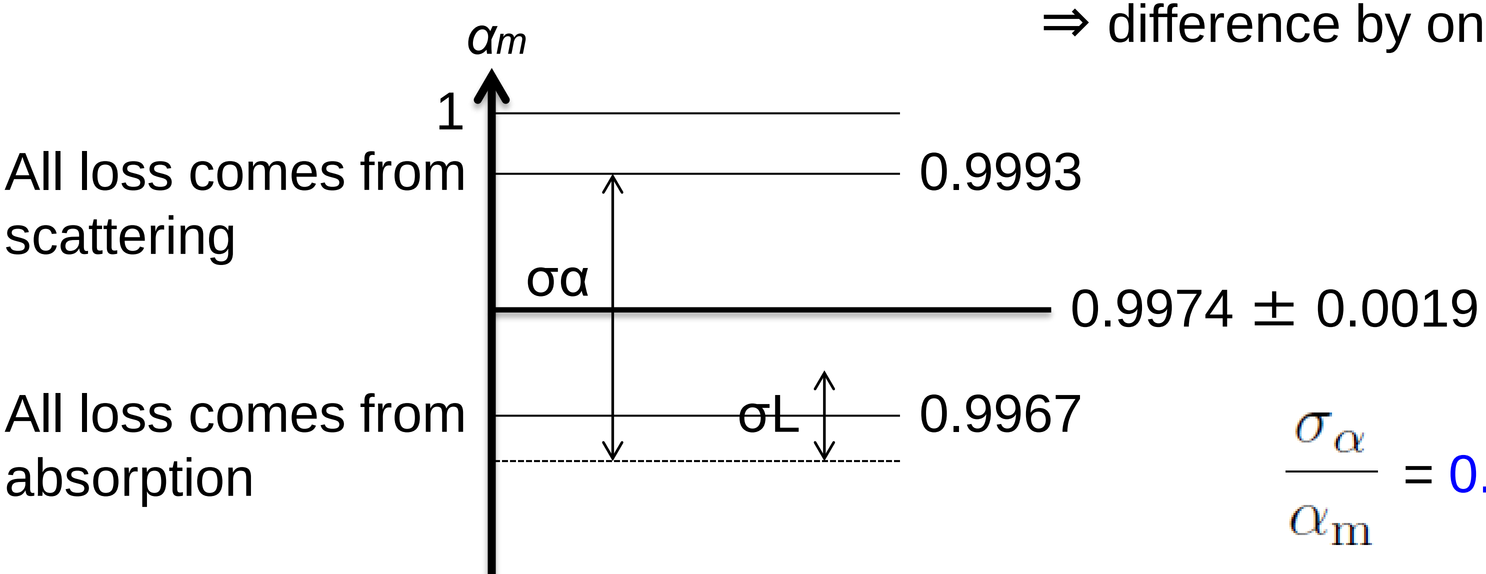
α_m: transfer efficiency of photon momentum

Incident, reflected, and transmitted laser power is measured
=> Reflectivity, Transmittance and Loss

Reflectivity : 0.9941 ± 0.0025
Transmittance : $(657 \pm 5) \times 10^{-6}$
Loss : 0.0052 ∓ 0.0025
($\sigma_L \equiv 0.0025$)

Loss
- ♦Scattering change momentum of mirror by $2\delta P$
- ♦Absorption change momentum of mirror by δP

⇒ difference by one photon



$$\frac{\sigma_\alpha}{\alpha_m} = 0.19\%$$

Summary

Accurate measurement of QE

We have demonstrated a power meter using radiation pressure

● Validity of experiment

- Calibrated laser power is consistent with that using commercial power meter within its uncertainty.
- Measured QE is consistent with the spec seat of our PDs.

● Evaluation of Uncertainty:

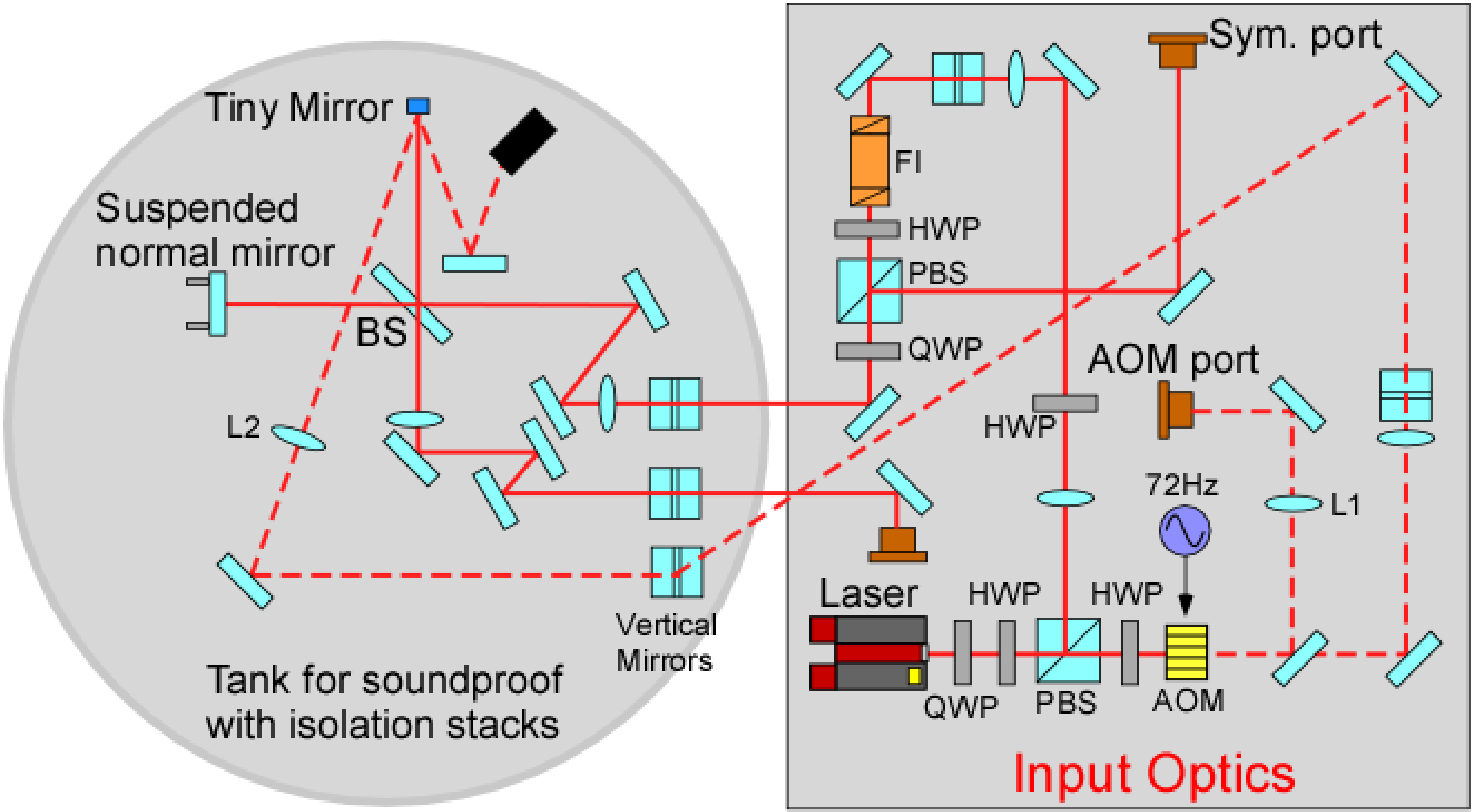
- Incident angle of small mirror： H 0.15%, V 0.01%
- Scattering and absorption of mirror: 0.19%
- CLG: 0.44%, Actuator efficiency: 0.49%, Feedback signal: 0.30%,
- Deviation from mid fringe: 0.05%, Floor noise: 0.06%
- Deviation from free mass: 0.07%, Mirror mass: 0.15%

$$\frac{\sigma_P}{P} = \sqrt{\left(\frac{\sigma_G}{G_{\mathrm{CL}}}\right)^2 + \left(\frac{\sigma_T}{T_{AH}}\right)^2 + \left(\frac{\sigma_{V_f}}{V_f}\right)^2 + \left(\frac{\sigma_H}{H_m}\right)^2 + \left(\frac{\sigma_{V_{pp}}}{V_{pp}}\right)^2 + \left(\frac{\sigma_\alpha}{\alpha_{\mathrm{m}}}\right)^2 + (\sigma_\phi \cdot \tan\phi)^2} = 0.79\ \%$$

Conclusion: We have achieved demonstration of an accurate QE measurement within 1% uncertainty.

Supplement slide

Schematic view



Stability of Laser Power

INNOLIGHT SCIENTIFIC LASERS

MEPHISTO

1064 nm, 500 mW

Coherent
PowerMax, PS19Q

Power drift
0.1 mW / 1 minute

AOM off/on