Introduction to radiation pressure noise squeezing and optomechanical interaction

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Introduction: Why Optomechanics is Amazing

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Outline

- **Optomechanics**
  - a simple model for GW detector
- **Standard Quantum Limit (SQL)**
- **Approaches for surpassing SQL**
  - Building up quantum correlation
- **Experimental evidence for quantum radiation pressure**

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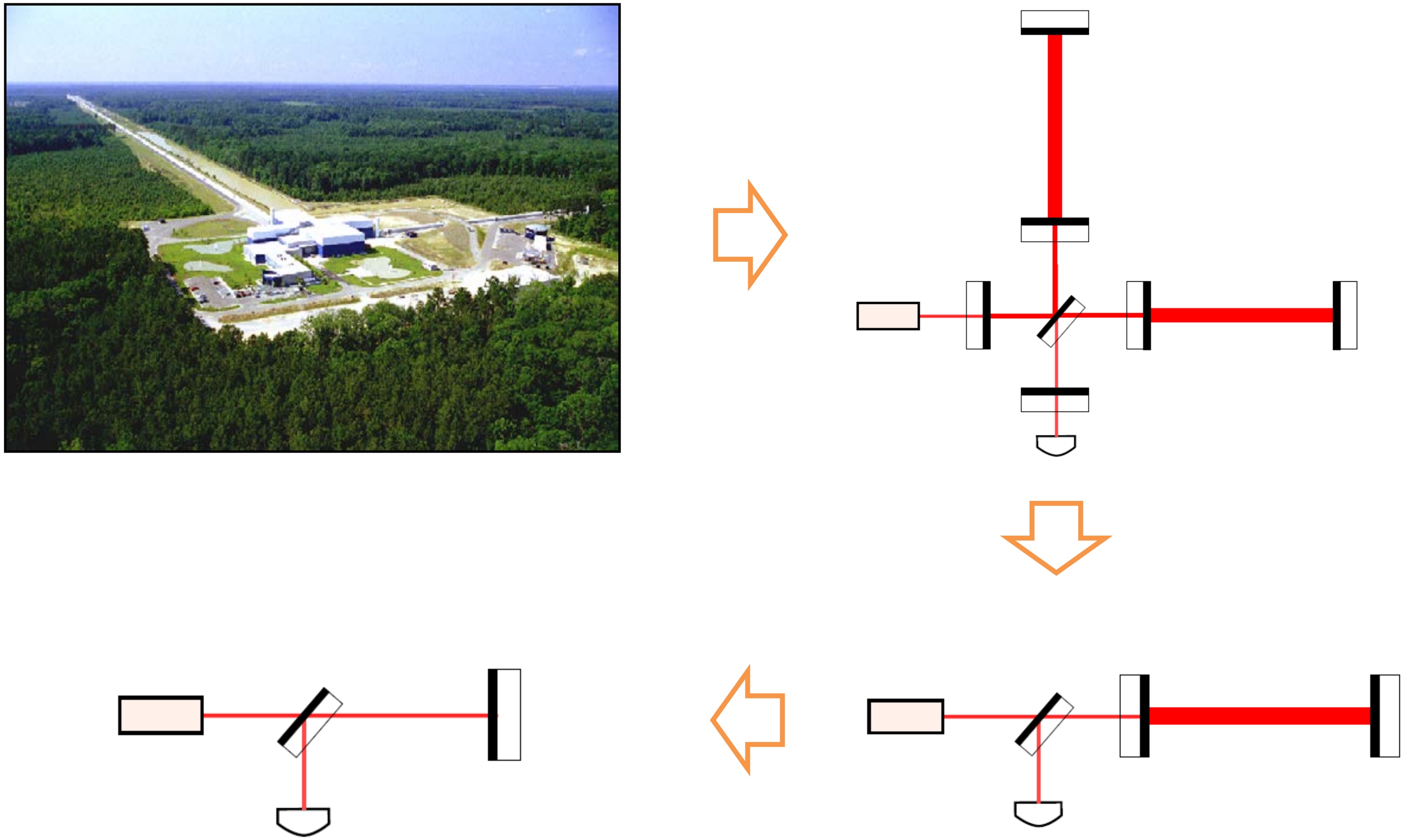
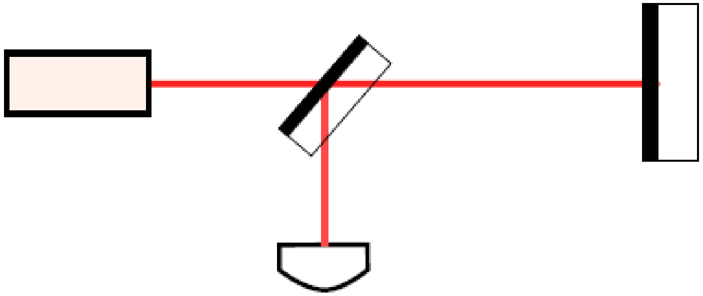
A simple^2 model for GW detector



Optomechanics



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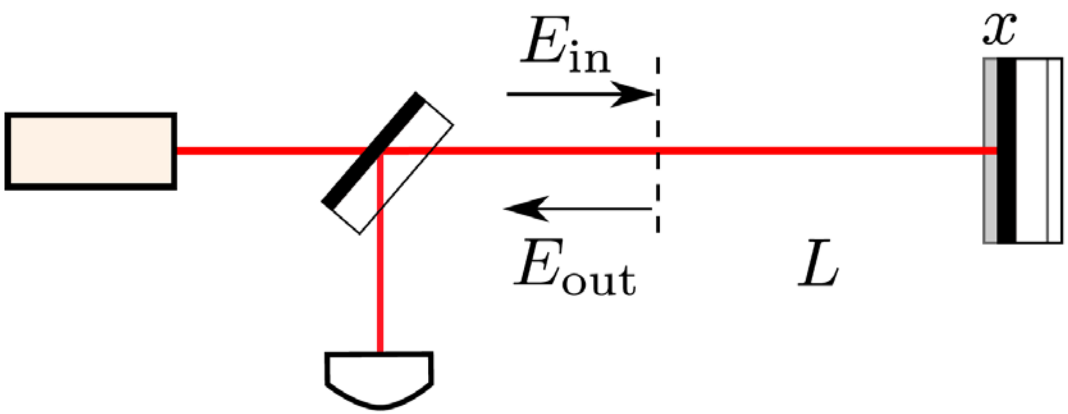
“White-board” section

Deriving SQL



$$E_{\text{out}} = E_{\text{in}}\left(t - \frac{2L}{c} - \frac{2x}{c}\right)$$

$$m\,\ddot{x}(t) = \frac{\hbar\omega_0}{c} E_{\text{in}}^2(t) + \frac{m}{2} L\ddot{h}(t)$$

Radiation pressure

GW tidal force

$$(a_1 \ll \bar{E}_0\,, \quad a_2 \ll \bar{E}_0)$$

$$E_{\rm in}(t) = [\bar{E}_0 + a_1(t)]\cos\omega_0 t + a_2(t)\sin\omega_0 t$$

$$\approx \bar{E}_0\left[1 + \frac{a_1(t)}{\bar{E}_0}\right]\cos\left[\omega_0 t - \frac{a_2(t)}{\bar{E}_0}\right]$$

Amplitude Modulation Phase Modulation

$$E_{\rm out}(t) = [\bar{E}_0' + b_1(t)]\cos\omega_0 t + b_2(t)\sin\omega_0 t$$

Linearization: $x \ll \lambda$

Input-output relation:

$$b_1(t) = a_1(t)$$

$$b_2(t) = a_2(t) + \frac{\alpha}{\hbar} x(t) \qquad (\alpha \equiv \hbar \omega_0 \bar{E}_0 / c)$$

$$m\ddot{x}(t) = \alpha\, a_1(t) + \frac{1}{2} m L \ddot{h}(t)$$

Radiation pressure GW tidal force

“One-step” quantization:

Input-output relation:

$$\hat{b}_1(t) = \hat{a}_1(t)$$

$$\hat{b}_2(t) = \hat{a}_2(t) + \frac{\alpha}{\hbar}\hat{x}(t) \qquad (\alpha \equiv \hbar\omega_0\bar{E}_0/c)$$

$$m\ddot{\hat{x}}(t) = \alpha\,\hat{a}_1(t) + \frac{1}{2}mL\ddot{h}(t)$$

Radiation pressure GW tidal force

Frequency domain:

$$\hat{b}_1(\Omega) = \hat{a}_1(\Omega)$$

$$\hat{b}_2(\Omega) = \hat{a}_2(\Omega) - \kappa\,\hat{a}_1(\Omega) + \sqrt{2\kappa}\,\frac{h(\Omega)}{h_{\rm SQL}}$$

$$\kappa = \frac{\alpha^2}{\hbar m \Omega^2} \qquad h_{\rm SQL} = \sqrt{\frac{2\hbar}{m\Omega^2 L^2}}$$

Measuring phase quadrature:

$$\hat{b}_2(\Omega) = \hat{a}_2(\Omega) - \kappa\,\hat{a}_1(\Omega) + \sqrt{2\kappa}\,\frac{h(\Omega)}{h_{\rm SQL}}$$

Shot noise — Radiation-pressure noise — GW signal

Noise spectrum (for vacuum):

$$S_{11}(\Omega) = S_{22}(\Omega) = 1, \quad S_{12}(\Omega) = 0$$

Measuring phase quadrature:

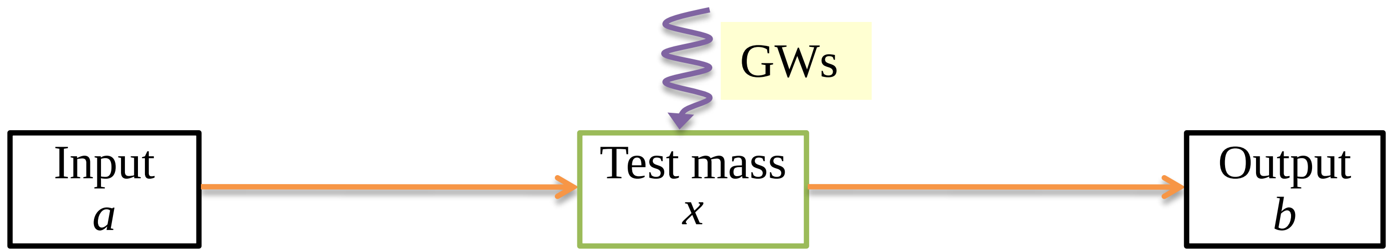
$$\hat{b}_2(\Omega) = \hat{a}_2(\Omega) - \kappa\,\hat{a}_1(\Omega) + \sqrt{2\kappa}\,\frac{h(\Omega)}{h_{\mathrm{SQL}}}$$

h-referred noise spectrum:

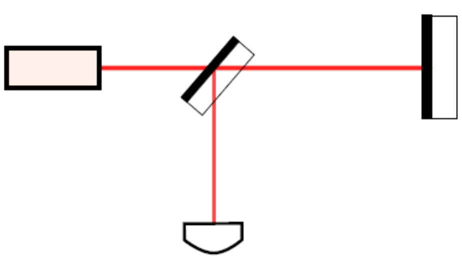
$$S_h(\Omega) = \left(\frac{1}{\kappa} + \kappa\right)\frac{h_{\mathrm{SQL}}^2}{2}$$

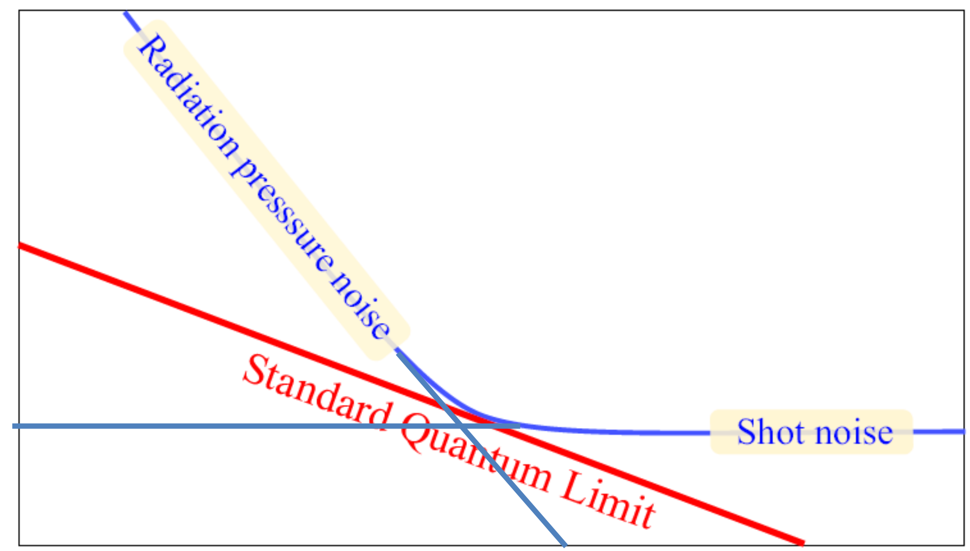
$$\geq h_{\mathrm{SQL}}^2 = \frac{2\hbar}{m\Omega^2 L^2}$$

Standard Quantum Limit (SQL)

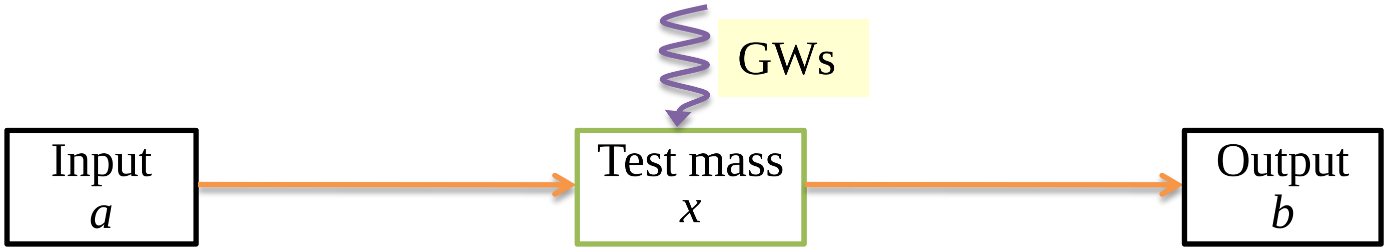


Noise spectrum: $S_h(\Omega) = \left[\frac{1}{\kappa} + \kappa\right] \frac{h_{\rm SQL}^2}{2} \geq h_{\rm SQL}^2 = \frac{2\hbar}{m\Omega^2 L^2}$

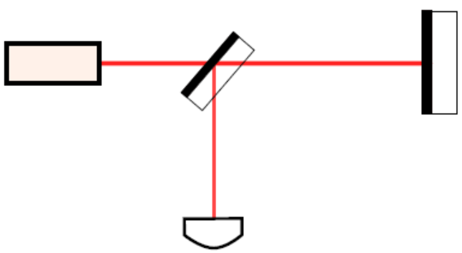


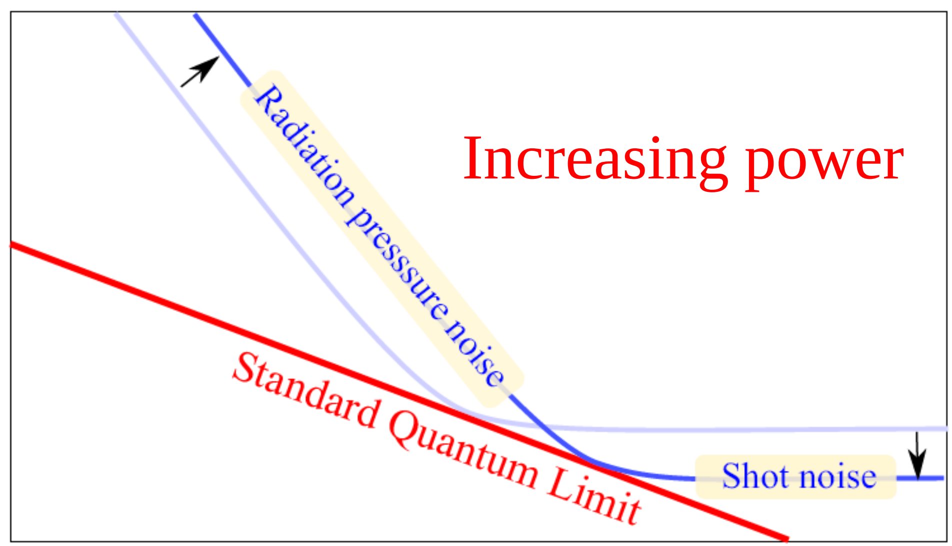


Standard Quantum Limit (SQL)

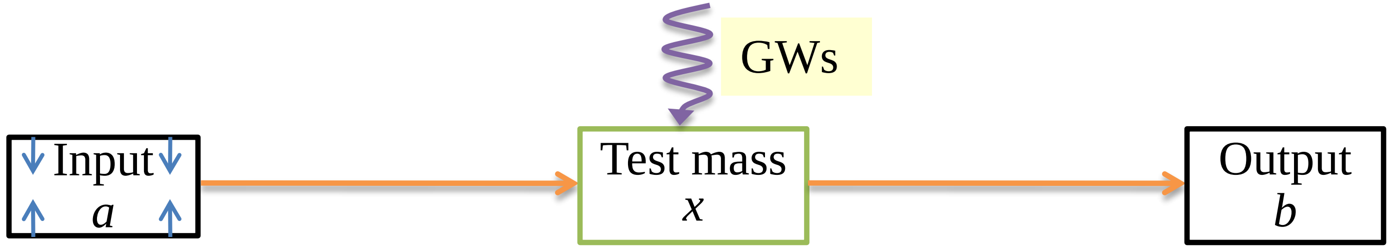


Noise spectrum: $S_h(\Omega) = \left[\frac{1}{\kappa} + \kappa\right] \frac{h_{\rm SQL}^2}{2} \geq h_{\rm SQL}^2 = \frac{2\hbar}{m\Omega^2 L^2}$

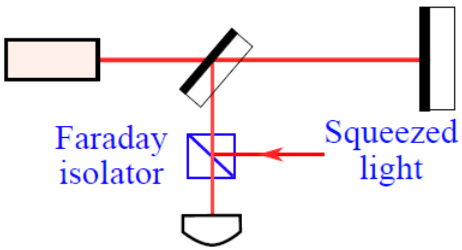


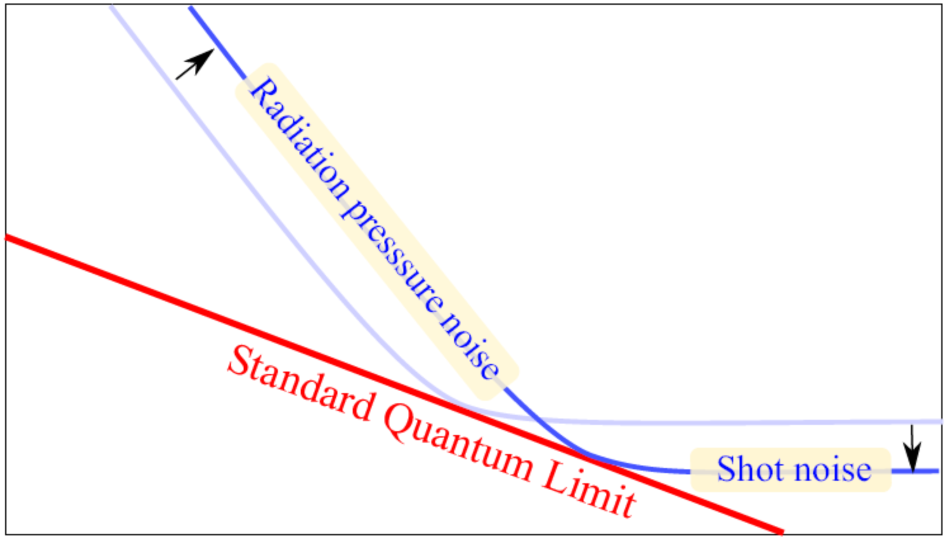


Standard Quantum Limit (SQL)



Squeezed light input:





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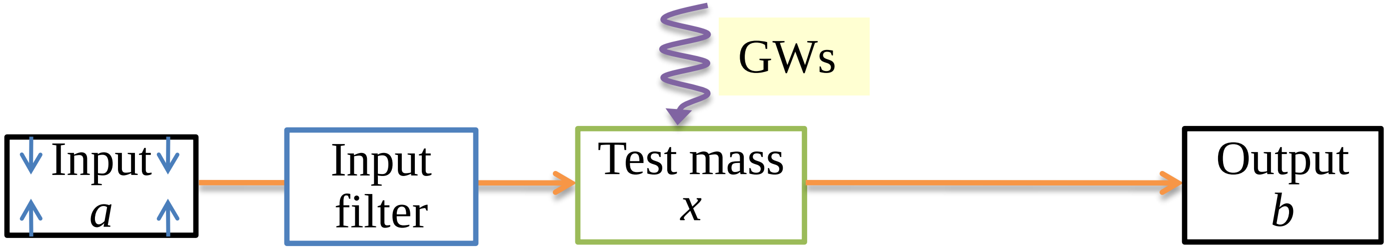
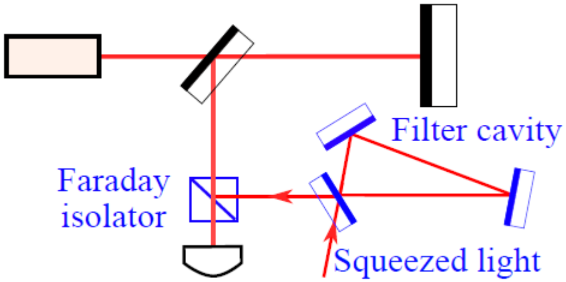
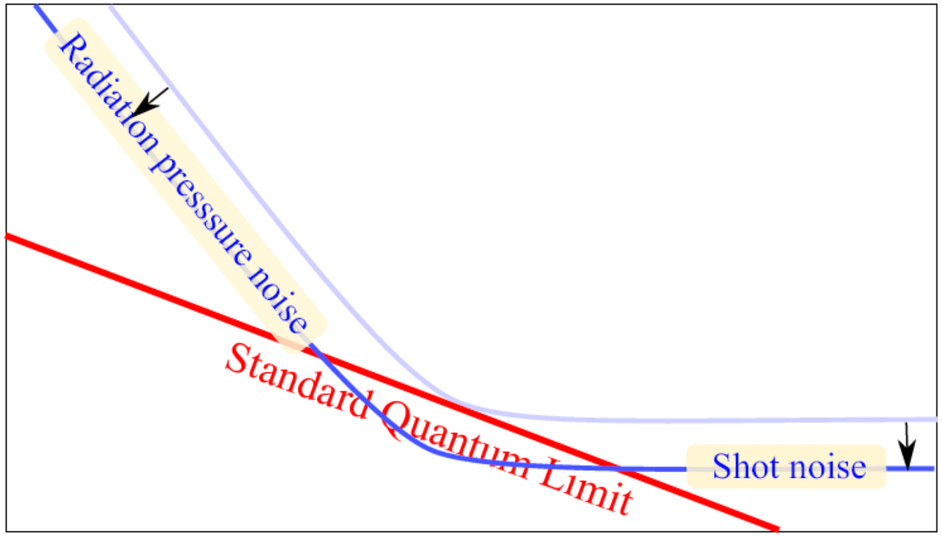
Surpassing SQL: Freq. dep. squeezing



Frequency-dependent squeezing: input filtering

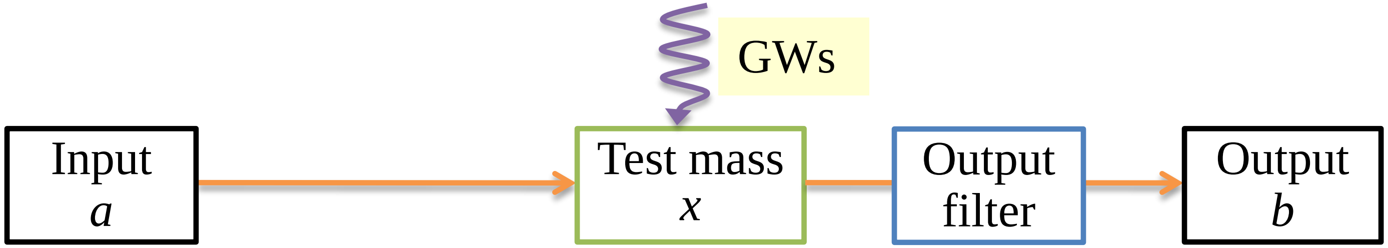




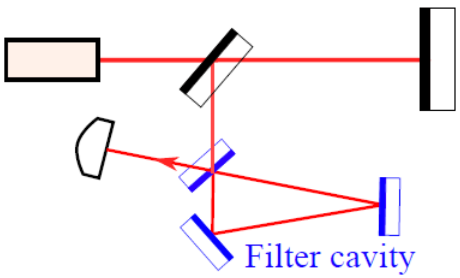
Filter cavity:
1. Passive optical cavity
2. Optomechanical cavity

Reference: H. J. Kimble *et al.*, Phys. Rev. D **65**, 022002 (2001).

Surpassing SQL: Variational readout



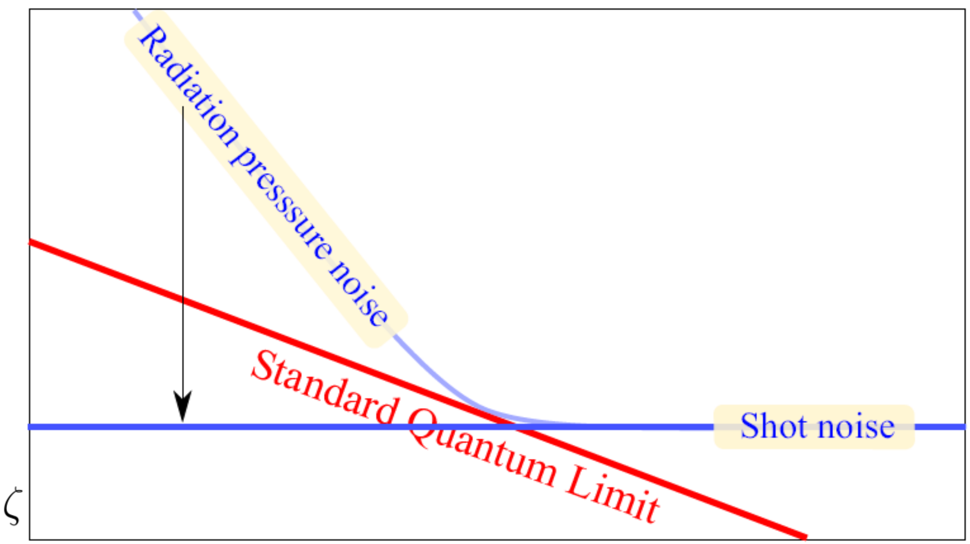
Variational readout: Output filtering



$$b_1 = a_1$$
$$b_2 = a_2 - \kappa\, a_1 + \text{signal}$$

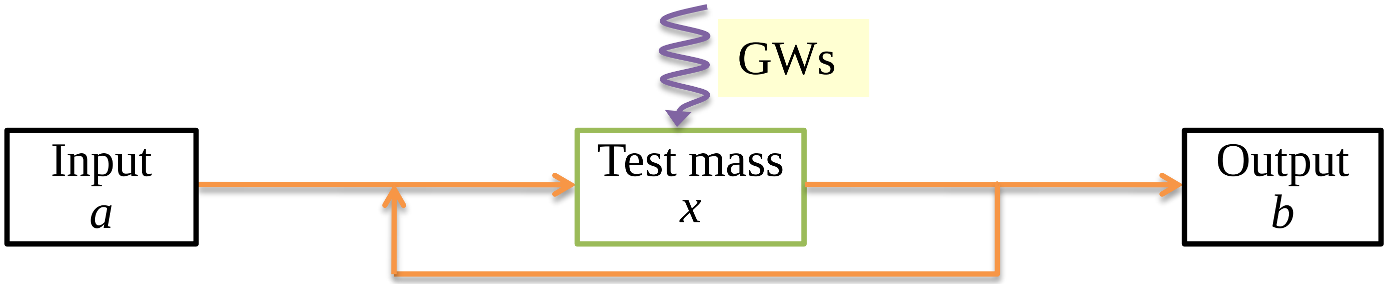
Measure: $b_\zeta = b_1 \sin\zeta + b_2 \cos\zeta$

$\tan\zeta = \kappa$ ⇨ Rad. noise cancelled

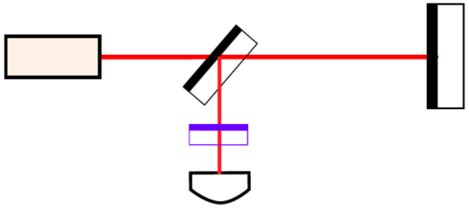


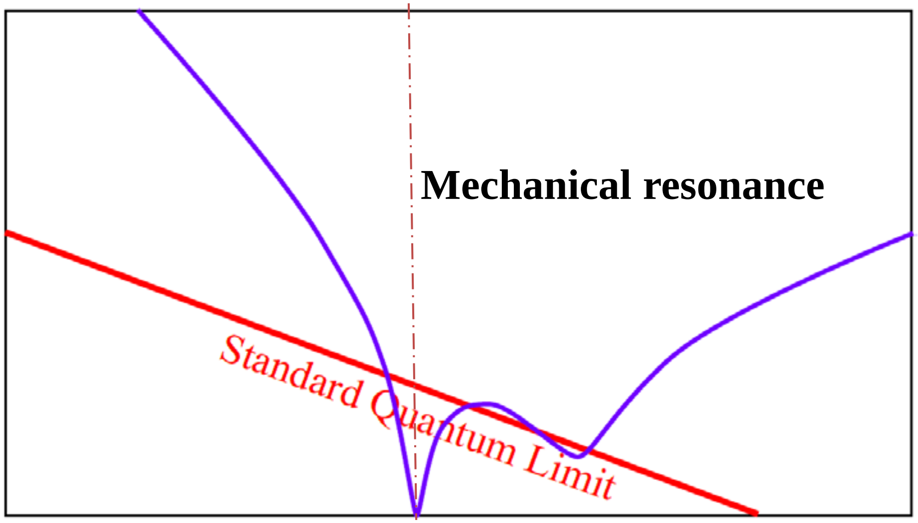
Reference: H. J. Kimble *et al.*, Phys. Rev. D **65**, 022002 (2001).

Surpassing SQL: Coherent feedback



Coherent feedback:





1. Modifying test-mass dynamics:
Optical-spring effect

2. Quantum correlation between
Shot and rad. pre. noise

Reference: A. Buonanno and Y. Chen, Phys. Rev. D **65**, 042001 (2002).

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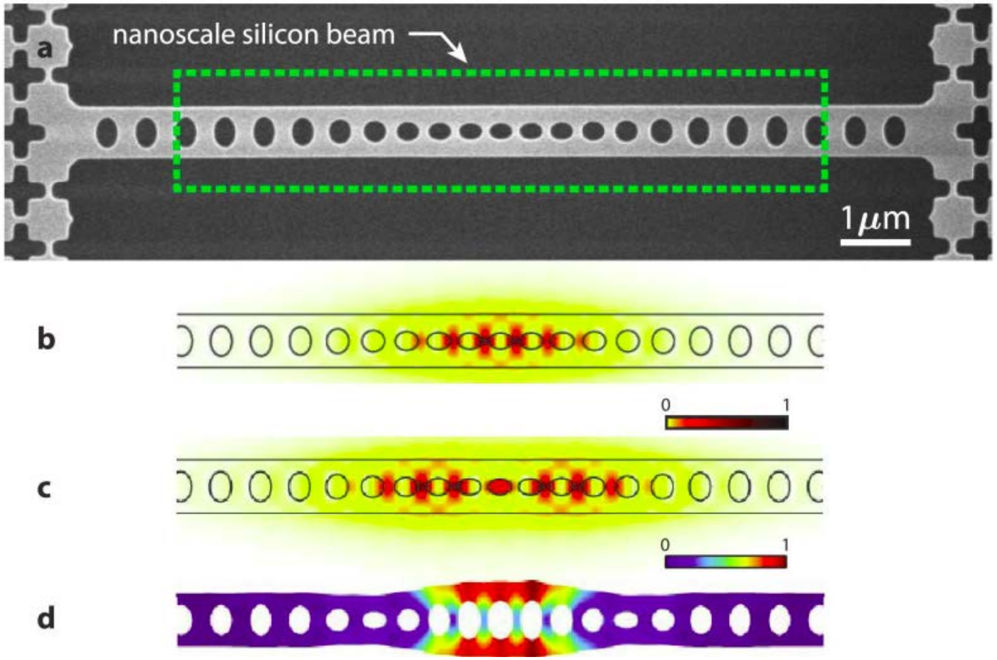
Experimental evidence finally comes

Setup:



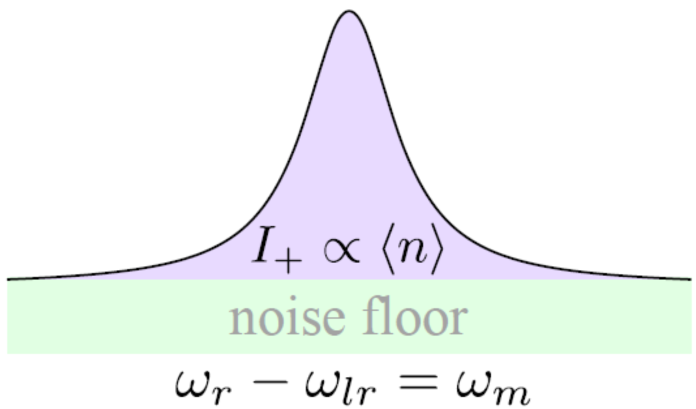
Cavity parameters

$$\omega_c/2\pi = 205.3\,\mathrm{THz}$$
$$\kappa_c/2\pi = 390\,\mathrm{MHz}$$
$$g_c/2\pi = 960\,\mathrm{KHz}$$
$$\omega_r/2\pi = 194.1\,\mathrm{THz}$$
$$\kappa_r/2\pi = 1\,\mathrm{GHz}$$
$$g_r/2\pi = 430\,\mathrm{KHz}$$
$$\omega_m/2\pi = 3.99\,\mathrm{GHz}$$

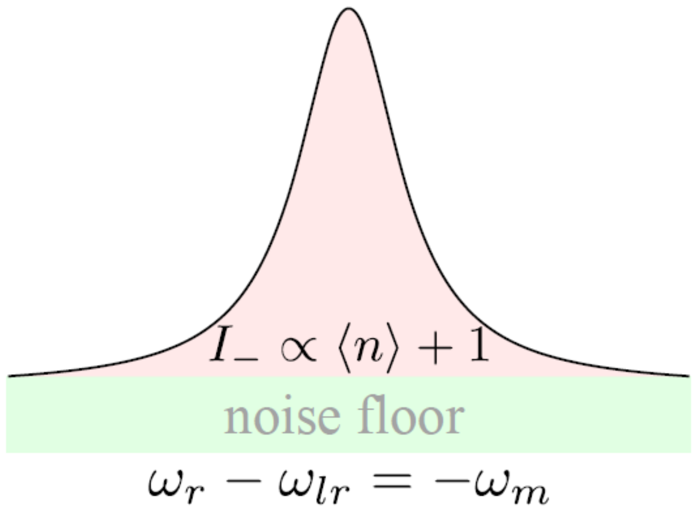
Reference: [1] A. Safavi-Naeini *et al.*, Phys. Rev. Lett. **108**, 033602 (2012).
[2] F. Khalili *et al.*, In preparation (2012)

Experimental evidence finally comes

Asymmetric Spectra:



Positive detuning



Negative detuning

Explanation:

$$\hat{y}_{\rm out} = \alpha^{-1}\hat{z} + \hat{x} = \alpha^{-1}\hat{z} + \chi[\alpha\hat{F}_{\rm rad} + F_{\rm th}]$$

Experimental evidence finally comes

Output spectrum:

$$S_{yy} = \alpha^{-2} S_{zz} + 2\Re\{\chi^* S_{zF}\} + |\chi|^2 (\alpha^2 S_{FF}^{\mathrm{rad}} + S_{FF}^{\mathrm{th}})$$

Shot noise **Correlation** **Rad. Pres. noise**

$$S_{zF} \approx -i\hbar \frac{\omega_m}{\Delta} |_{\Delta=\pm\omega_m} = \mp i\,\hbar$$

Correlation at the quantum level

Experimental evidence finally comes

Output spectrum:

$$S_{yy} = \alpha^{-2} S_{zz} + 2\Re\{\chi^* S_{zF}\} + |\chi|^2(\alpha^2 S_{FF}^{\rm rad} + S_{FF}^{\rm th})$$

$$S_{zF} \approx -i\hbar\frac{\omega_m}{\Delta}|_{\Delta=\pm\omega_m} = \mp i\,\hbar$$

Correlation at the quantum level