

# aLIGO Recycling Cavities Optical Layout

## ■ Revision History

### ■ Version -v1

2/7/2013, D. Coyne: initial release

## ■ Notes and Notation

Optica (a TM package for optical design in Mathematica) can be used to ray trace through an existing placement of optics, but it is cumbersome to use for determining the aligned positions of a number of wedged optics (as is the case with the recycling cavity). This notebook determines the positions and orientations of the 3rd recycling mirrors (PR3, SR3), BeamSplitter (BS) and Input Test Masses (ITM) given the following information/assumptions:

- 1) the X- and Y-arms are assumed to exactly 90 degrees apart. (T960176-C indicates that the deviation from 90 degrees is 1.2 microradian).
- 2) the x and y axes are aligned parallel to the Fabry-Perot arm cavity axes, with the origin defined at the intersection of the recycling cavity beams and the splitting surface of the BS (Note that this is later converted to the LIGO global coordinate system; The LIGO coordinate system has its origin at the projected intersection of the BT cavity axes.)

3) the notation is as follows:

$u_i$  = ray unit vector for the beam incident upon surface  $i$   
 $v_i$  = ray unit vector for the beam reflected from surface  $i$   
 $w_i$  = ray unit vector for the beam refracted through surface  $i$   
 $n_i$  = unit normal vector for surface  $i$

$i$	surface
1	ITMy (HR surface)
2	ITMy (AR surface)
3	CPy (surface adjacent to ITM)
4	CPy (other surface)
5	BS (50/50 surface)
6	PR3 (HR surface)
7	BS (AR surface, y-transmitted ray)
8	SR3 (HR surface)
9	ITMx (HR surface)
10	ITMx (AR surface)

- 11 CPx (surface adjacent to ITM)
- 12 CPx (other surface)
- 13 BS (AR surface, x-transmitted ray)
- 14 PR2 (HR surface)
- 15 PRM (HR surface)

ar = anti-reflectance surface  
 hr = high reflectance surface  
 bs = beamsplitting surface

d12 = ITMy thickness at center, mm  
 d23 = ITMy to CPy gap distance, mm  
 d34 = CPy thickness at center, mm  
 d45 = CPy to BS(hr) distance, mm  
 d56 = BS(hr) to PR3 distance, mm  
 dBS = BS thickness at center, mm  
 d57 = BS(hr) to BS(ar) for y-beam, mm  
 d78 = BS(ar), at y-beam intercept, to SR3, mm  
 d513 = BS(hr) to BS(ar) for x-beam, mm  
 d910 = ITMx thickness at center, mm  
 d1011 = ITMx to CPx gap distance, mm  
 d1112 = CPx thickness at center, mm  
 d1213 = CPx to BS(ar) distance, mm  
 d614 = PR3 to PR2 distance, mm  
 d1415 = PR2 to PRM distance, mm

The “given” parameters (from Zemax layout, etc.) are as follows:

- ITM (hr) center positions from Zemax
- ITM thickness
- ITM to CP gaps
- CP thickness
- BS thickness

## ■ Initialization

```
Off [General::"spell"]
Off [General::"spell1"]
```

```
Needs ["VectorAnalysis`"]
```

```
Needs ["Geometry`Rotations`"]
```

Needs["Geometry`Rotations`"]

should no longer be needed. Use built - in RotationTransform and RotationMatrix

RotationMatrix uses yaw-pitch-roll angles rather than the Euler angles used by RotationMatrix3D

HOWEVER, didn't Minimize function for determination of the BS orientation to work with RotateMatrix3D, so reverted to Rotate3D

```
RotationMatrix3D[phi_,theta_,psi_]:=RotationMatrix[Pi - psi, {0, 0, 1}].RotationMatrix[
  theta, {1, 0, 0}].RotationMatrix[Pi - phi, {0, 0, 1}]
```

```
RotationMatrix3D[phi, theta, psi]
```

```
{Cos[phi] Cos[psi] - Cos[theta] Sin[phi] Sin[psi], Cos[psi] Sin[phi] + Cos[phi] Cos[theta] Sin[psi], Sin[psi] Sin[theta]},
{-Cos[psi] Cos[theta] Sin[phi] - Cos[phi] Sin[psi], Cos[phi] Cos[psi] Cos[theta] - Sin[phi] Sin[psi], Cos[psi] Sin[theta]},
{Sin[phi] Sin[theta], -Cos[phi] Sin[theta], Cos[theta]}
```

Note: Per J. Wertz, "Spacecraft Attitude Determination and Control", D. Reidel Pub., 1985, pp.763-764, This is a Type 2 Euler Angle representation with a z-x-z rotation sequence.

Needs["Graphics`Shapes`"]

The functionality of RotateShape, TranslateShape, and AffineShape is provided by the newly added kernel functions [Rotate](#), [Translate](#), [Scale](#) and [GeometricTransformation](#)

```
pi = N[π, 10];
```

```
Clear[Reflect];
```

```
Reflect[u_, n_] := Block[{temp, i, j}, temp = CrossProduct[u, n]; i =  $\frac{\text{temp}}{\sqrt{\text{temp} \cdot \text{temp}}}$ ; j = CrossProduct[n, i]; Return[u.j j - u.n n];
```

```
Clear[Refract, ni, nt];
```

```
Refract[u_, n_, ni_, nt_] :=
```

```
Block[{temp, i, j}, temp = CrossProduct[u, n]; i =  $\frac{\text{temp}}{\sqrt{\text{temp} \cdot \text{temp}}}$ ; j = CrossProduct[n, i]; temp =  $\frac{\text{ni} \cdot \text{u} \cdot \text{j}}{\text{nt}}$ ; Return[temp j -  $\sqrt{1 - \text{temp}^2}$  n];
```

Needs["Optica`Optica`"]

SetOptions[DrawSystem, QuickTrace -> False];

```
DMS[rad_] :=
```

```
{IntegerPart[rad 180 / Pi], IntegerPart[FractionalPart[rad 180 / Pi] 60 + Sign[rad] 10^-18], Round[FractionalPart[(rad 180 / Pi) 60] 60]};
```

```
RAD[deg_, min_, sec_] := (deg + min / 60 + sec / 3600) Pi / 180;
```

## ■ Global to Local Coordinate Transformation Matrices

See Tables 10-14 and Tables 25-27 of T980044-v1(aka -10), "Determination of Local and Global Coordinate Axes for the LIGO Sites".

### ■ Rhc -> Hanford Corner station

```
xangle = 619.49 × 10-6;
yangle = 12.4832 × 10-6;
```

```
Rhc = RotationMatrix3D[0, yangle, 0].RotationMatrix3D[pi / 2, xangle, -pi / 2];
```

```
MatrixForm[Rhc]
```

$$\begin{pmatrix} 1. & 0. & -0.00061949 \\ 7.73322 \times 10^{-9} & 1. & 0.0000124832 \\ 0.00061949 & -0.0000124832 & 1. \end{pmatrix}$$

```
MatrixForm[Rhc - IdentityMatrix[3]]
```

$$\begin{pmatrix} -1.91884 \times 10^{-7} & 0. & -0.00061949 \\ 7.73322 \times 10^{-9} & -7.79151 \times 10^{-11} & 0.0000124832 \\ 0.00061949 & -0.0000124832 & -1.91962 \times 10^{-7} \end{pmatrix}$$

```
Rhcinv = RotationMatrix3D[Pi / 2, -xangle, -Pi / 2].RotationMatrix3D[0, -yangle, 0];
```

```
MatrixForm[Rhc.Rhcinv]
```

$$\begin{pmatrix} 1. & 1.65436 \times 10^{-24} & 1.0842 \times 10^{-19} \\ 1.65436 \times 10^{-24} & 1. & 0. \\ 1.0842 \times 10^{-19} & 0. & 1. \end{pmatrix}$$

```
MatrixForm[Rhcinv]
```

$$\begin{pmatrix} 1. & 7.73322 \times 10^{-9} & 0.00061949 \\ 0. & 1. & -0.0000124832 \\ -0.00061949 & 0.0000124832 & 1. \end{pmatrix}$$

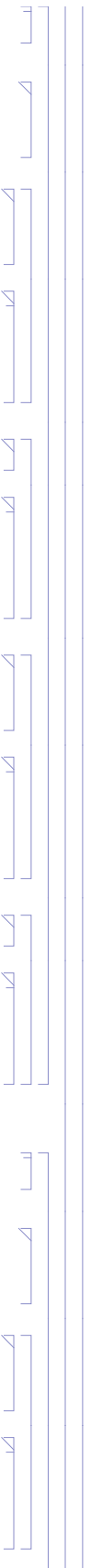
### ■ Rhxm -> Hanford x-mid station

```
xangle = 305.827 × 10-6;
yangle = 12.0075 × 10-6;
```

```
Rhxm = RotationMatrix3D[0, yangle, 0].RotationMatrix3D[Pi / 2, xangle, -Pi / 2];
```

```
MatrixForm[Rhxm]
```

$$\begin{pmatrix} 1. & 0. & -0.000305827 \\ 3.67222 \times 10^{-9} & 1. & 0.0000120075 \\ 0.000305827 & -0.0000120075 & 1. \end{pmatrix}$$



**MatrixForm[Rhxm - IdentityMatrix[3]]**

$$\begin{pmatrix} -4.67651 \times 10^{-8} & 0. & -0.000305827 \\ 3.67222 \times 10^{-9} & -7.209 \times 10^{-11} & 0.0000120075 \\ 0.000305827 & -0.0000120075 & -4.68372 \times 10^{-8} \end{pmatrix}$$

**Rhxminv = RotationMatrix3D[Pi / 2, -xangle, -Pi / 2].RotationMatrix3D[0, -yangle, 0];**

**MatrixForm[Rhxm.Rhxminv]**

$$\begin{pmatrix} 1. & 4.1359 \times 10^{-25} & 0. \\ 4.1359 \times 10^{-25} & 1. & 0. \\ 0. & 0. & 1. \end{pmatrix}$$

### ■ Rhxe -> Hanford x-end station

**xangle = -7.8389 10^-6;**

**yangle = 11.5318 x 10^-6;**

**Rhxe = RotationMatrix3D[0, yangle, 0].RotationMatrix3D[Pi / 2, xangle, -Pi / 2];**

**MatrixForm[Rhxe]**

$$\begin{pmatrix} 1. & 0. & 7.8389 \times 10^{-6} \\ -9.03966 \times 10^{-11} & 1. & 0.0000115318 \\ -7.8389 \times 10^{-6} & -0.0000115318 & 1. \end{pmatrix}$$

**MatrixForm[Rhxe - IdentityMatrix[3]]**

$$\begin{pmatrix} -3.07242 \times 10^{-11} & 0. & 7.8389 \times 10^{-6} \\ -9.03966 \times 10^{-11} & -6.64913 \times 10^{-11} & 0.0000115318 \\ -7.8389 \times 10^{-6} & -0.0000115318 & -9.72155 \times 10^{-11} \end{pmatrix}$$

**Rhxeinv = RotationMatrix3D[Pi / 2, -xangle, -Pi / 2].RotationMatrix3D[0, -yangle, 0];**

**MatrixForm[Rhxe.Rhxeinv]**

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & 1. & 1.69407 \times 10^{-21} \\ 0. & 1.69407 \times 10^{-21} & 1. \end{pmatrix}$$

### ■ Rhym -> Hanford y-mid station

**xangle = 619.97 x 10^-6;**

**yangle = 325.84 x 10^-6;**



```
Rhym = RotationMatrix3D[0, yangle, 0].RotationMatrix3D[Pi / 2, xangle, -Pi / 2];
```

```
MatrixForm[Rhym]
```

$$\begin{pmatrix} 1. & 0. & -0.00061997 \\ 2.02011 \times 10^{-7} & 1. & 0.00032584 \\ 0.00061997 & -0.00032584 & 1. \end{pmatrix}$$

```
MatrixForm[Rhym - IdentityMatrix[3]]
```

$$\begin{pmatrix} -1.92181 \times 10^{-7} & 0. & -0.00061997 \\ 2.02011 \times 10^{-7} & -5.30859 \times 10^{-8} & 0.00032584 \\ 0.00061997 & -0.00032584 & -2.45267 \times 10^{-7} \end{pmatrix}$$

```
Rhyminv = RotationMatrix3D[Pi / 2, -xangle, -Pi / 2].RotationMatrix3D[0, -yangle, 0];
```

```
MatrixForm[Rhym.Rhyminv]
```

$$\begin{pmatrix} 1. & 2.64698 \times 10^{-23} & 0. \\ 2.64698 \times 10^{-23} & 1. & 0. \\ 0. & 0. & 1. \end{pmatrix}$$

#### ■ Rhye -> Hanford y-end station

```
xangle = 620.45 × 10-6;
```

```
yangle = 639.20 × 10-6;
```

```
Rhye = RotationMatrix3D[0, yangle, 0].RotationMatrix3D[Pi / 2, xangle, -Pi / 2];
```

```
MatrixForm[Rhye]
```

$$\begin{pmatrix} 1. & 0. & 7.8389 \times 10^{-6} \\ -9.03966 \times 10^{-11} & 1. & 0.0000115318 \\ -7.8389 \times 10^{-6} & -0.0000115318 & 1. \end{pmatrix}$$

```
MatrixForm[Rhye - IdentityMatrix[3]]
```

$$\begin{pmatrix} -1.92479 \times 10^{-7} & 0. & -0.00062045 \\ 3.96592 \times 10^{-7} & -2.04288 \times 10^{-7} & 0.0006392 \\ 0.00062045 & -0.0006392 & -3.96767 \times 10^{-7} \end{pmatrix}$$

```
Rhyeinv = RotationMatrix3D[Pi / 2, -xangle, -Pi / 2].RotationMatrix3D[0, -yangle, 0];
```

```
MatrixForm[Rhye.Rhyeinv]
```

$$\begin{pmatrix} 1. & -5.29396 \times 10^{-23} & 0. \\ -5.29396 \times 10^{-23} & 1. & -1.0842 \times 10^{-19} \\ 0. & -1.0842 \times 10^{-19} & 1. \end{pmatrix}$$

```
(180 / Pi) 619 × 10-6 // N
```

```
0.0354661
```

### ■ Rlc -> Livingston Corner station

```
xangle = 312.0 × 10-6;
```

```
yangle = -611.0 10-6;
```

```
Rlc = RotationMatrix3D[0, yangle, 0].RotationMatrix3D[Pi / 2, xangle, -Pi / 2];
```

```
MatrixForm[Rlc]
```

$$\begin{pmatrix} 1. & 0. & -0.000312 \\ -1.90632 \times 10^{-7} & 1. & -0.000611 \\ 0.000312 & 0.000611 & 1. \end{pmatrix}$$

```
MatrixForm[Rlc - IdentityMatrix[3]]
```

$$\begin{pmatrix} -4.8672 \times 10^{-8} & 0. & -0.000312 \\ -1.90632 \times 10^{-7} & -1.8666 \times 10^{-7} & -0.000611 \\ 0.000312 & 0.000611 & -2.35332 \times 10^{-7} \end{pmatrix}$$

```
Rlcinv = RotationMatrix3D[Pi / 2, -xangle, -Pi / 2].RotationMatrix3D[0, -yangle, 0];
```

```
MatrixForm[Rlc.Rlcinv]
```

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & 1. & 1.0842 \times 10^{-19} \\ 0. & 1.0842 \times 10^{-19} & 1. \end{pmatrix}$$

### ■ Rlxe -> Livingston x-end station

```
xangle = -315.0 10-6;
```

```
yangle = -610.0 10-6;
```

```
Rlxe = RotationMatrix3D[0, yangle, 0].RotationMatrix3D[Pi / 2, xangle, -Pi / 2];
```

```
MatrixForm[Rlxe]
```

$$\begin{pmatrix} 1. & 0. & 0.000315 \\ 1.9215 \times 10^{-7} & 1. & -0.00061 \\ -0.000315 & 0.00061 & 1. \end{pmatrix}$$



**MatrixForm[Rlxe - IdentityMatrix[3]]**

$$\begin{pmatrix} -4.96125 \times 10^{-8} & 0. & 0.000315 \\ 1.9215 \times 10^{-7} & -1.8605 \times 10^{-7} & -0.00061 \\ -0.000315 & 0.00061 & -2.35662 \times 10^{-7} \end{pmatrix}$$

**Rlxeinv = RotationMatrix3D[Pi / 2, -xangle, -Pi / 2].RotationMatrix3D[0, -yangle, 0];**

**MatrixForm[Rlxe.Rlxeinv]**

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & 1. & 0. \\ 0. & 0. & 1. \end{pmatrix}$$

**MatrixForm[Rlxeinv - Transpose[Rlxe]]**

$$\begin{pmatrix} 0. & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0. \end{pmatrix}$$

#### ■ Rlye -> Livingston y-end station

**xangle = 311.0 × 10<sup>-6</sup>;**

**yangle = 18.8 × 10<sup>-6</sup>;**

**Rlye = RotationMatrix3D[0, yangle, 0].RotationMatrix3D[Pi / 2, xangle, -Pi / 2];**

**MatrixForm[Rlye]**

$$\begin{pmatrix} 1. & 0. & -0.000311 \\ 5.8468 \times 10^{-9} & 1. & 0.0000188 \\ 0.000311 & -0.0000188 & 1. \end{pmatrix}$$

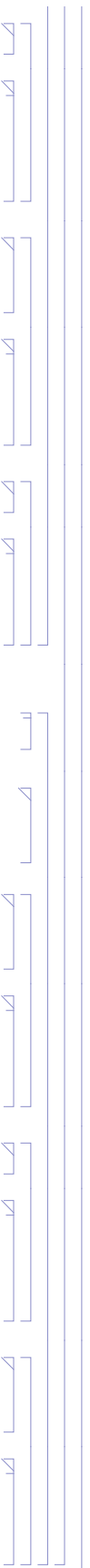
**MatrixForm[Rlye - IdentityMatrix[3]]**

$$\begin{pmatrix} -4.83605 \times 10^{-8} & 0. & -0.000311 \\ 5.8468 \times 10^{-9} & -1.7672 \times 10^{-10} & 0.0000188 \\ 0.000311 & -0.0000188 & -4.85372 \times 10^{-8} \end{pmatrix}$$

**Rlyeinv = RotationMatrix3D[Pi / 2, -xangle, -Pi / 2].RotationMatrix3D[0, -yangle, 0];**

**MatrixForm[Rlye.Rlyeinv]**

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & 1. & 0. \\ 0. & 0. & 1. \end{pmatrix}$$





# Vector Analysis for Wedge Position & Orientation Determination

## ■ Parameters

```
wl = 1.064;  
nOptic = ModelRefractiveIndex[FusedSilica][WaveLength -> wl]  
nAir = ModelRefractiveIndex[Air][WaveLength -> wl]  
nVacuum = ModelRefractiveIndex[Vacuum][WaveLength -> wl]
```

```
nOptic = 1.44963;  
nAir = 1.0;  
nVacuum = 1.0;
```

Note : D0901920-v12 &-v13 pdf file reports nOptic = 1.44963. In fact Zemax uses Suprasil-Ext index for 20C, zero pressure at 1064 nm which is 1.4500310  
According to

- I. H. Malitson. Interspecimen Comparison of the Refractive Index of Fused Silica, J. Opt. Soc. Am. 55, 1205-1208 (1965) doi:10.1364/JOSA.55.001205  
- Handbook of Optics, 3rd edition, Vol. 4. McGraw-Hill 2009

as calculated at this URL:

[http://refractiveindex.info/wiki/Citing\\_RefractiveIndex.INFO](http://refractiveindex.info/wiki/Citing_RefractiveIndex.INFO)  
the index at 1064 nm is 1.44963

## ■ H1

## ■ Notes

1) The ITM optic assignments of specific serial numbered optics is given in T1200324-v2. The assignments of the CP and BS optics, and the parameters of each COC optic is given by serial number at <https://nebula.ligo.caltech.edu/optics/>, with the exception of CPy. GariLynn states that CP09 must be replaced (currently part of H2-ITMy destined to become H1-ITMy) and has chosen CP02 for its replacement.

```
ITMx is ITM09  
ITMy is ITM06  
CPx is CP01  
CPy is CP02  
BS is BS06
```

2) The magnitude and orientation of the wedge angles are as follows::

```
ITMx = 0.077 deg, vertical, thick side down  
ITMy = 0.078 deg, vertical, thick side down  
CPx = 0.069 deg, horizontal, thick side -Y  
CPy = 0.069 deg, horizontal, thick side -X
```

BS = 0.076 deg, horizontal, thick side +X/+Y

Notes:

a) The wedge angle values given in <https://nebula.ligo.caltech.edu/optics/> are the "final" values reported by the polishing contractor. However there is some round-off/approx. in this data. GariLynn Billingsley suggests using the values reported in C1107164-v1.

b) The wedge angle magnitudes (but not orientations/signs) reported in the Zemax optical layout, D0901920-v13 differ, as follows:

ITM<sub>x</sub> = ITM10? = 0.077 deg (same value but serial number designation is wrong)

ITM<sub>y</sub> = ITM11? = 0.076 deg (value and serial number incorrect)

CP<sub>x</sub> = CP01 = 0.070 deg (serial number correct, used "final" wedge angle instead of C1107164-v1 value)

CP<sub>y</sub> = CP02 = 0.069 deg (serial number is incorrect)

BS = BS06 = 0.076 deg (same)

ITM<sub>x</sub> = vertical, thick side down

ITM<sub>y</sub> = vertical, thick side down

CP<sub>x</sub> = horizontal, thick side - Y

CP<sub>y</sub> = horizontal, thick side - X

BS = horizontal, thick side + X/+Y

## ■ H1 Unique Parameters

```
wedgeITMx = 0.077*pi/180;
wedgeITMy = 0.078*pi/180;
wedgeCPx = 0.069*pi/180;
wedgeCPy = 0.069*pi/180;
wedgeBS = 0.076*pi/180;
```

```
ITMxThick = 200.22;
```

```
ITMxDiameter = 340.13;
```

```
ITMyThick = 199.64;
```

```
ITMyDiameter = 340.06;
```

```
CPxThick = 99.82;
```

```
CPxDiameter = 340.13;
```

```
CPyThick = 99.91;
```

```
CPyDiameter = 340.22;
```

```
BSThick = 60.41;
```

```
BSDiameter = 369.85;
```

```

p1 = {-200, 4983.1, -80};
p9 = {5013, -200, -80};
d12 = ITMyThick - (ITMyDiameter / 2) Tan[wedgeITMy];
d910 = ITMxThick - (ITMxDiameter / 3) Tan[wedgeITMx];
d23 = 20;
d1011 = 20;
d34 = CPyThick - (CPyDiameter / 2) Tan[wedgeCPy];
d1112 = CPxThick - (CPxDiameter / 2) Tan[wedgeCPx];
dBS = BSThick - (BSDiameter / 2) Tan[wedgeBS];

```

Note that the distances d513 and d57 are approximate -- they do not take into account the wedge angle of the BS

### ■ optic surface orientation determination

#### ■ normal vectors for ITMs and CPs

```

n1 = {0, 1, 0};
n2=RotationMatrix3D[0,wedgeITMy,0].n1
{0., 0.999999, -0.00136136}

n3 = n2;
n4 = RotationMatrix3D[0, 0, -wedgeCPy].n3
{-0.00120428, 0.999998, -0.00136136}

n9 = {1, 0, 0};
n10 = RotationMatrix3D[pi/2, -wedgeITMx, -(pi/2)].n9
{0.999999, 0., -0.0013439}

n11 = n10;
n12 = RotationMatrix3D[0, 0, wedgeCPx].n11
{0.999998, -0.00120428, -0.0013439}

```



Note also that I have assumed a zero pitch angle. This is approximately correct because the BS has no vertical wedge component. However the reflected and refracted rays have a pitch angle difference of ~8 microrad (very small).

```

Clear [yaw];
soln =
  Maximize[Reflect[u5y, Rotate3D[{0, 1, 0}, -yaw, 0, 0]].Refract[Refract[u13, Rotate3D[-Rotate3D[{0, 1, 0}, -yaw, 0, 0], wedgeBS, 0, 0],
    nVacuum, nOptic], -Rotate3D[{0, 1, 0}, -yaw, 0, 0], nOptic, nVacuum], yaw]
{1., {yaw -> 0.784873}}

BSyaw = -yaw /. soln[[2]];
BSyawDeg = BSwaw 180 / pi
-44.9699

```

Compare to Zemax BS yaw angle (45.02885 deg) -- only 22 microradian difference:

```

(90 - 45.02885 + BSwawDeg) pi / 180
0.0000211772

n5y = RotationMatrix3D[BSyaw, 0, 0].{0, 1, 0}
n5x = -n5y;
{-0.706736, 0.707478, 0.}

v5 = Reflect[u5y, n5y]
{-1., 0.000507929, -0.000612108}

w5y = Refract[u5y, n5y, nVacuum, nOptic]
{0.271849, -0.96234, -0.000422251}

n13 = RotationMatrix3D[wedgeBS, 0, 0].n5x;
n7 = -n13
{-0.705797, 0.708415, 0.}

u7 = w5y;
w7 = Refract[u7, n7, nOptic, nVacuum]
{0.00050405, -1., -0.000612108}

w13 = Refract[u13, n13, nVacuum, nOptic];
u5x = w13;
w5x = Refract[u5x, n5x, nOptic, nVacuum]
{-1., 0.000507926, -0.000604261}

```

## ■ BS position determination

```
d513 = dBS / (-w13.n5x)
```

```
68.9204
```

```
d57 = dBS / (w5y.n5x)
```

```
68.9204
```

```
p2 = p1 + d12 w1
```

```
{-200., 4783.69, -80.}
```

```
p3 = p2 + d23 w2
```

```
{-200., 4763.69, -80.0122}
```

```
p4 = p3 + d34 w3
```

```
{-200., 4663.99, -80.0122}
```

```
p10 = p9 + d910 w9
```

```
{4812.93, -200., -80.}
```

```
p11 = p10 + d1011 w10
```

```
{4792.93, -200., -80.0121}
```

```
p12 = p11 + d1112 w11
```

```
{4693.32, -200., -80.0121}
```

```
Clear[dBSy, dBSx]
```

```
soln = Minimize[EuclideanDistance[p4 + dBSy w4, p12 + dBSx w12 + d513 w13], {dBSy, dBSx}]
```

```
{0.0204444, {dBSy → 4847.8, dBSx → 4829.64}}
```

```
d45 = dBSy /. soln[[2]];
```

```
d1213 = dBSx /. soln[[2]];
```

```
p5 = p4 + d45 w4
```

```
p13 = p12 + d1213 w12
```

```
p13 + d513 w13
```

```
{-202.625, -183.81, -82.9796}
```

```
{-136.32, -202.615, -82.9304}
```

```
{-202.625, -183.81, -82.9592}
```

Compare to Zemax (H1 : D0901920 - v12) global coordinates for p5 (BS (hr)) =  $\{-202.6, -183.9, -82.9\}$ , so Zemax is correct to within  $\sim 0.1$  mm

$$\mathbf{SchnuppAssy} = (\mathbf{d910} \mathbf{nOptic} + \mathbf{d1011} + \mathbf{d1112} \mathbf{nOptic} + \mathbf{d1213} + \mathbf{d513} \mathbf{nOptic}) - (\mathbf{d12} \mathbf{nOptic} + \mathbf{d23} + \mathbf{d34} \mathbf{nOptic} + \mathbf{d45})$$

82.5752

Compare to Zemax (H1 : D0901920 - v12) Schnupp Assymetry of 80 mm, so 2.6 mm too large ... but this is OK.

$$\mathbf{p7} = \mathbf{p5} + \mathbf{d57} \mathbf{u7}$$

$\{-183.889, -250.134, -83.0087\}$

### ■ PR3 position determination

Take as a given the global x - coordinate of the PR3 from Zemax & IO layout.

pPR3 from Zemax should equal p6

$$\mathbf{pPR3} = \{-19\,741, -174.4, -94.6\};$$

$$\mathbf{xPR3} = \mathbf{pPR3}[[1]];$$

$$\mathbf{d56} = (\mathbf{xPR3} - \mathbf{p5}[[1]]) / \mathbf{v5}[[1]]$$

19 538.4

$$\mathbf{p6} = \mathbf{p5} + \mathbf{d56} \mathbf{v5}$$

$\{-19\,741., -173.885, -94.9392\}$

$$\mathbf{p6} - \mathbf{pPR3}$$

$\{0., 0.514529, -0.339221\}$

So Zemax position for pPR3 is in error by  $\sim 0.5$  mm ... which is OK

### ■ SR3 position determination

Take as a given the global y - coordinate of the SR3 from Zemax & IO layout.

pSR3 from Zemax should equal p8

$$\mathbf{pSR3} = \{-174.6, -19\,615.9, -94.5\};$$

$$\mathbf{ySR3} = \mathbf{pSR3}[[2]];$$

$$\mathbf{d78} = (\mathbf{ySR3} - \mathbf{p7}[[2]]) / \mathbf{w7}[[2]]$$

19 365.8

$$\mathbf{p8} = \mathbf{p7} + \mathbf{d78} \mathbf{w7}$$

$$\{-174.128, -19615.9, -94.8627\}$$

$$\mathbf{p8} - \mathbf{pSR3}$$

$$\{0.472292, 0., -0.362667\}$$

So Zemax position for pSR3 is in error by ~0.5 mm ... which is OK

$$\mathbf{n15} = -\mathbf{v14};$$

## ■ L1

### ■ Notes

1) The ITM optic assignments of specific serial numbered optics is given in T1200324-v2. The assignments of the CP and BS optics, and the parameters of each COC optic is given by serial number at <https://nebula.ligo.caltech.edu/optics/>, with the exception of CPy. GariLynn states that CP09 must be replaced (currently part of H2-ITMy destined to become H1-ITMy) and has chosen CP02 for its replacement.

ITMx is ITM04

ITMy is ITM08

CPx is CP06

CPy is CP08

BS is BS02

2) The magnitude and orientation of the wedge angles are as follows::

ITMx = 0.0725 deg, vertical, thick side down

ITMy = 0.074 deg, vertical, thick side down

CPx = 0.073 deg, horizontal, thick side -Y

CPy = 0.066 deg, horizontal, thick side -X

BS = 0.070 deg, horizontal, thick side +X/+Y

Notes:

a) The wedge angle values given in <https://nebula.ligo.caltech.edu/optics/> are the "final" values reported by the polishing contractor. However there is some round-off/approx. in this data. GariLynn Billingsley suggests using the values reported in C1107164-v1.

b) The wedge angle magnitudes (but not orientations/signs) reported in the Zemax optical layout, D0902216-v8 match the serial numbers and wedge angle values given above.

ITMx = vertical, thick side down

ITMy = vertical, thick side down

CPx = horizontal, thick side - Y

CPy = horizontal, thick side - X

BS = horizontal, thick side + X/+Y



## ■ L1 Unique Parameters

optic thicknesses are the as-built values reported in C1107164-v1 for the assigned optics

```
wedgeITMx = 0.0725*pi/180;
wedgeITMy = 0.074*pi/180;
wedgeCPx = 0.073*pi/180;
wedgeCPy = 0.066*pi/180;
wedgeBS = 0.070*pi/180;

ITMxThick = 200.27;
ITMxDiameter = 340.0;
ITMyThick = 199.61;
ITMyDiameter = 339.92;
CPxThick = 100.31;
CPxDiameter = 339.94;
CPyThick = 100.32;
CPyDiameter = 340.11;
BSthick = 59.88;
BSDiameter = 369.98;

p1 = {-200, 4983.1, -80};
p9 = {5013, -200, -80};
d12 = ITMyThick - (ITMyDiameter / 2) Tan[wedgeITMy];
d910 = ITMxThick - (ITMxDiameter / 3) Tan[wedgeITMx];
d23 = 20;
d1011 = 20;
d34 = CPyThick - (CPyDiameter / 2) Tan[wedgeCPy];
d1112 = CPxThick - (CPxDiameter / 2) Tan[wedgeCPx];
dBS = BSthick - (BSDiameter / 2) Tan[wedgeBS];
```

Note that the distances d513 and d57 are approximate -- they do not take into account the wedge angle of the BS

## ■ optic surface orientation determination

### ■ normal vectors for ITMs and CPs

```
n1 = {0, 1, 0};
n2=RotationMatrix3D[0,wedgeITMy,0].n1
{0., 0.999999, -0.00129154}
```

```

n3 = n2;
n4 = RotationMatrix3D[0, 0, -wedgeCPy].n3
{-0.00115192, 0.999999, -0.00129154}

n9 = {1, 0, 0};
n10 = RotationMatrix3D[pi/2, -wedgeITMx, -(pi/2)].n9
{0.999999, 0., -0.00126536}

n11 = n10;
n12 = RotationMatrix3D[0, 0, wedgeCPx].n11
{0.999998, -0.00127409, -0.00126536}

```

#### ■ ITMy path to BS

```

u1 = -n1;
w1 = u1;
u2 = w1;
w2 = Refract[u2, n2, nOptic, nVacuum]
{0., -1., -0.000580717}

u3 = w2;
w3 = Refract[u3, n3, nVacuum, nOptic]
{0., -1., 0.}

u4 = w3;
w4 = Refract[u4, n4, nOptic, nVacuum]
{-0.000517937, -1., -0.000580718}

u5y = w4;

```

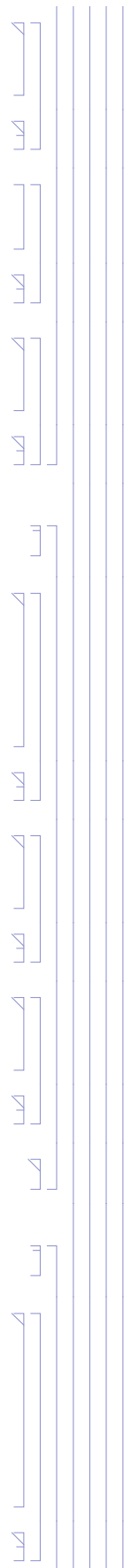
#### ■ ITMx path to BS

```

u9 = -n9;
w9 = u9;
u10 = w9;
w10 = Refract[u10, n10, nOptic, nVacuum]

{-1., 0., -0.000568946}

```



```

u11 = w10;
w11 = Refract[u11, n11, nVacuum, nOptic]
{-1., 0., 0.}

```

```

u12 = w11;
w12 = Refract[u12, n12, nOptic, nVacuum]
{-1., -0.00057287, -0.000568947}

```

```

u13 = w12;

```

Tried to convert from Rotate3D to RotationMatrix3D and didn't get this to work, so reverting to old function Rotate3D in package "Geometry`Rotations`"

```

soln = Minimize[Abs[Reflect[u5y, RotationMatrix3D[-yaw, 0, 0].{0, 1, 0}].Refract[Refract[u13, RotatationMatrix3D[wedgeBS, 0, 0].-RotationMatrix3D[yaw, 0, 0].{0, 1, 0}, nVacuum, nOptic],
-RotationMatrix3D[yaw, 0, 0].{0, 1, 0}, nOptic, nVacuum]], yaw]

```

Note also that I have assumed a zero pitch angle. This is approximately correct because the BS has no vertical wedge component. However the reflected and refracted rays have a pitch angle difference of ~8 microrad (very small).

```

Clear[yaw];
soln =
  Maximize[Reflect[u5y, Rotate3D[{0, 1, 0}, -yaw, 0, 0]].Refract[Refract[u13, Rotate3D[-Rotate3D[{0, 1, 0}, -yaw, 0, 0], wedgeBS, 0, 0],
  nVacuum, nOptic], -Rotate3D[{0, 1, 0}, -yaw, 0, 0], nOptic, nVacuum], yaw]
{1., {yaw -> 0.784942}}

```

```

BSyaw = -yaw /. soln[[2]]
-0.784942

```

```

BSyawDeg = BSwaw 180 / pi
-44.9739

```

Compare to Zemax BS yaw angle (45.02885 deg) -- only 22 microradian difference:

```

(90 - 45.02885 + BSwawDeg) pi / 180
-0.0000477476

```

```

n5y = RotationMatrix3D[BSyaw, 0, 0].{0, 1, 0}
n5x = -n5y;
{-0.706784, 0.707429, 0.}

```

```

v5 = Reflect[u5y, n5y]
{-1., 0.000393622, -0.000580718}

```

```

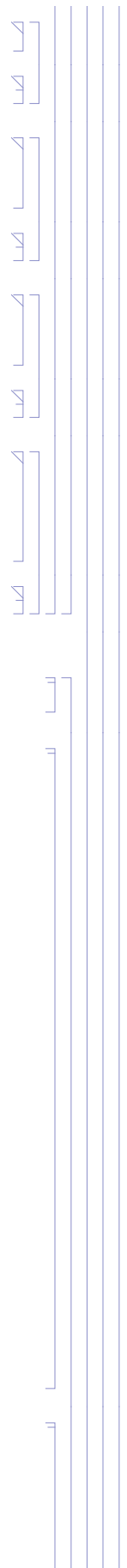
w5y = Refract[u5y, n5y, nVacuum, nOptic]
{0.271891, -0.962328, -0.000400597}

n13 = RotationMatrix3D[wedgeBS, 0, 0].n5x;
n7 = -n13
{-0.70592, 0.708292, 0.}

u7 = w5y;
w7 = Refract[u7, n7, nOptic, nVacuum]
{0.000445259, -1., -0.000580718}

w13 = Refract[u13, n13, nVacuum, nOptic];
u5x = w13;
w5x = Refract[u5x, n5x, nOptic, nVacuum]
{-1., 0.000393615, -0.000568947}

```



#### ■ BS position determination

ar = anti-reflectance surface  
hr = high reflectance surface  
bs = beamsplitting surface

d12 = ITMy thickness at center, mm  
d23 = ITMy to CPy gap distance, mm  
d34 = CPy thickness at center, mm  
d45 = CPy to BS(hr) distance, mm  
d56 = BS(hr) to PR3 distance, mm  
dBS = BS thickness at center, mm  
d57 = BS(hr) to BS(ar) for y-beam, mm  
d78 = BS(ar), at y-beam intercept, to SR3, mm  
d513 = BS(hr) to BS(ar) for x-beam, mm  
d910 = ITMx thickness at center, mm  
d1011 = ITMx to CPx gap distance, mm  
d1112 = CPx thickness at center, mm  
d1213 = CPx to BS(ar) distance, mm

The “given” parameters (from Zemax layout, etc.) are as follows:

- ITM (hr) center positions from Zemax
- ITM thickness
- ITM to CP gaps

- CP thickness
- BS thickness

```

p1 = {-200, 4983.1, -80};
p9 = {5013, -200, -80};
d12 = 199.64 - (340.06 / 2) Tan[wedgeITMy];
d910 = 200.22 - (340.13 / 3) Tan[wedgeITMx];
d23 = 20;
d1011 = 20;
d34 = 99.91 - (340.22 / 2) Tan[wedgeCPy];
d1112 = 99.82 - (340.13 / 2) Tan[wedgeCPx];
dBS = 60.41 - (369.85 / 2) Tan[wedgeBS];

```

Note that the distances d513 and d57 are approximate -- they do not take into account the wedge angle of the BS

```

d513 = dBS / (-w13.n5x)
68.9435

d57 = dBS / (w5y.n5x)
68.9435

p2 = p1 + d12 w1
{-200., 4783.68, -80.}

p3 = p2 + d23 w2
{-200., 4763.68, -80.0116}

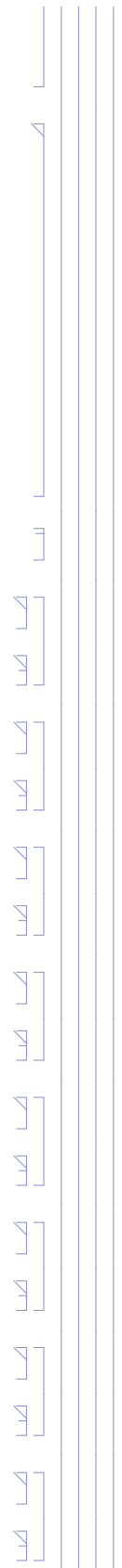
p4 = p3 + d34 w3
{-200., 4663.97, -80.0116}

p10 = p9 + d910 w9
{4812.92, -200., -80.}

p11 = p10 + d1011 w10
{4792.92, -200., -80.0114}

p12 = p11 + d1112 w11
{4693.32, -200., -80.0114}

```



```

Clear[dBSy, dBSx]
soln = Minimize[EuclideanDistance[p4 + dBSy w4, p12 + dBSx w12 + d513 w13], {dBSy, dBSx}]
{0.0407252, {dBSy → 4847.93, dBSx → 4829.5}}

```

```

d45 = dBSy /. soln[[2]];
d1213 = dBSx /. soln[[2]];

```

```

p5 = p4 + d45 w4
p13 = p12 + d1213 w12
p13 + d513 w13
{-202.511, -183.961, -82.8269}
{-136.182, -202.767, -82.7591}
{-202.511, -183.961, -82.7862}

```

Compare to Zemax (L1 : D0902216 - v8) global coordinates for p5 (BS (hr)) = {-202.5, -184.0, -82.8}, so Zemax is correct to within ~0.1 mm

```

p0 = p5 + n5x BStThick;
SchnuppAssy = (d910 nOptic + d1011 + d1112 nOptic + d1213 + d513 nOptic) - (d12 nOptic + d23 + d34 nOptic + d45)
82.3087

```

Compare to Zemax (H1 : D0901920 - v12) Schnupp Assymetry of 80 mm, so 2.6 mm too large ... but this is OK.

```

p7 = p5 + d57 u7
{-183.766, -250.307, -82.8545}

```

### ■ PR3 position determination

Take as a given the global x - coordinate of the PR3 from the IO layout (because the IO layout position is used in the SolidWorks layouts used to physically place the suspensions pPR3 from Zemax (D0902216-v8) & IO layout (E1100492-v11, E1100493-v9) should equal p6

```

pPR3zemax = {-19 740.5, -176.3, -93.9};
pPR3io = {-19 740.0, -177.4, -94.5};
xPR3 = pPR3io[[1]];
d56 = (xPR3 - p5[[1]]) / v5[[1]]
19 537.5

```

```
p6 = p5 + d56 v5
```

```
{-19 740., -176.271, -94.1727}
```

```
p6 - pPR3zemax
```

```
p6 - pPR3io
```

```
{0.5, 0.0293646, -0.272665}
```

```
{0., 1.12936, 0.327335}
```

So Zemax position for pPR3 is in error by ~0.3 mm ... which is OK

IO position for PR3 is in error by ~1.1mm

### ■ PR2 position determination

PR3 is constrained to be vertical (zero pitch angle) in local coordinates (to minimize vertical bounce mode-to-length coupling). Consequently the normal vector for this optic must be pitched in global coordinates.

Take as a given the global x and y coordinates of the PR2 from the IO layout (because the IO layout position is used in the SolidWorks layouts used to physically place the suspensions pPR2 from Zemax (D0902216-v8) & IO layout (E1100492-v11, E1100493-v9) should equal p14

```
pPR2zemax = {-3579.6, -530.4, -93.3};
```

```
pPR2io = {-3579.2, -530.4, -94.1};
```

```
u6 = v5;
```

```
Clear[s, yaw]
```

```
soln = Minimize[EuclideanDistance[p6 + s Reflect[u6, Rotate3D[Rlc.{1, 0, 0}], -yaw, 0, 0]], pPR2io], {s, yaw}]
```

```
{0.626391, {s → 16 164.7, yaw → -0.0111513}}
```

```
PR3yaw = yaw /. soln[[2]];
```

```
d614 = s /. soln[[2]];
```

```
n6 = Rotate3D[Rlc.{1, 0, 0}], -PR3yaw, 0, 0]
```

```
v6 = Reflect[u6, n6];
```

```
p14 = p6 + d614 v6
```

```
{0.999938, -0.0111513, 0.000312}
```

```
{-3579.2, -530.4, -93.4736}
```

**p14 - pPR2io**

**p14 - pPR2zemax**

$\{-0.000025692, -2.44644 \times 10^{-6}, 0.626391\}$

$\{0.399974, -2.44644 \times 10^{-6}, -0.173609\}$

## ■ PRM position determination

PR2 is constrained to be vertical (zero pitch angle) in local coordinates (to minimize vertical bounce mode-to-length coupling). Consequently the normal vector for this optic must be pitched in global coordinates.

Take as a given the global x and y coordinates of the PRM from the IO layout (because the IO layout position is used in the SolidWorks layouts used to physically place the suspensions pPRM from Zemax (D0902216-v8) & IO layout (E1100492-v11, E1100493-v9) should equal p15

**pPRMzemax =  $\{-20190.0, -628.0, -102.8\}$ ;**

**pPRMio =  $\{-20189.6, -628.0, -104.1\}$ ;**

**u14 = v6;**

**Clear[s, yaw]**

**soln = Minimize[EuclideanDistance[p14 + s Reflect[u14, Rotate3D[Rlc. $\{-1, 0, 0\}$ , -yaw, 0, 0]], pPRMio], {s, yaw}]**

$\{0.980667, \{s \rightarrow 16610.7, \text{yaw} \rightarrow -0.0080166\}\}$

**PR2yaw = yaw /. soln[[2]];**

**d1415 = s /. soln[[2]];**

**n14 = Rotate3D[Rlc. $\{-1, 0, 0\}$ , -PR2yaw, 0, 0]**

**v14 = Reflect[u14, n14];**

**p15 = p14 + d1415 v14**

$\{-0.999968, 0.00801671, -0.000312\}$

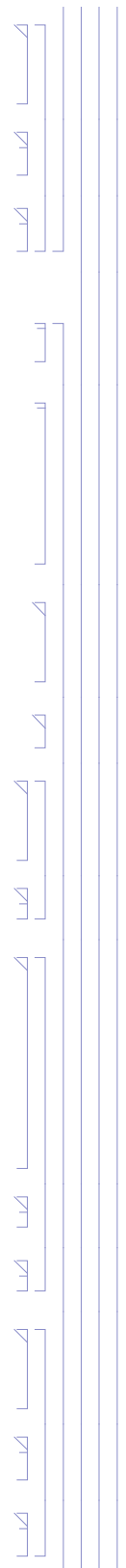
$\{-20189.6, -628., -103.119\}$

**p15 - pPRMio**

**p15 - pPRMzemax**

$\{-0.00057253, 8.42312 \times 10^{-7}, 0.980667\}$

$\{0.399427, 8.42312 \times 10^{-7}, -0.319333\}$





```

u15 = v14;
n15 = -v14;
Clear[yaw]
soln = Maximize[n15.Rotate3D[{1, 0, 0}, -yaw, 0, 0], {yaw}];
PRMyaw = yaw /. soln[[2]]

0.00587577

```

### ■ SR3 position determination

Take as a given the global y - coordinate of the SR3 from Zemax & IO layout.

pSR3 from Zemax should equal p8

```

pSR3 = {-175.2, -19 615.9, -94.1};
ySR3 = pSR3[[2]];

d78 = (ySR3 - p7[[2]]) / w7[[2]]

19 365.6

p8 = p7 + d78 w7

{-175.143, -19 615.9, -94.1005}

p8 - pSR3

{0.0569283, 0., -0.000460114}

```

So Zemax position for pSR3 is in error by ~0.1 mm ... which is OK

### ■ SR2 position determination

SR3 is constrained to be vertical (zero pitch angle) in local coordinates (to minimize vertical bounce mode-to-length coupling). Consequently the normal vector for this optic must be pitched in global coordinates.

Take as a given the global x and y coordinates of the SR2 from the Zemax layout (because the Zemax layout positions for the SRC are used in the SolidWorks layouts used to physically place the suspensions pSR2 from Zemax (D0902216-v8) should equal p16

```

pSR2zemax = {-594.1, -4178.1, -84.4};

u8 = w7;

Clear[s, yaw]
soln = Minimize[EuclideanDistance[p8 + s Reflect[u8, Rotate3D[Rlc.{0, 1, 0}, -yaw, 0, 0]], pSR2zemax], {s, yaw}]

{0.201491, {s → 15 443.5, yaw → 0.0137885}}

```

```

SR3yaw = yaw /. soln[[2]];
d816 = s /. soln[[2]];
n8 = Rotate3D[Rlc.{0, 1, 0}, -SR3yaw, 0, 0]
v8 = Reflect[u8, n8];
p16 = p8 + d816 v8

{-0.013788, 0.999905, 0.000611}

{-594.1, -4178.1, -84.1985}

p16 - pSR2zemax

{0.000223243, 0.000194702, 0.201491}

```

### ■ SRM position determination

SR2 is constrained to be vertical (zero pitch angle) in local coordinates (to minimize vertical bounce mode-to-length coupling). Consequently the normal vector for this optic must be pitched in global coordinates.

Take as a given the global x and y coordinates of the PRM from the Zemax layout (because the Zemax SRC layout positions are used in the SolidWorks layouts used to physically place the suspensions pSRM from Zemax (D0902216-v8) should equal p17

```

pSRMzemax = {305.0, -19 908.6, -93.2};

u16 = v8;

Clear[s, yaw]
soln = Minimize[EuclideanDistance[p16 + s Reflect[u16, Rotate3D[Rlc.{0, -1, 0}, -yaw, 0, 0]], pSRMzemax], {s, yaw}]

{0.147955, {s → 15 756.2, yaw → 0.042113}}

SR2yaw = yaw /. soln[[2]];
d1617 = s /. soln[[2]];
n16 = Rotate3D[Rlc.{0, -1, 0}, -SR2yaw, 0, 0]
v16 = Reflect[u16, n16];
p17 = p16 + d1617 v16

{0.0421006, -0.999113, -0.000611}

{305., -19 908.6, -93.348}

p17 - pSRMzemax

{-8.93539 × 10-7, 0.0000358173, -0.147955}

```

```

u17 = v16;
n17 = -v16;
Clear[yaw]
soln = Maximize[n17.Rotate3D[{0, 1, 0}, -yaw, 0, 0], {yaw}];
SRMyaw = yaw /. soln[[2]]

0.0570943

```

### ■ PRC, SRC & Schnupp Assymetry Lengths

According to T0900043-v11, the recycling cavity lengths are supposed to be:

PRC length = 57656.0 mm

SRC length = 56008.0 mm

The Schnupp Assymetry is supposed to be 80 mm, according to RODA M1200276-v1

```

dPRC = d1415 + d614 + d56 + ((nOptic d12 + d23 + nOptic d34 + d45) + (nOptic d910 + d1011 + nOptic d1112 + d1213 + nOptic d513)) / 2

57 655.6

```

```

dSRC = d1617 + d816 + d78 + nOptic d57 + ((nOptic d12 + d23 + nOptic d34 + d45) + (nOptic d910 + d1011 + nOptic d1112 + d1213 + nOptic d513)) / 2

56 007.9

```

```

schnuppAssy = Abs[(nOptic d12 + d23 + nOptic d34 + d45) - (nOptic d910 + d1011 + nOptic d1112 + d1213 + nOptic d513)]

82.3087

```

### ■ Summary

```

opticLabel = {"PRM HR", "PR2 HR", "PR3 HR", "BS HR", "BS Arc", "SRM HR", "SR2 HR", "SR3 HR", "CPx", "ITMx", "CPy", "ITMy"};
nOptics = Length[opticLabel];
opticPositionG = {p15, p14, p6, p5, p0, p17, p16, p8, p11, p9, p3, p1};
opticPositionL = opticPositionG.Rlc;
opticNormal = {n15, n14, n6, n5y, n13, n17, n16, n8, n11, n9, n3, n1};
opticYaw = {PRMyaw, PR2yaw, PR3yaw, BSyaw, 0, SRMyaw, SR2yaw, SR3yaw, 0, 0, 0, 0};

```

```

formattedTable := Grid[Flatten[{{
  {"Optic", "Global Coordinates\n(mm)", SpanFromLeft, SpanFromLeft, "Local Coordinates\n(mm)", SpanFromLeft, SpanFromLeft,
  "Normal Unit Vector (global)\n(mm)", SpanFromLeft, SpanFromLeft, "Yaw", "Yaw", SpanFromLeft, SpanFromLeft},
  {SpanFromAbove, "Xg", "Yg", "Zg", "Xl", "Yl", "Zl", "Ug", "Vg", "Wg", "rad", "deg", "min", "sec"}},
Table[{opticLabel[[i]], NumberForm[opticPositionG[[i, 1]], {10, 1}},
  NumberForm[opticPositionG[[i, 2]], {10, 1}], NumberForm[opticPositionG[[i, 3]], {10, 1}], \
  NumberForm[opticPositionL[[i, 1]], {10, 1}], NumberForm[opticPositionL[[i, 2]], {10, 1}}, \
  NumberForm[opticPositionL[[i, 3]], {10, 1}], \
  NumberForm[opticNormal[[i, 1]], {10, 6}], NumberForm[opticNormal[[i, 2]], {10, 6}], NumberForm[opticNormal[[i, 3]], {10, 6}], \
  NumberForm[opticYaw[[i]], {10, 6}], NumberForm[DMS[opticYaw[[i]]][[1]], {10, 0}}, \
  NumberForm[DMS[opticYaw[[i]]][[2]], {10, 0}], NumberForm[DMS[opticYaw[[i]]][[3]], {10, 0}}],
{i, 1, nOptics}]], 1],
Frame →
All];

```

formattedTable

Optic	Global Coordinates (mm)			Local Coordinates (mm)			Normal Unit Vector (global) (mm)			Yaw	Yaw		
	Xg	Yg	Zg	Xl	Yl	Zl	Ug	Vg	Wg	rad	deg	min	sec
PRM HR	-20189.6	-628.0	-103.1	-20189.6	-628.1	-96.4	0.999983	0.005876	0.000581	0.005876	0.	20.	12.
PR2 HR	-3579.2	-530.4	-93.5	-3579.2	-530.5	-92.0	-0.999968	0.008017	-0.000312	-0.008017	0.	-27.	-34.
PR3 HR	-19740.0	-176.3	-94.2	-19740.0	-176.3	-87.9	0.999938	-0.011151	0.000312	-0.011151	0.	-38.	-20.
BS HR	-202.5	-184.0	-82.8	-202.5	-184.0	-82.7	-0.706784	0.707429	0.000000	-0.784942	-44.	-58.	-26.
BS ARC	-160.2	-226.3	-82.8	-160.2	-226.4	-82.6	0.705920	-0.708292	0.000000	0.000000	0.	0.	0.
SRM HR	305.0	-19908.6	-93.3	305.0	-19908.7	-81.3	-0.057063	0.998370	0.000581	0.057094	3.	16.	17.
SR2 HR	-594.1	-4178.1	-84.2	-594.1	-4178.2	-81.5	0.042101	-0.999113	-0.000611	0.042113	2.	24.	46.
SR3 HR	-175.1	-19615.9	-94.1	-175.2	-19616.0	-82.1	-0.013788	0.999905	0.000611	0.013788	0.	47.	24.
CPx	4792.9	-200.0	-80.0	4792.9	-200.0	-81.4	0.999999	0.000000	-0.001265	0.000000	0.	0.	0.
ITMx	5013.0	-200.0	-80.0	5013.0	-200.0	-81.4	1.000000	0.000000	0.000000	0.000000	0.	0.	0.
CPy	-200.0	4763.7	-80.0	-200.0	4763.6	-82.9	0.000000	0.999999	-0.001292	0.000000	0.	0.	0.
ITMy	-200.0	4983.1	-80.0	-200.0	4983.1	-83.0	0.000000	1.000000	0.000000	0.000000	0.	0.	0.