LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

Electro-Static Driver Range and Noise

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1 Introduction

The UK high voltage drivers for the end test mass electro-static (ES) actuators have more range and also more noise than is ultimately desired for low-noise operation of the interferometer. However, the impact of their voltage noise can be controlled to some degree by reducing the bias voltage, and thereby the control range. This note investigates how much range will be required of the ES drivers, the resulting displacement noise due to the driver noise, and the impact on the interferometer sensitivity. In particular we consider the interferometer configuration with 25 W of input laser power, and a signal recycling mirror transmission of 35%. Limited by quantum and thermal noise, this configuration has a NS-NS inspiral range of 160 Mpc.

In short, we find that there are two approaches to driving the electro-static actuators with sufficiently low actuation noise to maintain the above sensitivity. One approach is to use the UK drivers with a lowered bias voltage (relative to the maximum). This would also require a (switchable) noise-shaping filter between the digital-to-analog converter (DAC) and the UK driver to control the DAC noise. This filter module does not currently exist, but it would be a very straightforward design. The other approach would be to employ a new driver that would have lower noise and lower range than the UK driver, along with a means to switch between the UK driver and the low noise driver while the interferometer is locked. The latter solution will ultimately be needed to achieve the final actuation noise goals, but the former may be simpler to implement in the interim.

2 Electro-static actuation: useful facts

The following features and specifications of the ETM electro-static actuation are largely taken from John Miller's PhD thesis, LIGO-P1000032. The force applied by the ESD is:

$$F = \alpha \cdot (V_b - V_s)^2,$$

where V_b is the bias voltage and V_s is the signal voltage. We also have the maximum peak-to-peak force, and the force noise:

$$F_{max} = 4\alpha \cdot V_b{}^2 \quad , \quad F_n = 2\alpha \cdot V_b \cdot v_n$$

where v_n is the noise voltage, and we assume that the bias is low-pass filtered so that its noise is negligible.

The UK ESD has an output range of ±400 V, and an output voltage noise of $1 \, \mu V/\sqrt{Hz}$ (see test data in LIGO-Q1100005). The voltage gain is 40, thus the input referred noise is $25 \, nV/\sqrt{Hz}$. Additional noise characterization was done by Weiss, and is documented in LIGO-T1200020. This showed that the noise could increase above 1 uV/rtHz if the bias voltage was above 50 V or so; this underscores the importance of low-pass filtering the bias channel.

The end reaction mass (ERM) electrode pattern (LIGO-D0900949-v2), coupled with a ERM-ETM gap of 5 mm, gives a force coefficient of $\alpha = 4\times10^{-10} N/V^2$; see LIGO-G0900956[1]. This gives a force noise of $F_n = 8\times10^{-16} \cdot V_b \, N/\sqrt{Hz}$.

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[1] Value is rounded compared to the number found in G0900956. Also, pattern modeled in that document is not exactly the same as the actual pattern given by D0900949.

3 DARM noise model

To calculate the required ES actuation range, we need a model for the noise in the DARM (differential arm) degree-of-freedom, which can then be propagated through a DARM feedback loop model. The DARM noise model is made by combining two pieces:

- GWINC, $f > 5$ Hz. GWINC is used to calculate the strain noise with the following parameters: 25 W input laser power; 35% SRM transmission; no SRC detuning; homodyne detection phase of zero. The seismic contribution is removed from the total noise, as that is calculated separately.

- BSC ISI requirement + quad suspension model, $f = 0.1$—20 Hz. The seismic contribution to DARM is calculated by using the BSC ISI displacement requirement (including the increase around 10 Hz), and propagating it through the quad suspension model. This procedure is the same as given in LIGO-T1200155, except that I use the BSC ISI requirements curve, and that document uses some other model of the ISI performance.

The seismic noise estimate (second piece above) is added to the GWINC output (in the 5-20 Hz band), to give the total DARM noise model. The result is shown in Figure 1.
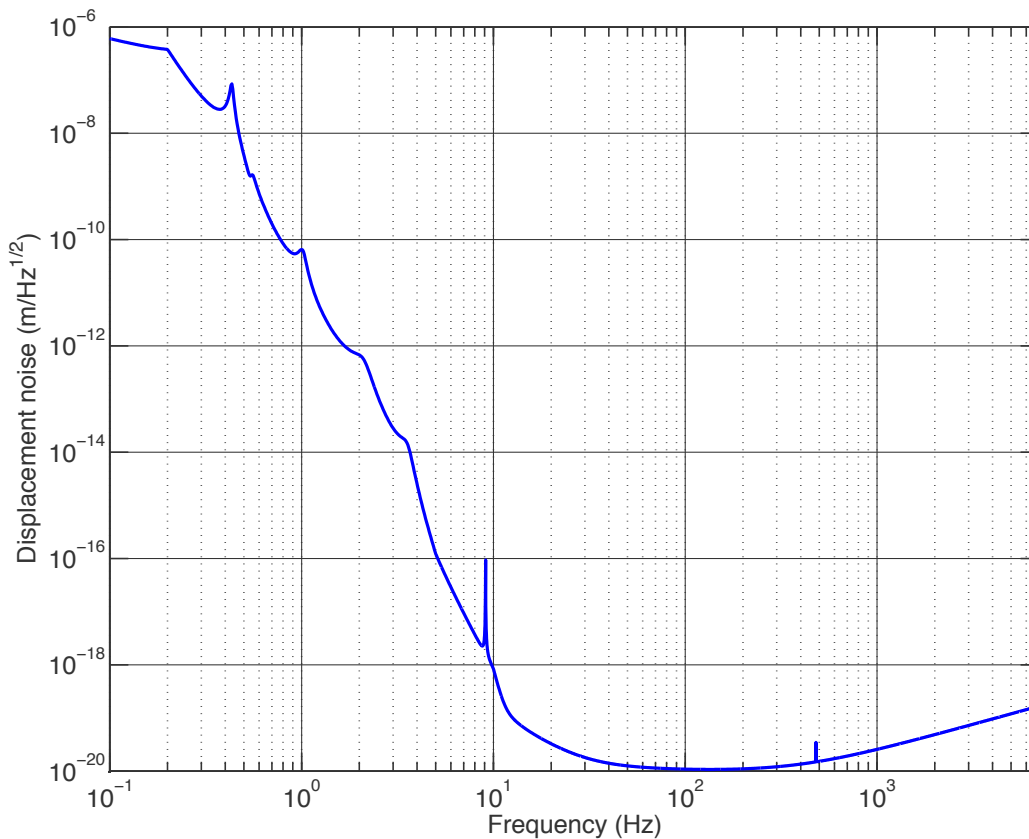

Figure 1. Model spectrum for the noise in the DARM degree-of-freedom. Details explained in the text.

4   DARM loop model

The next ingredient we need is the design of the DARM servo. This is designed based on the following guidelines:

- The unity gain frequency is taken to be 200 Hz, as it was in i/eLIGO. It is possible that it could be somewhat lower and still provide adequate suppression, but we won't assume so for now.

- Phase margin should be 50 degrees or more.

- The loop should suppress the residual DARM fluctuations to 10^{-15} m-rms or less. This is about 10^{-3} of the DARM offset (for DC readout) and about $10\times$ smaller than in i/eLIGO.

Using these guidelines, the following DARM loop design was arrived at:

- 4 poles at DC

- 1 real zero at 3 Hz. Thus, loop is $1/f^4$ below 3 Hz.

- 1 complex zero pair at 30 Hz, phase angle of 60 degrees. Thus, loop is $1/f^3$ between 3 Hz and 30 Hz.

- Cut-off filter: Second-order Butterworth low-pass, at 800 Hz. This is the filter that cuts off the drive to the ESD.

- Cavity pole compensation. The arm cavity pole is at 400 Hz. This is compensated with a real zero at 400 Hz, plus a real pole at 4 kHz. The net effect is a pole in the DARM loop at 4 kHz.

- ESD pole. The frequency response of the UK driver has a real pole at 2 kHz. This is included in the loop model, without any compensation (too high a frequency to compensate digitally).

- Suspension mode resonant gain. The loop model includes a resonant gain for the first quad suspension mode at 0.43 Hz. This is needed because the model does not actually include the suspension dynamics, but the noise model does include this mode (see above). In practice, the extra gain at this frequency would be provided by the suspension response itself.

- The model does not include time delays or the phase responses of the anti-alias and anti-image hardware filters. These will add a few degrees of phase lag at the u.g.f. in practice.

- The model does not include notch filters for the violin modes of the suspension (which are around 500 Hz). These will be needed, but can be high Q and shouldn't add more than a couple of degrees at the u.g.f.

The bode plot of the DARM servo is shown in Figure 2, and the loop suppressed DARM spectrum is shown in Figure 3.
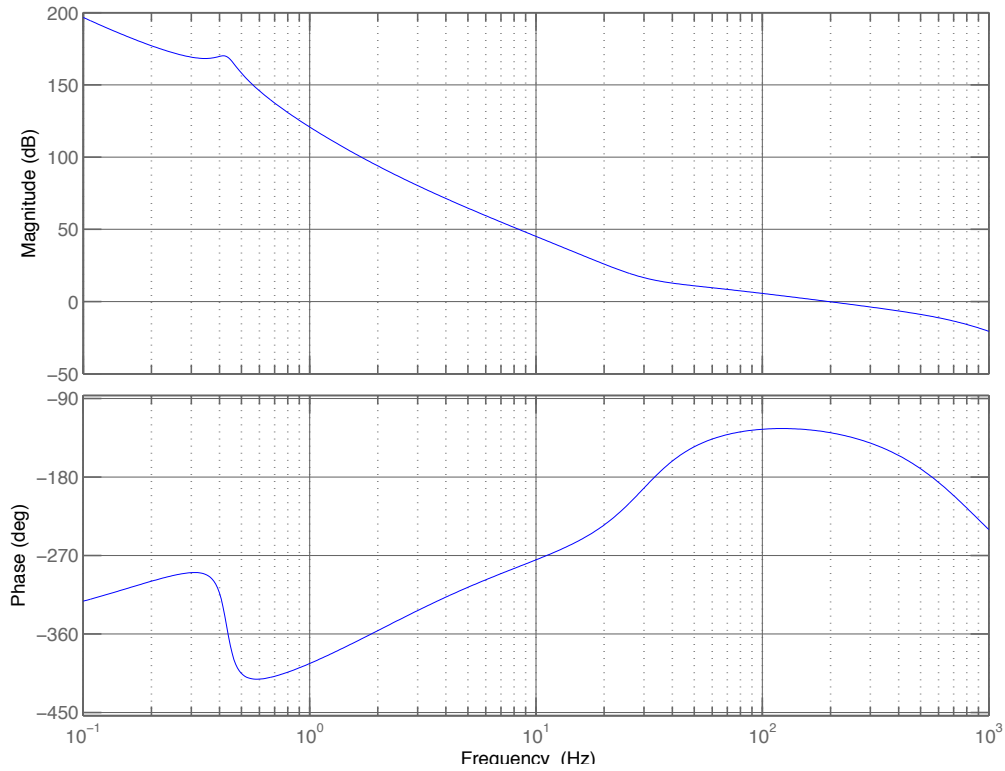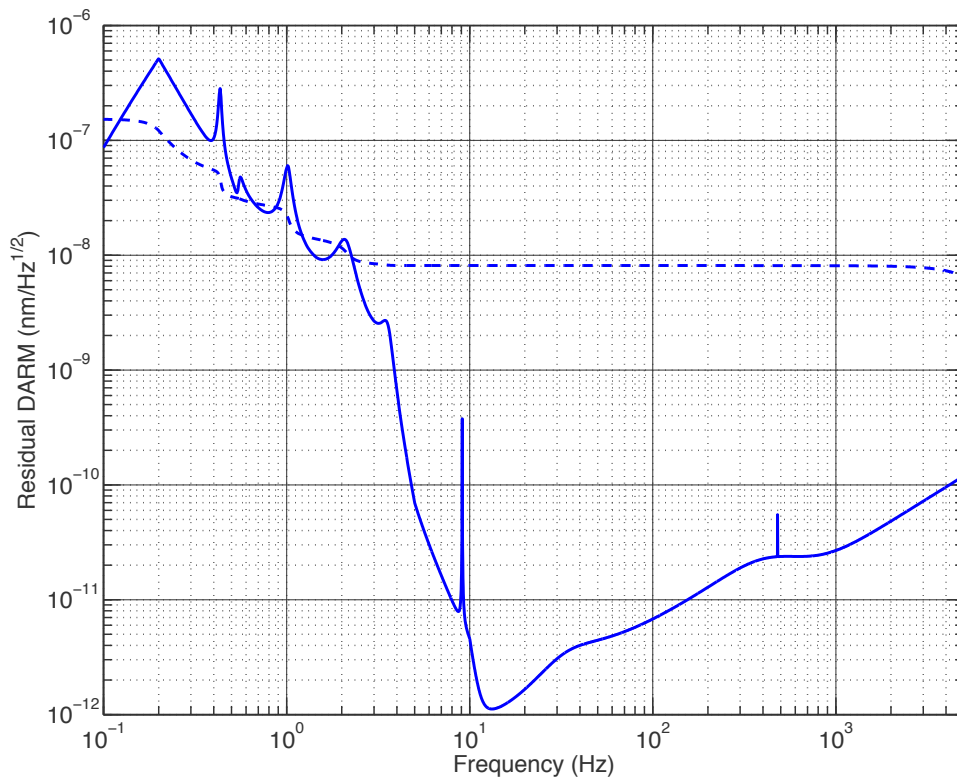
Figure 2. Open loop gain of the DARM servo, with filters as describe above. The phase margin at the u.g.f. of 200 Hz is 51 degrees.

Figure 3. Residual DARM spectrum (solid) and integrated rms (dashed) for the DARM noise model and loop design given above. The integrated rms is just over 10^{-16} m-rms.

5 Electro-static actuation range

To calculate the force required from the electro-static drive, we calculate the ETM displacement, x_{ETM} , arising from the DARM noise model, x_{DARM}, and the DARM open loop gain, G, shown in Figure 2:

$$x_{\text{ETM}} = x_{\text{DARM}} \cdot \frac{G}{1 + G}$$

Note: the DARM noise model, x_{DARM}, corresponds to the displacement noise of a single arm cavity.

This is then converted to force on the ETM via: $F_{\text{ETM}} = 40\text{kg} \cdot x_{\text{ETM}} \cdot \omega^2$. The calculation is restricted to frequencies $f > 5$ Hz, under the assumption that no lower frequencies will be significant in the ESD feedback path (i.e., they are suitable high-pass filtered). The result is shown in Figure 4.
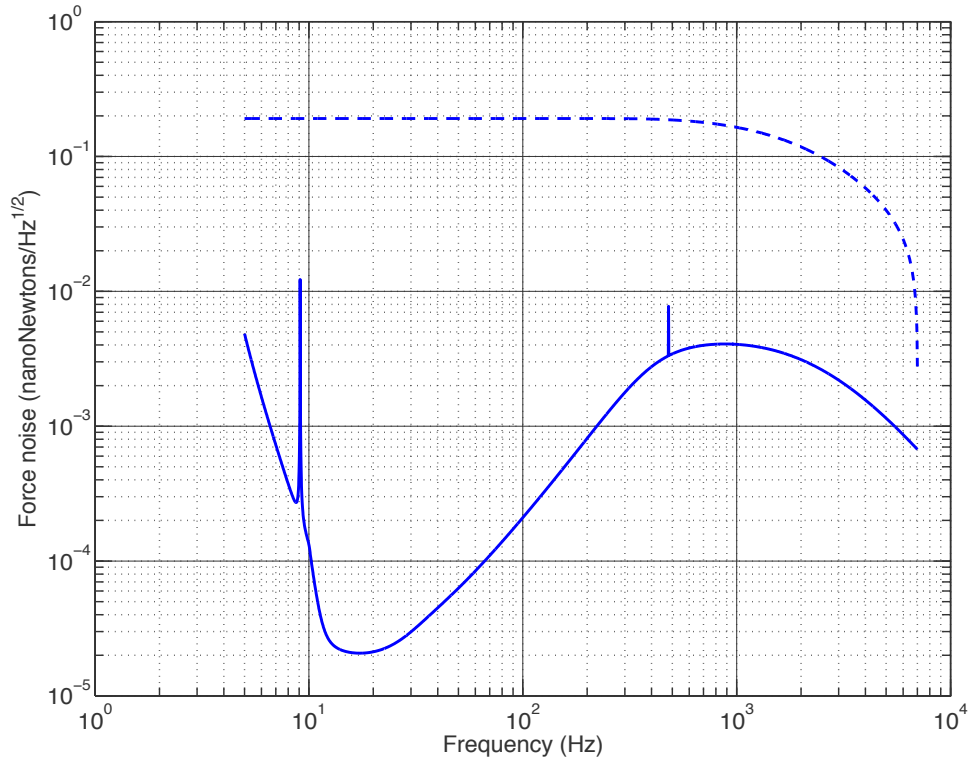

Figure 4. Force noise (solid) and integrated rms of force (dashed) applied to an ETM, given the DARM noise and loop models above. The rms force noise, in the 5—7000 Hz band, is 0.2 nanoNewton.

We would like at least a factor of 50 headroom between rms force and peak-to-peak force; that is, $F_{\text{max}} > 10$ nN. This translates to a bias voltage of $V_b > 2.5$ V. Furthermore, for a given force range, a higher bias voltage also means smaller non-linearity (which is nominally compensated for, but it

won't be perfect). Thus we will proceed assuming a bias voltage of $V_b = 11$ V, which gives a maximum force swing of 200 nN (1000 times the rms noise force).

6 Impact on interferometer sensitivity

The force noise for the bias voltage of 11 V is $8.7\times10^{-15}\ N/\sqrt{Hz}$. The resulting displacement noise is shown in Figure 5, along with the interferometer noise for low power operation. With the ESD noise present, the NS-NS inspiral range is reduced from 160.7 Mpc to 158.4 Mpc.
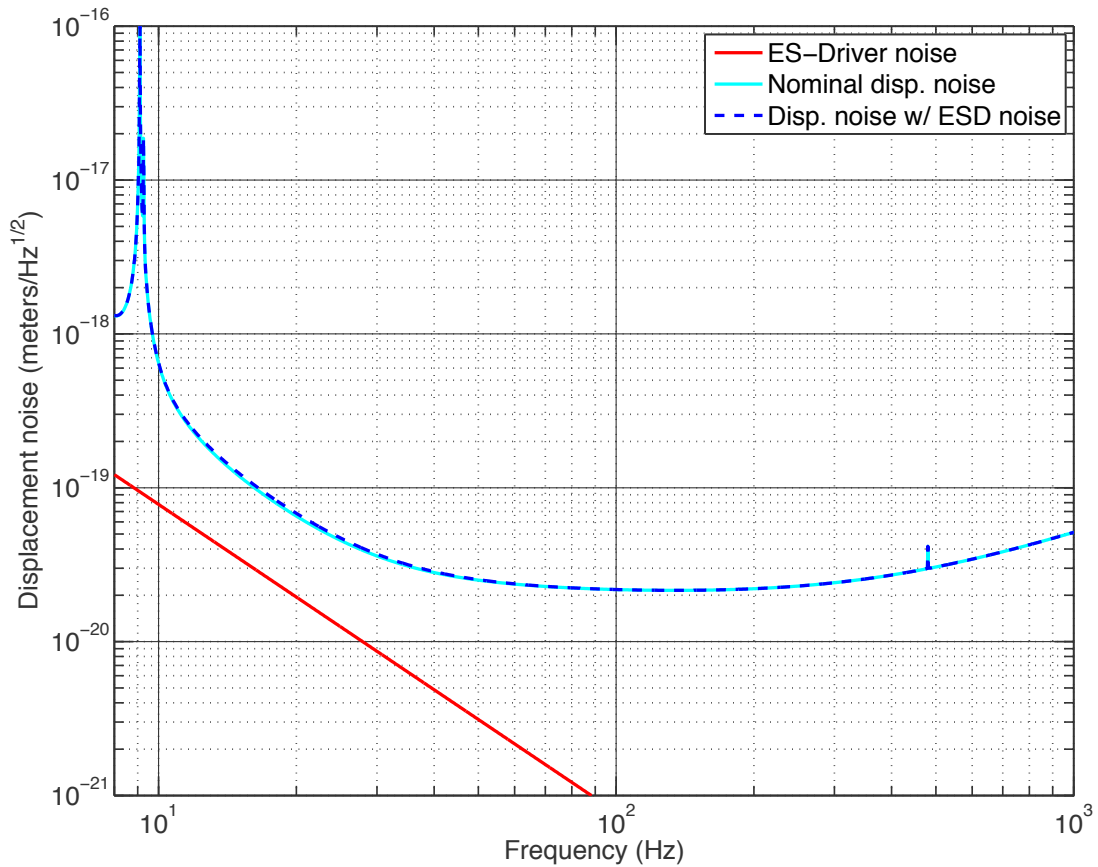

Figure 5. DARM displacement noise from the UK ESD with a bias voltage of 15 V, and the GWINC interferometer noise for 25 W laser power, SRM transmission of 35%, zero SRC detuning, and homodyne phase of zero. The cyan curve is the straight output of GWINC, and the dashed blue curve has the ESD noise added in. The read curve includes the noise from two ES drivers (one for each ETM). At this level of ESD noise/range, the effect on the interferometer noise is negligible.

7 DAC noise

The UK ESD has a differential input, with an input range that matches the General Standards 18-bit DAC output. However, the output noise of the DAC (150 nV/Hz$^{1/2}$), is significantly higher than the input noise of the ESD (25 nV/Hz$^{1/2}$). Therefore, to realize the displacement noise given in the

preceding section (i.e., as shown in Figure 5), a noise-shaping filter is needed between the DAC and the ESD. This is feasible because we require the noise to be low only in the 10—100 Hz band, whereas the full range of the ESD is required only above a few hundred Hz.

A simple, first-order high-pass filter between the DAC and the ESD should be adequate. Referring to Figure 5, we should have a factor of 10 or more filtering of the DAC noise for frequencies below about 40 Hz (a factor of 10 reduces DAC noise to 15 nV/ $Hz^{1/2}$, which would raise the ESD output noise by17%). Thus an AC-coupling frequency of 400 or 500 Hz would do the job. There is probably no need to have the path be DC coupled.

8   Low noise driver option

The alternative to simply reducing the bias voltage of the UK drivers is to have separate low-voltage drivers that are switched in (and the UK drivers switched out) for low-noise operation. This approach could clearly achieve lower noise performance. For example, with a +/- 15 V driver, we could achieve the same range as discussed above, but the output noise could easily be at the DAC noise level of 150 nV/ $Hz^{1/2}$ – a factor of 6 lower than with the UK drivers. The trade-off is more complication in the channel switching. (No design for doing this exists at present, but one approach would be to switch two ESD quadrants at a time, leaving the ETM DARM feedback active only on the other two quadrants; this way, the synchronization of the two quadrants being switched is not critical.)