T1300414 Faraday Isolator Blade Creep
6/17/13

acceleration of gravity, m/s^2 $g := 9.8$

Faraday Isolator upper blade spring

E correction factor (see p. 4) $\rho := 0.9896$

new modulus of elasticity, Pa $E := 186 \cdot 10^9 \cdot \rho$ $E = 1.84066 \times 10^{11}$

modulus of elasticity, psi $E_{psi} := \frac{E}{6895}$ $E_{psi} = 2.66955 \times 10^7$

Weight suspended

OFI without balance wts, lb $m_{wtlb} := 38.93 - 2$ $m_{wtlb} = 36.93$

variable balance wt, lbs $m_v := 2$

design weight, lbs $m_{bslb} := m_{wtlb} + m_v$

$m_{bslb} = 38.93$

suspended mass, kg $m_{mp}(m_{bslb}) := \frac{m_{bslb}}{2.205}$ $m_{mp}(m_{bslb}) = 17.65533$

yield stress of C-250 steel, Pa $S_{yieldms} := 1800 \cdot 10^6$

yield stress of C-250 steel, psi $S_{yieldmspsi} := S_{yieldms} \cdot \left(1.45 \cdot 10^{-4}\right)$

$S_{yieldmspsi} = 2.61 \times 10^5$

factor of safety (see p. 4) $FS := 3.58328$

working stress of C-250 steel, Pa

$$S_{wms}(FS) := \frac{S_{yieldms}}{FS} \qquad S_{wms}(FS) = 5.02333 \times 10^{8}$$

working stress of C-250 steel, psi

$$S_{wspsi} := S_{wms}(FS)\cdot 1.45\cdot 10^{-4} \qquad S_{wspsi} = 7.28383 \times 10^{4}$$

number of springs

$$N := 2$$

mass supported by each blade spring, kg

$$m_{bs}(m_{bslb}) := \frac{m_{mp}(m_{bslb})}{N} \qquad m_{bs}(m_{bslb}) = 8.82766$$

load on blade spring, N

$$P(m_{bslb}) := m_{bs}(m_{bslb})\cdot 9.8 \qquad P(m_{bslb}) = 86.51111$$

arc of blade spring, rad

$$\theta_m := \frac{\pi}{4} \qquad \theta_m = 0.7854$$

blade arc angle, deg

$$\theta_{mdeg}(\theta_m) := \theta_m\cdot\frac{180}{\pi} \qquad \theta_{mdeg}(\theta_m) = 45$$

horizontal distance of suspension point from blade spring mount, in

$$x_{bsin} := 9.918$$

mounting location of blade spring
left of center, m

$$x_{bs} := x_{bsin}\cdot .0254 \qquad x_{bs} = 0.25192$$

radius of blade spring, m

$$R_{bs}(\theta_m, x_{bs}) := \frac{x_{bs}}{\sin(\theta_m)} \qquad R_{bs}(\theta_m, x_{bs}) = 0.35626$$

radius of blade spring, in

$$R_{bsin}(\theta_m, x_{bs}) := \frac{R_{bs}(\theta_m, x_{bs})}{.0254}$$

$$R_{bsin}(\theta_m, x_{bs}) = 14.02617$$

length of blade spring, m

$$l_{bs}(\theta_m, x_{bs}) := R_{bs}(\theta_m, x_{bs})\cdot\theta_m$$

length of blade spring, in

$$l_{bsin}(\theta_m, x_{bs}) := \frac{l_{bs}(\theta_m, x_{bs})}{.0254}$$

design width, in $b_{in} := 2.83$

Calculate thickness

$$t(m_{bslb}) := \left(\frac{12 \cdot P(m_{bslb}) \cdot R_{bs}(\theta_m, x_{bs})^2}{0.0254 \cdot E \cdot b_{in}} \cdot \sin\left(\frac{l_{bs}(\theta_m, x_{bs})}{R_{bs}(\theta_m, x_{bs})} \right) \right)^{\frac{1}{3}}$$

$$t(m_{bslb}) = 1.91674 \times 10^{-3}$$

thickness of blade spring, in $t_{in}(m_{bslb}) := \frac{t(m_{bslb})}{.0254}$ $t_{in}(m_{bslb}) = 0.07546$

incremental weight change with δt inch increase in thickness, lbs

$$\delta m_{\delta tbslb}(\delta t) := \frac{m_{bslb}}{N} \cdot \left[\left(\frac{t_{in}(m_{bslb}) + \delta t}{t_{in}(m_{bslb})} \right)^3 - 1 \right]$$

$\delta t := 0.0005$

$\delta m_{\delta tbslb}(\delta t) = 0.38948$

maximum stress, Pa $S_{wms} := \frac{E \cdot t(m_{bslb})}{2 \cdot R_{bs}(\theta_m, x_{bs})}$

$S_{wms} = 4.95146 \times 10^{8}$

maximum stress, psi $S_{wspsi} := S_{wms} \cdot 1.45 \cdot 10^{-4}$ $S_{wspsi} = 7.17962 \times 10^{4}$

factor of safety $FS := \frac{S_{yieldms}}{S_{wms}}$ $FS = 3.63529$

Vertical Bounce Frequency

vertical height of suspension from blade spring mount, m

$$y_{bs}(\theta_m) := R_{bs}(\theta_m, x_{bs}) \cdot (1 - \cos(\theta_m))$$

$$y_{bs}(\theta_m) = 0.10435$$

vertical height of suspension from blade spring mount, in

$$y_{bsin}(\theta_m) := \frac{y_{bs}(\theta_m)}{0.0254}$$

$$y_{bsin}(\theta_m) = 4.10817$$

unloaded height of blade spring, m

$$y_{max} := l_{bs}(\theta_m, x_{bs}) \cdot \sin(\theta_m)$$

vertical distance blade moves, m

$$\Delta_y(\theta_m) := y_{max} - y_{bs}(\theta_m)$$

vertical distance blade moves, in

$$\Delta_{yin}(\theta_m) := \frac{\Delta_y(\theta_m)}{0.0254} \qquad \Delta_{yin}(\theta_m) = 3.68141$$

vertical resonant frequency based on blade depression, Hz

$$f_{0v}(\theta_m) := \frac{\sqrt{\frac{g}{\Delta_y(\theta_m)}}}{2 \cdot \pi} \qquad f_{0v}(\theta_m) = 1.62933$$

effective spring constant, N/m

$$k := (2 \cdot \pi \cdot f_{0v}(\theta_m))^2 \cdot m_{mp}(m_{bslb})$$

effective spring constant, N/m

$$k = 1.85035 \times 10^3$$

incremental force for 0.25 lb weight change, N

$$\delta F := \frac{0.25}{2.205} \cdot g$$

height change with 0.25 lb added weight, m

$$\delta h := \frac{\delta F}{k}$$

$$\delta h = 6.00487 \times 10^{-4}$$

volume of suspended OFI, in^3

$$V_{OFIin} := 288.2$$

volume of suspended OFI, m^3

$$V_{OFI} := 288.2 \cdot (.0254)^3 \qquad V_{OFI} = 4.72275 \times 10^{-3}$$

density of air, kg/m^3 $\rho_{air} := 1.2$

effective reduction in mass during pumpdown, kg $\Delta m := \rho_{air} \cdot V_{OFI}$ $\Delta m = 5.6673 \times 10^{-3}$

height change due to change in effective mass, m $\Delta h := \Delta m \cdot \frac{g}{k}$

$\Delta h = 3.00157 \times 10^{-5}$

height change vs temperature

Modulus variation with temperature, Pa/degC
(ref: Lisa Bates, et al; p.9 Vol 18, #1 Journal of Undergraduate Researach in Physics, and De Salvo P070095)

$R_{Et} := 2 \cdot 10^{-4} \cdot E$

$R_{Et} = 3.68131 \times 10^{7}$

Effective spring constant variation with temp, N/m-degC

$$R_{kt} := \frac{g \cdot m_{mp}(m_{bslb}) \cdot t(m_{bslb}) \cdot FS \cdot R_{Et}}{R_{bs}(\theta_m, x_{bs}) \cdot (1 - \cos(\theta_m)) \cdot 2 \cdot S_{yieldms}}$$

$R_{kt} = 0.11815$

Effective height variation with temp, m/degC

$$R_{ht} := \frac{-m_{mp}(m_{bslb}) \cdot g \cdot R_{kt}}{k^2}$$

$R_{ht} = -5.97058 \times 10^{-6}$

Blade height change with long-term creep

ref: De Salvo P070095

blade spring elongation under load, m $\Delta_y(\theta_m) = 0.09351$

long-term creep elongation, m $y_{creep} := 0.0045 \cdot \Delta_y(\theta_m)$

$y_{creep} = 4.20785 \times 10^{-4}$

effective balance weight loss of blade due to initial creep aging, lbs

$\delta F_{lb} := k \cdot y_{creep} \cdot \frac{2.205}{g}$

$\delta F_{lb} = 0.17519$

Pendulum Frequency

length of pendulum, m

$l_{fiw} := 24.5 \cdot 0.0254$

$l_{fiw} = 0.6223$

pendulum frequency, Hz

$f_{0p} := \frac{\sqrt{\frac{g}{l_{fiw}}}}{2 \cdot \pi}$

$f_{0p} = 0.63159$

$L_v := 0$ $E_a := 0$ $T := 0$ $t := 0$

$y_m := 0$ $a := 0$ $n_0 := 0$ $\delta_y := 0$

CREEP RATE THEORY

Boltzmann's constant1.38*10^-23, J/K $k_B := 1.38 \cdot 10^{-23}$

Dislocation activation energy, J E_a

Temperature of blade, deg C T

vertical deflection of blade under load, m L_v

maximum vertical creep, m y_m

maximum creep strain, m/m $\varepsilon_m := \frac{y_m}{L_v}$

based on the DeSalvo-SURF data $\varepsilon_m = 0.004$

probability of dislocation activation $f_{Boltz}(T) := \exp\left[\frac{-E_a}{k_B \cdot (T + 273)}\right]$

time interval of applied load, day t

activation rate constant, day^-1 a

total number of available dislocations per unit vertical deflection of blade n_0

activation rate of dislocations $\frac{dn}{dt} = -a \cdot n \cdot f_{Boltz}(T)$

$$\int \frac{1}{n}\, dn = \int -a \cdot f_{Boltz}(T)\, dt$$

$$\ln\left(\frac{n(t)}{n_0}\right) = -a \cdot f_{Boltz}(T) \cdot t$$

number of dislocation events per unit vertical deflection of blade after interval t

$$n(t,T) = n_0 \cdot \exp\left(-a \cdot f_{Boltz}(T) \cdot t\right)$$

vertical deflection of blade per dislocation event δ_y

integrated vertical creep of blade after time t, m

$$y(t,T) = L_v \cdot \left(n_0 - n(t)\right) \cdot \delta_y$$

maximum vertical creep, m $y_m := L_v \cdot n_0 \cdot \delta_y$

maximum vertical creep strain, m/m ε_c

$$y(t,T) = y_m \cdot \left(1 - \exp\left(-a \cdot f_{Boltz}(T) \cdot t\right)\right)$$

Then, integrated vertical creep of blade after time t, m

$$y\left(t, T, E_a, a, y_m\right) := y_m \cdot \left[1 - \exp\left[-a \cdot \exp\left[\frac{-E_a}{k_B} \cdot \left[\frac{1}{(T + 273)}\right]\right] \cdot t\right]\right]$$

initial creep rate, m/day

$$\sigma_0\left(T, E_a, a, y_m\right) := y_m \cdot a \cdot \exp\left[\frac{-E_a}{k_B} \cdot \left[\frac{1}{(T + 273)}\right]\right]$$

Riccardo-SURF data (Ref: LIGO P070095-02-Z

initial blade deflection under load, m $L_{vsurf} := 0.336$

maximum stress, Pa $S_{vms} := 680 \cdot 10^6$

SURF Creep data

$T := 60$	$t := 41$	$y(41, 60) = 0.26 \cdot 10^{-3}$
$T := 90$	$t := 20$	$y(20, 90) = 0.56 \cdot 10^{-3}$
$T := 150$	$t := 19$	$y(19, 150) = 1.17 \cdot 10^{-3}$
$T := 170$	$t := 27$	$y(27, 170) = 1.33 \cdot 10^{-3}$
$T := 190$	$t := 14$	$y(14, 190) = 1.51 \cdot 10^{-3}$

$E_a := 0.448 \cdot 1.6 \times 10^{-19}$

$a := 3 \cdot 10^4$

$$y_{data} := \begin{pmatrix} 0.26 \cdot 10^{-3} \\ 0.56 \cdot 10^{-3} \\ 1.17 \cdot 10^{-3} \\ 1.33 \cdot 10^{-3} \\ 1.51 \cdot 10^{-3} \end{pmatrix}$$

least squares fit of activation energy, activation rate, maximum creep to creep data

First Iteration

Guess activation energy, J $E_a := 6.19401 \times 10^{-20}$

Guess activation rate constant, day^-1 $a := 4.61895 \times 10^3$

Guess maximum creep, m $y_m := 1.41077 \times 10^{-3}$

Creep theory, m

$$y(t, T, E_a, a, y_m) := y_m \cdot \left[1 - \exp\left[-a \cdot \exp\left[\frac{-E_a}{k_B} \cdot \left[\frac{1}{(T + 273)}\right]\right] \cdot t\right]\right]$$

difference between data and theory at each data point

$$\Delta_0(E_a, a, y_m) := y_{data_0} - y(41, 60, E_a, a, y_m)$$

$$\Delta_1(E_a, a, y_m) := y_{data_1} - y(20, 90, E_a, a, y_m)$$

$$\Delta_2(E_a, a, y_m) := y_{data_2} - y(19, 150, E_a, a, y_m)$$

$$\Delta_3(E_a, a, y_m) := y_{data_3} - y(27, 170, E_a, a, y_m)$$

$$\Delta_4(E_a, a, y_m) := y_{data_4} - y(14, 190, E_a, a, y_m)$$

Given

$$\left[\left(\Delta_0(E_a,a,y_m)\right)^2 + \left(\Delta_1(E_a,a,y_m)\right)^2 + \left(\Delta_2(E_a,a,y_m)\right)^2 + \left(\Delta_3(E_a,a,y_m)\right)^2 + \left(\Delta_4(E_a,a,y_m)\right)^2\right]^{0.5} = 0$$

$$\begin{pmatrix} E_{aval1} \\ a_{val1} \\ y_{mval1} \end{pmatrix} := \mathrm{Find}(E_a, a, y_m)$$

Results

$$E_{aval1} = 6.20624 \times 10^{-20}$$

$$a_{val1} = 4.73594 \times 10^{3}$$

$$y_{mval1} = 1.41003 \times 10^{-3}$$

$E_a := E_{aval1}$ $\quad E_a = 6.20624 \times 10^{-20}$

$a := a_{val1}$ $\quad a = 4.73594 \times 10^{3}$

$y_m := y_{mval1}$ $\quad y_m = 1.41003 \times 10^{-3}$

$$\left[\left(\Delta_0(E_a,a,y_m)\right)^2 + \left(\Delta_1(E_a,a,y_m)\right)^2 + \left(\Delta_2(E_a,a,y_m)\right)^2 + \left(\Delta_3(E_a,a,y_m)\right)^2 + \left(\Delta_4(E_a,a,y_m)\right)^2\right]^{0.5} = 2$$

$$y_{data_0} - y(41, 60, E_a, a, y_m) = -6.80176 \times 10^{-5}$$

Second Iteration

Guess activation energy, J $\quad E_a := E_{aval1}$

Guess activation rate constant, day^-1 $a := a_{val1}$

Guess maximum creep, m $y_m := y_{mval1}$

Given

$$\left[\left(\Delta_0(E_a,a,y_m)\right)^2 + \left(\Delta_1(E_a,a,y_m)\right)^2 + \left(\Delta_2(E_a,a,y_m)\right)^2 + \left(\Delta_3(E_a,a,y_m)\right)^2 + \left(\Delta_4(E_a,a,y_m)\right)^2\right]^{0.5} = 0$$

Results

$$\begin{pmatrix} E_{aval2} \\ a_{val2} \\ y_{mval2} \end{pmatrix} := \text{Find}(E_a, a, y_m)$$

$$E_{aval2} = 6.20628 \times 10^{-20}$$

$$a_{val2} = 4.73629 \times 10^{3}$$

$$y_{mval2} = 1.41003 \times 10^{-3}$$

$$\left[\left(\Delta_0(E_a,a,y_m)\right)^2 + \left(\Delta_1(E_a,a,y_m)\right)^2 + \left(\Delta_2(E_a,a,y_m)\right)^2 + \left(\Delta_3(E_a,a,y_m)\right)^2 + \left(\Delta_4(E_a,a,y_m)\right)^2\right]^{0.5} = 2$$

$$y_{data_0} - y(41, 60, E_a, a, y_m) = -6.80176 \times 10^{-5}$$

$$E_{asurfy} := E_{aval2}$$

$$E_{asurfy\_ev} := \frac{E_{asurfy}}{\left(1.6 \times 10^{-19}\right)}$$

$$a_{surfy} := a_{val2}$$

$$y_{msurfy} := y_{mval2}$$

$$\varepsilon_{msurfy} := \frac{y_{msurfy}}{L_{vsurf}}$$

activation energy, J $E_{asurfy} = 6.20628 \times 10^{-20}$

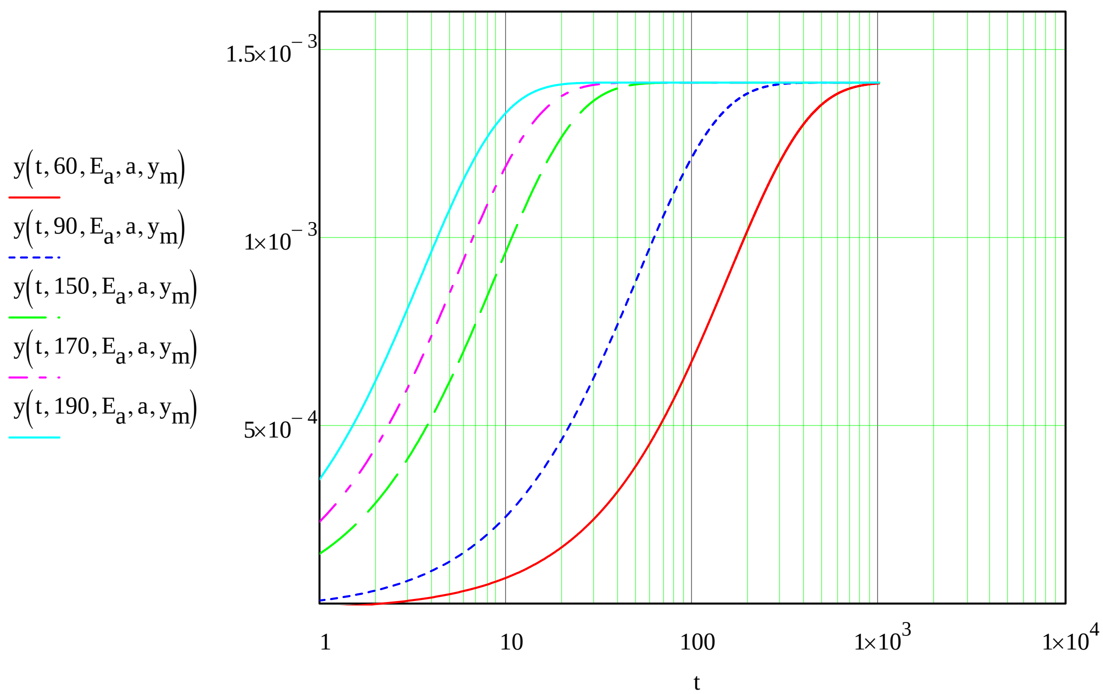
activation energy, eV $E_{asurfy\_ev} = 0.38789$

activation rate constant, day^-1 $a_{surfy} = 4.73629 \times 10^{3}$

maximum creep, m $y_{msurfy} = 1.41003 \times 10^{-3}$

maximum vertical creep strain, m/m $\varepsilon_{msurfy} = 4.19651 \times 10^{-3}$

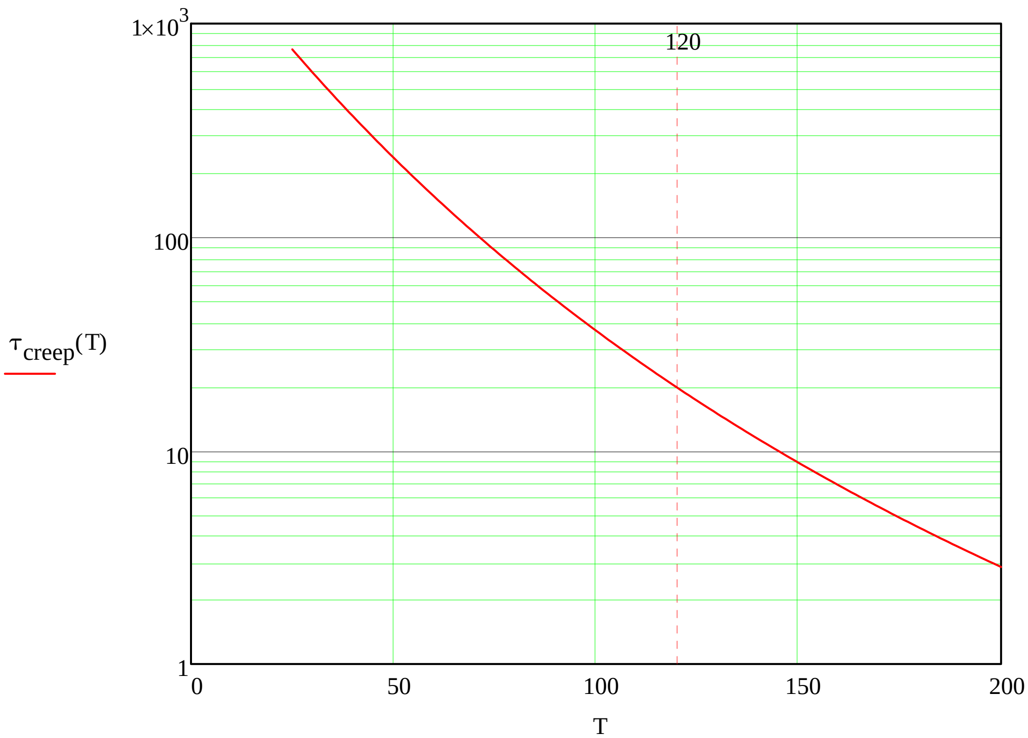
$t := 1, 1.1 .. 1000$



$e^{-1} = 0.36788$ $\quad T := 30$

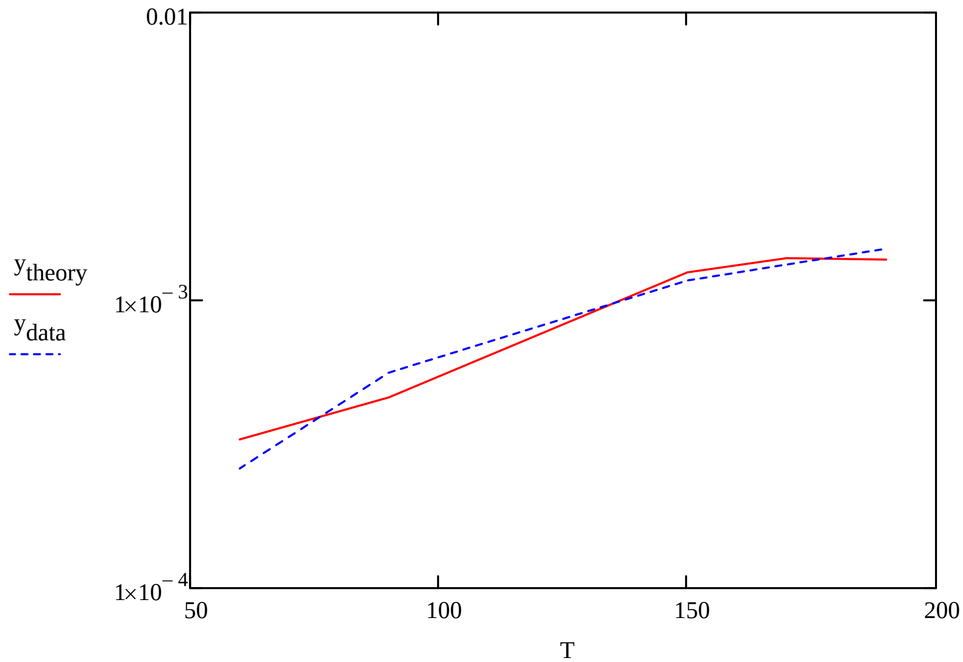
Creep relaxation time, days $\quad \tau_{creep}(T) := \dfrac{1}{\left[a \cdot \exp\left[\dfrac{-E_a}{k_B} \cdot \left[\dfrac{1}{(T + 273)}\right]\right]\right]}$

$T := 25, 26 .. 200$



Riccardo-Surf Data compared with theory for each temperature run

$$T := \begin{pmatrix} 60 \\ 90 \\ 150 \\ 170 \\ 190 \end{pmatrix} \qquad y_{data} := \begin{pmatrix} 0.26 \cdot 10^{-3} \\ 0.56 \cdot 10^{-3} \\ 1.17 \cdot 10^{-3} \\ 1.33 \cdot 10^{-3} \\ 1.51 \cdot 10^{-3} \end{pmatrix} \qquad y_{theory} := \begin{pmatrix} y(41, 60, E_a, a, y_m) \\ y(20, 90, E_a, a, y_m) \\ y(19, 150, E_a, a, y_m) \\ y(27, 170, E_a, a, y_m) \\ y(14, 190, E_a, a, y_m) \end{pmatrix}$$



VIRGO data

initial deflection of blade under load, m $L_{vvirgo} := 0.1$

maximum stress, N/mm^2 $S_{Nmm2} := 1250$

maximum stress, Pa $S_{wms} := S_{Nmm2} \cdot 10^6$

$S_{wms} = 1.25 \times 10^9$

VIRGO Creep data

$T := 35$ $t := 12.5$ $y(12.5, 35) = 90 \cdot 10^{-6}$

$T := 50$ $t := 12.5$ $y(12.5, 50) = 180 \cdot 10^{-6}$

$T := 65$ $t := 12.5$ $y(12.5, 65) = 210 \cdot 10^{-6}$

$T := 80$ $t := 12.5$ $y(12.5, 80) = 190 \cdot 10^{-6}$

First Iteration

least squares fit of activation energy, activation rate, maximum creep to creep data

theoretical creep vs time, m/day

$$y(t, T, E_a, a, y_m) := y_m \cdot \left[1 - \exp\left[-a \cdot \exp\left[\frac{-E_a}{k_B} \cdot \left[\frac{1}{(T + 273)}\right]\right] \cdot t\right]\right]$$

difference between data and theory at each data point

$$\Delta_0(E_a, a, y_m) := 90 \cdot 10^{-6} - y(12.5, 35, E_a, a, y_m)$$

$$\Delta_1(E_a, a, y_m) := 180 \cdot 10^{-6} - y(12.5, 50, E_a, a, y_m)$$

$$\Delta_2(E_a, a, y_m) := 210 \cdot 10^{-6} - y(12.5, 65, E_a, a, y_m)$$

$$\Delta_3(E_a, a, y_m) := 190 \cdot 10^{-6} - y(12.5, 80, E_a, a, y_m)$$

Guess activation energy, J $E_a := 6.66316 \times 10^{-20}$

Guess activation rate constant, day^-1 $a := 3.85973 \times 10^{5}$

Guess maximum creep, m $y_m := 2.06746 \cdot 10^{-4}$

Given

$$\left[\left(\Delta_0(E_a,a,y_m)\right)^2 + \left(\Delta_1(E_a,a,y_m)\right)^2 + \left(\Delta_2(E_a,a,y_m)\right)^2 + \left(\Delta_3(E_a,a,y_m)\right)^2\right]^{0.5} = 0$$

Results

$$\begin{pmatrix} E_{aval1} \\ a_{val1} \\ y_{mval1} \end{pmatrix} := \text{Find}(E_a, a, y_m)$$

$E_{aval1} = 6.72444 \times 10^{-20}$

$a_{val1} = 4.45078 \times 10^{5}$

$y_{mval1} = 2.06646 \times 10^{-4}$

$$\left[\left(\Delta_0(E_a,a,y_m)\right)^2 + \left(\Delta_1(E_a,a,y_m)\right)^2 + \left(\Delta_2(E_a,a,y_m)\right)^2 + \left(\Delta_3(E_a,a,y_m)\right)^2\right]^{0.5} = 3.2955 \times 10^{-5}$$

$$90\cdot 10^{-6} - y(12.5, 35, E_a, a, y_m) = -1.91143 \times 10^{-5}$$

Second Iteration

Guess activation energy, J $E_a := E_{aval1}$

Guess activation rate constant, day^-1 $a := a_{val1}$

Guess maximum creep, m $y_m := y_{mval1}$

Given

$$\left[\left(\Delta_0(E_a,a,y_m)\right)^2 + \left(\Delta_1(E_a,a,y_m)\right)^2 + \left(\Delta_2(E_a,a,y_m)\right)^2 + \left(\Delta_3(E_a,a,y_m)\right)^2\right]^{0.5} = 0$$

Results

$$\begin{pmatrix} E_{aval2} \\ a_{val2} \\ y_{mval2} \end{pmatrix} := \text{Find}(E_a, a, y_m)$$

$E_{aval2} = 6.78326 \times 10^{-20}$

$a_{val2} = 5.09497 \times 10^{5}$

$y_{mval2} = 2.06456 \times 10^{-4}$

$$\left[\left(\Delta_0(E_a, a, y_m)\right)^2 + \left(\Delta_1(E_a, a, y_m)\right)^2 + \left(\Delta_2(E_a, a, y_m)\right)^2 + \left(\Delta_3(E_a, a, y_m)\right)^2\right]^{0.5} = 3.26085 \times 10^{-5}$$

$$90 \cdot 10^{-6} - y(12.5, 35, E_a, a, y_m) = -1.89376 \times 10^{-5}$$

$E_a := E_{aval2}$ $\quad E_{avirgoy} := E_{aval2}$

$$E_{avirgoy\_ev} := \frac{E_{avirgoy}}{\left(1.6 \times 10^{-19}\right)}$$

$a := a_{val2}$ $\quad a_{virgoy} := a_{val2}$

$y_m := y_{mval2}$ $\quad y_{mvirgoy} := y_{mval2}$

$$\varepsilon_{mvirgoy} := \frac{y_{mvirgoy}}{L_{vvirgo}}$$

activation energy, J $\quad E_{avirgoy} = 6.78326 \times 10^{-20}$

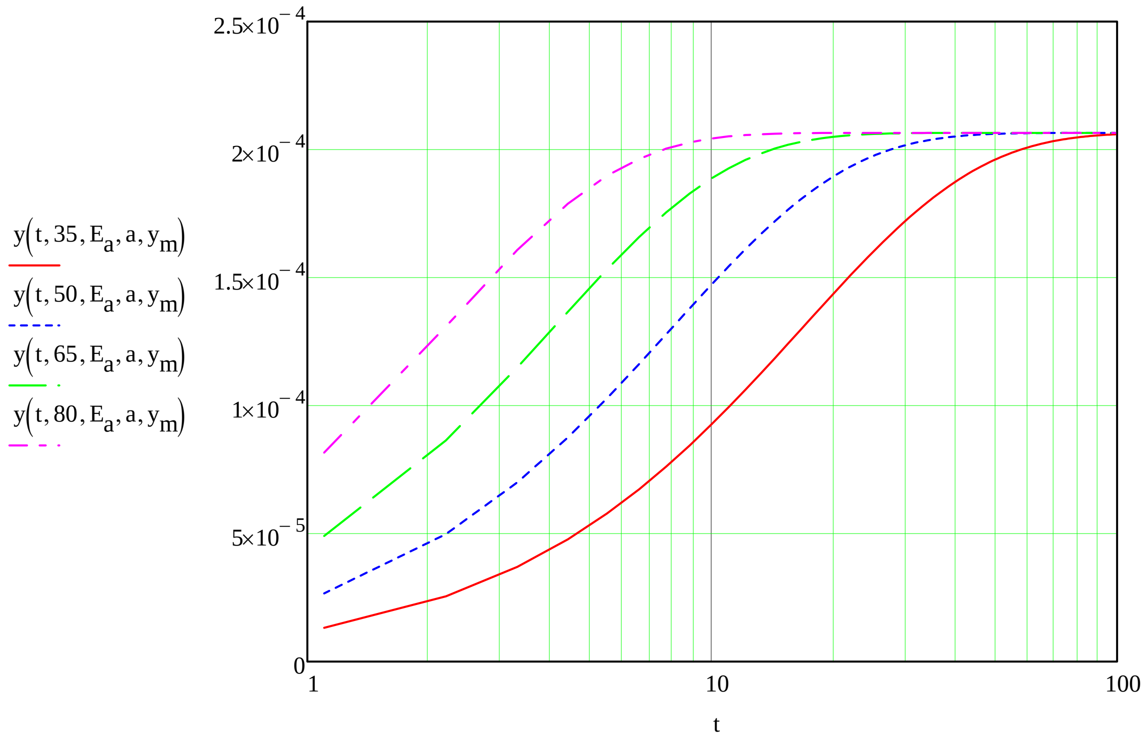
activation energy, eV $\quad E_{avirgoy\_ev} = 0.42395$

activation rate constant, day^-1 $\quad a_{virgoy} = 5.09497 \times 10^{5}$

maximum creep, m $\quad y_{mvirgoy} = 2.06456 \times 10^{-4}$

maximum vertical creep strain, m/m $\quad \varepsilon_{mvirgoy} = 2.06456 \times 10^{-3}$

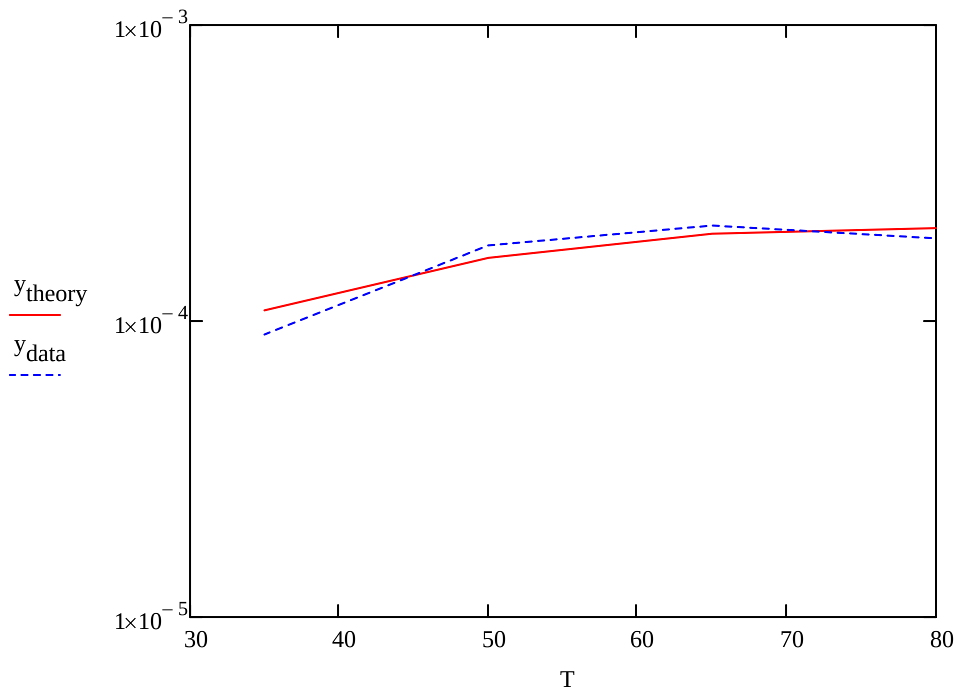
$t := 0, 1.1 .. 100$



VIRGO creep Data compared with theory for each temperature run

$$y(t,T,E_a,a,y_m) := y_m \cdot \left[1 - \exp\left[-a \cdot \exp\left[\frac{-E_a}{k_B} \cdot \left[\frac{1}{(T+273)}\right]\right] \cdot t\right]\right]$$

$$T := \begin{pmatrix} 35 \\ 50 \\ 65 \\ 80 \end{pmatrix} \qquad y_{theory} := \begin{pmatrix} y(12.5, 35, E_a, a, y_m) \\ y(12.5, 50, E_a, a, y_m) \\ y(12.5, 65, E_a, a, y_m) \\ y(12.5, 80, E_a, a, y_m) \end{pmatrix} \qquad y_{data} := \begin{pmatrix} 90 \cdot 10^{-6} \\ 180 \cdot 10^{-6} \\ 210 \cdot 10^{-6} \\ 190 \cdot 10^{-6} \end{pmatrix}$$



VIRGO Initial Creep Rate data

$T := 35 \qquad \sigma_{0.35} := \frac{200 \cdot 24 \cdot 10^{-6}}{255} \qquad \sigma_{0.35} = 1.88235 \times 10^{-5}$

$T := 50 \qquad \sigma_{0.50} := \frac{200 \cdot 24 \cdot 10^{-6}}{70} \qquad \sigma_{0.50} = 6.85714 \times 10^{-5}$

$T := 65 \qquad \sigma_{0.65} := \frac{200 \cdot 24 \cdot 10^{-6}}{60} \qquad \sigma_{0.65} = 8 \times 10^{-5}$

$T := 80$ $\sigma_{0.80} := \frac{200 \cdot 24 \cdot 10^{-6}}{40}$ $\sigma_{0.80} = 1.2 \times 10^{-4}$

$T := \begin{pmatrix} 35 \\ 50 \\ 65 \end{pmatrix}$ $\sigma_{0.data} := \begin{pmatrix} 1.88235 \times 10^{-5} \\ 6.85714 \times 10^{-5} \\ 8 \times 10^{-5} \end{pmatrix}$

First Iteration

least squares fit of activation energy and activation rateto creep data

maximum creep, m $y_m := y_{mvirgoy}$

$y_m = 2.06456 \times 10^{-4}$

theoretical initial creep rate, m/day

$$\sigma_0(T, E_a, a, y_m) := y_m \cdot a \cdot \exp\left[\frac{-E_a}{k_B} \cdot \left[\frac{1}{(T + 273)}\right]\right]$$

difference between data and theory at each data point

$\Delta_0(E_a, a, y_m) := \sigma_{0.data_0} - \sigma_0(35, E_a, a, y_m)$

$\Delta_1(E_a, a, y_m) := \sigma_{0.data_1} - \sigma_0(50, E_a, a, y_m)$

$\Delta_2(E_a, a, y_m) := \sigma_{0.data_2} - \sigma_0(65, E_a, a, y_m)$

Guess activation energy, J $E_a := 4.78712 \times 10^{-20}$

Guess activation rate constant, day^-1 $a := 1.1818 \times 10^4$

Given

$$\left[\left(\Delta_0(E_a, a, y_m)\right)^2 + \left(\Delta_1(E_a, a, y_m)\right)^2 + \left(\Delta_2(E_a, a, y_m)\right)^2 + \left(\Delta_3(E_a, a, y_m)\right)^2\right]^{0.5} = 0$$

$$\begin{pmatrix} E_{aval1} \\ a_{val1} \end{pmatrix} := \mathrm{Find}(E_a, a)$$

$$E_{aval1} = 4.7627 \times 10^{-20}$$

$$a_{val1} = 1.11938 \times 10^{4}$$

$$\left[\left(\Delta_0(E_a, a, y_m)\right)^2 + \left(\Delta_1(E_a, a, y_m)\right)^2 + \left(\Delta_2(E_a, a, y_m)\right)^2 + \left(\Delta_3(E_a, a, y_m)\right)^2\right]^{0.5} = 2.64224 \times 10^{-5}$$

Second Iteration

Guess activation energy, J $E_a := E_{aval1}$

Guess activation rate constant, day^-1 $a := a_{val1}$

Given

$$\left[\left(\Delta_0(E_a, a, y_m)\right)^2 + \left(\Delta_1(E_a, a, y_m)\right)^2 + \left(\Delta_2(E_a, a, y_m)\right)^2 + \left(\Delta_3(E_a, a, y_m)\right)^2\right]^{0.5} = 0$$

Results

$$\begin{pmatrix} E_{aval2} \\ a_{val2} \end{pmatrix} := \mathrm{Find}(E_a, a)$$

$$E_{aval2} = 4.75926 \times 10^{-20}$$

$$a_{val2} = 1.11096 \times 10^{4}$$

$$\left[\left(\Delta_0(E_a, a, y_m)\right)^2 + \left(\Delta_1(E_a, a, y_m)\right)^2 + \left(\Delta_2(E_a, a, y_m)\right)^2 + \left(\Delta_3(E_a, a, y_m)\right)^2\right]^{0.5} = 2.6421 \times 10^{-5}$$

$E_a := E_{aval2}$ $E_{avirgo\sigma} := E_{aval2}$

$$E_{avirgo\sigma\_ev} := \frac{E_{avirgo\sigma}}{\left(1.6 \times 10^{-19}\right)}$$

$a := a_{val2}$ $\quad a_{virgo\sigma} := a_{val2}$

$y_m := y_{mval2}$ $\quad y_{mvirgo\sigma} := y_{mval2}$

maximum strain, m/m $\quad \varepsilon_{mvirgo\sigma} := \frac{y_{mvirgo\sigma}}{L_{vvirgo}}$ $\quad \varepsilon_{mvirgo\sigma} = 2.06456 \times 10^{-3}$

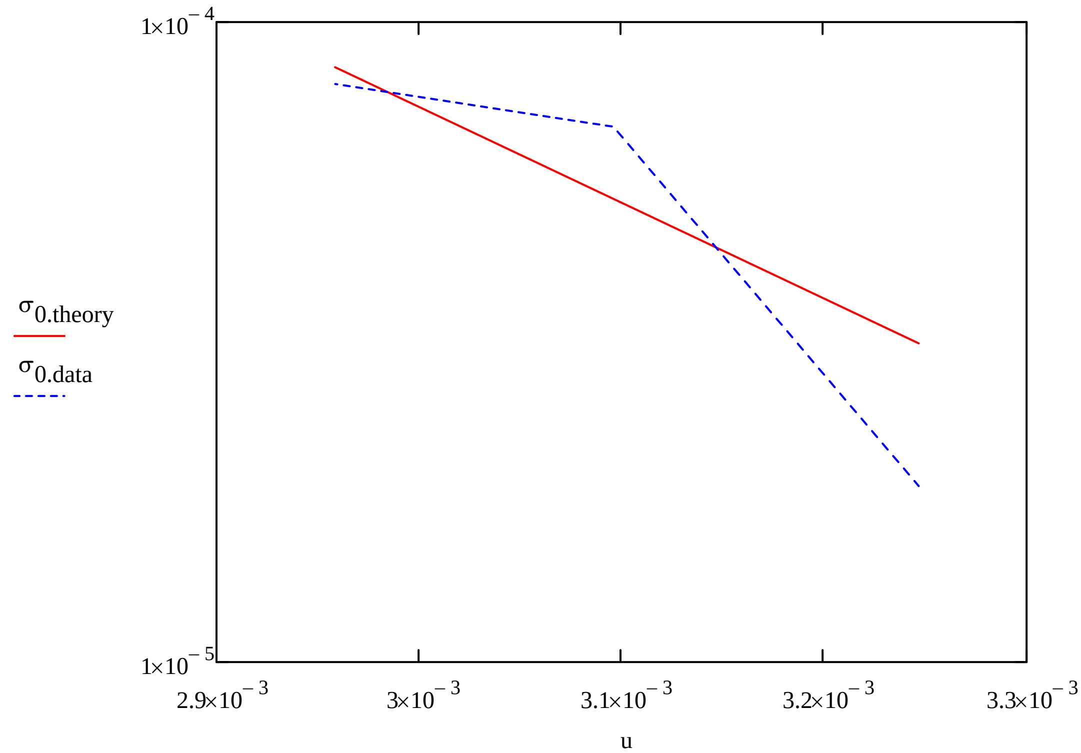
activation energy, J $\quad E_{avirgo\sigma} = 4.75926 \times 10^{-20}$

activation energy, eV $\quad E_{avirgo\sigma\_ev} = 0.29745$

activation rate constant, day^-1 $\quad a_{virgo\sigma} = 1.11096 \times 10^{4}$

$$\sigma_{0.theory} := \begin{pmatrix} \sigma_0\left(35, E_a, a, y_m\right) \\ \sigma_0\left(50, E_a, a, y_m\right) \\ \sigma_0\left(65, E_a, a, y_m\right) \end{pmatrix}$$

inverse absolute temp, K^-1 $\quad u := \frac{1}{T + 273}$



Summary of SURF and VIRGO parameters

$a_{virgoy} = 5.09497 \times 10^{5}$

$a_{virgo\sigma} = 1.11096 \times 10^{4}$

$a_{surfy} = 4.73629 \times 10^{3}$

$E_{avirgoy} = 6.78326 \times 10^{-20}$

$E_{avirgoy\_ev} = 0.42395$

$E_{avirgo\sigma} = 4.75926 \times 10^{-20}$

$E_{avirgo\sigma\_ev} = 0.29745$

$E_{asurfy} = 6.20628 \times 10^{-20}$

$E_{asurfy\_ev} = 0.38789$

$y_{mvirgoy} = 2.06456 \times 10^{-4}$

$y_{msurfy} = 1.41003 \times 10^{-3}$

$\varepsilon_{mvirgoy} = 2.06456 \times 10^{-3}$

$\varepsilon_{msurfy} = 4.19651 \times 10^{-3}$

SLC Data

maximum stress, Pa $S_{wms} = 1.25 \times 10^{9}$

maximum stress, N/mm^2 $S_{Nmm2} := \frac{S_{wms}}{10^{6}}$

$S_{Nmm2} = 1.25 \times 10^{3}$

loaded deflection of blade, m $Lvofi := 0.09351$

creep parameters based on VIRGO Data

maximum strain, m/m $\varepsilon_{mofi} := \varepsilon_{mvirgoy}$ $\varepsilon_{mofi} = 2.06456 \times 10^{-3}$

maximum creep, m $y_m := Lvofi \cdot \varepsilon_{mofi}$ $y_m = 1.93057 \times 10^{-4}$

activation energy, J $E_a := E_{avirgoy}$ $E_a = 6.78326 \times 10^{-20}$

activation rate constant, day^-1 $a := a_{virgoy}$ $a = 5.09497 \times 10^{5}$

theoretical creep vs time, m/day

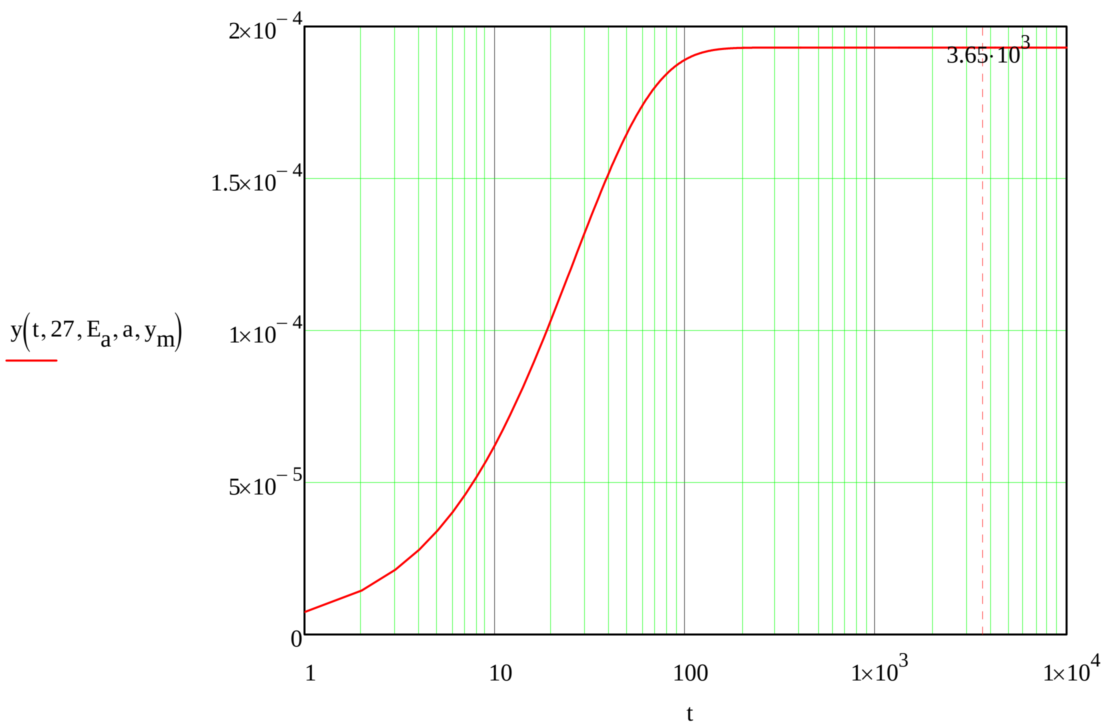
use the VIRGO parameters

$$y(t, T, E_a, a, y_m) := y_m \cdot \left[1 - \exp\left[-a \cdot \exp\left[\frac{-E_a}{k_B} \cdot \left[\frac{1}{(T + 273)}\right]\right] \cdot t\right]\right]$$

10 year time period, days $t := 10 \cdot 365$ $t = 3.65 \times 10^3$

maximum creep @ 27 deg C for 10 years, m $y(3.65 \times 10^3, 27, E_a, a, y_m) = 1.93057 \times 10^{-4}$

$t := 1, 2 .. 10000$



$.04298 \times 10^{-4}$

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