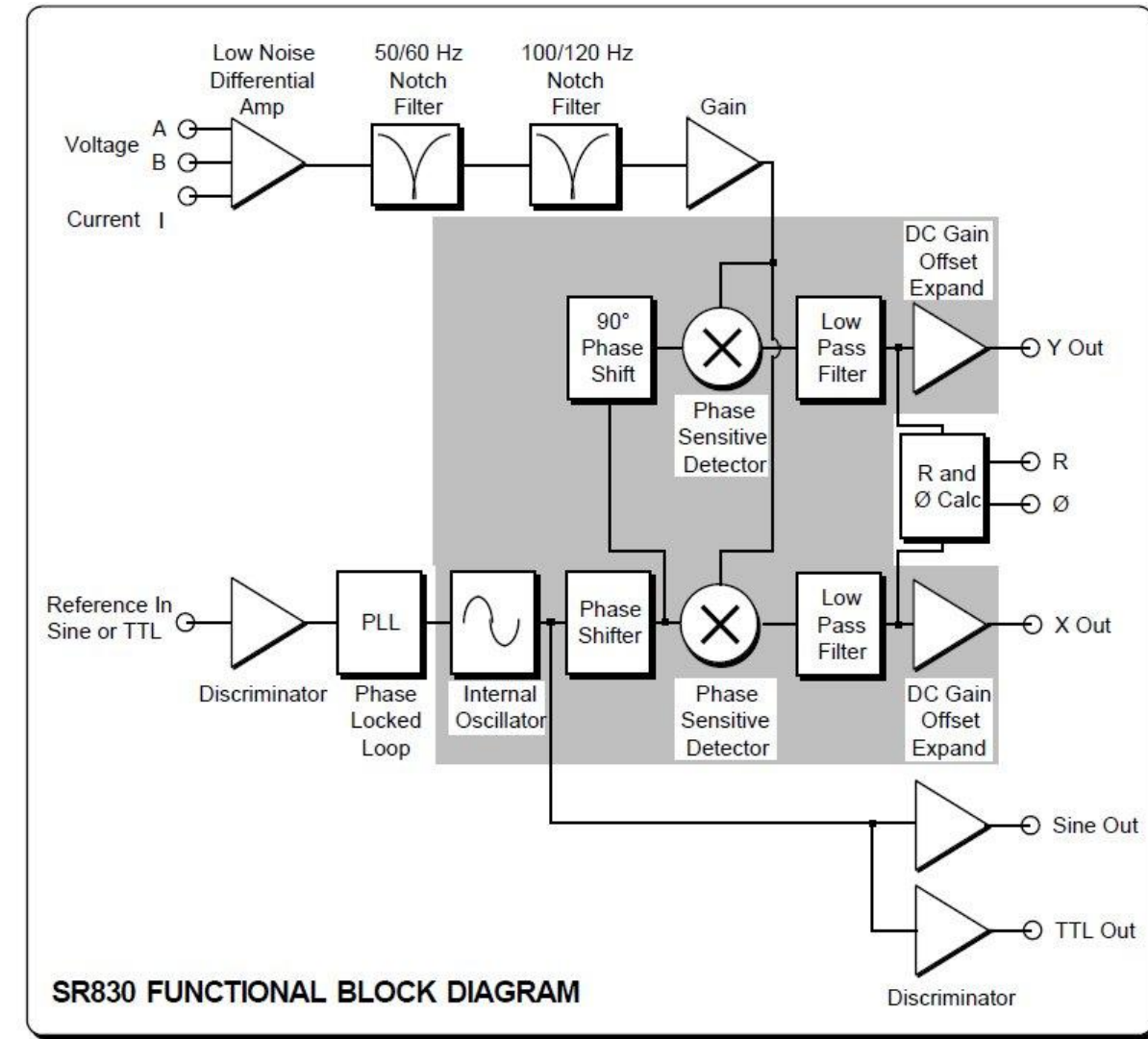


Lock-in Amplifier Measurements for the Intermediate Lab

by Jonathan Newport
American University, Washington DC

Introduction

Electronic Instrumentation students learn to measure AC waveforms with a digital multimeter and an oscilloscope, but often fail to realize that often the measurements given by these devices provide values that are not at the frequency of interest. This laboratory exercise is intended to introduce students to the powerful technique of phase sensitive detection with a lock-in amplifier.



Experiment 0 – Theory

The goal is to measure an AC signal with known angular frequency ω_{sig} , and an unknown amplitude A_{sig} phase ϕ_{sig} . If we apply some known gain G_{sig} to this signal, this wave may be represented by,

$$V_{sig}(t) = G_{sig} A_{sig} \sin(\omega_{sig} t + \phi_{sig})$$

Given a reference wave of known frequency ω_{ref} , amplitude A_{ref} and phase ϕ_{ref} ,

$$V_{ref}(t) = A_{ref} \sin(\omega_{ref} t + \phi_{ref})$$

The expression for the voltage output of a phase sensitive detector is found by multiplying the two waves and using a trig identity,

$$V_{psd}(t) = \frac{1}{2} G_{sig} A_{sig} A_{ref} [\cos((\omega_{sig} - \omega_{ref})t + \phi_{sig} - \phi_{ref}) - \cos((\omega_{sig} + \omega_{ref})t + \phi_{sig} + \phi_{ref})]$$

If the signal waveform is of the same frequency as the reference waveform, i.e. $\omega_{sig} = \omega_{ref}$, and the “reference phase” is zero $\phi_{ref} = 0$, the resulting waveform $V_X(t)$ is an AC waveform at twice the frequency with a DC offset that is proportional to the input amplitude A_{sig} and the input phase ϕ_{sig} . This is the “in-phase” component of the input signal,

$$V_X(t) = \frac{1}{2} G_{sig} A_{sig} A_{ref} [\cos(\phi_{sig}) - \cos(2\omega_{sig} t + \phi_{sig})]$$

Similarly, if the “reference phase” is shifted by ninety degrees $\phi_{ref} = -90^\circ$, the resulting waveform $V_Y(t)$ is the “quadrature component” of the input signal,

$$V_Y(t) = \frac{1}{2} G_{sig} A_{sig} A_{ref} [\sin(\phi_{sig}) + \cos(2\omega_{sig} t + \phi_{sig})]$$

If these two signals are subjected to a low pass filter with a sufficiently low time constant $\tau = 1/\omega_c$, then two DC voltages proportional *only* to the input signal amplitude and phase result. From these in phase and quadrature components, one can solve for the input signal amplitude A_{sig} and the input phase ϕ_{sig} with respect to a reference phase of zero,

$$A_{sig} = \frac{2\sqrt{V_X^2 + V_Y^2}}{G_{sig} A_{ref}} \quad \text{and} \quad \phi_{sig} = \arctan\left(\frac{V_Y}{V_X}\right)$$



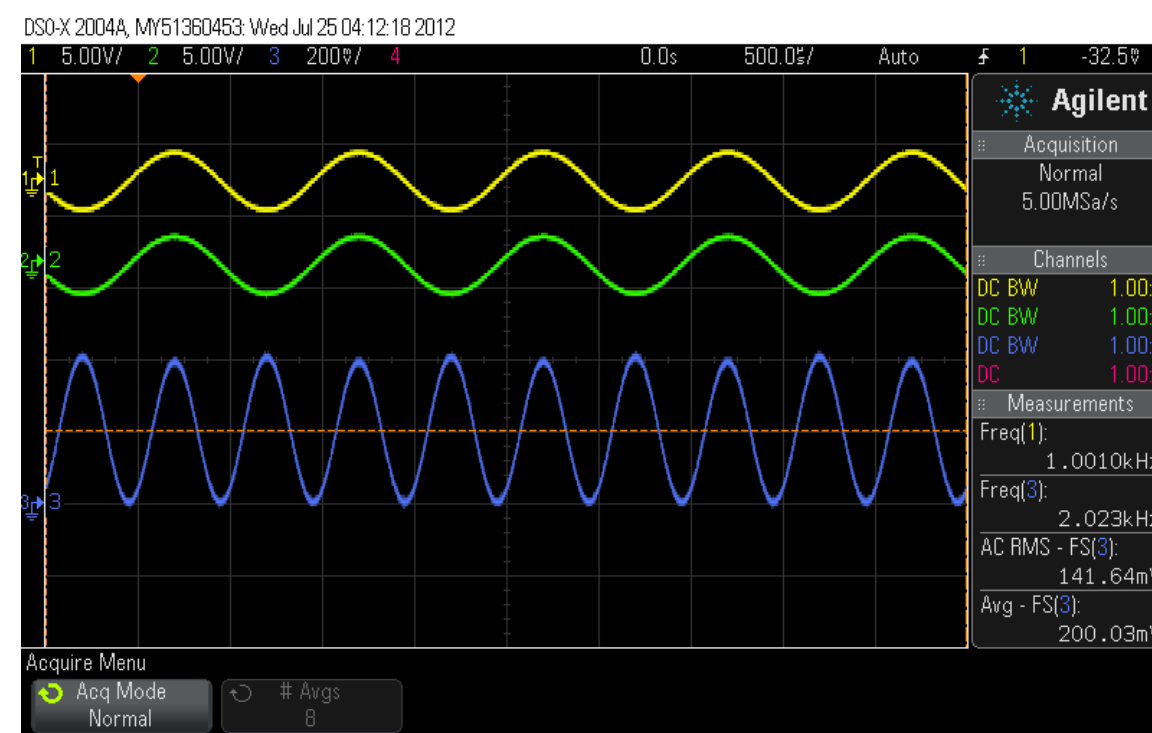
The Rigol DG1022 (~\$375) is a dual-channel function generator that can be phase aligned and computer controlled via USB. This cheap generator is also great for producing lissajous patterns.

Experiment 1 – Hardware Multiplier

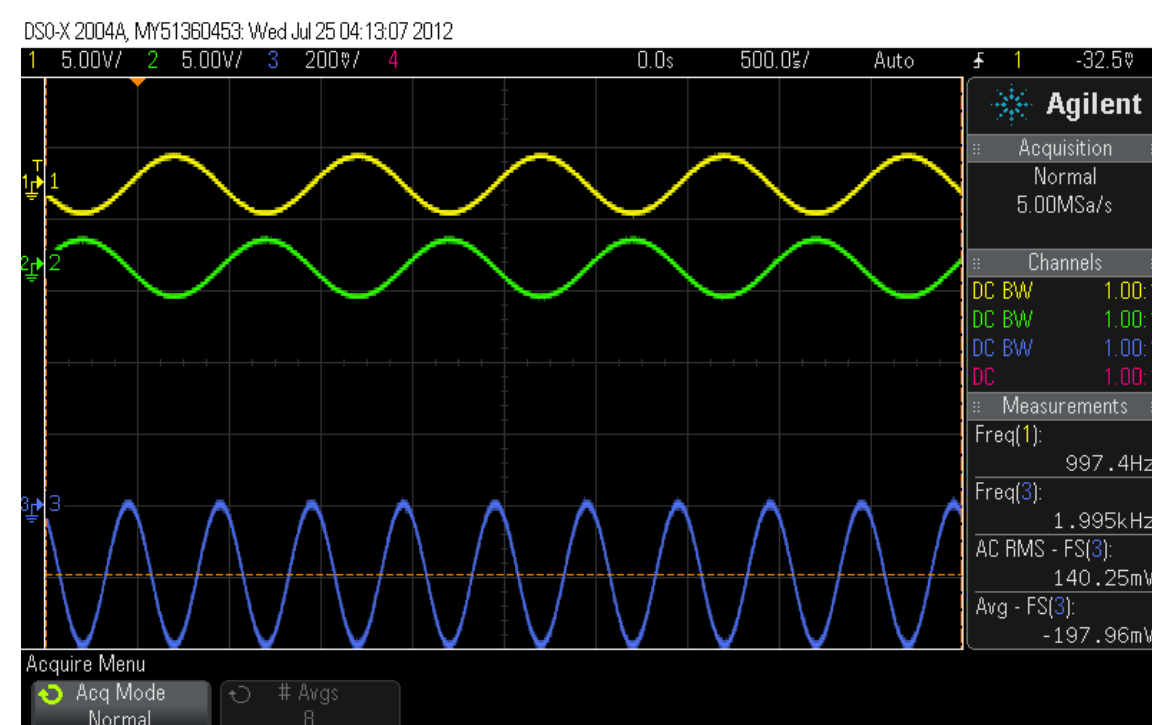
To get a “hands-on” feel for how the phase-sensitive detector operates, students construct a phase-sensitive detector with an AD633 four-quadrant hardware multiplier and a dual-channel function generator with relative phase control. A “reference” waveform is provided to one of the chip’s inputs and is considered to have zero phase offset. A “signal” waveform is fed to the other input and the output waveform is displayed along with both signals on a four-channel oscilloscope.

Students are asked to describe what occurs at the output when the phase difference, input amplitudes and input frequencies are changed. Furthermore, they are asked to describe the frequency content of the multiplied waveform if it were passed through low-pass filters of various corner frequencies.

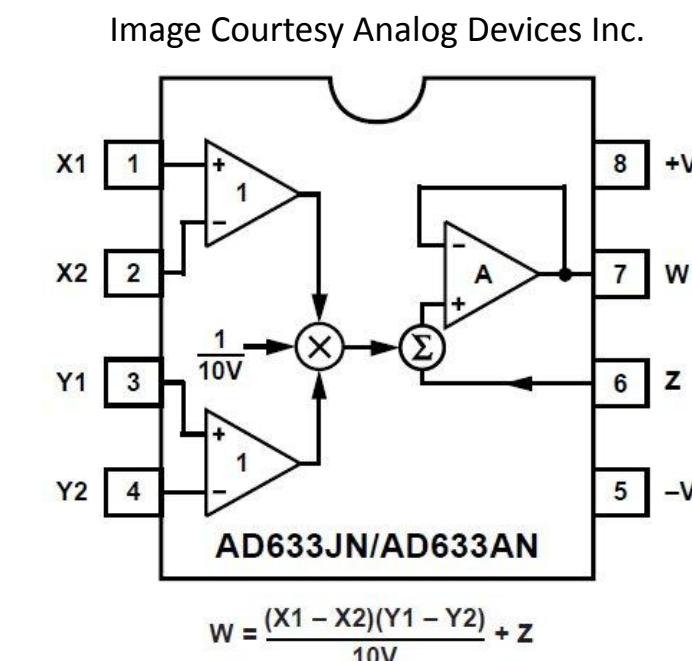
Input amplification and calibration of the output are not performed, but could easily be implemented if a fully functional, student-constructed discrete lock-in amplifier were desired.



Input signal (top trace) and reference signal (middle trace) of the same frequency and phase aligned. Output of hardware multiplier is the bottom trace. Note the DC offset - this is proportional to the input signal amplitude, the reference signal amplitude and the phase difference between them.



Input signal (top trace) and reference signal (middle trace) of the same frequency and 180° out of phase. Output of hardware multiplier is the bottom trace. Note that the DC offset is now negative.

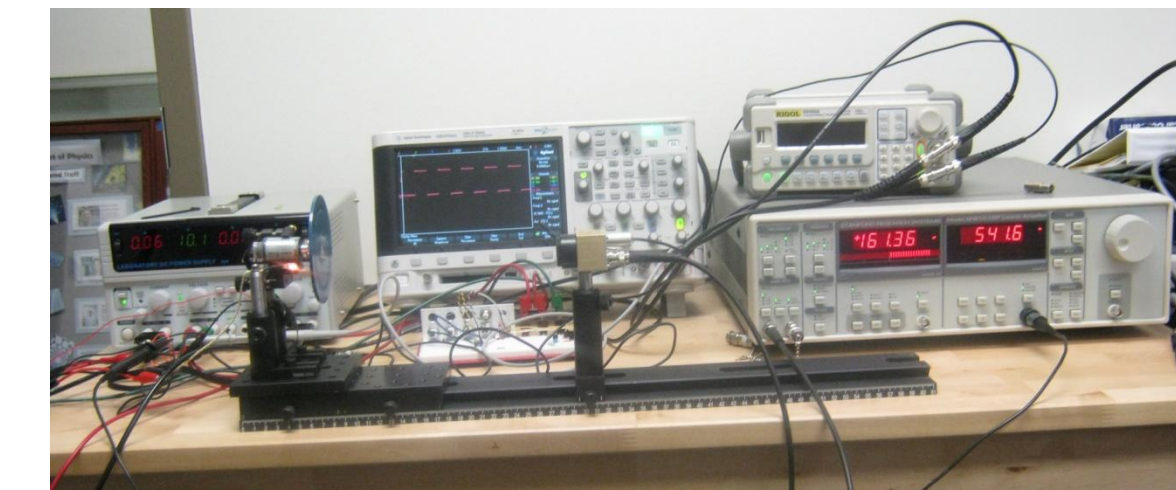


AD633 Four Quadrant Hardware multiplier. Useful for many of one’s AC measurement needs!

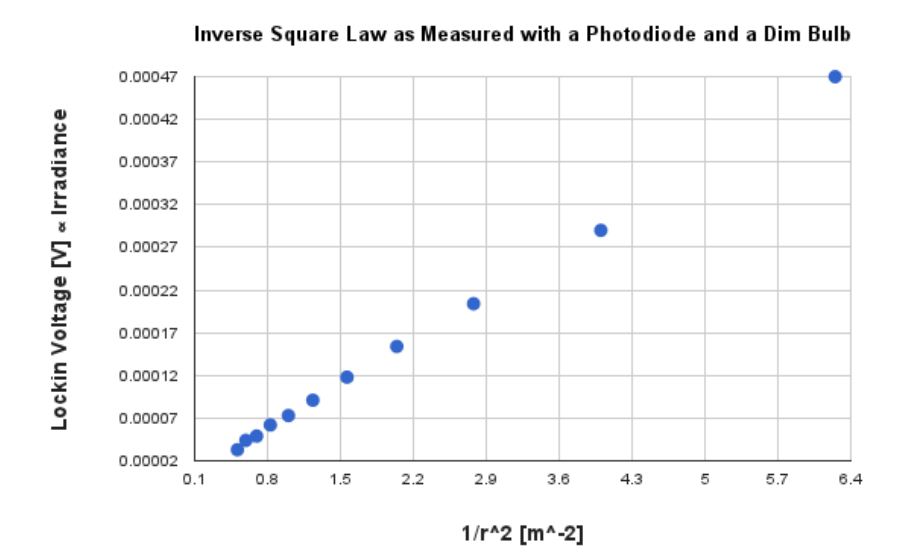
Experiment 2 – The SR830 as an AC Voltmeter

Students are given a *small* ac waveform to the voltage input and the sync signal from their function generator to the reference input of the SR830 lock-in amplifier. The waveform is also displayed on an oscilloscope and the RMS voltage read from a Fluke Multimeter.

The same signal is then “buried” in noise. Students are asked to make the measurements with the three instruments again, and it is very clear which instrument is providing the desired measurement! The time constant is also altered and its effect is also considered.



Lab station setup. Quad power supply is useful for supplying split power supply in addition to providing power for optical chopper and lamp. Breadboard with hardware multiplier can be seen behind optical rail.



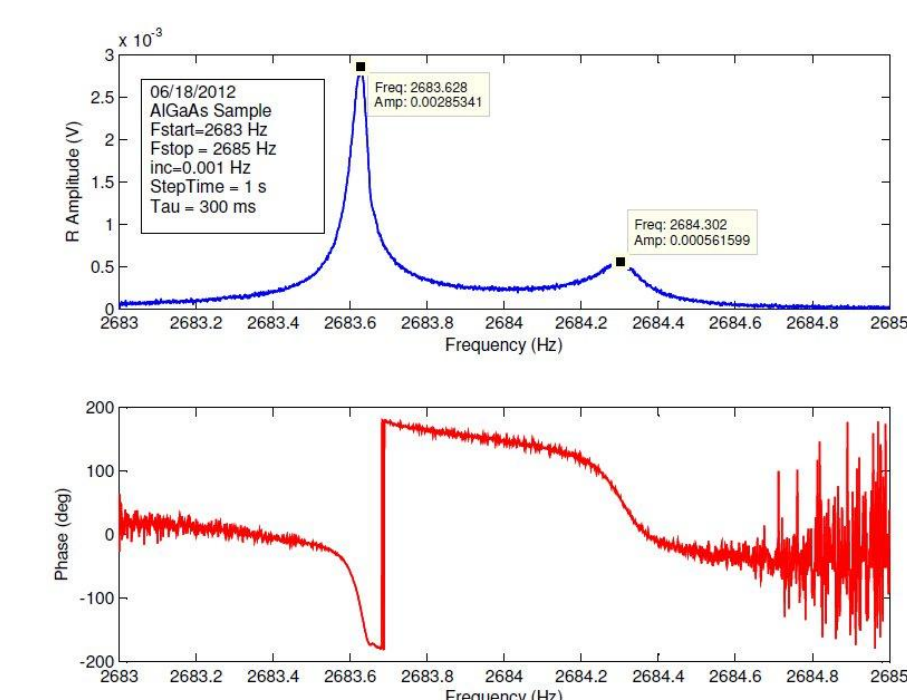
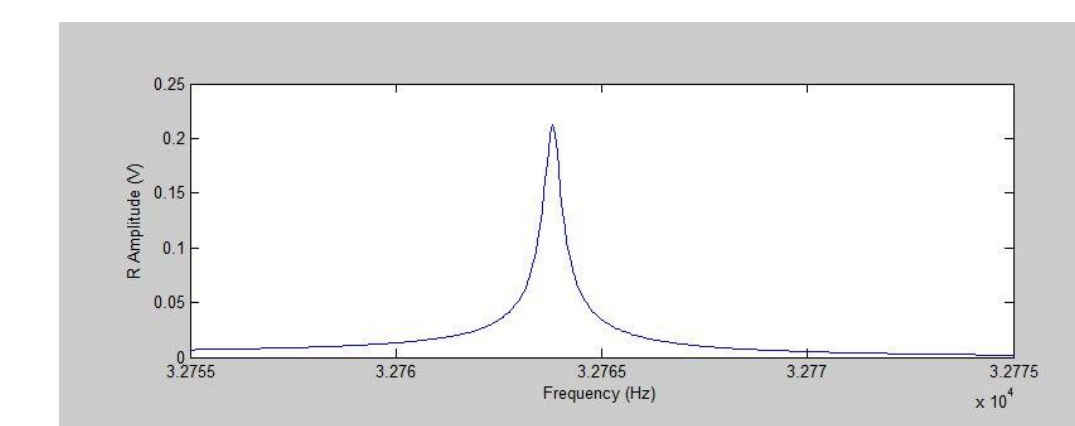
Students acquire inverse-square law data from a dim bulb in a lighted room with an optical chopper. Depending on chopper jitter (among other things), valid data can be obtained from across the room.

Experiment 3 – Optical Chopping

The classic optical chopping experiment of demonstrating the inverse square law is then performed with a homemade optical chopper. Students are asked to attempt the collection of inverse-square law data with and without chopping. It is very clear which technique is actually detecting the bulb’s light! Students are also surprised at the distance one can take data (often this is limited by cable length) and in light of a wobbly chopper.

Experiment 4 – Bode Plots

A MATLAB script is used to effectively make a signal analyzer out of a lock-in amplifier and a function generator. Frequency sweeps and associated amplitude and phase response curves can be obtained for systems of interest. Students are asked to produce Bode Plots for low-pass, high-pass and bandpass filters using this tool. Furthermore, the quality factor of resonant circuits can easily be found using the FWHM of the appropriate peaks.



Crystal Q – the resonant peak of a tuning-fork watch crystal is gathered using a spectrum analysis technique. Note the “parallel resonance” is not shown to avoid confusion.

LIGO Research – the normal modes of a suspended silica flat are found using a lock-in spectrum analysis technique.

Research Experiments – LIGO Q Measurements

The function generator and lock-in amplifier combination can be a powerful spectrum analysis tool when brought under computer control. The mechanical strain of silica flats is inversely proportional to the quality factor of the normal modes of this silica flat. Because the Q is so high, the normal modes can be challenging to ring up and find, even for a dynamic signal analyzer. If a computer is used to control the frequency and readout of the system, then arbitrarily small step sizes and long integration times can be achieved without human supervision. Furthermore, the quality factor can be estimated from the width of this peak with suitable input parameters. This has proven itself to be a useful tool in determining mechanical loss in various materials in the lab.