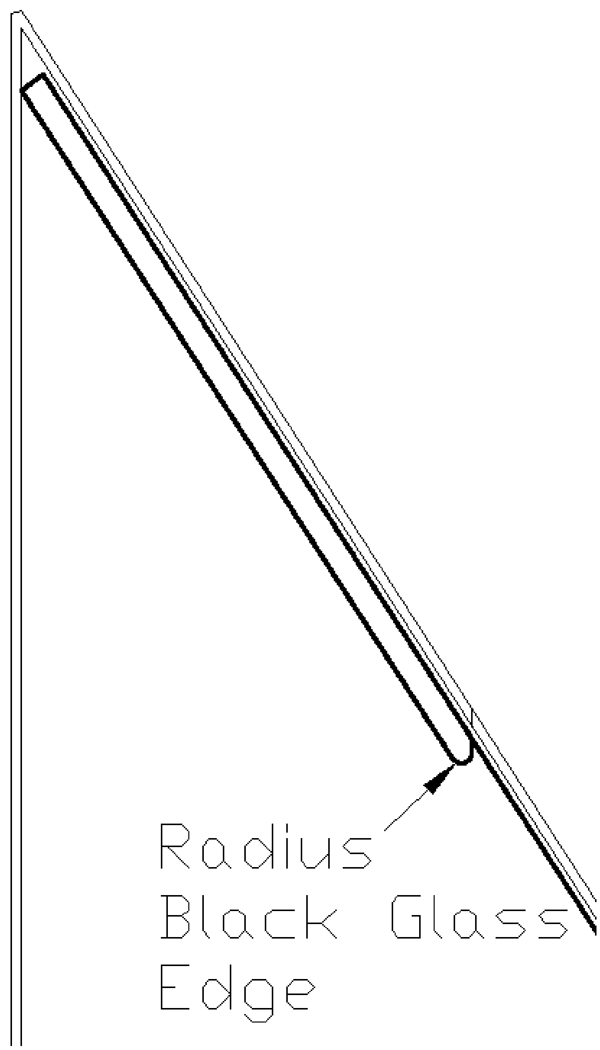
T1300689 ITM Elliptical Baffle Scatter_Black Glass Hybrid
8/15/13



BRDF of ellip baf SS, sr^-1 $BRDF_{ellbafss} := 0.030$

BRDF of ellip baf black glass sr^-1 $BRDF_{ellbafbg} := 5 \cdot 10^{-6}$

BRDF of SS edge, sr^-1 $BRDF_{edgess} := 0.1$

BRDF of rough cut black glass edge, sr^-1 $BRDF_{edgebgr} := 0.1$

BRDF of fire-polish black glass edge, sr^-1 $BRDF_{edgebgfp} := 0.0001$

BRDF of BD, sr^-1 $BRDF_{bd} := 0.030$

BRDF of chamber wall, sr^-1 $BRDF_{wall} := 0.1$

Motion of suspended baffle @ 100 Hz, m/rt Hz $x_{baf} := 3 \cdot 10^{-14}$

Motion of BSC chamber @ 100 Hz, m/rt Hz $x_{bscchamber} := 2 \cdot 10^{-11}$

laser wavelength, m $\lambda := 1.064 \cdot 10^{-6}$

wave number, m^-1 $k := 2 \cdot \frac{\pi}{\lambda}$ $k = 5.9052 \times 10^{6}$

Transfer function @ 100 Hz, ITM AR $TF_{itmar} := 3.16 \cdot 10^{-11}$

ITM beam radius, m $w_{itm} := 0.053168$

virtual beam waist looking toward ITM AR
(see H1 Signal Recycling Cavity beam
size_8-12-13) $w_{itmar0} := 0.008342$

distance from ITM AR to virtual beam waist, m $l_{itmar0} := 1.293 \times 10^{3}$

solid angle of ITM AR virtual beam
waist, sr $\Delta_{itmar} := \frac{\lambda^2}{\pi \cdot w_{itmar0}^{\;2}}$

$\Delta_{itmar} = 5.1784 \times 10^{-9}$

Ref: T1000090-v5, aLIGO Baffle Design using SIS

elliptical baffle minor
semi-axis, m $a := \frac{0.21 + 0.014}{2}$ $a = 0.112$

elliptical baffle major
semi-axis, m $b := \frac{0.260 + 0.014}{2}$ $b = 0.137$

thickness of SS baffle plate, m $t_{ss} := 0.047 \cdot 0.0254$

radius of SS edge, m $r_{edgess} := 0.001 \cdot 0.0254$

$r_{edgess} = 2.54 \times 10^{-5}$

thickness of black glass baffle plate, m $t_{bg} := 0.128 \cdot 0.0254$

radius of BG baffle rough-cut edge, m $r_{edgebgr} := 0.01 \cdot 0.0254$

$r_{edgebgr} = 2.54 \times 10^{-4}$

Reflectivity of Elliptical Baffle $R_{ellbaf} := 2.4e\text{-}5$

Transmissivity of ITM HR $T_{itmhr} := 0.0140$

Reflectivity of ITM HR $R_{itmhr} := 0.9951 - T_{itmhr}$ $R_{itmhr} = 0.9811$

Reflectivity of ITM AR $R_{itmar} := 50 \cdot 10^{-6}$

Transmissivity of ETM HR $T_{etm} := 5 \cdot 10^{-6}$

Transmissivity of FM HR $T_{FMhr} := 10 \times 10^{-5}$

Ref. T070247

input laser power, W $P_{psl} := 125$

arm cavity gain $G_{ac} := 13000$

arm cavity power, W $P_a := \frac{P_{psl}}{2} \cdot G_{ac}$

arm cavity power, W $P_a = 8.125 \times 10^{5}$

Ref. Hiro e-mail 8/29/11

power in power recycling cavity arm, W

$$P_{rca} := \frac{P_a \cdot T_{itmhr}}{4} \qquad P_{rca} = 2.8438 \times 10^3$$

Gaussian power parameter in recycling cavity

$$P_{0rc} := P_{rca}$$

Power recycling cavity gain

$$G_{rc} := \frac{2 \cdot P_{rca}}{P_{psl}} \qquad G_{rc} = 45.5$$

Gaussian beam equation in recycling cavity

$$I_{rc}(x, y) := 2 \cdot \frac{P_{0rc}}{\pi \cdot w_{itm}^2} \cdot e^{-2 \cdot \left(\frac{x^2 + y^2}{w_{itm}^2} \right)}$$

total integrated power in recycling cavity arm, W

$$P_{rca} := 4 \cdot \int_0^b \int_0^{a \cdot \sqrt{1 - \frac{y^2}{b^2}}} I_{rc}(x, y) \, dx \, dy$$

$$P_{rca} = 2.8436 \times 10^3$$

radius of ITM, m

$$r_{itm} := 0.170$$

horizontal displacement of ITM Elliptical Baffle, m

$$\delta x := 0$$

vertical displacement of ITM Elliptical Baffle, m

$$\delta y := 0$$

exitance function from ITM

$$I_{itm}(x, y) := 2 \cdot \frac{4 \cdot P_{0rc}}{\pi \cdot w_{itm}^2} \cdot e^{-2 \cdot \left(\frac{x^2 + y^2}{w_{itm}^2} \right)}$$

power exiting from ITM toward elliptical baffle, W

$$P_{itm} := 4 \cdot \int_0^{r_{itm}} \int_0^{r_{itm} \cdot \sqrt{1 - \frac{y^2}{r_{itm}^2}}} I_{itm}(x, y) \, dx \, dy$$

$$P_{itm} = 1.1375 \times 10^{4}$$

ITM ELLIPTICAL BAFFLE INCIDENT POWER

Power passing through the elliptical baffle hole, W

$$P_{itmellbaftran}(\delta x, \delta y) := \left(\int_{\delta y - b}^{\delta y + b} \int_{\delta x - a\cdot\sqrt{1 - \frac{y^2}{b^2}}}^{\delta x + a\cdot\sqrt{1 - \frac{y^2}{b^2}}} I_{itm}(x, y)\, dx\, dy \right)$$

$$P_{itmellbaftran}(0, 0) = 1.1374 \times 10^{4}$$

power hitting the ITM elliptical baffle from ITM side, W

$$P_{itmellbaf}(\delta x, \delta y) := P_{itm} - P_{itmellbaftran}(\delta x, \delta y)$$

$$P_{itmellbaf}(0, 0) = 0.5342$$

power hitting the ITM elliptical baffle from ITM side, W

$$P_{itmellbaf}(0, 0) = 0.5342$$

intercepton efficiency for ITM power

$$\eta_{itmellbaf} := \frac{P_{itmellbaf}(0, 0)}{P_{itm}}$$

$$\eta_{itmellbaf} = 4.6958 \times 10^{-5}$$

SCATTER FROM OXIDIZED SS ITM ELLIP BAFFLE SURFACE

Power scattered into IFO mode from both arms, W

$$P_{itmellbafsss} := \sqrt{2}\cdot P_{itmellbaf}(0, 0)\cdot BRDF_{ellbafss}\cdot \Delta_{itmar}$$

$$P_{itmellbafsss} = 1.1735 \times 10^{-10}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmellbafss} := TF_{itmar} \cdot \left(\frac{P_{itmellbafsss}}{P_{psl}} \right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmellbafss} = 7.671 \times 10^{-24}$$

SCATTER FROM SS ITM ELLIP BAFFLE HOLE EDGE

horizontal edge, m $\quad x := a$

vertical edge, m $\quad y := 0$

exitance function from ITM at edge, W/m^2

$$I_{itm}(x,y) := 2 \cdot \frac{4 \cdot P_{0rc}}{\pi \cdot w_{itm}^{2}} \cdot e^{-2 \cdot \left(\frac{x^2+y^2}{w_{itm}^{2}} \right)}$$

$$I_{itm}(x,y) = 358.2562$$

maximum width of exposed edge, m $\quad w_{itmbafe} := 2 \cdot r_{edgess}$

Radius of baffle hole, m $\quad R_{itmbaf} := a$

exposed area of baffle hole edge, m^2

$$A_{itmbafess} := \int_{-R_{itmbaf}}^{0} 2 \cdot \sqrt{R_{itmbaf}^{2} - x^2}\, dx - \int_{-R_{itmbaf}+w_{itmbafe}}^{0} 2 \cdot \sqrt{R_{itmbaf}^{2} - (x - w_{itmbafe})^2}\, dx$$

$$A_{itmbafess} = 1.1379 \times 10^{-5}$$

power incident on ITM Baf hole edge, W

$$P_{itmbafedgess} := I_{itm}(a,0) \cdot A_{itmbafess}$$

$$P_{itmbafedgess} = 4.077 \times 10^{-3}$$

intercepton efficiency for ITM power

$$\eta_{itmellbafedgess} := \frac{P_{itmbafedgess}}{P_{itm}}$$

$$\eta_{itmellbafedgess} = 3.5839 \times 10^{-7}$$

power scattered from two ITM Ellip Baf hole edge toward ITM, W

$$P_{itmbafedgesss} := \sqrt{2} \cdot P_{itmbafedgess} \cdot BRDF_{edgess} \cdot \Delta_{itmar}$$

$$P_{itmbafedgesss} = 2.9855 \times 10^{-12}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbafedgess} := TF_{itmar} \cdot \left(\frac{P_{itmbafedgesss}}{P_{psl}}\right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbafedgess} = 1.2235 \times 10^{-24}$$

ratio of edge scatter to baf scatter

$$\frac{DN_{itmbafedgess}}{DN_{itmellbafss}} = 0.1595$$

Total SS baf scatter power, W

$$P_{itmbafssts} := P_{itmellbafsss} + P_{itmbafedgesss}$$

$$P_{itmbafssts} = 1.2034 \times 10^{-10}$$

total SS baffle displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbafsst} := TF_{itmar} \cdot \left(\frac{P_{itmbafssts}}{P_{psl}}\right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbafsst} = 7.768 \times 10^{-24}$$

SCATTER FROM BLACK GLASS ITM ELLIP BAFFLE SURFACE

Power scattered into IFO mode
from both arms, W

$$P_{itmellbafbgs} := \sqrt{2} \cdot P_{itmellbaf}(0,0) \cdot BRDF_{ellbafbg} \cdot \Delta_{itmar}$$

$$P_{itmellbafbgs} = 1.9559 \times 10^{-14}$$

displacement noise @ 100 Hz,
m/rtHz

$$DN_{itmellbafbg} := TF_{itmar} \cdot \left(\frac{P_{itmellbafbgs}}{P_{psl}} \right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmellbafbg} = 9.9032 \times 10^{-26}$$

SCATTER FROM ROUGH CUT BG ITM ELLIP BAFFLE HOLE EDGE

maximum width of exposed edge, m

$$w_{itmbafe} := 2 \cdot r_{edgebgr} \qquad r_{edgebgr} = 2.54 \times 10^{-4}$$

Radius of baffle hole, m

$$R_{itmbaf} = 0.112$$

exposed area of baffle hole edge, m^2

$$A_{itmbafebg} := \int_{-R_{itmbaf}}^{0} 2 \cdot \sqrt{R_{itmbaf}^{2} - x^{2}}\, dx - \int_{-R_{itmbaf}+w_{itmbafe}}^{0} 2 \cdot \sqrt{R_{itmbaf}^{2} - \left(x - w_{itmbafe}\right)^{2}}\, dx$$

$$A_{itmbafebg} = 1.1379 \times 10^{-4}$$

power incident on ITM Baf hole edge, W

$$P_{itmbafedgebg} := I_{itm}(a,0) \cdot A_{itmbafebg}$$

$$P_{itmbafedgebg} = 0.041$$

power scattered from two BG ITM Ellip Baf hole
edge toward ITM, W

$$P_{itmbafedgebgrs} := \sqrt{2} \cdot P_{itmbafedgebg} \cdot BRDF_{edgebgr} \cdot \Delta_{itmar}$$

$$P_{itmbafedgebgrs} = 2.9855 \times 10^{-11}$$

displacement noise @ 100 Hz,
m/rtHz

$$DN_{itmbafedgebgr} := TF_{itmar} \cdot \left(\frac{P_{itmbafedgebgrs}}{P_{psl}} \right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbafedgebgr} = 3.8691 \times 10^{-24}$$

ratio of rough cut edge scatter to baf scatter

$$\frac{DN_{itmbafedgebgr}}{DN_{itmellbafbg}} = 39.0693$$

SCATTER FROM FIRE-POLISHED BG ITM ELLIP BAFFLE HOLE EDGE

max input angle, rad $\theta_{xymaxedge} := \frac{\pi}{2} = 1.5708$

max input angle, deg $\theta_{xymaxpdeg} := \theta_{xymaxedge} \cdot \frac{180}{\pi} = 90$

BRDF Black Glass BRDF @ 5 deg incidence

$$BRDF_0 := 0.12$$

$$\beta := .75$$

$$C_{mr} := 1.655 \times 10^4$$

$$BRDF_{\theta 2} := 1 \cdot 10^{-5}$$

BRDF function, sr^-1

$$BRDF_{bg5}(\theta_i) := \frac{BRDF_0}{\left(1 + C_{mr} \cdot \theta_i^{\,2}\right)^{\beta}} + BRDF_{\theta 2}$$

BRDF Black Glass fire polish (empirical estimate)

break-over angle, rad $\theta_1 := 0.5 \cdot \frac{\pi}{180} = 8.7266 \times 10^{-3}$

micro-roughness angle, rad $\theta_2 := 5 \cdot \frac{\pi}{180} = 0.0873$

max BRDF, sr^-1 $BRDF_0 := 0.1$

final slope modifier $\beta := 0.85$

micro-roughness constant $C_{mr} := \frac{2^{\frac{1}{(\beta)}} - 1}{\theta_1^{\,2}}$

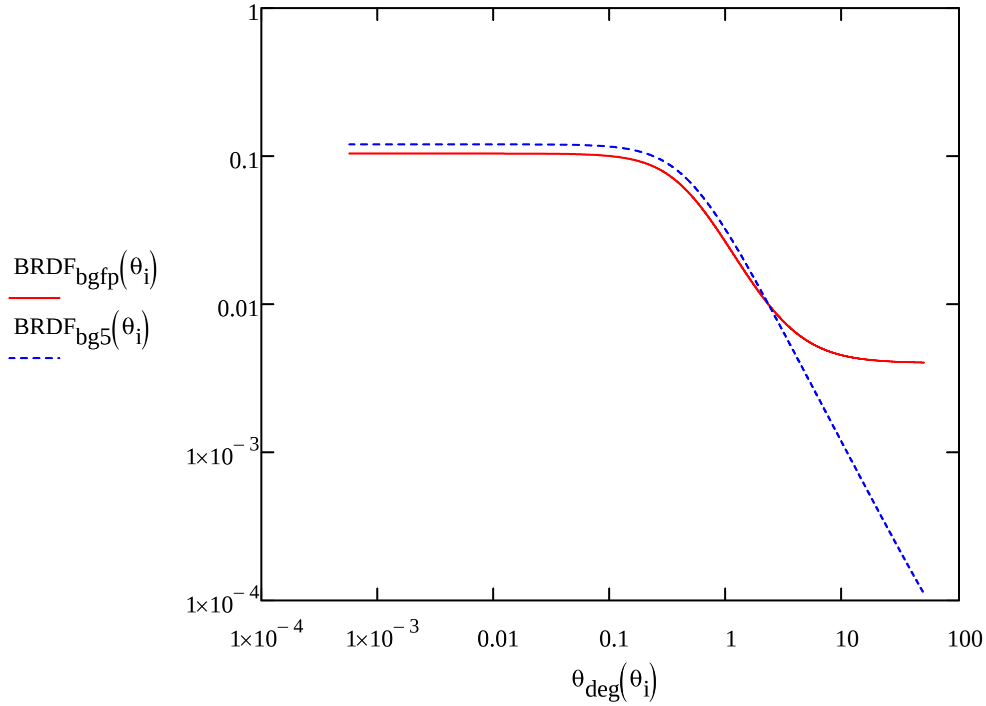
$C_{mr} = 1.6548 \times 10^4$

large angle BRDF, fire polish, sr^-1 $BRDF_{bgfp\theta2} := 4 \cdot 10^{-3}$

BRDF function, sr^-1 $BRDF_{bgfp}(\theta_i) := \frac{BRDF_0}{\left(1 + C_{mr} \cdot \theta_i^{\,2}\right)^{\beta}} + BRDF_{bgfp\theta2}$

$\theta_{deg}(\theta_i) := \theta_i \cdot \frac{180}{\pi}$

$\theta_i := 0, 0.00001 .. 10 \cdot \theta_2$



$$\theta_t := 0$$

$$\theta_i(\theta_t, \theta_{xy}) := \mathrm{acos}(\cos(\theta_{xy}) \cdot \cos(\theta_t))$$

$$S_{edgebg}(\theta_t, BRDF_{bgfp}) := \int_0^{\theta_{xymaxedge}} \left[\int_{2\cdot\theta_i(\theta_t,\theta_{xy}) - \frac{w_{itmar0}}{l_{itmar0}}}^{2\cdot\theta_i(\theta_t,\theta_{xy}) + \frac{w_{itmar0}}{l_{itmar0}}} BRDF_{bgfp}(\theta_s + 2\cdot\theta_i(\theta_t,\theta_{xy}))\cdot\sqrt{w_{itmar}} \right.$$

$$S_{edgebg}(\theta_t, BRDF_{bgfp}) = 2.883 \times 10^{-13}$$

$$P_{itmbafedgebgfps} := I_{itm}(a, 0) \cdot A_{itmbafebg} \cdot S_{edgebg}(\theta_t, BRDF_{bgfp})$$

$$P_{itmbafedgebgfps} = 1.1753 \times 10^{-14}$$

intercepton efficiency for ITM power

$$\eta_{itmellbafedgebg} := \frac{P_{itmbafedgebgfps}}{P_{itm}}$$

$$\eta_{itmellbafedgebg} = 1.0332 \times 10^{-18}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbafedgebgfp} := TF_{itmar} \cdot \left(\frac{P_{itmbafedgebgfps}}{P_{psl}}\right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbafedgebgfp} = 7.6767 \times 10^{-26}$$

ratio of edge scatter to baf scatter $\frac{DN_{itmbafedgebgfp}}{DN_{itmellbafbg}} = 0.7752$

Total BG baf scatter power, W

$$P_{itmbafbgfpts} := P_{itmellbafbgs} + P_{itmbafedgebgfps}$$

$$P_{itmbafbgfpts} = 3.1311 \times 10^{-14}$$

total black glass baffle displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbafbgfpt} := TF_{itmar} \cdot \left(\frac{P_{itmbafbgfpts}}{P_{psl}}\right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbafbgfpt} = 1.253 \times 10^{-25}$$

ratio of SS to BG baf scatter $\frac{DN_{itmellbafss}}{DN_{itmbafbgfpt}} = 61.2202$

ITM ELLIPTICAL BAFFLE REFL

Power reflected from baffle, W $P_{itmellbafrefl} := R_{ellbaf} \cdot P_{itmellbaf}(0,0)$

$$P_{itmellbafrefl} = 1.282 \times 10^{-5}$$

Power scattered into IFO mode
from both arms, W

$$P_{itmellbafrefls} := \sqrt{2} \cdot P_{itmellbafrefl} \cdot R_{ellbaf} \cdot BRDF_{wall} \cdot \Delta_{itmar}$$

$$P_{itmellbafrefls} = 2.2532 \times 10^{-19}$$

Motion of BSC chamber @ 100 Hz, m/rt Hz

displacement noise @ 100 Hz,
m/rtHz

$$DN_{itmellbafrefl} := TF_{itmar} \cdot \left(\frac{P_{itmellbafrefls}}{P_{psl}} \right)^{0.5} \cdot x_{bscchamber} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmellbafrefl} = 2.2408 \times 10^{-25}$$

$$\overline{0^2 - \left[l_{itmar0}\cdot\left(\theta_s - 2\cdot\theta_i\left(\theta_t,\theta_{xy}\right)\right)\right]^2}\cdot\frac{l_{itmar0}}{l_{itmar0}^2}\,d\theta_s\Bigg]\cdot\cos\left(\theta_{xy}\right)\,d\theta_{xy}$$