BS Elliptical Baffle Overlap Intetral Scattering Calculation
6/20/12

BRDF, sr^-1; CSIRO, surface 2, S/N 2 $BRDF_1(\theta) := \frac{2755.12}{\left(1 + 8.50787\cdot10^{8}\cdot\theta^{2}\right)^{1.23597}}$

incidence angle at COC, rad $\theta_{coc} := \frac{120}{4\cdot10^{6}}$ $\theta_{coc} = 3 \times 10^{-5}$

BRDF of COC, sr^-1 $BRDF_1\left(\theta_{coc}\right) = 1.364 \times 10^{3}$

BRDF of ellip baf, sr^-1 $BRDF_{ellbaf} := 0.030$

Motion of suspended baffle @ 100 Hz, m/rt Hz $x_{baf} := 1\cdot10^{-12}$

Motion of FM frame @ 100 Hz, m/rtHz $x_{sus} := 3.1\cdot10^{-14}$

laser wavelength, m $\lambda := 1.064\cdot10^{-6}$

wave number, m^-1 $k := 2\cdot\frac{\pi}{\lambda}$ $k = 5.905 \times 10^{6}$

ITM beam radius, m $w_{itm} := 0.053168$

IFO waist size, m $w_{ifo} := 0.0120$

IFO arm length, m $L := 4000.0$

radius of ITM, m $r_{itm} := 0.170$

solid angle of IFO mode, sr $\Delta_{ifo} := \pi\cdot\left(\frac{\lambda}{\pi\cdot w_{ifo}}\right)^{2}$ $\Delta_{ifo} = 2.502 \times 10^{-9}$

transformed beam waist after ITM AR surface

$$w_{itmar0} := 0.008342$$

solid angle of ITM AR beam waist, sr

$$\Delta_{itmar} := \pi \cdot \left(\frac{\lambda}{\pi \cdot w_{itmar0}} \right)^2 \qquad \Delta_{itmar} = 5.178 \times 10^{-9}$$

Transfer function @ 100 Hz, ITM AR

$$TF_{itmar} := 3.16 \cdot 10^{-11}$$

Transmissivity of ITM HR

$$T_{itmhr} := 0.0140$$

Ref. T070247

input laser power, W

$$P_{psl} := 125$$

arm cavity gain

$$G_{ac} := 13000$$

arm cavity power, W

$$P_a := \frac{P_{psl}}{2} \cdot G_{ac} \qquad P_a = 8.125 \times 10^5$$

Ref. Hiro e-mail 8/29/11

power in power recycling cavity arm, W

$$P_{rca} := \frac{P_a \cdot T_{itmhr}}{4} \qquad P_{rca} = 2.8438 \times 10^3$$

exitance function from ITM AR

$$I_{itm}(x, y) := 2 \cdot \frac{4 \cdot P_{rca}}{\pi \cdot w_{itm}^2} \cdot e^{-2 \cdot \left(\frac{x^2 + y^2}{w_{itm}^2} \right)}$$

power exiting from ITM toward elliptical baffle, W

$$P_{itm} := 4 \cdot \int_0^{r_{itm}} \int_0^{r_{itm} \cdot \sqrt{1 - \frac{y^2}{r_{itm}^2}}} I_{itm}(x, y) \, dx \, dy$$

$$P_{itm} = 1.1375 \times 10^4$$

also check

$$P_{itm} := P_a \cdot T_{itmhr} \qquad P_{itm} = 1.1375 \times 10^4$$

Beam curvature radius at ITM HR, m $R_{itm} := -1.326 \times 10^{6}$

vertical aperture in ITM ellip baf, m $r_{itmellipy} := 0.137$

horizontal aperture in ITM ellip baf, m $r_{itmellipx} := 0.112$

vertical aperture in BS ellip baf, m $r_{bsellipx} := 0.105$

horizontal aperture in BS ellip baf, m $r_{bsellipy} := 0.130$

X coordinate of BS Ellip Baf, m $x_s := 0$

Y coordinate of BS Ellip Baf,, m $y_s := 0$

axial coordinate of BS Ellip Baf,, m $z_s := -4.89$

X coordinate of ITMHR surface, m $x := 0$

Y coordinate of ITMHR surface, m $y := 0$

axial coordinate of ITMHR surface, m $z := 0$

outer radius of ITM ellip baf, m $r_{itmellipmax} := 0.170$

Arm Beam Reference Field

Arm field at ITM AR

$$E_{aitmar}(x,y) := e^{-i \cdot k \cdot \frac{x^2+y^2}{2 \cdot R_{itm}} - \frac{x^2+y^2}{w_{itm}^{2}}}$$

normalize arm cavity field

$$E_{aitmar0} := \sqrt{\frac{\pi}{2} \cdot w_{itm}^{2}}$$

Field coupling for arm cavity beam

$$OVI := 4 \cdot \int_0^{r_{itm}} \int_0^{r_{itm} \cdot \sqrt{1 - \frac{y^2}{r_{itm}^2}}} \frac{E_{aitmar}(x, y)}{E_{aitmar0}} \cdot \frac{\overline{E_{aitmar}(x, y)}}{E_{aitmar0}} \, dx \, dy$$

$$OVI = 1$$

Power coupling coupling factor for arm cavity beam

$$PCF := \left| OVI^2 \right|$$

$$PCF = 1$$

BS ELLIPTICAL BAFFLE

distance from BS Ellip Baf scattering surface to
ITM HR surface

$$z_{bsellipbafitm}(x_s, y_s, z_s, x, y, z) := \left[(x_s - x)^2 + (y_s - y)^2 + (z_s - z)^2 \right]^{0.5}$$

$$z_{bsellipbafitm}(x_s, y_s, z_s, x, y, z) = 4.89$$

arm power exiting from ITMAR passing through
itm elliptical baffle, W

$$P_{itmaritmellbaf} := 4 \cdot \int_0^{r_{itmellipy}} \int_0^{r_{itmellipx} \cdot \sqrt{1 - \frac{y^2}{r_{itmellipy}^2}}} I_{itm}(x, y) \, dx \, dy$$

$$P_{itmaritmellbaf} = 1.1374 \times 10^4$$

arm power exiting from ITMAR passing through
bs elliptical baffle, W

$$P_{itmarbsellbaf} := 4\cdot\int_{0}^{r_{bsellipy}}\int_{0}^{r_{bsellipx}\cdot\sqrt{1-\frac{y^2}{r_{bsellipy}^2}}} I_{itm}(x,y)\,dx\,dy$$

$$P_{itmarbsellbaf} = 1.1373\times 10^{4}$$

Power incident on BS baffle, W

$$P_{bsbaf} := P_{itmaritmellbaf} - P_{itmarbsellbaf}$$

$$P_{bsbaf} = 1.093$$

scattered power from BS Ellip Baf onto ITMAR surface, W

$$P_{bsellbafitm}(z_s, P_{bsbaf}) := P_{bsbaf}\cdot BRDF_{ellbaf}\cdot\frac{\pi\cdot r_{itm}^2}{z_s^2}$$

$$\frac{\pi\cdot r_{itm}^2}{z_s^2} = 3.797\times 10^{-3}$$

$$P_{bsellbafitm}(z_s, P_{bsbaf}) = 1.245\times 10^{-4}$$

Point Source Field

constant distance of scatter source from ITM AR

$$z_s = -4.89 \qquad z_c(z_s) := z_s$$

assume that phase factor is unity

$$z_c(z_s) := 0 \qquad z_c(z_s) := z_s$$

constant phase factor for scattered field

$$\Phi_{bsellbaf}(z_s) := e^{-i\cdot k\cdot z_c(z_s)}$$

$$\Phi_{bsellbaf}(z_s) = -0.527 - 0.85i$$

Normalized Scattered field at ITM AR

$x_s := 0$ $y_s := 0$ $z_s = -4.89$

$x := 0$ $y := 0$ $z := 0$

$$E_{bsellbafitm}\left(x_s, y_s, z_s, x, y, z, P_{bsbaf}\right) := \Phi_{bsellbaf}\left(z_s\right)\cdot e^{-i\cdot k\cdot\frac{\left(x-x_s\right)^2+\left(y-y_s\right)^2}{2\cdot z_{bsellipbafitm}\left(x_s, y_s, z_s, x, y, z\right)}}$$

$$E_{bsellbafitm}\left(x_s, y_s, z_s, x, y, z, P_{bsbaf}\right) = -0.527 - 0.85i$$

normalize arm cavity field

$$P_{bsellbafifo1ptnormalize}\left(z_s, P_{bsbaf}\right) := \frac{4}{\pi\cdot r_{itm}^{2}}\cdot\int_{0}^{r_{itm}}\int_{0}^{r_{itm}\cdot\sqrt{1-\frac{y^2}{r_{itm}^2}}} E_{bsellbafitm}\left(x_s, y_s, z_s, x, y, z, P_{b}\right.$$

$$P_{bsellbafifo1ptnormalize}\left(z_s, P_{bsbaf}\right) = 1$$

Field coupling for point source on-axis

$$E_{bsellbafifo1pt}\left(z_s, P_{bsbaf}\right) := \frac{4}{\sqrt{\pi\cdot r_{itm}^{2}}}\cdot\int_{0}^{r_{itm}}\int_{0}^{r_{itm}\cdot\sqrt{1-\frac{y^2}{r_{itm}^2}}} \sqrt{P_{bsellbafitm}\left(z_s, P_{bsbaf}\right)}\cdot E_{bsellbafitm}\left(0, 0\right.$$

$$E_{bsellbafifo1pt}\left(z_s, P_{bsbaf}\right) = 3.122\times 10^{-5} + 1.044i\times 10^{-5}$$

Coupled power, W

$$P_{bsellbafifo1pt}\left(z_s, P_{bsbaf}\right) := E_{bsellbafifo1pt}\left(z_s, P_{bsbaf}\right)\cdot\overline{E_{bsellbafifo1pt}\left(z_s, P_{bsbaf}\right)}$$

$$P_{bsellbafifo1pt}(z_s, P_{bsbaf}) = 1.084 \times 10^{-9}$$

Power coupling factor

$$PCF_{bsellbafifo1pt}(z_s, P_{bsbaf}) := \frac{P_{bsellbafifo1pt}(z_s, P_{bsbaf})}{P_{bsellbafitm}(z_s, P_{bsbaf})}$$

$$PCF_{bsellbafifo1pt}(z_s, P_{bsbaf}) = 8.703 \times 10^{-6}$$

effective scattering solid angle

$$\Delta\omega_{effbsellbaf1ptonaxis}(z_s, P_{bsbaf}) := \frac{P_{bsellbafifo1pt}(z_s, P_{bsbaf})}{P_{bsbaf} \cdot BRDF_{ellbaf}}$$

$$\Delta\omega_{effbsellbaf1ptonaxis}(z_s, P_{bsbaf}) = 3.304 \times 10^{-8}$$

$$DN_{bsellbafifo1ptonaxis}(z_s, P_{bsbaf}) := TF_{itmar} \cdot \left(\frac{\sqrt{2} \cdot P_{bsellbafifo1pt}(z_s, P_{bsbaf})}{P_{psl}} \right)^{0.5} \cdot x_{sus} \cdot 2 \cdot k$$

$$DN_{bsellbafifo1ptonaxis}(z_s, P_{bsbaf}) = 4.051 \times 10^{-23}$$

FOUR POINT
ANNULAR SOURCE

$x_s := r_{bsellipx}$ $\quad$ $y_s := r_{bsellipy}$ $\quad$ $z_s = -4.89$ $\quad$ $z_s(z_s) := z_s$

Tilt of baffle surface, deg $\quad$ $\theta_{tbaf} := 45$

baffle distance increment, m $\quad$ $\Delta z_s(y_s) := \frac{y_s}{\tan\left(\theta_{tbaf} \cdot \frac{\pi}{180}\right)}$ $\quad$ $\Delta z_s(y_s) = 0.13$

Coupled field, rtW

annular source field

field 1 @ 0, +ys, +Δzs

$$E_{bsellbafitm1}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) := \Phi_{bsellbaf}(z_c(z_s) + \Delta z_s(y_s)) \cdot e^{-i \cdot k \cdot \frac{(x-0)^2 + (y-y_s)^2}{2 \cdot z_{bsellipbafitm}(x_s, y_s, z_s + \Delta z_s}}$$

$$E_{bsellbafitm1}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) = 0.352 - 0.936i$$

field 2 @ 0, -ys, -Δzs

$$E_{bsellbafitm2}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) := \Phi_{bsellbaf}(z_c(z_s) - \Delta z_s(y_s)) \cdot e^{-i \cdot k \cdot \frac{(x-0)^2 + (y+y_s)^2}{2 \cdot z_{bsellipbafitm}(x_s, -y_s, z_s - \Delta}}$$

$$E_{bsellbafitm2}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) = 0.983 - 0.182i$$

field 3 @ +xs, 0, Δzs=0

$$E_{bsellbafitm3}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) := \Phi_{bsellbaf}(z_c(z_s)) \cdot e^{-i \cdot k \cdot \frac{(x-x_s)^2 + (y+0)^2}{2 \cdot z_{bsellipbafitm}(x_s, 0, z_s, x, y, z)}}$$

$$E_{bsellbafitm3}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) = -0.853 + 0.521i$$

field 4 @ -xs, 0 Δzs=0

$$E_{bsellbafitm4}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) := \Phi_{bsellbaf}(z_c(z_s)) \cdot e^{-i \cdot k \cdot \frac{(x+x_s)^2 + (y+0)^2}{2 \cdot z_{bsellipbafitm}(x_s, 0, z_s, x, y, z)}}$$

$$E_{bsellbafitm4}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) = -0.853 + 0.521i$$

$$F_{ann}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) := \frac{E_{bsellbafitm1}(x_s, y_s, z_s, x, y, z, P_{bsbaf})}{4} + \frac{E_{bsellbafitm2}(x_s, y_s, z_s, x, y, z}{4}$$

$$F_{ann}(x_s, y_s, z_s, x, y, z, P_{bsbaf}) = -0.093 - 0.019i$$

$$E_{bsellbafifo4pt}(x_s, y_s, z_s, P_{bsbaf}) := \frac{4}{\sqrt{\pi \cdot r_{itm}^{2}}} \cdot \int_{0}^{r_{itm}} \int_{0}^{r_{itm} \cdot \sqrt{1 - \frac{y^2}{r_{itm}^{2}}}} \sqrt{P_{bsellbafitm}(z_s, P_{bsbaf})} \cdot F_{ann}(x_s, y$$

$$E_{bsellbafifo4pt}(x_s, y_s, z_s, P_{bsbaf}) = -9.501 \times 10^{-6} - 6.033i \times 10^{-6}$$

Coupled power, W

$$P_{bsellbafifo4pt}(x_s, y_s, z_s, P_{bsbaf}) := E_{bsellbafifo4pt}(x_s, y_s, z_s, P_{bsbaf}) \cdot \overline{E_{bsellbafifo4pt}(x_s, y_s, z_s, P_{bsbaf})}$$

$$P_{bsellbafifo4pt}(x_s, y_s, z_s, P_{bsbaf}) = 1.267 \times 10^{-10}$$

$$P_{bsellbafifo4pt}(0, 0, z_s, P_{bsbaf}) = 1.084 \times 10^{-9}$$

Coupled power, W

$$P_{bsellbafifo1pt}(z_s, P_{bsbaf}) := E_{bsellbafifo1pt}(z_s, P_{bsbaf}) \cdot \overline{E_{bsellbafifo1pt}(z_s, P_{bsbaf})}$$

$$P_{bsellbafifo1pt}(z_s, P_{bsbaf}) = 1.084 \times 10^{-9}$$

Power coupling factor

$$PCF_{bsellbafifo4pt}(x_s, y_s, z_s, P_{bsbaf}) := \frac{P_{bsellbafifo4pt}(x_s, y_s, z_s, P_{bsbaf})}{P_{bsellbafitm}(z_s, P_{bsbaf})}$$

$$PCF_{bsellbafifo4pt}(x_s, y_s, z_s, P_{bsbaf}) = 1.017 \times 10^{-6}$$

$$PCF_{bsellbafifo4pt}(0,0,z_s,P_{bsbaf}) = 8.703 \times 10^{-6}$$

effective scattering solid angle

$$\Delta\omega_{effbsellipbaf4pt}(x_s,y_s,z_s,P_{bsbaf}) := \frac{P_{bsellbafifo4pt}(x_s,y_s,z_s,P_{bsbaf})}{P_{bsbaf}\cdot BRDF_{ellbaf}}$$

$$\Delta\omega_{effbsellipbaf4pt}(x_s,y_s,z_s,P_{bsbaf}) = 3.863 \times 10^{-9}$$

$$\Delta\omega_{effbsellbaf1ptonaxis}(z_s,P_{bsbaf}) = 3.304 \times 10^{-8}$$

displacement noise @ 100 Hz,
m/rtHz

$$DN_{bsellbaf4pt}(x_s,y_s,z_s,P_{bsbaf}) := TF_{itmar}\cdot\left(\frac{\sqrt{2}\cdot P_{bsellbafifo4pt}(x_s,y_s,z_s,P_{bsbaf})}{P_{psl}}\right)^{0.5}\cdot x_{sus}\cdot 2\cdot k$$

$$DN_{bsellbaf4pt}(x_s,y_s,z_s,P_{bsbaf}) = 1.385 \times 10^{-23}$$

$$DN_{bsellbafifo1ptonaxis}(z_s,P_{bsbaf}) = 4.051 \times 10^{-23}$$

Compare the effective solid angles

$$\Delta\omega_{effsmith} := \frac{\pi\cdot w_{ifo}^{2}}{L^2}\cdot BRDF_1\left(30\cdot 10^{-6}\right)\cdot\Delta_{ifo}$$

$$\Delta\omega_{effsmith} = 9.654 \times 10^{-17}$$

$$\Delta\omega_{effbsellbaf1ptonaxis}(z_s,P_{bsbaf}) = 3.304 \times 10^{-8}$$

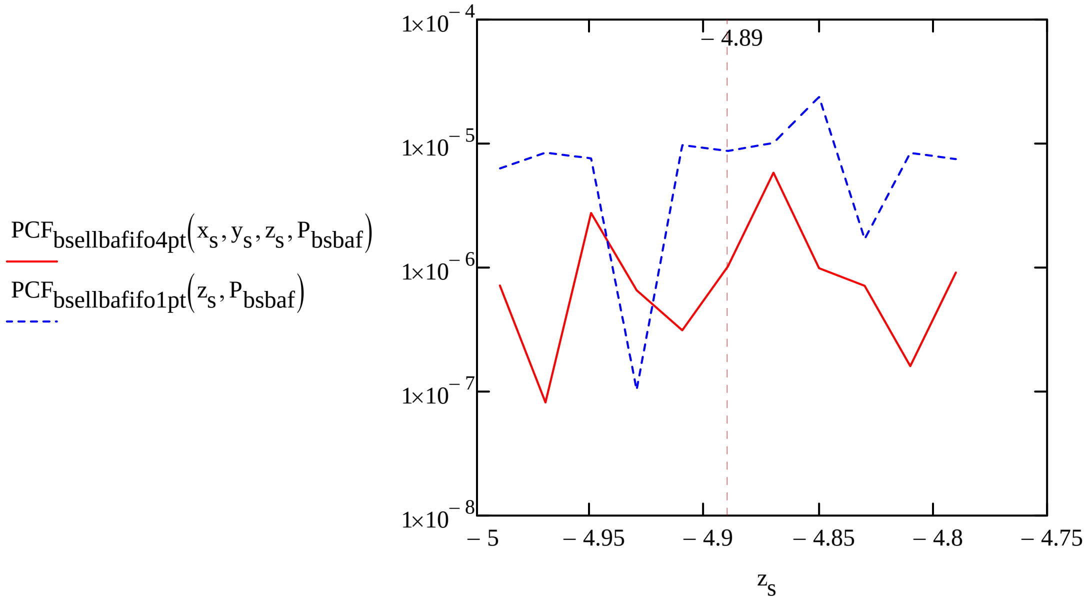
$$\Delta\omega_{effbsellipbaf4pt}(x_s,y_s,z_s,P_{bsbaf}) = 3.863 \times 10^{-9}$$

solid angle of IFO mode, sr $\Delta_{ifo} = 2.502 \times 10^{-9}$

solid angle of ITM AR beam waist, sr

$\Delta_{itmar} = 5.178 \times 10^{-9}$

$z_s := -4.79, -4.81 .. -5$



$$\text{bsbaf}\Big)\cdot\overline{E_{bsellbafitm}\Big(x_s, y_s, z_s, x, y, z, P_{bsbaf}\Big)}\,dx\,dy$$

$$, z_s, x, y, z, P_{bsbaf}\Big)\cdot\frac{\overline{E_{aitmar}(x,y)}}{E_{aitmar0}}\,dx\,dy$$

$$\overline{{}_{s}\left(y_{S}\right), x, y, z)}$$

$$\overline{\mathrm{a}z_{S}\left(y_{S}\right), x, y, z)}$$

$$\ldots z, P_{bsbaf}) + \frac{E_{bsellbafitm3}(x_s, y_s, z_s, x, y, z, P_{bsbaf})}{4} + \frac{E_{bsellbafitm4}(x_s, y_s, z_s, x, y, z, P_{bsbaf})}{4}$$

$$\ldots y_s, z_s, x, y, z, P_{bsbaf}) \cdot \frac{\overline{E_{aitmar}(x, y)}}{E_{aitmar0}} \, dx \, dy$$