Math trick for modeling platforms with input from ground motion

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1 Summary

We do a derivation which allows us to model an isolation system with damping, and only track the input from ground displacement, while including the effects of input ground velocity. The dynamics equation is

$$s \cdot \begin{bmatrix} x \\ v' \end{bmatrix} = \begin{bmatrix} -b/m & 1 \\ -k/m & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ v' \end{bmatrix} + \begin{bmatrix} b/m & 0 \\ k/m & 1/m \end{bmatrix} \cdot \begin{bmatrix} x_g \\ F \end{bmatrix} \tag{1}$$

The state variable x is the mass position, but v' is **not** velocity.

2 Derivation

We begin with a horizontal mass-spring system with a dash-pot. This system has a mass, m, on a spring, k, with damping b. The base of the spring and dash-pot are attached to the ground. The mass location is x, and the ground location is x_g

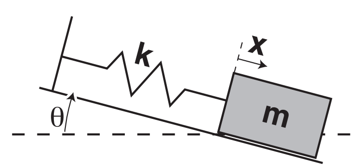

Figure 1: Model of a tilted mass-spring system

The basic equation describing this system is

$$m\,\ddot{x} = -k(x - x_g) - b(\dot{x} - \dot{x_g}) + F. \tag{2}$$

We Laplace transform the equation, so equation 2 becomes

$$ms^2 x = -kx + kx_g - bx + bsx_g + F. \tag{3}$$

$$s^2 x = -\frac{k}{m}x + \frac{k}{m}x_g - s\frac{b}{m}x + s\frac{b}{m}x_g + \frac{1}{m}F. \tag{4}$$

$$s(sx + \frac{b}{m}x - \frac{b}{m}x_g) = -\frac{k}{m}x + \frac{k}{m}x_g + \frac{1}{m}F. \tag{5}$$

We now define v' to be

$$v' = sx + \frac{b}{m}x - \frac{b}{m}x_g \tag{6}$$

so the first order coupled equations become

$$sv' = -\frac{k}{m}x + \frac{k}{m}x_g + \frac{1}{m}F. \tag{7}$$

and

$$sx = -\frac{b}{m}x + v' + \frac{b}{m}x_g \tag{8}$$

This can be rewritten into matrix form which and results in our original equation 1.

$$s\begin{bmatrix} x \\ v' \end{bmatrix} = \begin{bmatrix} -b/m & 1 \\ -k/m & 0 \end{bmatrix}\begin{bmatrix} x \\ v' \end{bmatrix} + \begin{bmatrix} b/m & 0 \\ k/m & 1/m \end{bmatrix}\begin{bmatrix} x_g \\ F \end{bmatrix} \tag{9}$$