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Abstract

An extreme class of core-collapse supernovae, so-called “hypernovae”, are hypothesized to be driven magnetorotationally by a combination of rapid rotation and ultra-strong magnetic fields and may explode extremely asymmetrically. The extreme conditions in the engine of such hypernovae are expected to lead to copious gravitational wave emission, which may perhaps be detectable out to megaparsec-distances with the upcoming generation of Advanced LIGO interferometers. This document is the final progress report on the inclusion of electromagnetic stresses in the extraction of transient gravitational wave signatures from simulations of magnetorotationally-driven core-collapse supernovae.

1 Introduction

The main goal of TAPIR's participation in the LIGO collaboration is to determine and study possible sources of gravitational radiation as well as make theoretical predictions based on large-scale numerical relativity simulations. Caltech has lead the world in the construction of such astrophysical simulations and possesses a wide variety of codes and computing resources taking into account various levels of modelling approaches. The current code under development uses an ideal fluid model to incorporate magnetohydrodynamics into core-collapse simulations.

The central relativistic characterization of any system consisting of mass-energy is the stress-energy tensor, from which the curvature of spacetime and hence the kinematics of test-particles may be computed. The stress-energy tensor for an ideal fluid, with ideal magnetohydrodynamics, is given by

$$T^{\mu\nu} = \rho h_* u^\mu u^\nu + P_* g^{\mu\nu} - b^\mu b^\nu, \tag{1}$$

where $h_* \equiv h + b^2/\rho$ is a modified enthalpy, and $P_* \equiv P + b^2/2$ a modified pressure. The magnetic field is defined in the comoving frame which for the simulation is identical to the Lagrange frame of the fluid. The magnetic field components in this frame are

$$b^i = B^i/W + W(B^k v_k)(v^i - \beta^i/\alpha), \tag{2}$$

where W is the Lorentz factor and β^i and α are the shift vector and lapse of the metric resulting from the ADM formalism. As usual the norm-squared of the comoving field is $b^2 \equiv b^k b_k$.

The quadrupole formula is the standard prescription for extracting gravitational wave signatures. If the amplitude of the gravitational wave, $h_{\mu\nu}$, is defined to be the deviation of the metric from a locally Minkowski spacetime, then the quadrupole formula can be stated as

$$h^{TT}_{ij}(R) = \frac{2G}{c^4}\frac{1}{R}\frac{d^2}{dt^2}I^{TT}_{ij}(t - R/c) = \frac{2}{R}\frac{d^2}{dt^2}I^{TT}_{ij}(t - R), \tag{3}$$

where the TT superscript denotes a projection onto the transverse-traceless gauge, and the `Zelmani` evolution code uses units of $c = G = M_\odot = 1$. The central aspect of incorporating new physics into this signature is to find an expression for $\ddot{I}_{ij}$, the second time derivative of the quadrupole moment defined as

$$I^{ij} \equiv \int d^3x T^{00} x^i x^j. \tag{4}$$

We have determined two possible models for the extension of the quadrupole moment to include magnetohydrodynamic effects. In the remainder of this note we shall first describe the progress on standardizing the code with previous TAPIR codes, the discuss the process of model determination, the newly added `QuadIntegrateM.cc` module of the `ZelmaniQuadWaveAnalysis` thorn and conclude with an update on current simulation progress of the modified code.

2 Standardization with Existing Code

Before new simulations can be run on a new supercomputing cluster or with a different compiled configuration of the `Cactus` framework or specific arrangements and thorns within a given framework configuration, several benchmarks are usually taken first. The current code has previously passed a series of general benchmarks in order to reproduce results from other core-collapse codes. As a first step for the current project, we standardized results from the core-collapse runs against the previously published simulation data [1].

3 Model Determination

3.1 T^{ij} Model

A first approach to model determination was to include a magnetohydrodynamic term into T^{00}. According to the previous prescription for the quadrupole formula, a first derivative of the quadrupole could be computed analytically via the continuity equation and the second derivative could then be computed via a finite-difference scheme during post-processing.

This was mainly taken due to the Newtonian approximation of the energy, which shows $T^{00} \sim \rho_*$ under the standard assumptions of the quadrupole formula. Since this is no longer true for a magnetohydrodynamic model, the standard form of the quadrupole moment is not necessarily inherently more efficient that any other form. An alternative form can be derived trivially from the tensor virial theorem, which states that for a closed system in which typical conservation laws hold (viz. $T^{\mu\nu}_{,\nu} = 0$),

$$\frac{\partial^2}{\partial t^2} \int d^3x T^{00} x^i x^j = 2 \int d^3x T^{ij}, \tag{5}$$

the proof of which is elementary. Here we assume locally Newtonian physics, meaning that covariant and contravariant indices are identical. Since the left side of (5) is defined as the quadrupole in the context of the theory of gravitational waves, and since we make a Newtonian approximation in which $I^{ij} \sim I_{ij}$ at the point of extraction,

$$\ddot{I}_{ij} \sim \ddot{I}^{ij} = 2 \int d^3x T^{ij}. \tag{6}$$

Equation (6) eliminates the need for the use of an analytic first derivative of the quadrupole moment, and also improves on the overall numerical noise due to the finite-differencing of the quantity during post-processing. Since the implementation of this model was considered to be the easiest and least time consuming, it was decided that (6) should be used as a first attempt to extract the gravitational wave signature due to magnetohydrodynamical effects.

3.2 b^2 Model

A second model would be to rescale the density parameter to take magnetic field into account. It can easily be shown that this corresponds to a variable substitution $\rho \to \rho_* \equiv \rho + b^2$ is the simplest way of including a first order magnetic correction into the hydrodynamics. These sort of variable substitutions lead to a number of different possible models. For instance, while the trace of the covariant magnetic tensor $M_{ij} = -b_i b_j$ is the only magnetohydrodynamic contribution to the first order model, higher fidelity models may be obtained by the inclusion of off-diagonal terms in the hydrodynamics or enthalpy.

4 The QuadIntegrateM.cc Module

A new module has been added to the `ZelmaniQuadWaveExtract` thorn to allow for the computation of (6) rather than extraction without magnetohydrodynamics. It is activated by a boolean parameter 'Mphys' which runs the MHD extraction by default. The module itself consists of two routines. The first is a local routine to compute the stress-energy tensor over the so-called 'Cartesian patch' of the mesh, given by

$$T_{ij} = \begin{cases} T_{ij}^{\text{TmunuBase}} & \text{for } \mathbf{x} \in \mathcal{N}_R(\mathcal{O}) \\ 0 & \text{else} \end{cases}, \tag{7}$$

where R is a user-specified radius inside which to include MHD effects in the extraction and the stress-energy tensor inside is taken from the TmunuBase module. It is then stored as a grid function internal to the framework.

The second part of the module is a global routine to integrate and return the $\ddot{I}_{ij}$ given by (6), using `Cactus` macros, and is patterned after the typical integration routines in the code and framework altogether. The routines were debugged and compiled with a new executable.

5 Simulations and Preliminary Results

A series of two-hour testing runs were completed using 96 cores on Zwicky. This allowed us to evaluate preliminary results and compare the two models in the early stages of evolution. Results we obtained to show that the T^{ij} model performed extremely poorly to correctly capture the morphology of the extraction without the use of gravitational waves, while the model which simply mapped $\rho \to \rho_* \equiv \rho + b^2$ performed much more closely to the expectation. This lead to speculation as to why the T^{ij} model does not appear to function. Possible explanations are causalic arguments, where the position of the observer, being removed from the immediate vicinity of the distribution, would see a Greens Function transformed stress-energy distribution which would need to be taken into account to correctly extract the wave signature. Similar suggestions and an in-depth analysis are provided in [2].

A 68 hour simulation was run on the CACR Zwicky cluster in order to ascertain the complete contribution, but the simulation was found to be corrupted by numerical noise. This noise was also found to exist in a simulation which did not include MHD effects in the gravitational extraction. Further debugging runs suggested the issue was with the lower level adaptive mesh refinement drivers in the Carpet package. A stable version of this code was obtained and recompiled.

Another issue suggested by the second round of debugging runs was the possibility that the b^2 model was not implemented correctly. If this was the case than either the calculation or writing of the value contained a bug. We determined first that there was no issue with the writing, and then determined that there was no issue eliminating the calculation of the magnetic field effects by using an initial field strength approximately 200 times larger than would be physically accurate. This would cause magnetic effects to become more pronounced and show if the calculation was implemented correctly. We received results indicating that even within the first few iterations there were points with pronounced differences from the version without magnetic effects in the extract. This implied that it was not actually a bug, but rather a result showing that the magnetic field does not contribute in an appreciable manner to the gravitational wave signature in the first moments of the collapse.

6 Final Results

Instabilities continued to plague the simulations until we reverted to a much older version, used to correctly simulate a CCSNe in [1]. This allowed a direct comparison between the non-MHD and MHD extractions. We successfully observed first bounce and morphology corresponding to gravitational wave emission after the occurrence of first bounce, and were able to compare simulations against results obtained from earlier simulations.

References

[1] Mösta, P. et al. (2013). GRHydro: A new open source general-relativistic magnetohydrodynamics code for the Einstein Toolkit. arXiv:1304.5544.

[2] Müller, B., et al. (2013). A New Multi-dimensional General Relativistic Neutrino Hydrodynamics Code of Core-collapse Supernovae. III. Gravitational Wave Signals from Supernova Explosion Models. *The Astrophysical Journal*, **766**, Issue 1, article id. 43, 21 pp.