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CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Thermal Noise Analysis in Coating-Less Optical Cavities

Sam Moore

California Institute of Technology
LIGO Project, MS 18-34
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project, Room NW22-295
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
Route 10, Mile Marker 2
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189
E-mail: info@ligo.caltech.edu

http://www.ligo.caltech.edu/

1 Motivation

Laser frequency stabilization is important for use in high-precision measurements (such as gravitational wave detection). The cavity from which the laser originates contains many sources of noise, such as thermal and mechanical fluctuations [3]. To achieve the precision necessary for gravitational wave detection, this project aims to reduce noise in these cavities. Optical cavities often contain highly reflective coatings, which have the problem of high mechanical (Brownian) noise. As a result, this project removes the reflective coatings in the resonator and instead uses total internal reflection. The goal will then be to minimize the thermal noise associated with this cavity.

2 TIR Cavity

Our optical cavity removes all of the multilayer coatings that are usually present. In doing so, losses upon reflection are reduced; as mentioned previously, Brownian noise is prevalent in multilayer coatings. The cavity operates by using total internal reflection (TIR). For this to work, it must be that $n_2 < n_1$ [1], where n_1 is the index of refraction of the cavity, and n_2 is the index of refraction of the surrounding medium.

Furthermore, the angle of incidence θ inside of the medium n_1 must satisfy $\sin\theta > n_2/n_1$ for TIR to occur. This means that the values by which n_1 and n_2 differ depends on the geometry of the cavity. For example, if we describe total internal reflection inside of a square, $\theta = 45°$, so $n_1/n_2 > 1.41$. For a triangle, meanwhile, $n_1/n_2 > 2$.

How does the light first get into and out of the cavity? It "leaks" into the cavity by use of frustrated total internal reflection. Here, another material is placed a distance of order λ (light wavelength) from the cavity. This allows an evanescent wave to travel into and out of cavity.

3 Thermal Noise

Gravitational wave interferometers deal with many sources of noise, which is why it is important to have a strong signal. The arms of the interferometer contain optical cavity "test masses," which are used to amplify the laser beam signal. For this optical cavity, thermal noise is the most relevant, and this project seeks to minimize this thermal noise. There are three sources of thermal noise: Brownian noise, Thermoelastic (TE) noise, and Thermorefractice (TR) noise.

Fluctuation Dissipation Theorem

The Fluctuation Dissipation theorem (FDT) will be the primary tool used to calculate the thermal noise. The central idea of the FDT is that fluctuations cause dissipation. Hence, if we have low fluctuations, there will also be low dissipation. Notice that this is relating a microscopic property to a macroscopic property. The dissipation (a macroscopic property) is usually the observed property, which means that it can be used to infer the thermal fluctuations.

Note that the power spectrum of a resonant cavity can be used to infer to the dissipations. For a cavity with very low dissipation, the energy will be localized near the resonant modes. For large dissipation, that energy spreads out.

Levin's Approach

Levin's approach utilizes the FDT to calculate thermal noise. The technique works for non-uniform dissipation and an arbitrary laser beam size. To calculate the thermal noise $S_x(f)$ at a frequency f, one applies an oscillatory generalized force $F_0 \cos(2\pi f t) f(\vec{r})$ to the geometry of interest (the "test mass") [4]. $f(\vec{r})$ indicates the shape of the laser beam on the surface of that geometry. In this process, one can calculate W_{diss}, the dissipation associated with the friction of the test mass. In the Levin paper, $S_x(f)$ can then be calculated via

$$S_x(f) = \frac{2k_B T}{\pi^2 f^2} \frac{W_{\text{diss}}}{F_0^2} \tag{1}$$

where T is the temperature of the test mass. Note that the F_0 term is not necessary to calculate because it cancels out with the F_0 in the expression for W_{diss}.

Brownian Noise

The first noise that is considered here is Brownian Noise. This is an effect that arises out of Brownian motion, where particles in a fluid are observed to jostle randomly while suspended in a fluid. It was first discovered in 1828, but remained a mystery until Einstein, in 1905, used the finding to demonstrate the existence of atoms. Brownian motion can be described using the Diffusion equation, where particles move from high to low concentrations. Brownian noise manifests itself as slight, fluctuating distortions in the shape of the cavity. Brownian noise can occur in an optical cavity's reflectors, and is especially prevalent in multilayer coatings.

Thermoelastic Noise

The second source of noise is thermoelastic (TE) noise, which arises from thermal fluctuations in a cavity's mirror and optical coatings. These thermal fluctuations cause the cavity to create small, fluctuating deformations throughout its surface. These geometry changes then cause fluctuations (i.e. noise) in the laser's frequency. TE noise is characterized by an expansion coefficient α.

Thermorefractive Noise

The third source source of noise is thermorefractive (TR) noise, caused by fluctuations in the index of refraction of the cavity. The result of these fluctuations is that radiation in the cavity develops random fluctuations in its phase. The parameter $\beta \equiv \frac{\partial n}{\partial T}$ characterizes this

TR noise (where n is the index of refraction). For Thermorefractive noise (see description below), the generalized force of the Levin Approach has the form of an oscillatory heat source [5]:

$$q(\vec{r},t) = T(\vec{r},t)F_0\cos(2\pi ft)\frac{\beta}{\pi r_0^2}e^{-r^2/r_0^2} \tag{2}$$

The dissipation of the heat source in the test masses is related to the temperature gradient via

$$W_{\text{diss}} = \frac{1}{2T_0}\int_V \kappa|\nabla T|^2\,dV \tag{3}$$

where κ is the thermal conductivity and T_0 is a homogeneous reference temperature of the test mass. From this, one can use the FDT (eq. 1) to calculate the thermal noise.

Thermo-optic Noise

TE and TR noise can be combined together, which is the aim of this project. Evans et. [4] al showed that in a cavity with multilayer coatings, the TE and TR mechanisms have a negative relative sign in the overall thermal noise ("thermo-optic" noise) spectrum, leading to possible thermal noise cancellation. However, this relative negative sign does not occur with coating-less cavities. As a result, the last year's project sought materials whose parameters α and β *themselves* had a relative sign difference (as opposed to a sign difference in the power spectrum) [3].

Since TE and TR noise both derive from the same source—thermal fluctuations—it is reasonable to suspect correlation between the two noise sources. Indeed, last year's project found that TE and TR noise are at least somewhat correlated. The more correlated these sources are, the more cancellation between α and β is possible. The goal of this project will be to determine this noise correlation.

4 Finite Element Analysis (Progress so Far)

COMSOL and MATLAB are being used to simulate the sources of noise. The COMSOL model will utilize Finite element analysis and the Fluctuation Dissipation theorem. The following is a discussion of what has been done so far, as well as what will be done in the future.

The current progress has been made in verifying the COMSOL model for TR noise in cylindrical test masses, as presented in the Heinert et al [5]. Here, the aim is to calculate the TR noise in a cylinder subject to adiabatic boundary conditions ($\nabla T = 0$ at the boundary). Heinert et al. derives a plot for this TR noise $\sqrt{S_z(f)}$, where f is the frequency of a heat injection, following Levin's approach. The heat injection is assumed to be of the form of equation (2). If one further assumes small temperature fluctuations, $T(\vec{r},t)$ in equation (2) can be taken as a constant ambient temperature T_0. This heat $q(\vec{r},t)$ enters into the heat equation as

$$C_p\frac{\partial T}{\partial t} - \kappa\nabla^2 T = \dot{q}(\vec{r},t), \tag{4}$$

where C_p is the heat capacity per volume at constant pressure. In last year's approach, the TR noise was calculated by solving for the time-dependent solution $T(\vec{r}, t)$. Here, the goal is a less computationally expensive approach. We assume a steady-state temperature $T(\vec{r}, t) = T(\vec{r})e^{i\omega t}$, which yields a stationary differential equation of the form

$$i\omega C_p T(\vec{r}) - \kappa \nabla^2 T(\vec{r}) = i\omega A e^{-r^2/r_0^2} \tag{5}$$

where $A \equiv \beta T_0 F_0/(\pi r_0^2)$. Once COMSOL calculates the temperature profile, the dissipation is calculated via equation (3), and the TR noise is calculated via equation (1). The following noise plot was obtained for a silicon test mass at $T_0 = 10$ K.

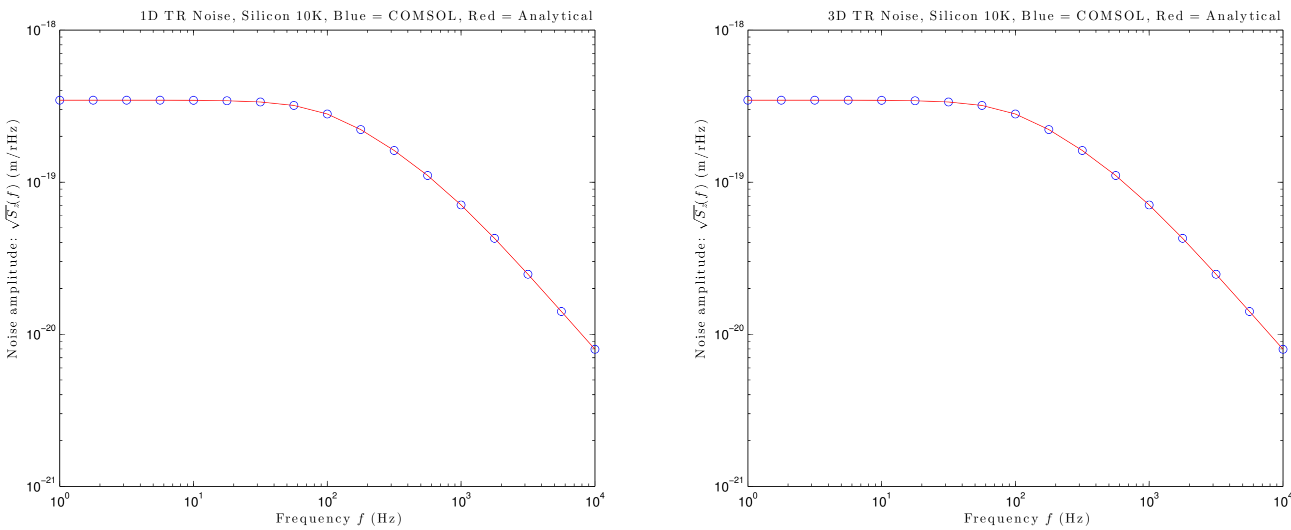

Figure 1: Displacement TR noise as a function of frequency f for silicon at 10 K. The verification is shown for two different types of models in COMSOL. The 3D model (right) is the most general model, and it still runs much faster than the previous time-dependent model.

The two plots show that a 1D axisymmetric model (left) and a full 3D model (right) in COMSOL both yield agreement with the red analytical curve. Furthermore, the steady-state temperature approach allowed these plots to be generated much faster than before.

The next step will be a similar verification for TE noise, using the same steady-state method as before. Once agreement has been achieved between the numerical and analytical plots, the goal will be to use this same steady-state method for our TIR cavity. The geometry in this case is not symmetric, meaning that a full 3D model will be used in COMSOL. With this geometry, the goal will be to find a material that minimizes thermo-optic noise. Last year, some cancellation was observed for Sapphire at 300 K, using the negative of its usual thermal expansion coefficient. In this case, it was found that the noises were correlated, which meant that cancellation of the noises was possible.

The following is a tentative time frame for the remainder of the project.

1. Extend the model to the desired geometry (5 weeks).

2. Explore the parameter space (2 weeks)

3. Find the optimum parameter set that reduces thermal noise (time permitting) Upon calculating the thermal noise, the goal is to find a material where this noise is minimized. The goal is to achieve cancellation of these two coefficients by finding the right material. If such a material can be found, the goal will be to build a setup for laser stabilization with that material. This could help identify unknown sources of noise.

References

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[5] Heinert, et al. *Thermorefractive noise of finite-sized cylindrical test masses.* Physical Review D84, 062001 (2011).