Modeling a Long Interferometer

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LIGO-G09xxxxx-v1

Why should we go bigger?

$$h = \frac{\Delta L}{L}$$

- Gravitational wave – h
- Displacement noise – ΔL

Preliminary noise budget

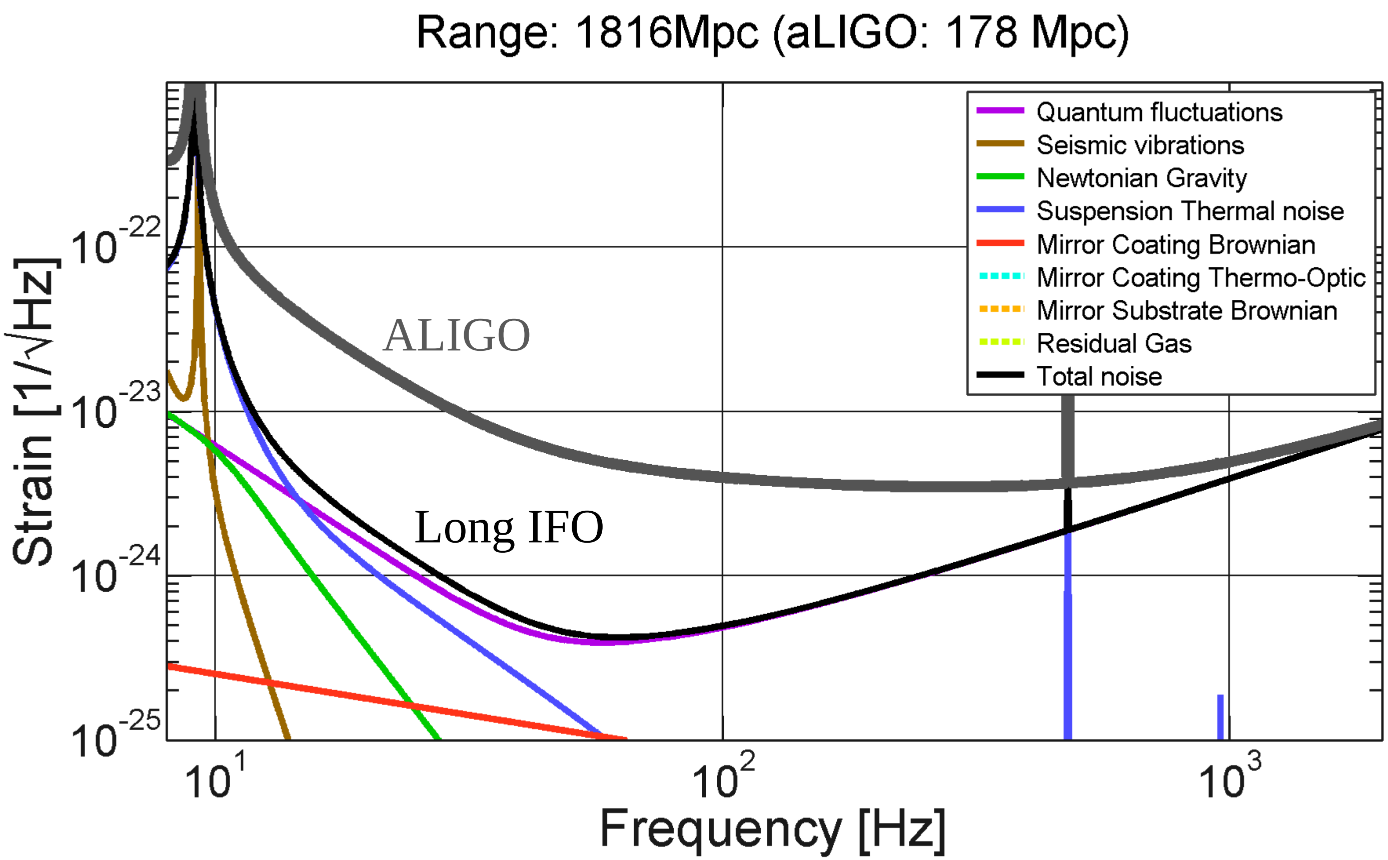


Image created by Stefan Ballmer using GWINC

What's the problem?

- Cost
- Finding a location
- Curvature of Earth

- Maintaining a narrow beam
- Solution: Add lenses to beam tube

Lens problems

- Find a lens configuration
  - » Study analytically
  - » Brute force computation
- Find the noise introduced by the lenses
  - » Misalignments
  - » Scattering
  - » Thermal noise
  - » Thermal lensing

Optics background

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad q_{out} = \frac{Aq_{in}+B}{Cq_{in}+D}$$

- Model optics as matrices
- Gaussian beam represented as q
- Sequences of optics can be calculated through matrix multiplication

See *Lasers* by A.E. Siegman

How to build a resonator

- Requirements:
  - $q_{in} = q_{out}$ after full roundtrip
  - q is a physical solution
  - Resonator is stable to input perturbations
- Can be described with m parameter
  - Similar to the g parameter, but more general
- $m \equiv \frac{A+D}{2}$
- $-1 \leq m \leq 1$

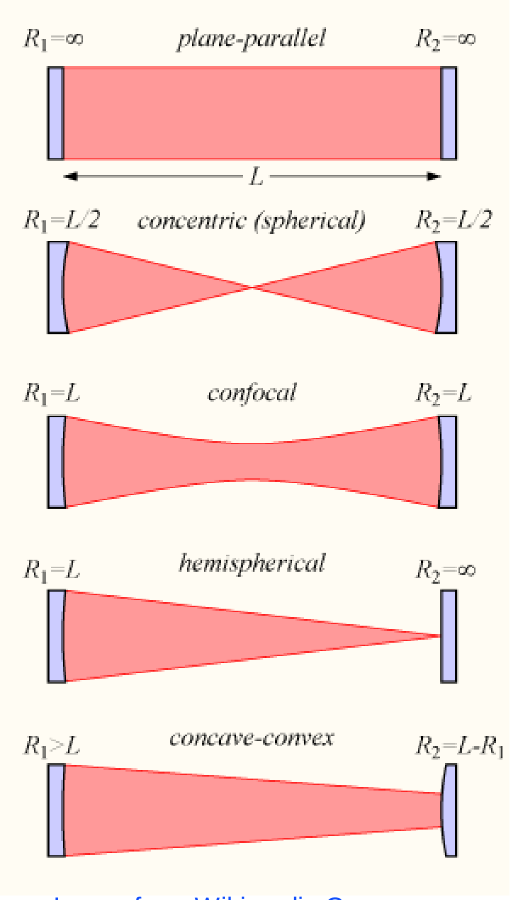


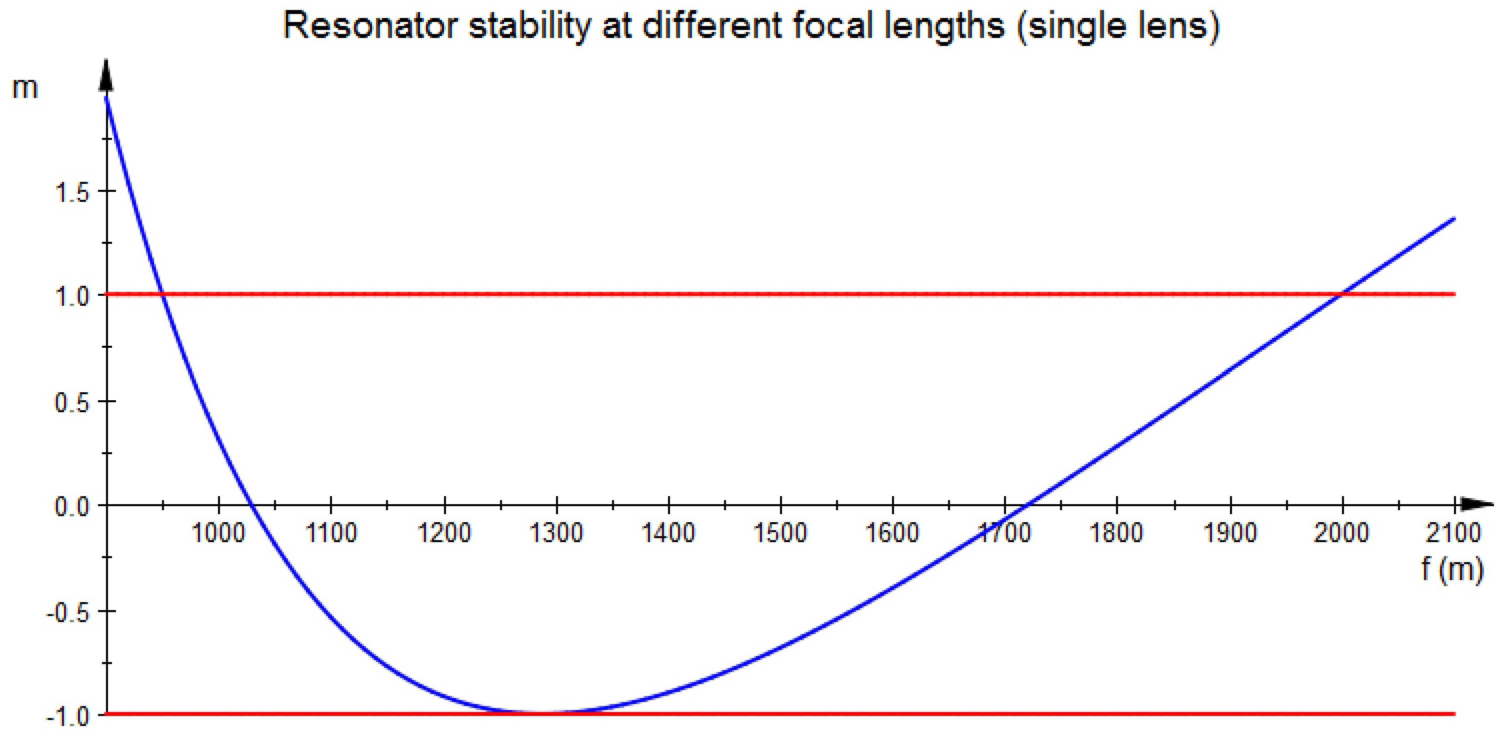
Image from Wikimedia Commons: http://upload.wikimedia.org/wikipedia/commons/c/ca/Optical-cavity1.png

The analytic approach

- Examine cavity with single lens
- Examine cavity with multiple lenses of equal focal length evenly spaced

How f affects m

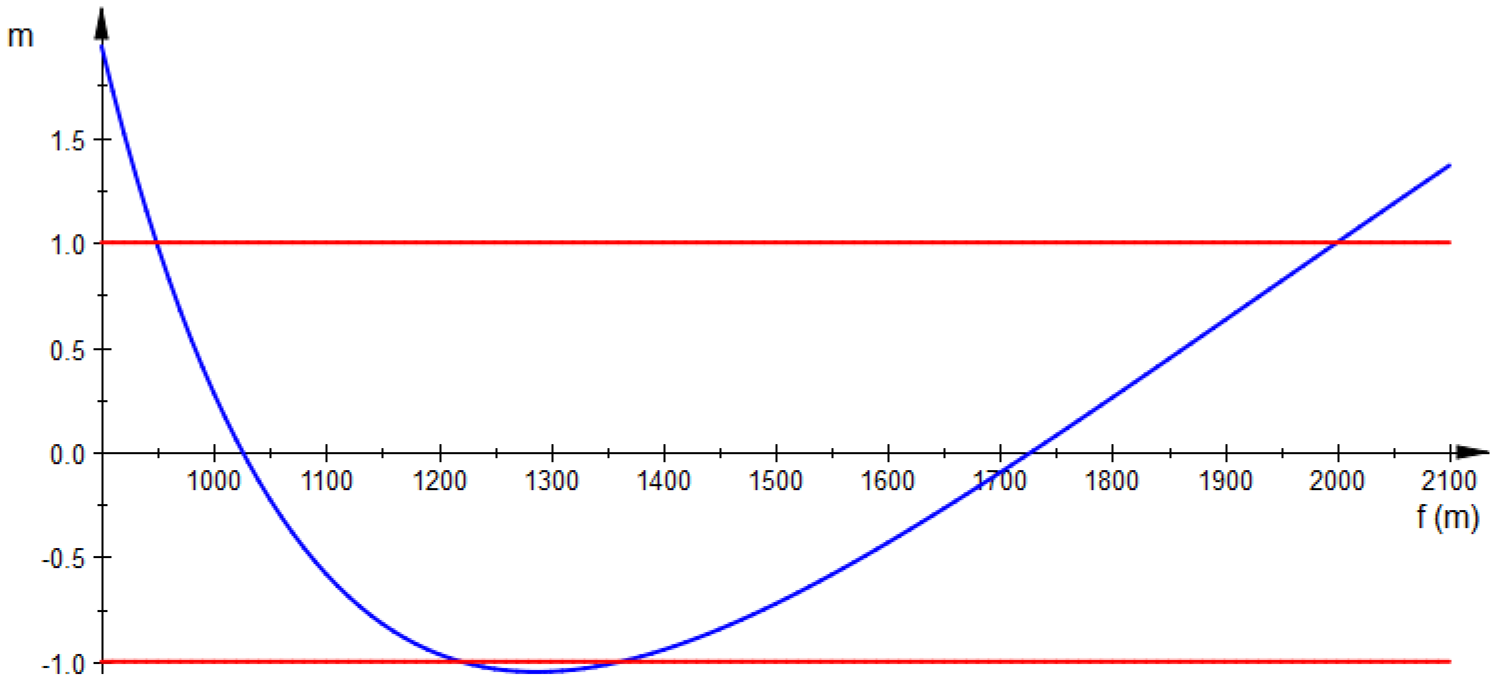
- 8km resonator, symmetric mirrors, lens at 4km



Asymmetric mirrors

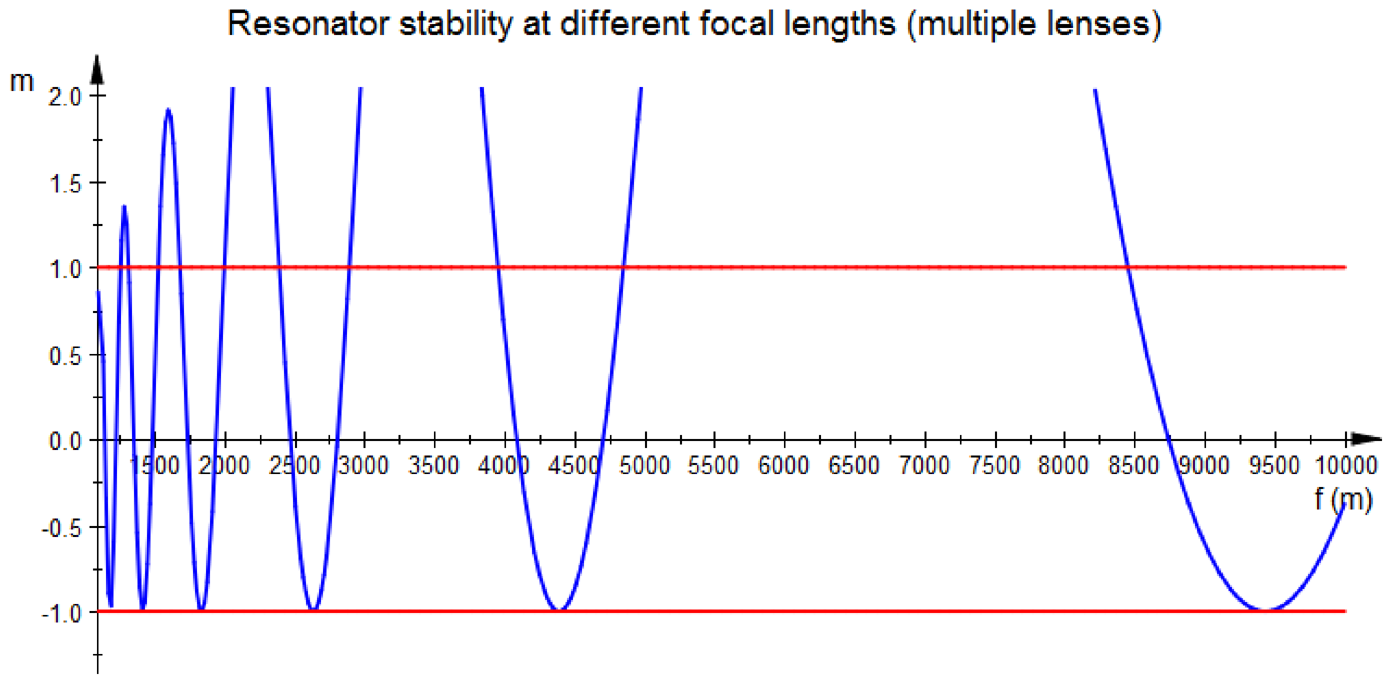
- Same situation, but aLIGO mirrors

Resonator stability at different focal lengths (single lens)



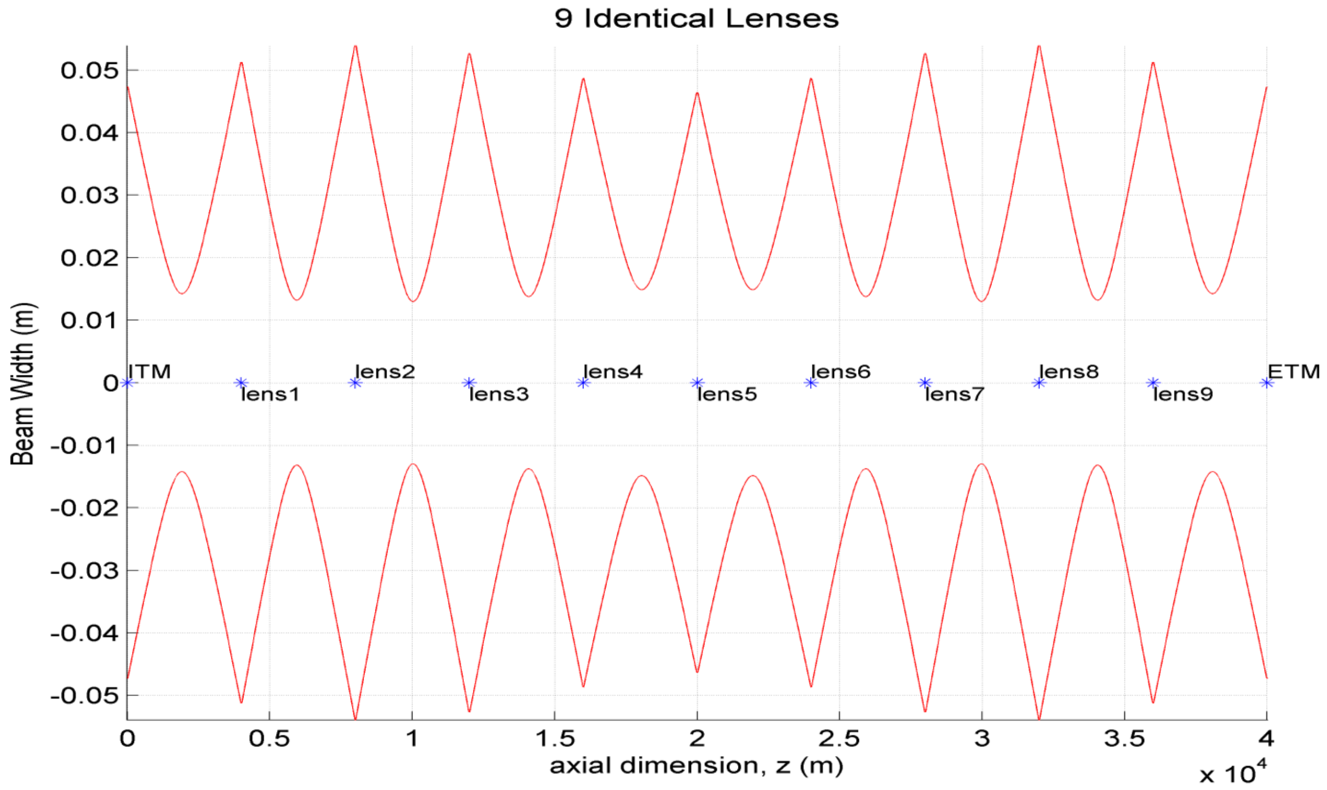
Multiple lenses

- Generalize to multiple identical lenses
- 40 km with 9 lenses 4km apart



A possible path

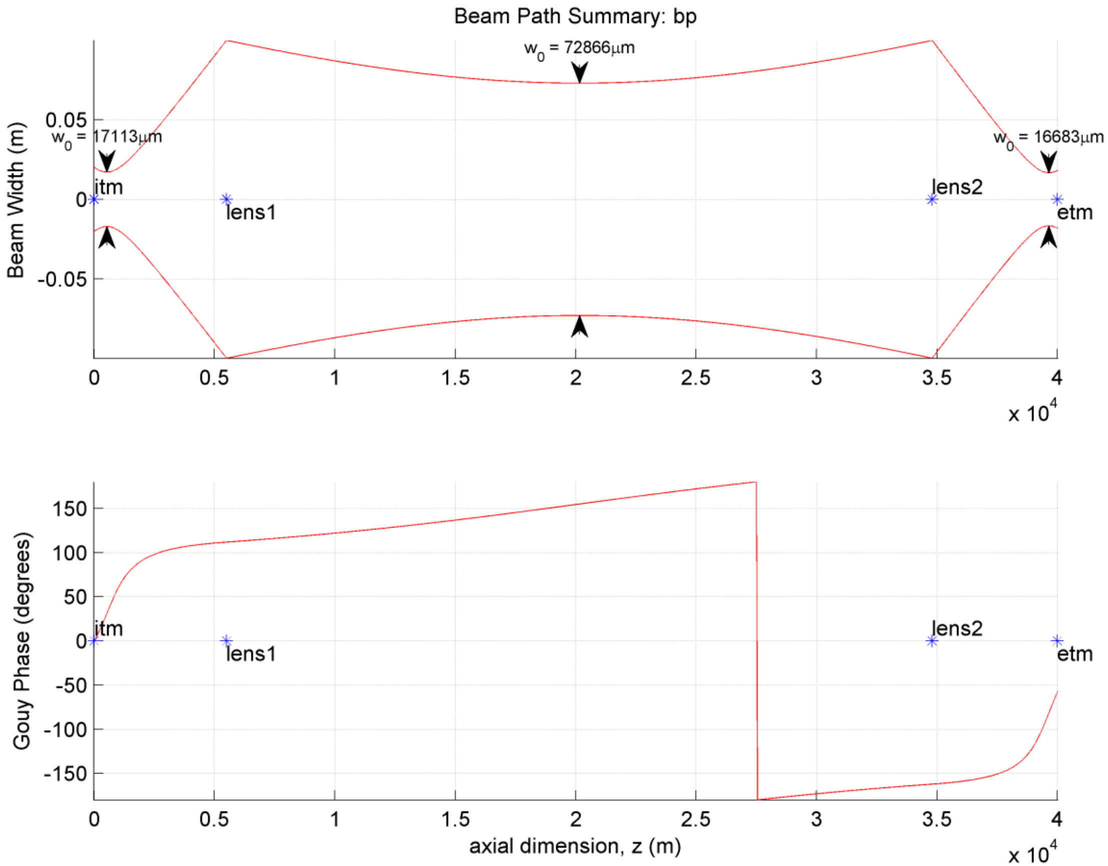
f=1083m, m=0



The brute force approach

- Used MATLAB to find configuration using few lenses
- Result:
  - » Lens 1: z=5.5km,   f=4400m
  - » Lens 2: z=34.8km, f=4300m

Brute force result



Misalignments

- Optics do not remain fixed in position
- Small misalignments can cause power losses
- Shot noise becomes more significant

Misalignment theory

- Cavity has eigenmodes TEM_{ij}
- Detector uses TEM_{00} mode
- Misalignments transfer power to higher order modes
- Misalignments represented as matrices

See Hefetz, Mavalvala, and Sigg. Principles of calculating alignment signals in complex resonant optical interferometers

Misalignment results

- Fractional power losses in 2 lens configuration:
  - Lens 1
    - Rotation: $0.19\theta^2$
    - Displacement: $48.1\frac{x^2}{w^2}$
  - Lens 2
    - Rotation: $3.7\theta^2$
    - Displacement: $50.3\frac{x^2}{w^2}$

- Rotation is negligible
- Transverse displacement could be significant

Further steps

- Scattering
- Thermal noise
- Thermal lensing
- Putting all these effects together

Questions?