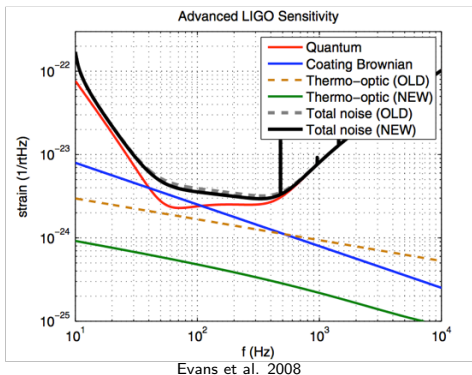


Thermal Noise Reduction in Coating-Less Optical Cavities

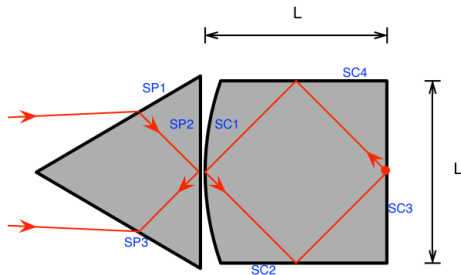
Sam Moore, Mentors: Matt Abernathy & Koji Arai

LIGO SURF Project

August 21, 2014



- LIGO laser cavity noise sources: seismic, thermal, optical (= quantum and technical laser noise)
- Coating brownian noise - significant noise source in AdvLIGO
- Question: Can we reduce thermal noise of the cavity by using a “coating-less optical cavity?”



Courtesy: Koji Arai

- Setup from Schiller et al. 1992
- Removes reflective coatings.
- Cryogenic Silicon: high Q ($\sim 10^9$).
- **Goal: reduce remaining thermal noise, TR and TE**

- Thermorefractive noise: Noise due to $\beta \equiv \frac{\partial n}{\partial T}$
- Thermoelastic noise: Noise due to $\alpha \equiv \frac{1}{L} \frac{\partial L}{\partial T}$
- Thermo-optic noise: Combined TE and TR noise.
- TE and TR correlation \Rightarrow possible cancellation

- 1 Developed frequency-domain FEA methods for calculating thermal noise
- 2 Verified methods on test cases
- 3 Calculated thermal noise for TIR cavity
- 4 **Finding: TIR cavity too noisy**

- Fluctuations cause dissipation
- Levin approach: Calculate W_{diss} by applying forced oscillations $F_0 f(\vec{r}) e^{i\omega t}$.
- Infer fluctuations $S_z(\omega)$ from dissipation W_{diss} :

$$S_z(\omega) = \frac{8k_B T_0}{\omega^2} \frac{W_{\text{diss}}}{F_0^2}$$

$\Rightarrow S_z(\omega)$ is a variance density of random variable z .

- 1 Frequency-domain method: assume $T(\vec{r}, t) = T(\vec{r})e^{i\omega t}$, given $q(\vec{r}, t) = q_{TR}(\vec{r})e^{i\omega t}$:

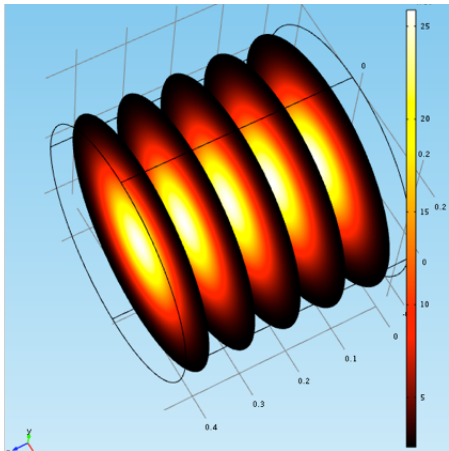
$$i\omega C_p T(\vec{r}) - \kappa \nabla^2 T(\vec{r}) = i\omega q_{TR}(\vec{r})$$

COMSOL: using stationary solver

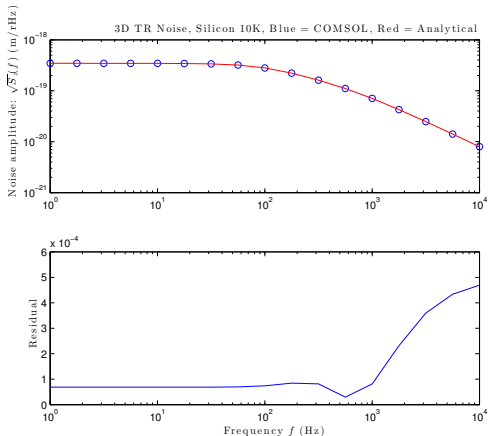
- 2 Calculate W_{diss} :

$$W_{\text{diss}} = \frac{\kappa}{2T_0} \int |\nabla T|^2 dV$$

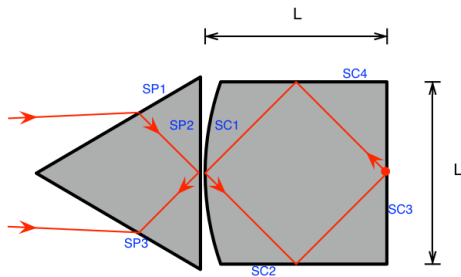
- 3 Then calculate $S_z = 8k_B T_0 W_{\text{diss}} / (\omega^2 F_0^2)$.



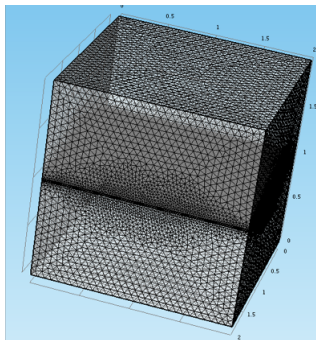
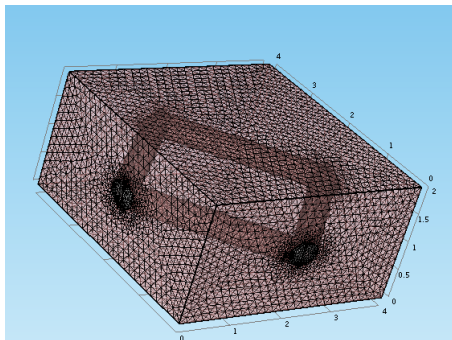
- Assume Gaussian laser ($f(\vec{r}) \propto e^{-r^2/r_0^2}$), adiabatic boundary conditions
- Compare COMSOL model with analytical model



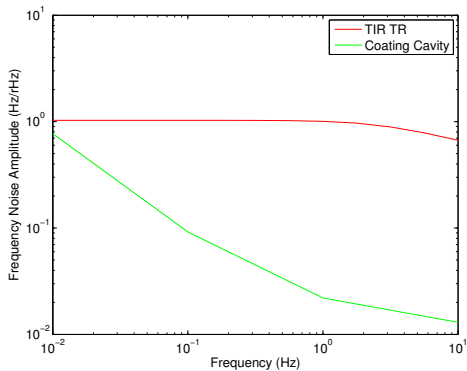
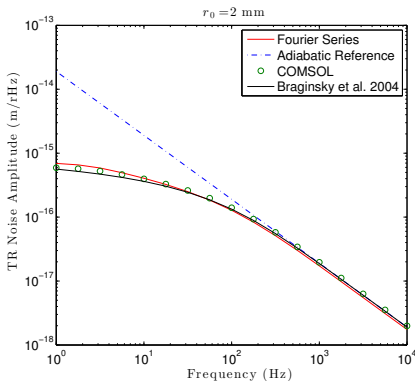
- Why does the elbow happen? At low f , the heat has time to escape. Heat flow can't increase.
- My frequency-domain study works, *and runs much faster!*



- Assumptions: rectangular cavity, no energy loss, Gaussian beams, no interference or beam waisting effects
- $L = 4$ cm
- Si at 120 K: $\alpha = -0.057 \times 10^{-6} \text{ K}^{-1}$, $\beta = 8.7 \times 10^{-5} \text{ K}^{-1}$.



- Source of error: meshing inaccuracies in COMSOL
- Which mesh to choose?
- Simplest possible: uniform mesh. Other choices use too much memory

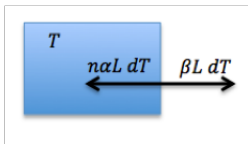


- Faster and more accurate convergence with larger beam size
- TIR TR noise is too high (comparison with Kessler et al. 2012)
- Next step: look for correlation with TE noise to achieve cancellation

- 1 Apply oscillating stress on faces (boundary conditions).
- 2 Solve for the elastic displacement via $(1 - 2\sigma)\nabla^2\vec{u} + \nabla(\nabla \cdot \vec{u}) = 0$.
- 3 Elastic strain $\nabla \cdot \vec{u}$ creates heat \Rightarrow solve heat eq. with TE heat injection

$$q_{TE}(\vec{r}) = -\frac{\alpha ET_0}{1 - 2\sigma} \nabla \cdot \vec{u}$$

- 4 Apply W_{diss} to get S_z .



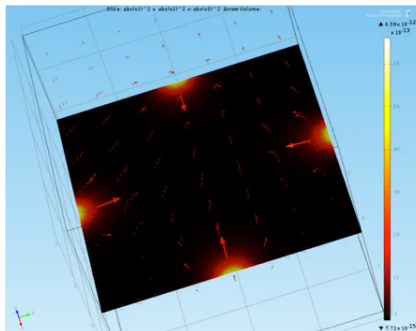
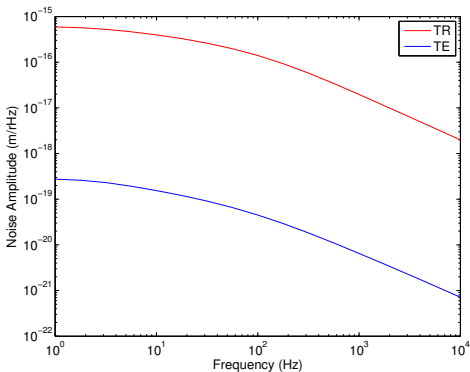
- Combine heat injections: $q(\vec{r}) = q_{TE}(\vec{r}) + q_{TR}(\vec{r})$.
- Thermal noise leads to optical path length $OPL = nL$ fluctuations.

$$d(OPL) = (\beta + n\alpha)L dT \quad (\text{uniform temperature})$$

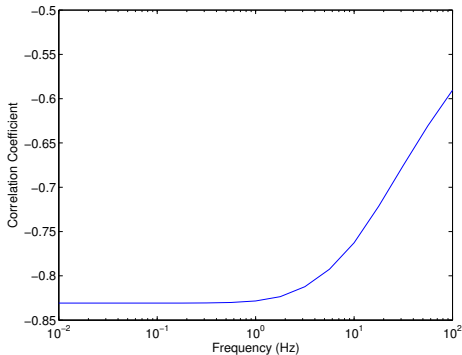
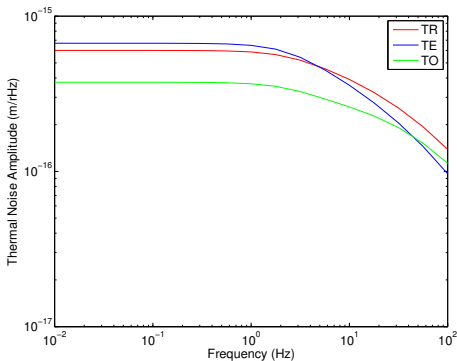
- Cancellation depends on TE/TR correlation,

$$S_{TO} = S_{TE} + S_{TR} + 2R\sqrt{S_{TE}S_{TR}}$$

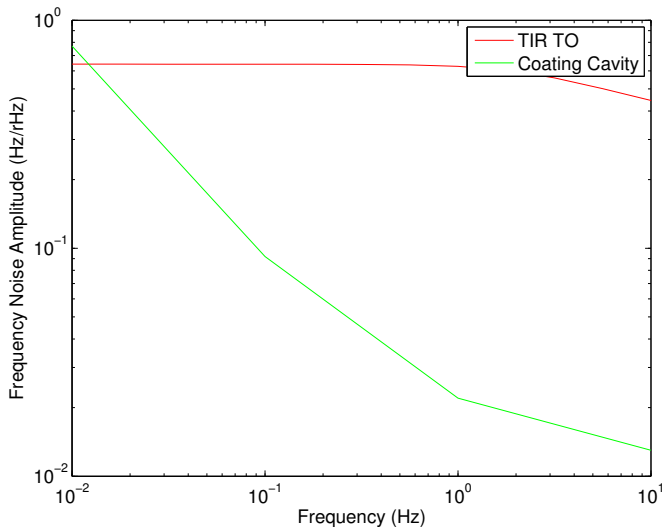
- Cancellation requires $S_{TE} = S_{TR}$ and $R = -1$.



TE is much different than TR. Cancellation is not possible with the current α and β .



- Adjusted α to $-1.3 \times 10^{-4} \text{ K}^{-1}$, so TE resembles TR at low frequency
- Calculation of correlation given TO, TE, TR.
- Partial correlation leads to partial cancellation



Noise is still too high

- Implemented faster frequency-domain method for calculating thermal noise
- Given TE/TR correlation, we obtained partial cancellation.
- TIR cavity: too much noise
- Next steps: different geometry, α optimization, mesh refinements

- Mentors: Matt & Koji
- LIGO SURFS
- Collaboration with Daniel Heinert
- NSF