LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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ITM transmission map and contrast defect

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1 Introduction

The effect of the non-uniformity of the ITM substrate in the contrast defect (CD) is discussed in this document. The CD is defined to be the ration of the power on the BS going toward the dark port and that going toward the bright port, or the dark fringe count to the bright fringe count in MI. The power term of the non-uniformity in ITM can be compensated by TCS, either CP or RH, but higher order aberration is not easy to be compensated and one of the main cause of the CD of PRMI or MI comes from this ITM non-uniformity. This quantity is used to find the better pairing of the ITM coating among the 4 uncoated mirrors.

2 Analytic formula of the CD

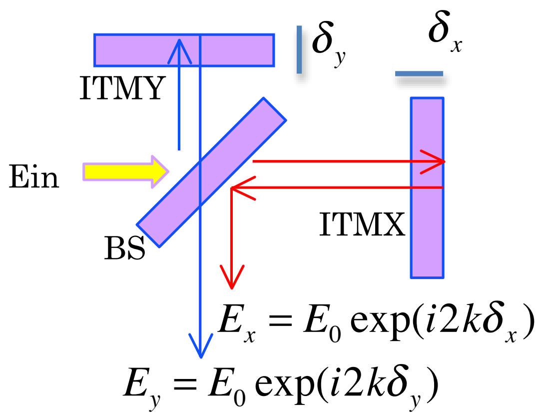

Figure 1 Near field propagation around BS

The cavity lengths around BS are short (~10m) compared to the Rayleigh range (~200m), and the change of the beam size is small (~0.5%). Because of this near field propagation characteristics, when the input field, Ein, goes to an ITM and comes back, only major change is the phase change induced at the reflection by the ITM.

In this crude, yet reasonably good approximation, the contrast defect can be express as follows.

$$CD = Power(E_x - E_y) / Power(E_x + E_y)$$
$$= \int dx\, dy (2k)^2 \frac{2}{\pi w^2} \exp(-2\frac{r^2}{w^2})(\delta_x(x,y) - \delta_y(x,y))^2 / 4 \qquad (1)$$

In this expression, w is the beam size (same on all mirrors) and $\delta_{x,y}$ is the one way optical path length of ITMX and ITMY substrates. With typical aLIGO ITMs, this integral comes out to be several 100ppm and the clipping due to finite mirror size is negligible. The integral goes over the ITM surface.

3 Lens in ITM substrate

When there is a lens with focal length f in one ITM and the other ITM does not have a static lens, δ and CD can be written as follows.

$$\delta = \frac{r^2}{2f} \tag{2}$$

$$\begin{aligned} CD &= \iint r\,dr\,d\varphi \frac{(2k)^2}{\pi w^2} \exp(-\frac{2r^2}{w^2}) \frac{1}{2} \frac{r^4}{(2f)^2} = \frac{1}{2}\alpha(f)^2 \\ \alpha(f) &= \frac{kw^2}{2f} \end{aligned} \tag{3}$$

In the modal calculation, when a LG00 field is reflected by a mirror with curvature error of dR (1/RoC of the reflecting mirror = 1/RoC of the incoming field + 1/dR), the reflected field can be approximated as the following in the lowest order approximation (T990081).

(1- i α(dR)) LG00 + i α(dR) LG10

The CD expression above is the case for the mixing of two beams on the BS, one LG00 and the other (1- i α(f)) LG00 + i α(f) LG10. The power on the dark port is 2 $\alpha(f)^2$, relative to the bright port power of 4.

Because α is proportional to w^2, the CD itself is proportional to w^4. As a numerical example, for w = 5.3cm and f = 80km, α =0.1 and CD = 5400ppm. When the beam size becomes larger in PRMI due to the static lens, the CD can become larger. E.g., for w = 6.6cm, CD becomes 1.3%.

4 SPTWE

Fig.2 shows the SPTWE (Single Path Transmission Wavefront Ertror) after subtracting tilt and power terms. This is the nonuniformity of ITM substrate after removing the power term. Table 1 shows the CDs of all 10 ITMs by using Eq.(1) with the other δ = 0. For the case of the lens effect, discussed in Sec.3, the dependence on the beam size can be calculated to be w^4 because the structure of the nonuniformity is known to be $r^2/(2\ f)$, but, for the remaining non-uniformity, there is no simple analytic formula about the beam size dependence. In Table 1, CDs are calculated for beam sizes of 5.3cm and 6cm. Together in the same table, P-V and RMS calculated in the central 160mm region after subtracting up to power and those after subtracting up to astigmatism and the focal lengths quoted from T1300954 are shown.

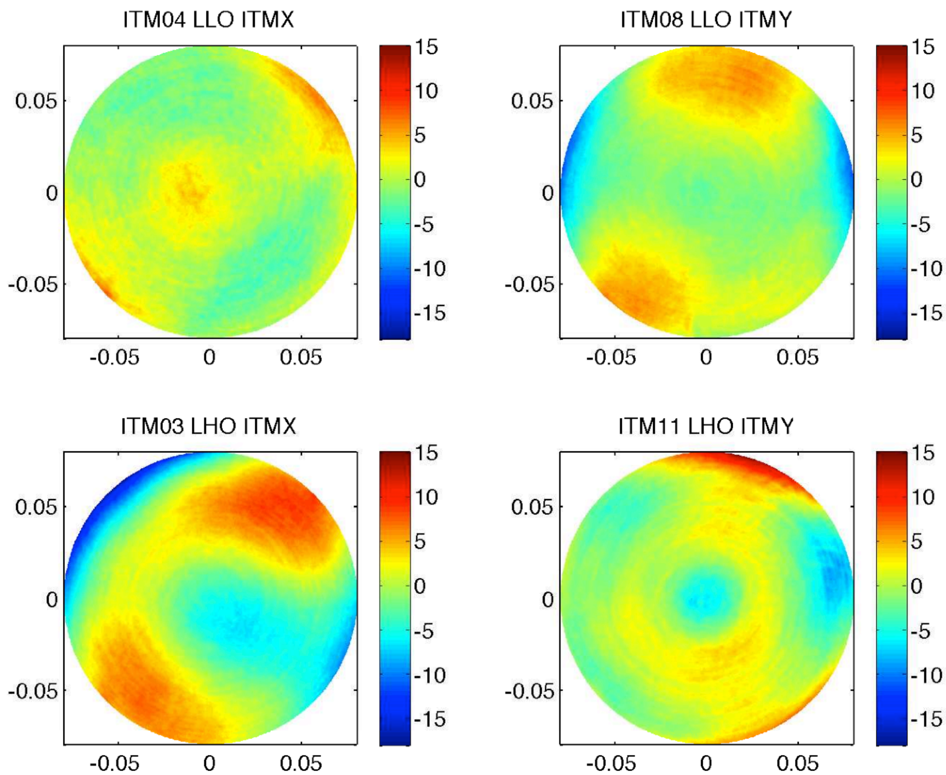

Figure 2 SPTWE of L1 and H1 ITMs after subtracting tilt and power

ITM #	CD (ppm) (5.3cm)	CD (ppm) (6.0cm)	P-V/RMS (nm) Up to power removed	P-V/RMS (nm) Up to astigmatism removed	Focal length (km)
1	189	399	27/4.2	11/1.7	-92.8
3	559	820	26/4.8	18/2.9	-80.2
4	115	234	12/1.9	8/1.3	305.0
5	390	630	18/3.3	12/2.2	78.3
6	154	207	14/2.1	12/1.9	381.1
7	83	116	16/2.2	9/1.3	-310.8
8	174	442	18/3.1	12/1.7	-82.4
9	142	344	11/2.1	9/1.5	229.4
10	210	411	22/3.1	13/2.3	-93.6
11	335	842	24/2.9	18/2.4	572.1

Table 1 ITM characteristics

As the equation of the CD shows, there are some interference effects of two transmission maps. The following table shows the CDs of pairs of ITMs using Eq.(1).

ITM pairs	CD (5.3cm)	CD (6cm)
04-08 (L1)	400	1046
03-11 (H1)	795	2013
01-05	800	1600
01-07	189	407
01-10	330	852
05-07	590	880
05-10	510	620
07-10	447	686
06-09	350	760
04-07	298	423

Table 2 CDs of pairs of ITMs