Quantum state tomography of cavities using highly squeezed light source and other methods

Bence Borcsok

Mentors: Rana Adhikari, Yanbei Chen, Nicolas Smith-Lefebvre

Abstract

Cancelling the quantum back action noise in time-dependent homodyne detection for quantum state tomography of a broadband cavity was discussed previously by Miao *et al.* [Phys. Rev. A 81, 012114 (2010)]. We extended the method for finite bandwidth cavities. The fact that the internal optical field in the cavity contains information about the state of the mirror at earlier times, thus neither the internal optical field nor the mirror are Markovian systems, was formulated. A general method was proposed for any system for which the same dynamics governs the preparation and the verification stage. The method involves causal factorisation of the connection matrix in frequency domain, which is only possible to carry out analytically for simple cases. A perturbative method was discussed, which treats the general connection matrix as first-order perturbation around a known one. The calculations were carried out for the finite bandwidth cavity. The calculations of the form of the optimal filtering functions were started. Future plans involve finishing calculating the additional noise from the presence of the cavity, comparing the perturbation method with the Davis routine for causal factorisation and examining the possibility of designing an experiment to check the theoretical findings.

1 Introduction

Measuring the state of a system is the key element of many experiments and often limits the precision of the whole measurement. Any measurement is limited by the Heisenberg uncertainty principal but there are also random noises which lower the accuracy of the measurement. In quantum optical measurements, noise can enter from numerous sources: thermal fluctuations, back-action (random fluctuation in the optical field results fluctuation in the radiation-pressure on the mirror), quantum inefficiency of the photodetector etc. To increase our precision, we need to carefully evade the most noise possible. One can cancel the back-action noise by feeding back the signal as discussed in Ref [2] or by measuring the appropriate combination of position and momentum.
During my project I study the second method in time-dependent homodyne detection. Measuring two quadratures and combining them might cancel some

of the noise. It can be shown that for a given quadrature, there is an optimal time-dependent filtering, which minimises the noise Ref [1]. My goal was to find this optimal filtering and measurement parameters for different setups.

2 Background

Quantum tomography using homodyne detection for broadband cavity was analysed in great details in Ref [1]. A conditional quantum state was achieved by measuring the position of the oscillator continuously, which causes the wave function to collapse to a classical trajectory. During preparation the quantum state of the system was prepared into a nearly pure state (with perfect measurement pure state can be achieved). After preparation, the verification process started, in which the aim was to determine the quantum state of the system introducing the least noise possible. For this reason, they mixed the outgoing optical field with a local oscillator and applied filtering. The optimal filtering functions were obtained for the assumed noise budget. They found that the back-action noise can be evaded entirely in the free-mass regime if there is no optical loss. This means that a piece of shot noise is used to cancel the back-action noise.

They also considered the general case, where evading the back-action noise fully might not minimise the total noise. To obtain the optimal filter function one needs to write the signal-to-noise ratio as a functional of the filtering functions and minimise it with the normalisation constraints using the variational method. The system of integral equations obtained from the Euler-Lagrange equations can be solved via the Wiener-Hopf method Ref [3].

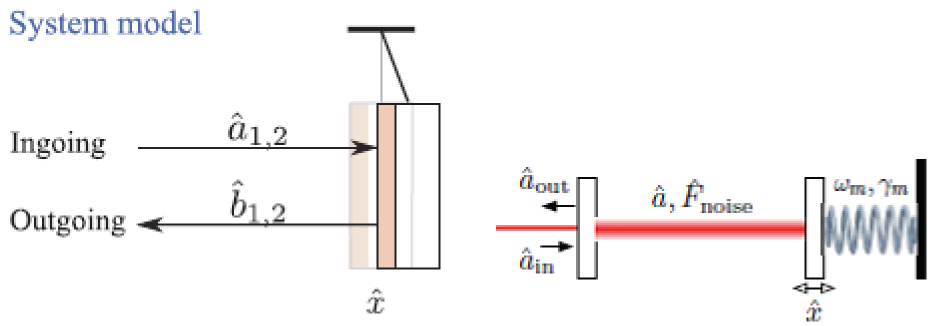
In Ref [1] the considered system consisted of one mirror and one optical field, Fig. 1.a. In this work, we extend the calculations for cavities with finite bandwidth.

3 Method

3.1 General case

In Ref [1] the mirror was considered as Markovian system. This is no longer true because the internal optical field couples to the mirror and contains information about the position of the mirror at earlier times. This means that the radiation pressure force on the mirror is correlated with the position of the mirror in the past, which results non-Markovian effects. However, the overall system (the mirror and the internal optical field) can be considered as a Markovian system, which results that the quantum tomography will investigate the state of the joint system.

For the purpose of this paper we assume the following form for the noise in the tomography measurement:



(a) (b)

Figure 1: schematic diagram broadband cavity. The optical field is coupled to the oscillator through the radiation-pressure.[1]
The internal field couples to the oscillator and the external field interacts with the internal field. The position of the movable mirror is measured in an indirect way. [2]

$$\delta\hat{c}_i(t) = \int_0^t \mathrm{d}t_1 G_{ij}(t - t_1)\hat{b}_j(t_1). \tag{1}$$

$$\delta\hat{c}_i(t) = \int_{-\infty}^t \mathrm{d}t_1 G_{ij}(t - t_1)\hat{b}_j(t_1) - \int_{-\infty}^0 \mathrm{d}t_1 G_{ij}(t - t_1)\hat{b}_1(t_1) \tag{2}$$

In Eq (2) the second term can be interpreted as the noise of the preparation while the first term is the noise of the verification process. It is important to note that even though these two terms are correlated, the self-correlation of $\delta\hat{c}_i(t)$ is the difference of the self-correlations of the two terms. This is due to the fact that $\delta\hat{c}_i(t)$ and the second term are uncorrelated.
In the most relevant cases the Green functions can be projected to the eigenbase of the system:

$$G_{ij}(x) = \sum_n M_{ij}^n e^{-i\Omega_n x} \tag{3}$$

where Ω_n is the nth eigenfrequency of the system. We expand $G_{ij}(t - t_1)$ in the second term but not the first term in Eq (2):

$$\delta\hat{c}_i(t) = \int_{-\infty}^t \mathrm{d}t_1 G_{ij}\hat{b}_j(t_1) - \int_{-\infty}^0 \mathrm{d}t_1 \sum_n M_{ij}^n e^{-i\Omega_n(t-t_1)}\hat{b}_j(t_1) \tag{4}$$

Because the joint system is Markovian the integrals for $t < 0$ and for $t > 0$ are uncorrelated so, applying the same homodyne detection method as in Ref [1], the noise in our measurement (summing over double indices):

$$\langle g_i(t)\delta\hat{c}_i(t)g_j(t')\delta\hat{c}_j(t')\rangle = (g_i|G_{ij}|g_j) + i\theta_{\alpha\alpha'}\frac{M_{i\alpha}^n M_{j\alpha'}^{m*}}{\Omega_m^* - \Omega_n}(g_i|e^{-i\Omega_n t + i\Omega_m^* t'}|g_j) \tag{5}$$

where

$$\langle \hat{b}_\alpha(t)\hat{b}_{\alpha'}(t')\rangle = \theta_{\alpha\alpha'}\delta_{\alpha\alpha'}\delta(t-t') \tag{6}$$

$$(A|B) = \int_0^T A(t)B(t)\mathrm{d}t \tag{7}$$

We have to apply normalisation constraints for our signal, which originates Lagrange multipliers in the functional we minimise:

$$I[g_i] = (g_i|C_{ij}|g_j) + i\theta_{\alpha\alpha'}\frac{M^n_{i\alpha}M^{m*}_{j\alpha'}}{\Omega^*_m - \Omega_n}(g_i|e^{-i\Omega_n t + i\Omega^*_m t'}|g_j) - \mu_i(g_i|f_i) \tag{8}$$

Minimisation of the functional leads us to the following matrix equation:

$$C_{ij}|g_j) + i\theta_{\alpha\alpha'}\frac{M^n_{i\alpha}M^{m*}_{j\alpha'}}{\Omega^*_m - \Omega_n}e^{-i\Omega_n t}g^{(m)}_j - \mu_i|f_i) = 0 \tag{9}$$

where

$$g^{(m)}_j = \int_0^\infty \mathrm{d}t e^{i\Omega^*_m t}g_j(t) \tag{10}$$

One can solve these equations by Fourier transforming it to the frequency domain

$$[S_{ij}(\Omega)g_j(\Omega) + \frac{\theta_{\alpha\alpha'}M^n_{i\alpha}M^{m*}_{j\alpha'}}{(\Omega-\Omega_n)(\Omega-\Omega^*_m)}g^{(m)}_j - \mu_i f_i(\Omega)]_+ = 0 \tag{11}$$

We only consider the causal part of the equation because the verification starts at $t = 0$ so $g_i(t)$ is only defined for time $t > 0$. We need to use the casual factorisation of S matrix to find g_j

$$\mathsf{S} = \psi_+\psi_- \tag{12}$$

where ψ_+ (ψ_-) and its inverse are causal (anticausal). Thus, g_l:

$$g_l(\Omega) = \psi_{+}{}^{-1}_{lk}\{\psi_{-}{}^{-1}_{ki}[\mu_i f_i(\Omega) - \frac{\theta_{\alpha\alpha'}M^n_{i\alpha}M^{m*}_{j\alpha'}}{(\Omega-\Omega_n)(\Omega-\Omega^*_m)}g^{(m)}_j]\}_+ \tag{13}$$

Here $g^{(m)}_j$ is a parameter. We solve Eq (13) with $g^{(m)}_j$ as parameters and then substitute into Eq (10) for each value of m and solve the system of linear equation.

3.2 Causal factorisation

In general causal factorisation is a complicated process and often it is impossible or unfeasible to carry out analytically. The casual factorisation of the broadband cavity is obtained in Ref [1] so we consider the finite bandwidth of the cavity as a perturbation and use the following method to factorise S:
If we know the factorisation of the stationary matrix:

$$\mathsf{S_s} = \mathsf{C_o}\mathsf{C_o}^\dagger \tag{14}$$

where $\mathsf{C_o}$ and its inverse are casual (and $\mathsf{C_o^\dagger}$ is anticausal). Then we can transform our matrix and expand it around the unit matrix

$$\mathsf{C_o^{-1} S (C_o^\dagger)^{-1}} = \mathbb{1} + \varepsilon \mathsf{S_1} = \mathsf{Z Z^\dagger} \tag{15}$$

$$\mathsf{Z} = \mathbb{1} + \varepsilon \mathsf{Z_1} \tag{16}$$

Keepeing only the first order terms and matching the multipliers of ε:

$$\mathsf{S_1} = \mathsf{Z_1} + \mathsf{Z_1^\dagger} \tag{17}$$

This means we need to causally decompose $\mathsf{S_1}$ to obtain the causal factorisation of S:

$$\mathrm{S} \approx (\mathrm{C_o} + \varepsilon \mathrm{C_o S_1})(\mathrm{C_o} + \varepsilon \mathrm{C_o S_1})^\dagger \tag{18}$$

3.3 Non-detuned cavity case

The Hamiltonian for our system is the following (c.f. Ref [2]):

$$\mathcal{H} = \frac{\hat{p}^2(t)}{2m} + \frac{1}{2} m \omega_m^2 \hat{x}^2(t) + \mathcal{H}_{dis} + \hbar G a \hat{a}_1(t) \hat{x}(t) + i\hbar\sqrt{2\gamma}(\hat{a}_{ext}\hat{a}^\dagger(t) - \hat{a}_{ext}^\dagger(t)\hat{a}(t)) \tag{19}$$

The first two terms describe the mechanical oscillator. $\mathcal{H}_{dis}$ contains the dissipative terms and noises. The forth term is the interaction between the internal field and the mirror, and the last term is the interaction of the external field with the cavity. From this we obtain the equations of motion:

$$\dot{\hat{x}} = \frac{\hat{p}}{m} \tag{20}$$

$$0 = m\ddot{\hat{x}} + m\omega_m^2 \hat{x} + \hbar G a \hat{a}_1(t) + \hat{\xi}_F(t) \tag{21}$$

$$\dot{\hat{a}} = -iGa\hat{x}(t) + \sqrt{2\gamma}\hat{a}_{ext}(t) \tag{22}$$

These equations of motion lead to the following amplitude and the phase quadratures of the outgoing field and the Fourier transform of the connection matrix:

$$\hat{c}_1(t) = -\hat{b}_1(t) + 2\gamma \int_0^t \mathrm{d}t_1 e^{-\gamma(t-t_1)} \hat{b}_1(t_1) \tag{23}$$

$$\hat{c}_2(t) = -\hat{b}_2(t) + 2\gamma \int_0^t \mathrm{d}t_1 e^{-\gamma(t-t_1)} \hat{b}_2(t_1) - 2Ga\sqrt{\gamma} \int_0^t \mathrm{d}t_1 e^{-\gamma(t-t_1)} \hat{x}(t_1) \tag{24}$$

$$\mathsf{S} = \begin{pmatrix} \frac{e^{2q}}{2} & \frac{\alpha^2 \hbar e^{2q}}{m(\gamma^2+\omega^2)((\gamma_m - i\omega)^2 - \omega_m^2)} \\ \frac{\alpha^2 \hbar e^{2q}}{m(\gamma^2+\omega^2)((\gamma_m + i\omega)^2 - \omega_m^2)} & \frac{e^{-2q}}{2} + \frac{2\alpha^4 \hbar^2 \gamma^2 e^{2q}}{m(\gamma^2+\omega^2)^2((\gamma_m^2 - \omega^2 + \omega_m^2)^2 + 4\gamma_m^2\omega^2)} \end{pmatrix} \tag{25}$$

4 Conslusion

In this paper a general treatment for optimising quantum state tomography was presented. If the system has finite degrees of freedom, the method shown in Section 3.1 provides a feasible way to minimise the additional noise.
We also evaluated the first steps of the calculation for the non-detuning cavity case.
In the future I plan to find the exact form of the filtering functions ($g_i(t)$) and estimate the additional noise the cavity causes in the measurement for realistic cavities. In this paper we did not consider optical loss or thermal noise. In the first order approximation the net noise is the sum of the noise from different sources. Thus we compare the additional noise from the cavity to the noise obtained in Ref [1] to see if the cavity makes the tomography impossible.

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