UNDREAMT BY EINSTEIN: PROSPECTS AND CHALLENGES IN GRAVITATIONAL WAVE ASTRONOMY

General Relativity and Gravitation: A Centennial Perspective
Penn State University, June 8-12 2015

SPECIAL THANKS FOR SLIDES AND DISCUSSIONS TO MANY COLLEAGUES,
IN PARTICULAR SARAH GOSSAN AND
CENTENNIAL VOLUME CO-AUTHOR ALESSANDRA BUONANNO

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School of Physics and Astronomy, Cardiff University, UK

CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD

PROGRESS FOR PAST 30 YEARS

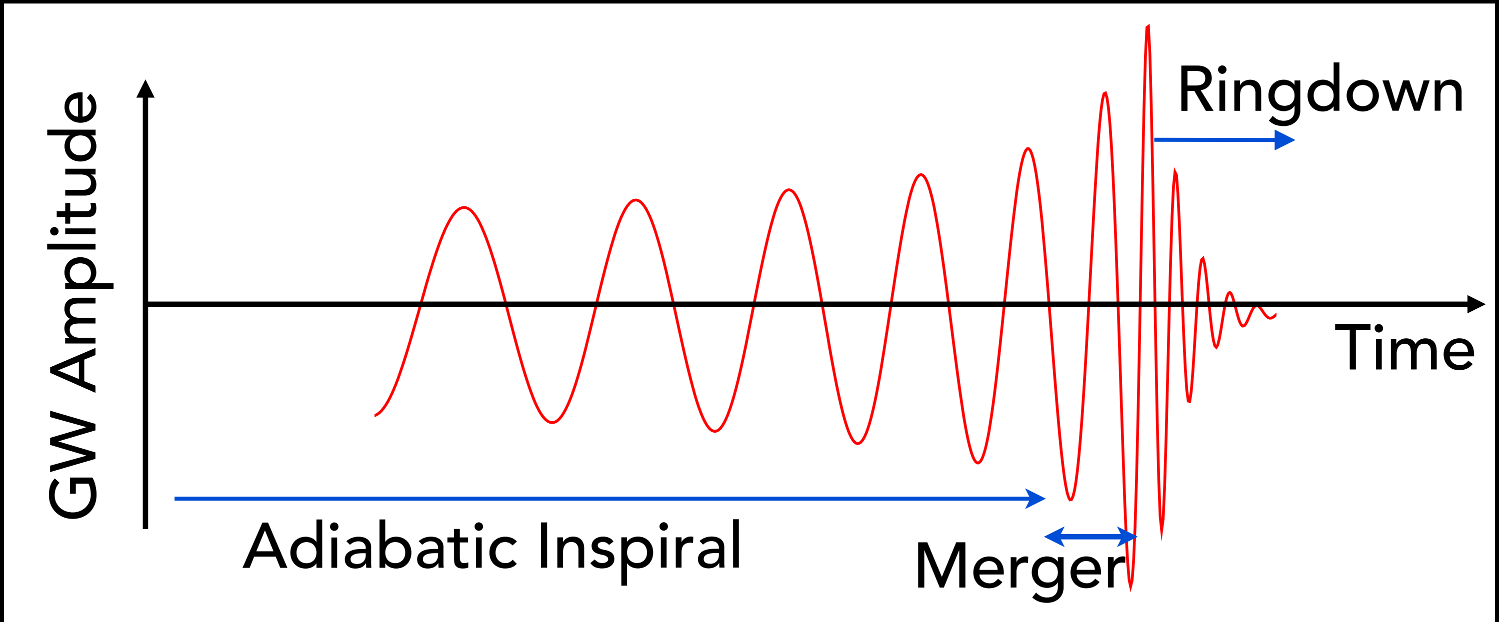
- impressive progress in analytical and numerical computation of source dynamics
  - compact binary dynamics and IMR waveforms and ejecta
  - full GR 3-D SN simulations with neutrino transport, magnetohydrodynamics;
  - GRB progenitor models including GRB afterglows
  - plethora of mechanics for production primordial gravitational waves
- many new potential sources
  - SMBBH, LMXBs, glitching pulsars, flaring magnetars, r-modes, …
- sophisticated search algorithms to dig signals out of noise
  - geometric formulation of signal analysis; wavelets; multi-variate analysis, …
  - comprehensive off-line searches and on-line searches that produce results within minutes of acquiring data
  - Bayesian parameter estimation and inference
- we are at the verge of making first detections

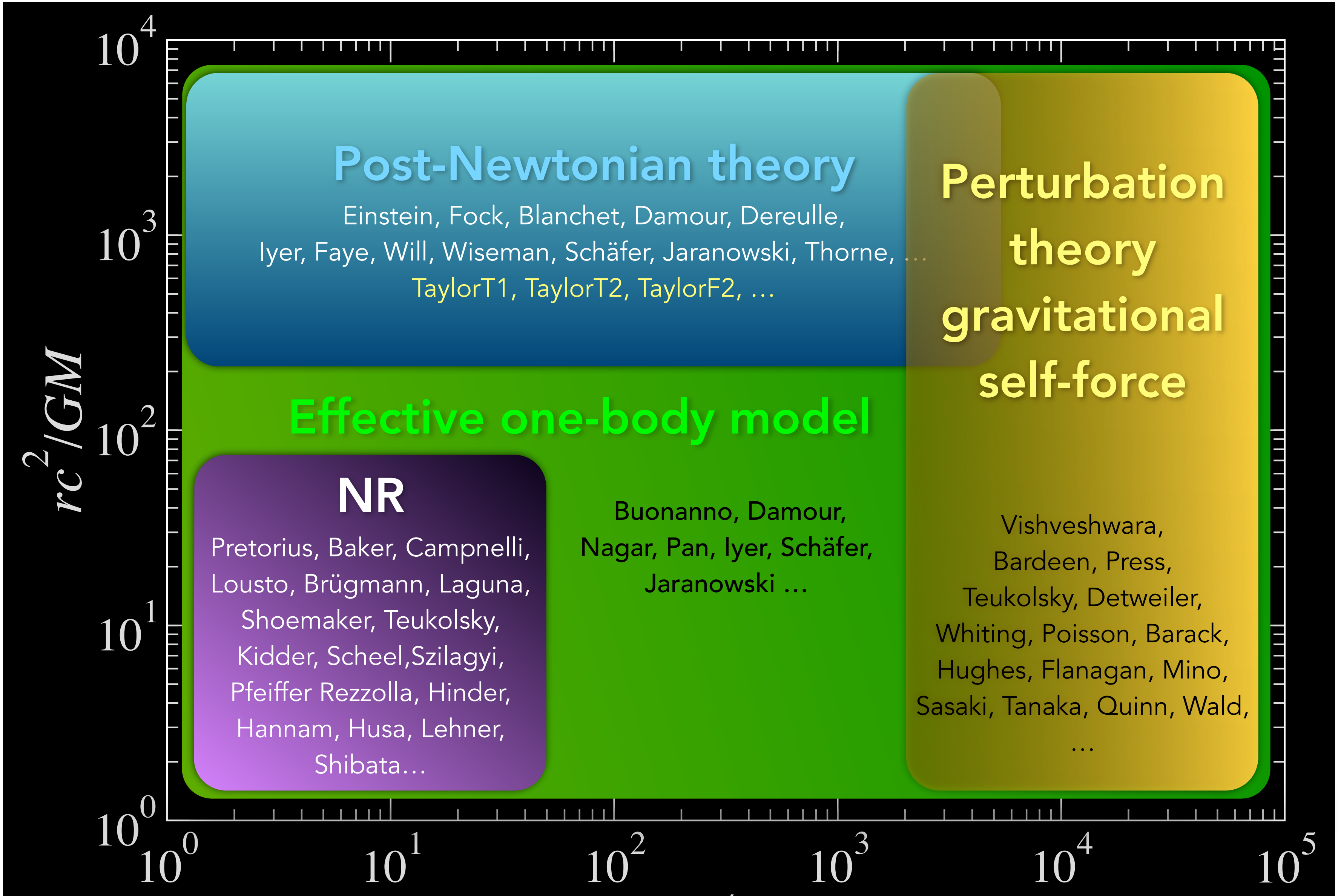
SOURCES OF GRAVITATIONAL WAVES

- binaries of compact objects
  - neutron star binaries, neutron star-black hole binaries, black hole binaries
- gravitational collapse and supernovae and other transients
  - SN, LMXBs, pulsar glitches, magnetars
- non-axisymmetric spinning compact objects
  - neutron stars, white dwarfs
- stochastic backgrounds
  - primordial gravitational waves, astrophysical backgrounds

BINARY BLACK HOLES

- waveform characterised by
  - slow adiabatic inspiral, fast and luminous merger, rapid ringdown
- very large parameter space
  - mass ratio, large BH spins misaligned with orbit, eccentricity
- waveform shape can tell us about component masses, spins and eccentricity
- waveform amplitude (in a detector network) can tell us about source's orientation, sky position, polarisation and distance

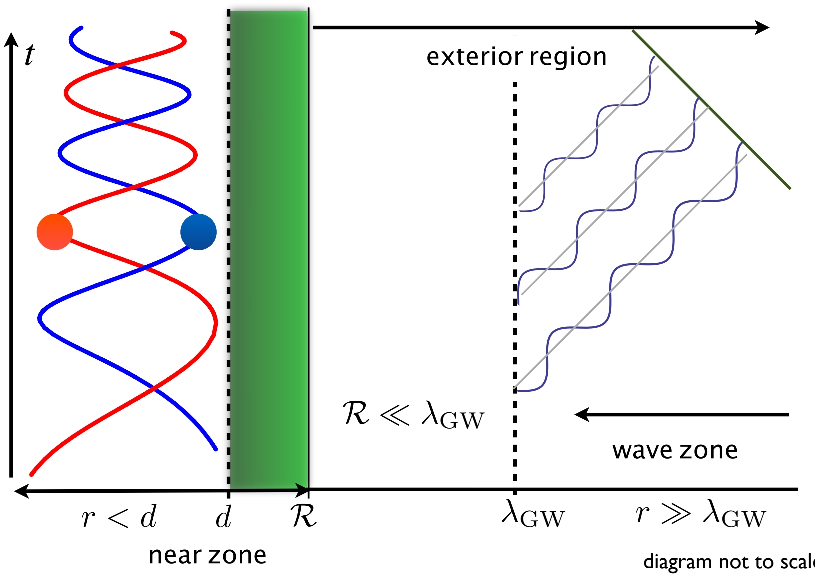




proximation Methods: Mon and Wed

PROGRESS IN TWO-BODY PROBLEM

- Caltech group pointed out the importance of computing phasing beyond leading order; followed by very impressive progress in post-Newtonian computation of two-body dynamics
- construction of LIGO, Virgo, GEO600 and TAMA brought theory and observations closer
- effective one-body approach developed: bold prediction for the late inspiral, merger and ringdown
- first successful NR simulations broke conventional wisdom - a far simpler merger than anyone predicted
- remarkable interactions between GW data analysts, astrophysicists and theorists to open a new observational window



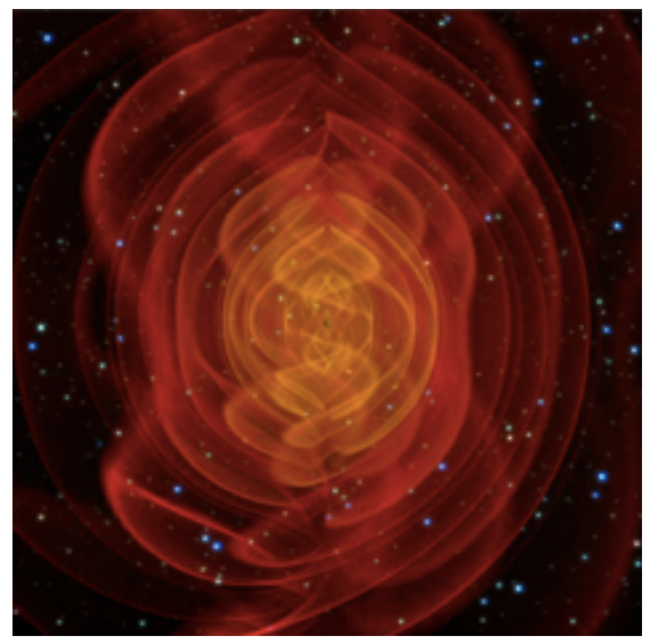


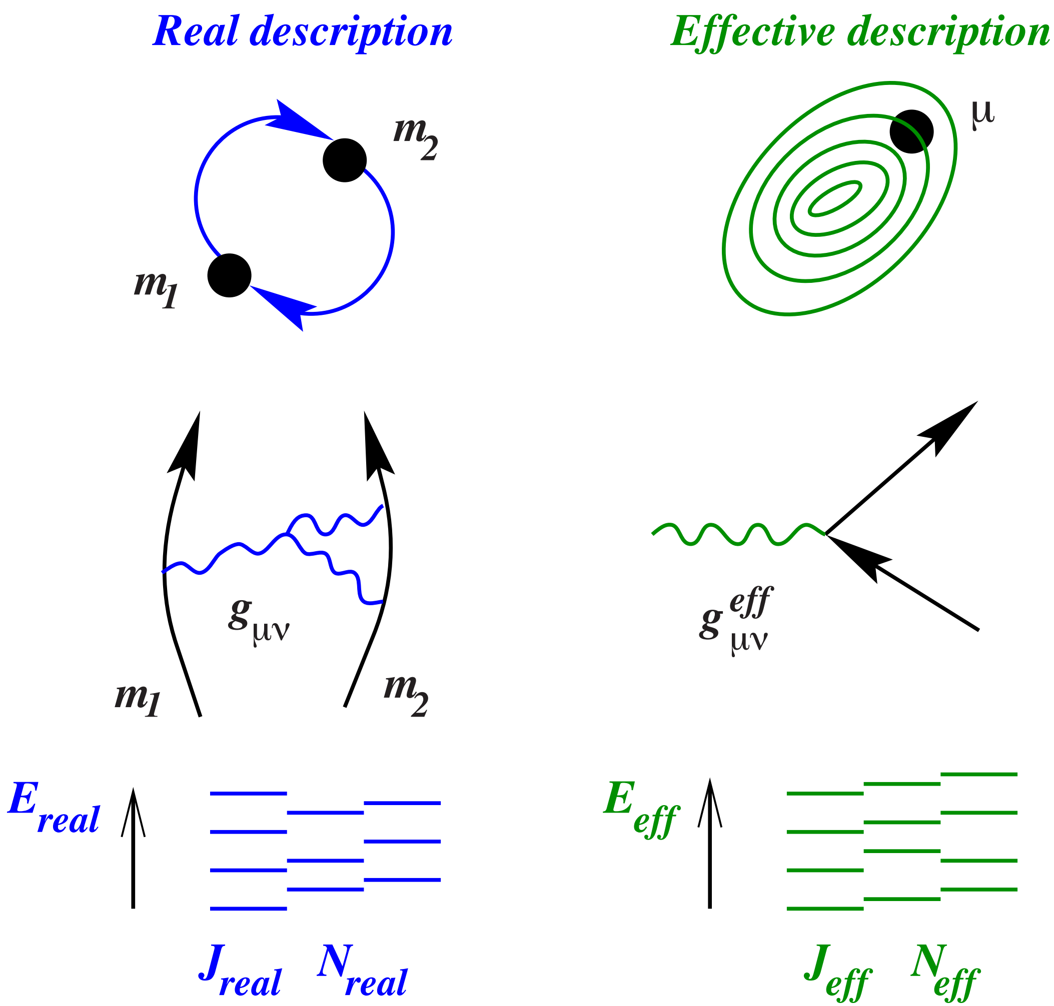
image curtsey NASA/C. Henze

CURRENT STATUS OF PN CALCULATIONS

	No Spin	Spin-Linear	Spin-Squared	Tidal
Conservative Dynamics	4PN	3.5PN	3PN	7PN
Energy Flux at Infinity	3.5PN	4PN	2PN	6PN
RR Force	4.5PN	4PN	4.5PN	6PN
Waveform Phase	3.5PN	4PN	2PN	6PN
Waveform Amplitude	3PN	2PN	2PN	6PN
BH Horizon Energy Flux	5PN	3.5PN	4PN	–

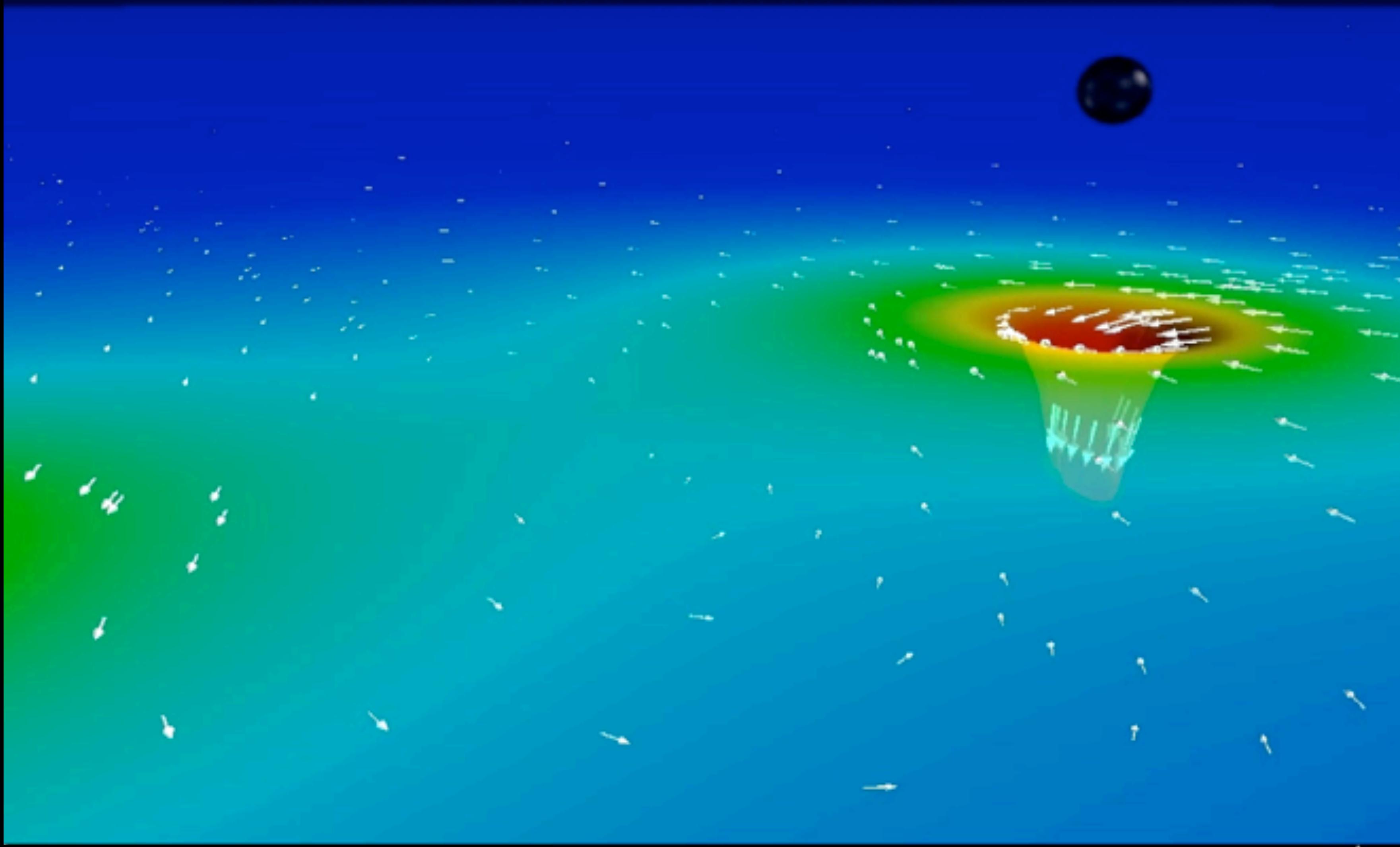
Table from Buonanno and BSS 2014

BEYOND INSPIRAL: EFFECTIVE ONE BODY FORMALISM

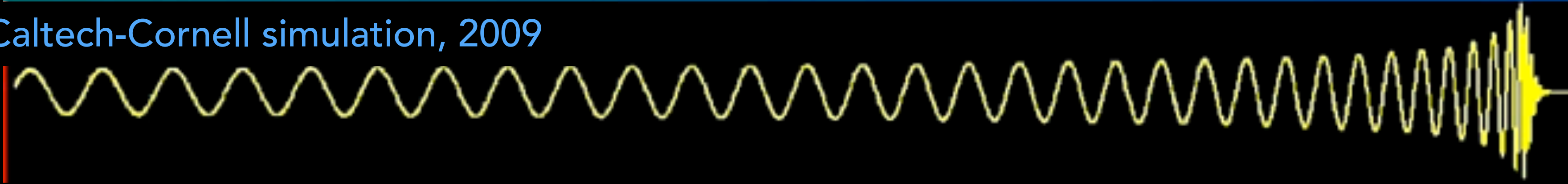


Buonanno and Damour 1999

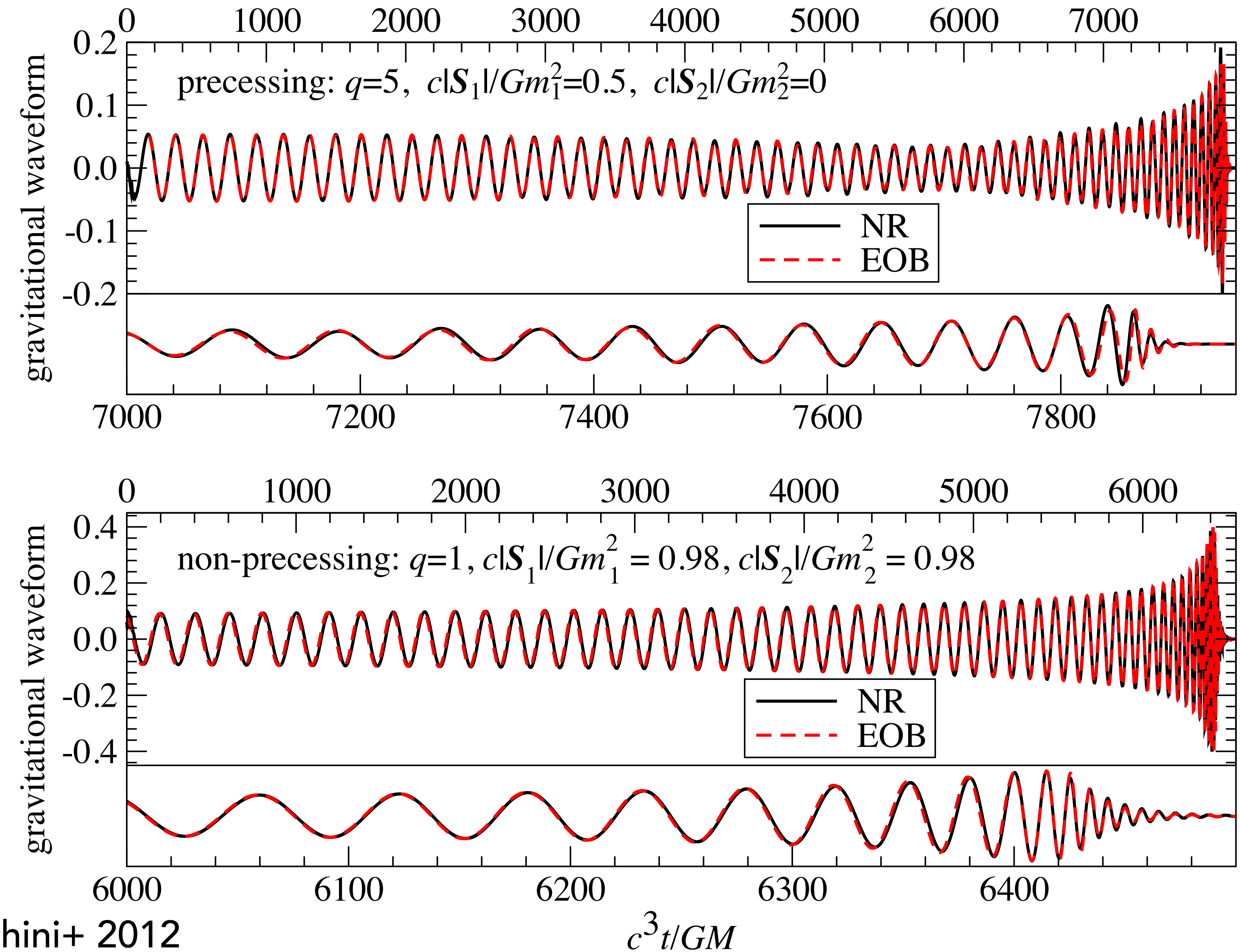
NUMERICAL SIMULATIONS OF BBH



Caltech-Cornell simulation, 2009

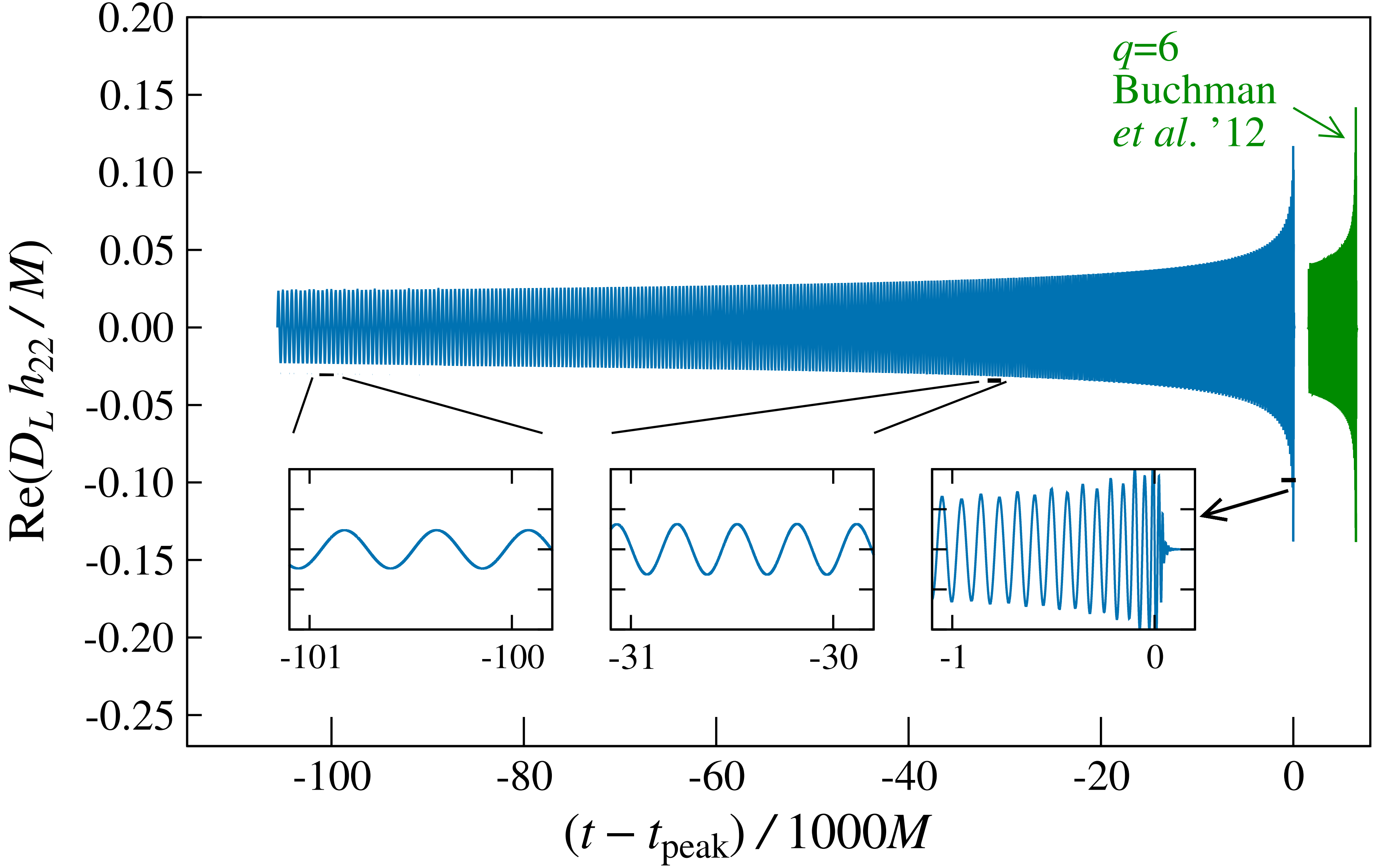


EOB VIS-A-VIS NR SIMULATIONS



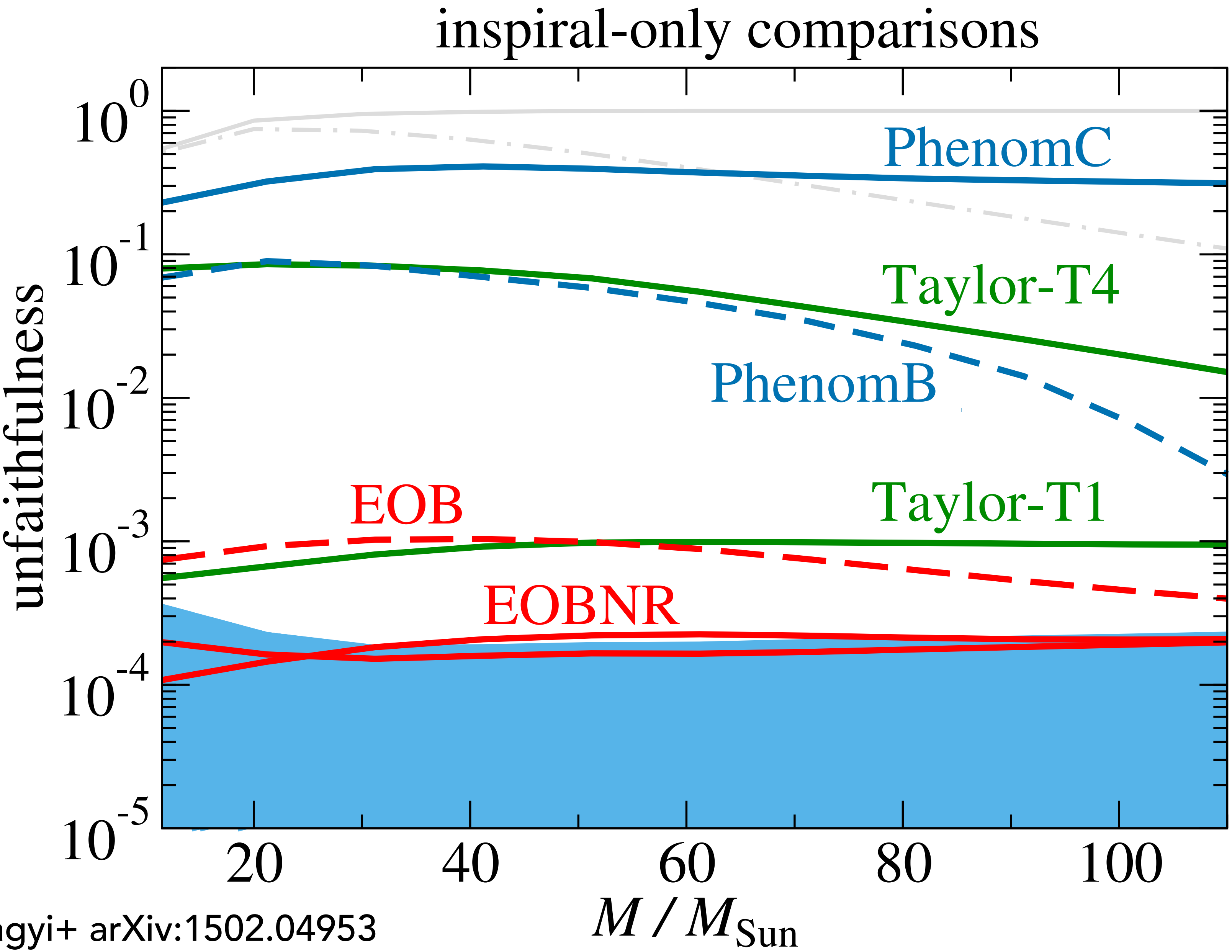
Taracchini+ 2012

LONGEST SO FAR: 170-ORBITS, MASS RATIO 1:7, NON-SPINNING



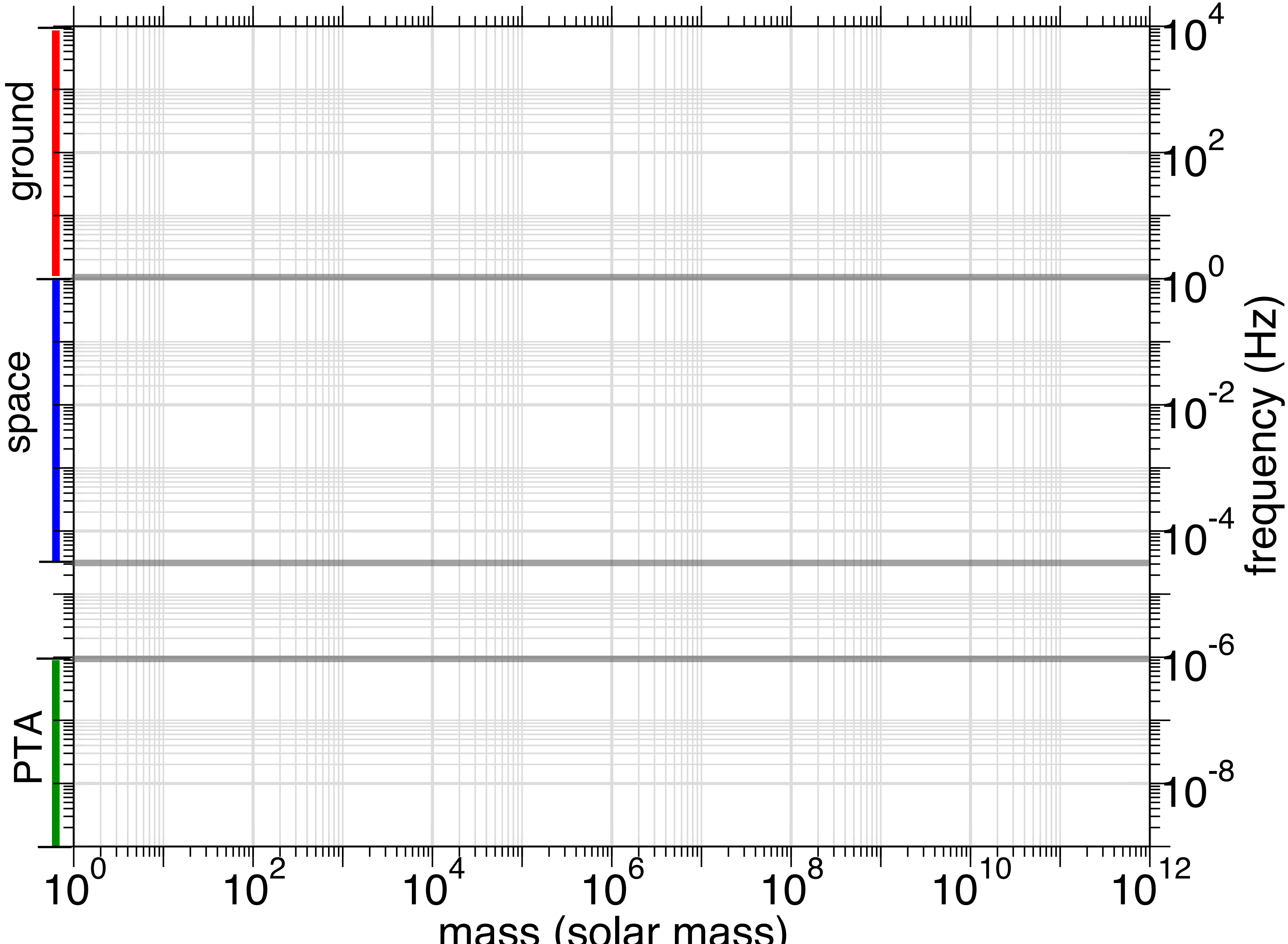
Szilagyi+ arXiv:1502.04953

UNFAITHFULNESS OF EOB < 0.1%

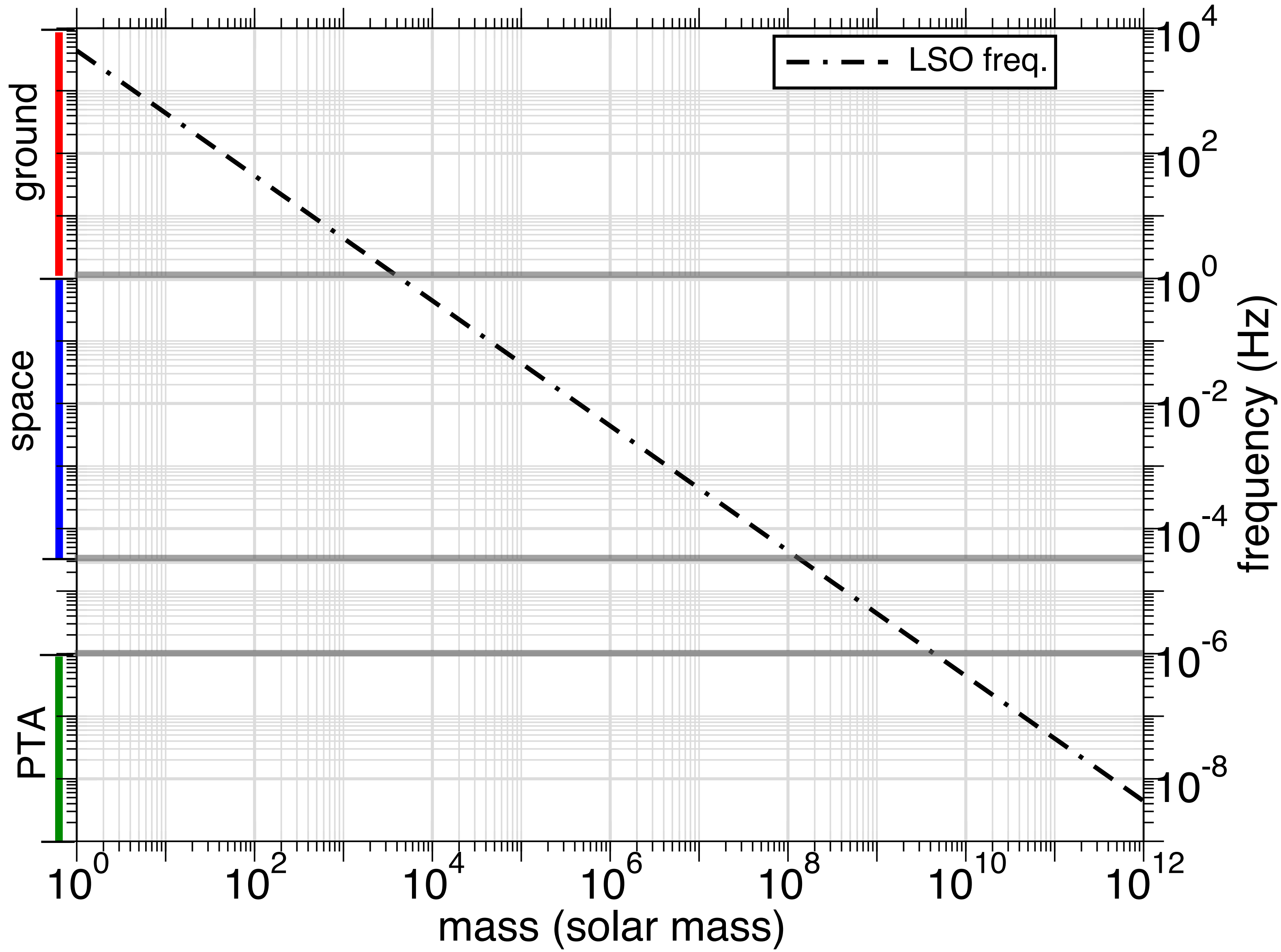


Szilagyi+ arXiv:1502.04953

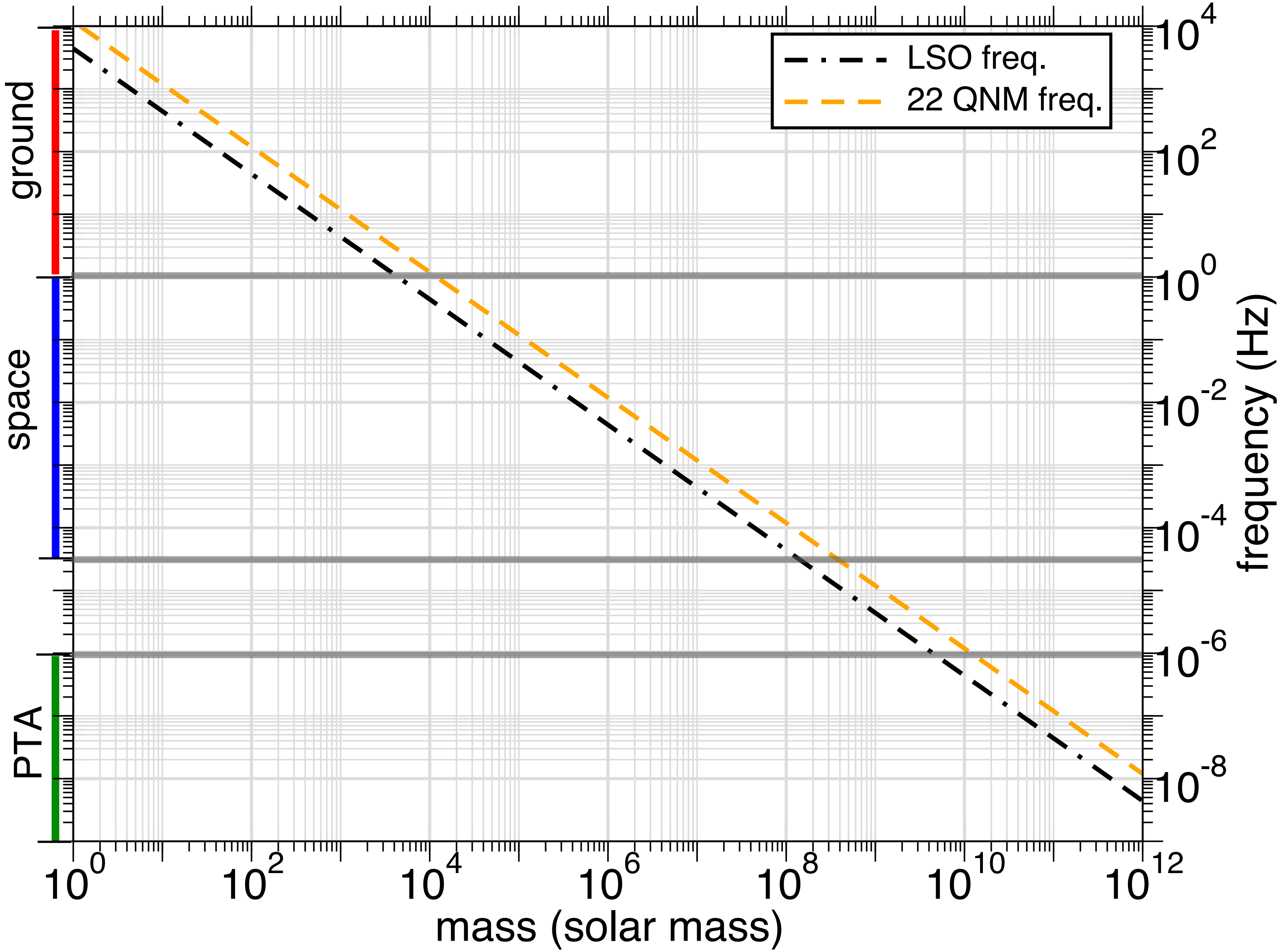
FREQUENCY SPAN OF VARIOUS DETECTORS



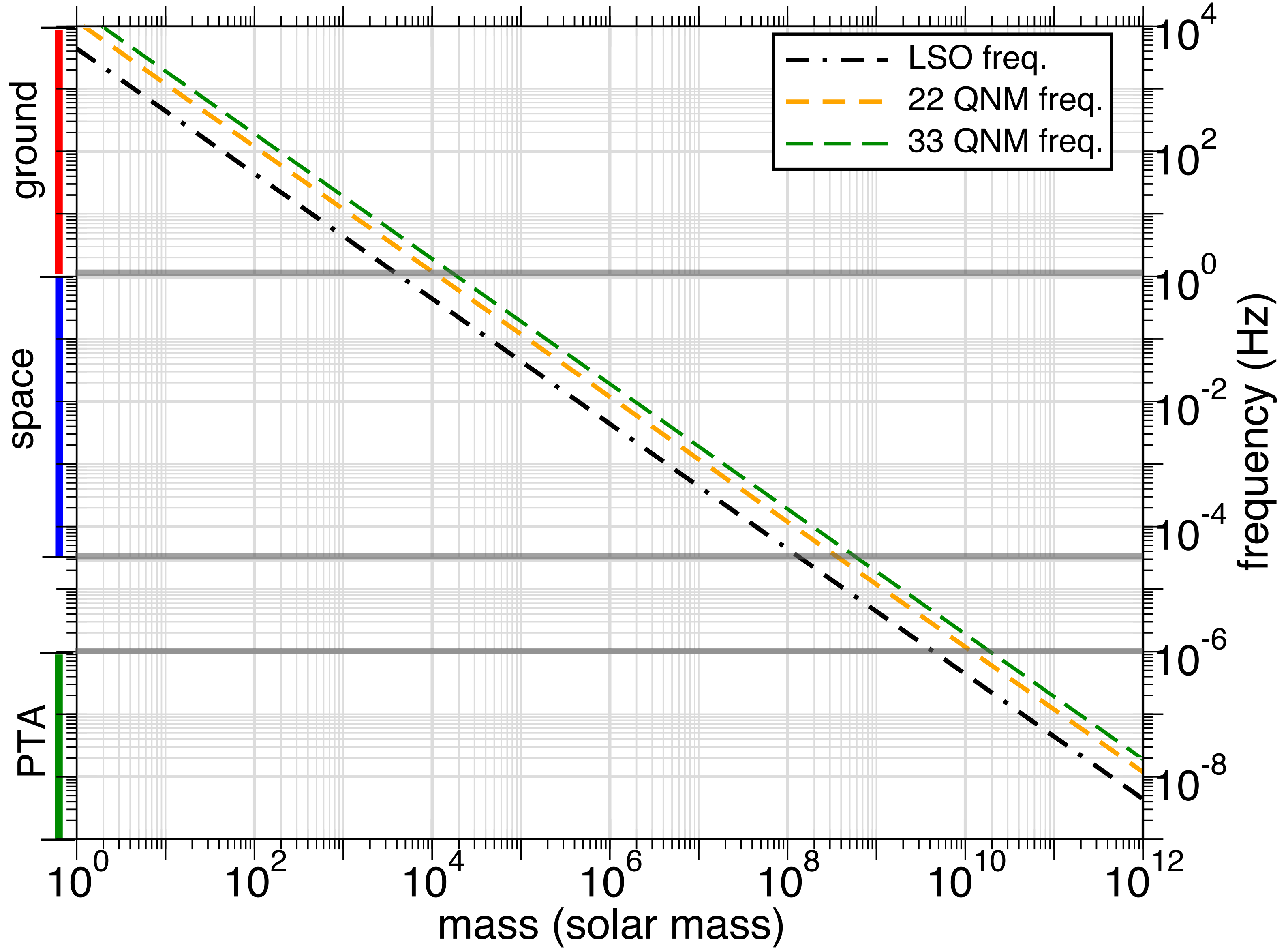
LAST STABLE ORBIT FREQUENCY: SCHWARZSCHILD BLACK HOLE



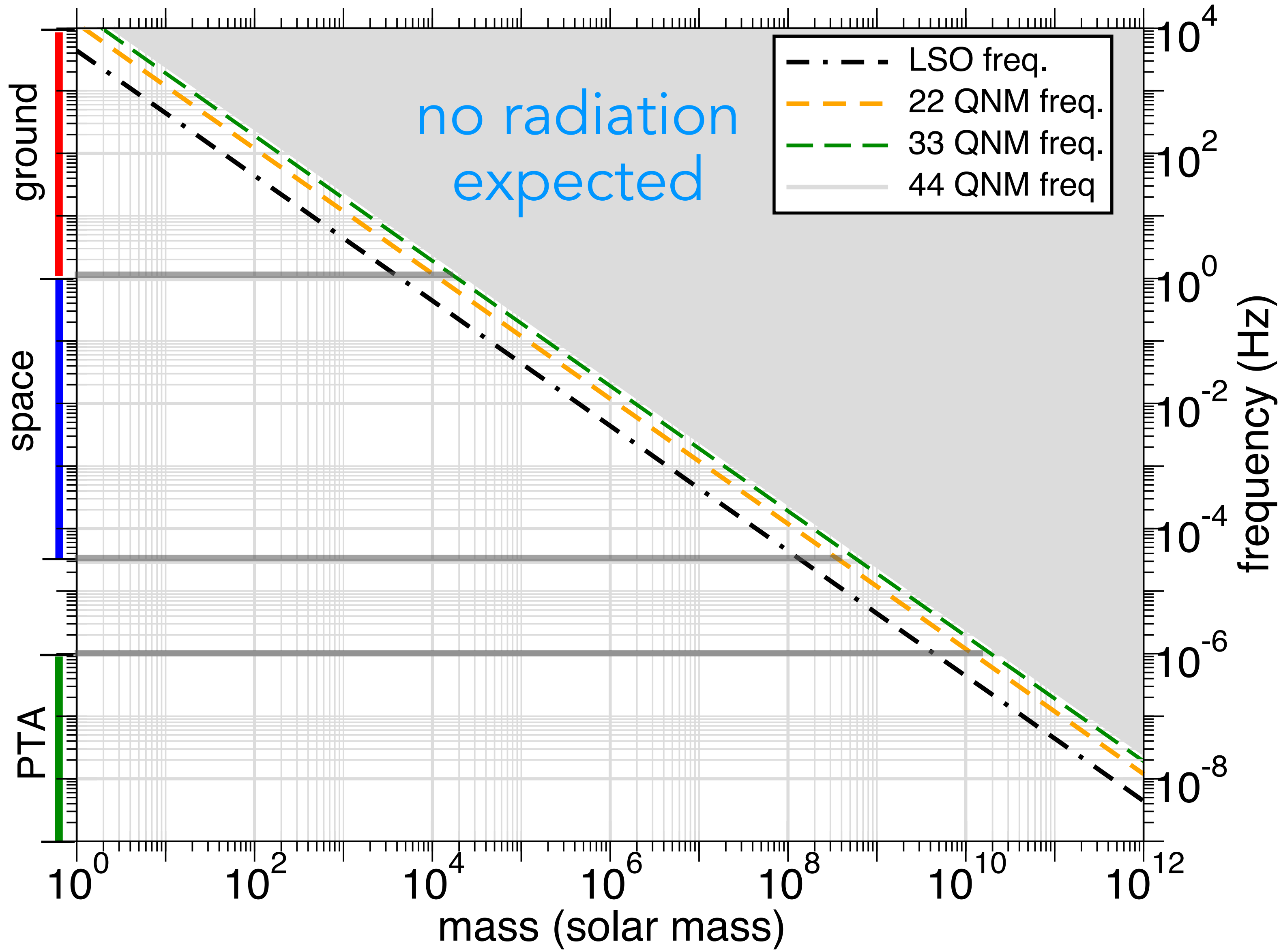
DOMINANT QUASI-NORMAL MODE FREQUENCY



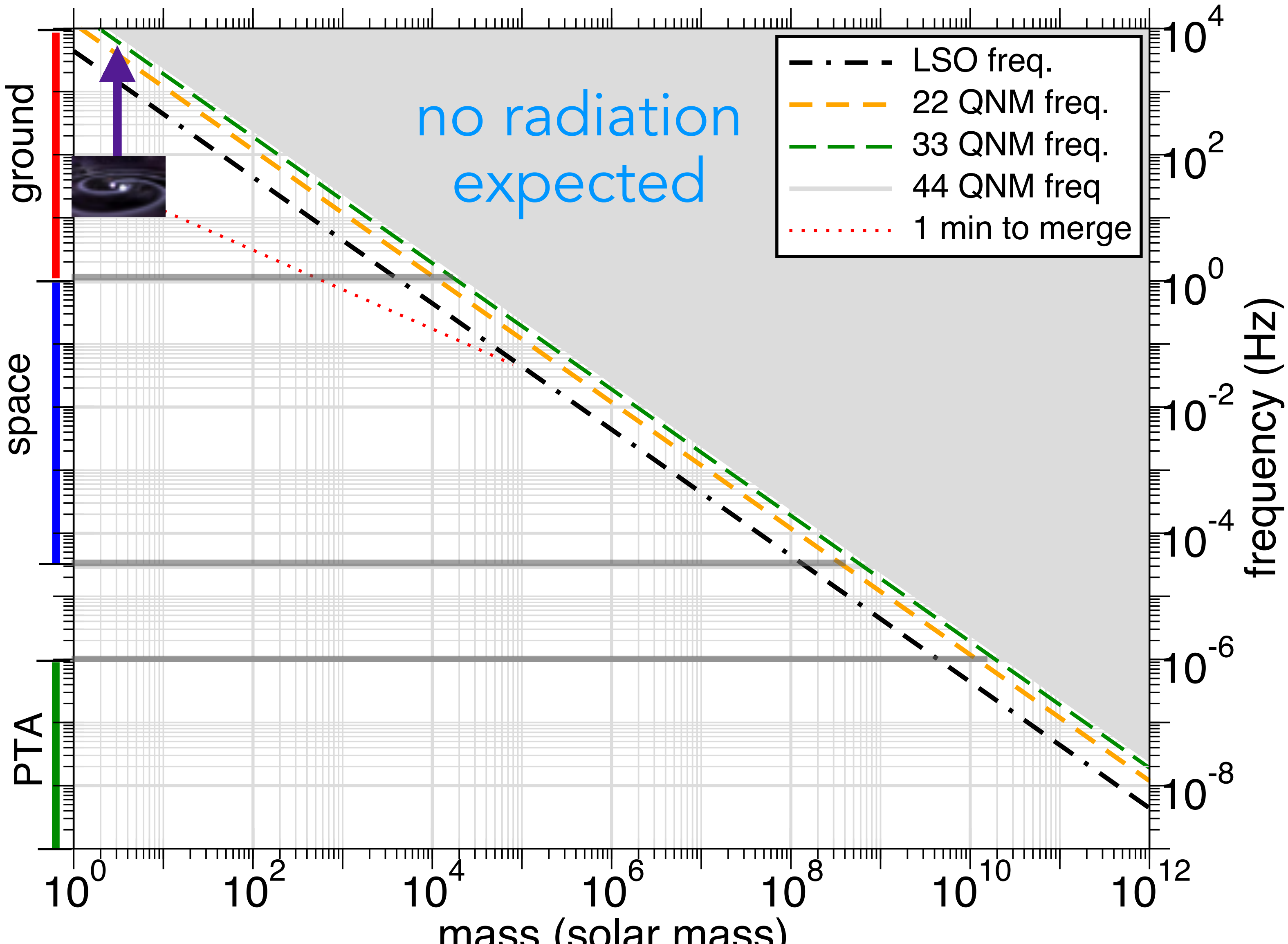
HIGHER ORDER QUASI-NORMAL MODES: SUB-DOMINANT



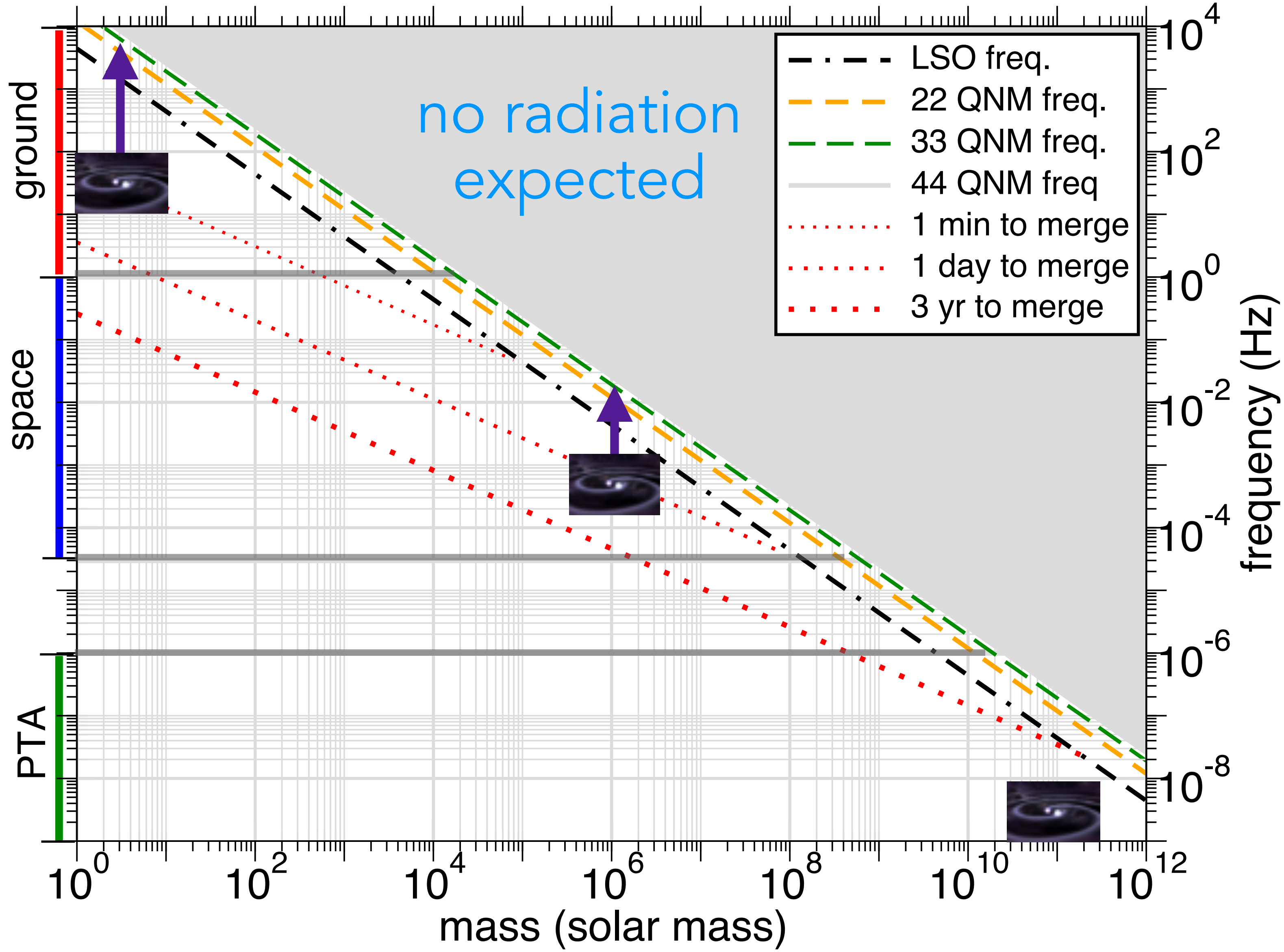
NO RADIATION FROM INSIDE A BLACK HOLE



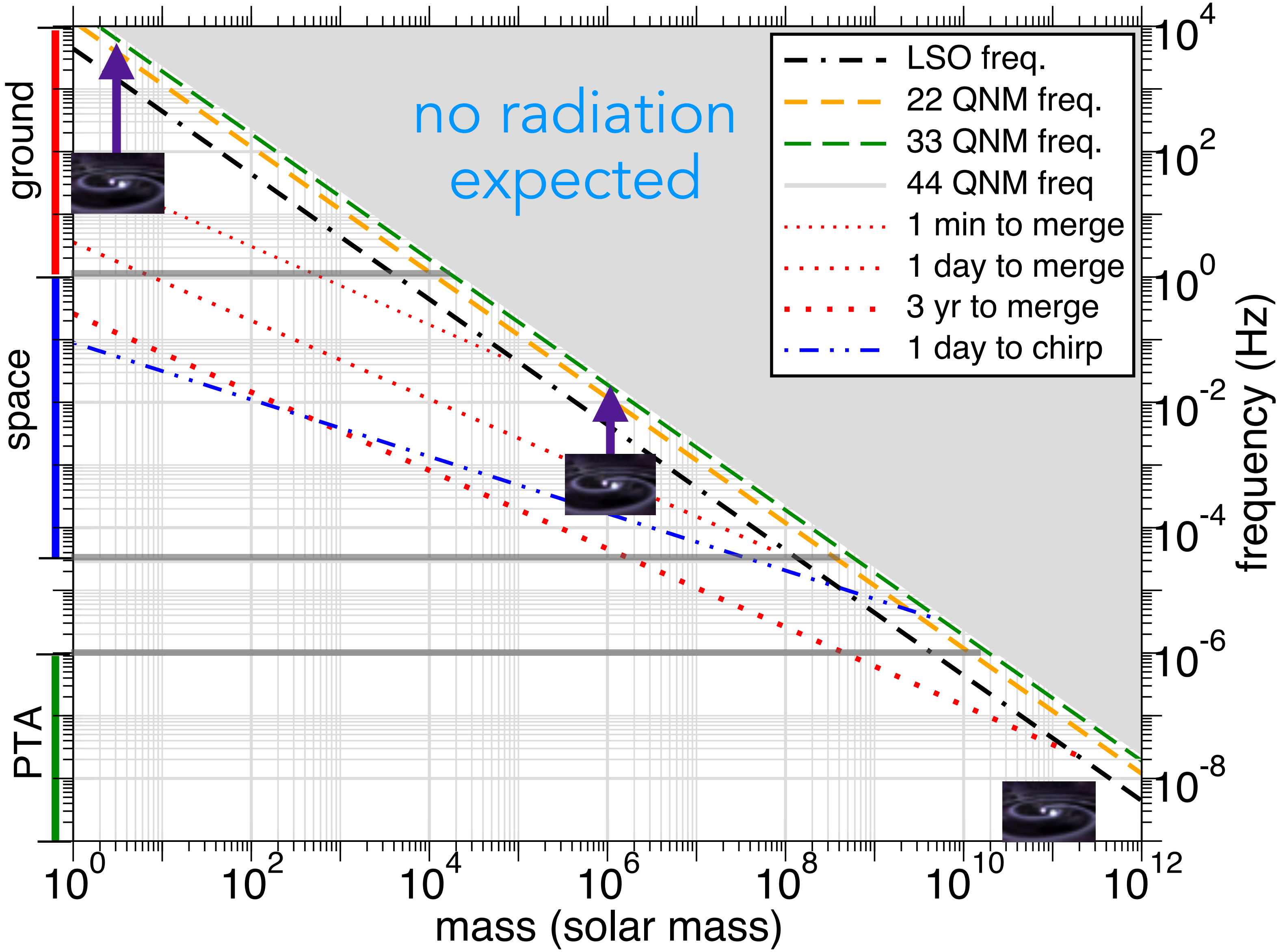
SYSTEMS OBSERVED GROUND BASED DETECTORS MERGE WITHIN A FEW DAYS



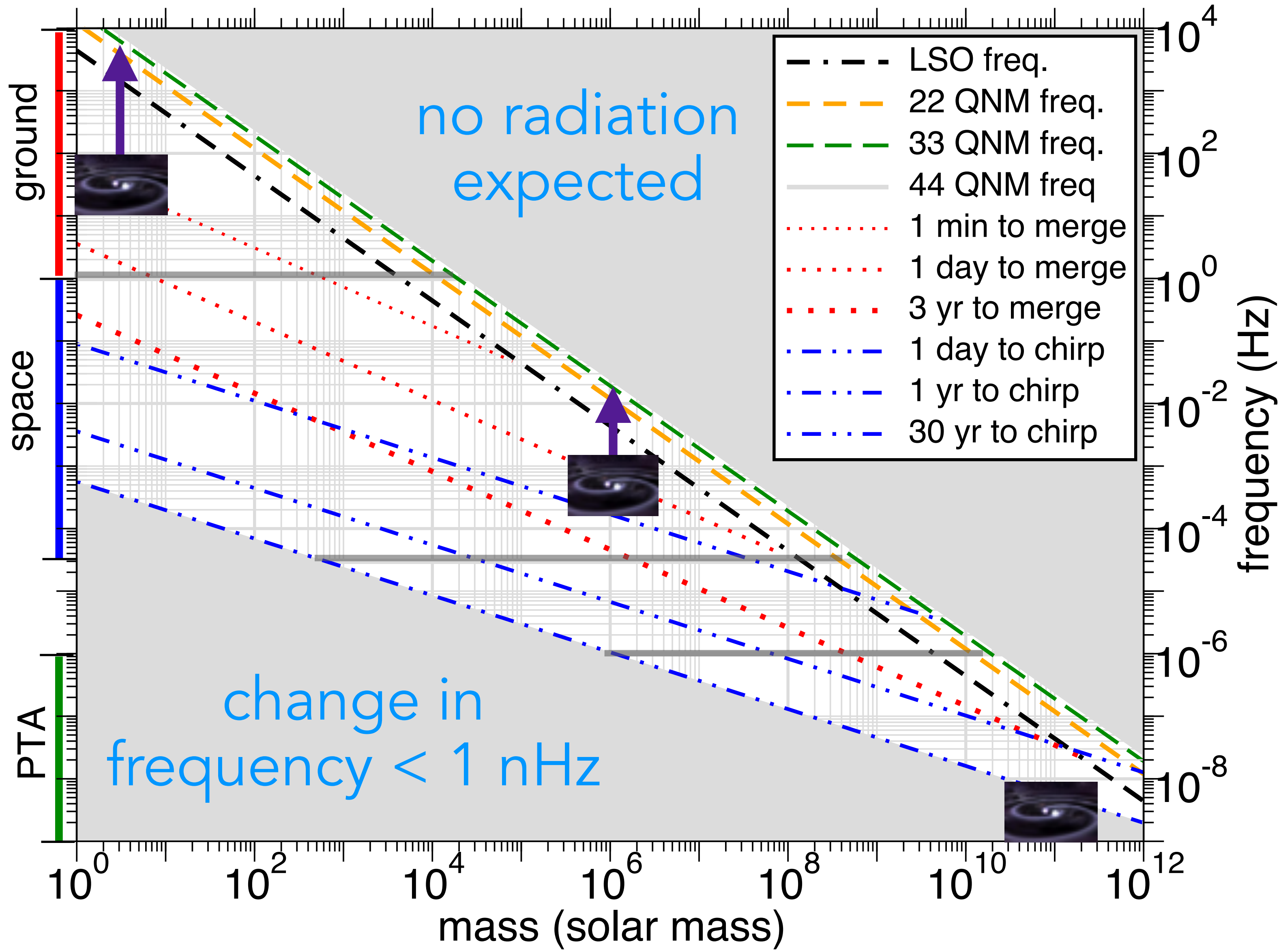
PULSAR TIMING ARRAYS COULD SEE MONOCHROMATIC WAVES



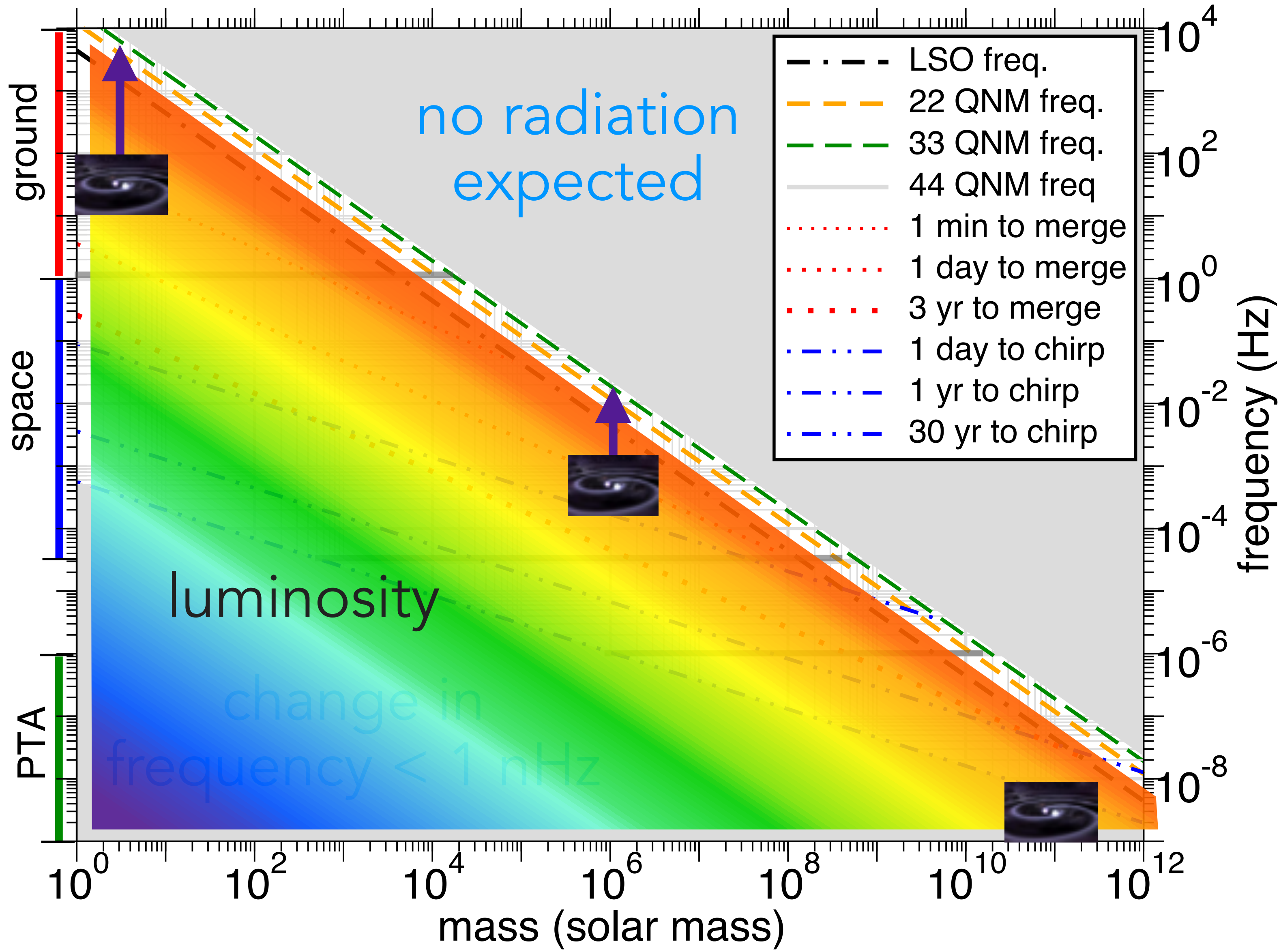
PARAMETER MEASUREMENTS ARE POSSIBLE FOR CHIRPING BINARIES



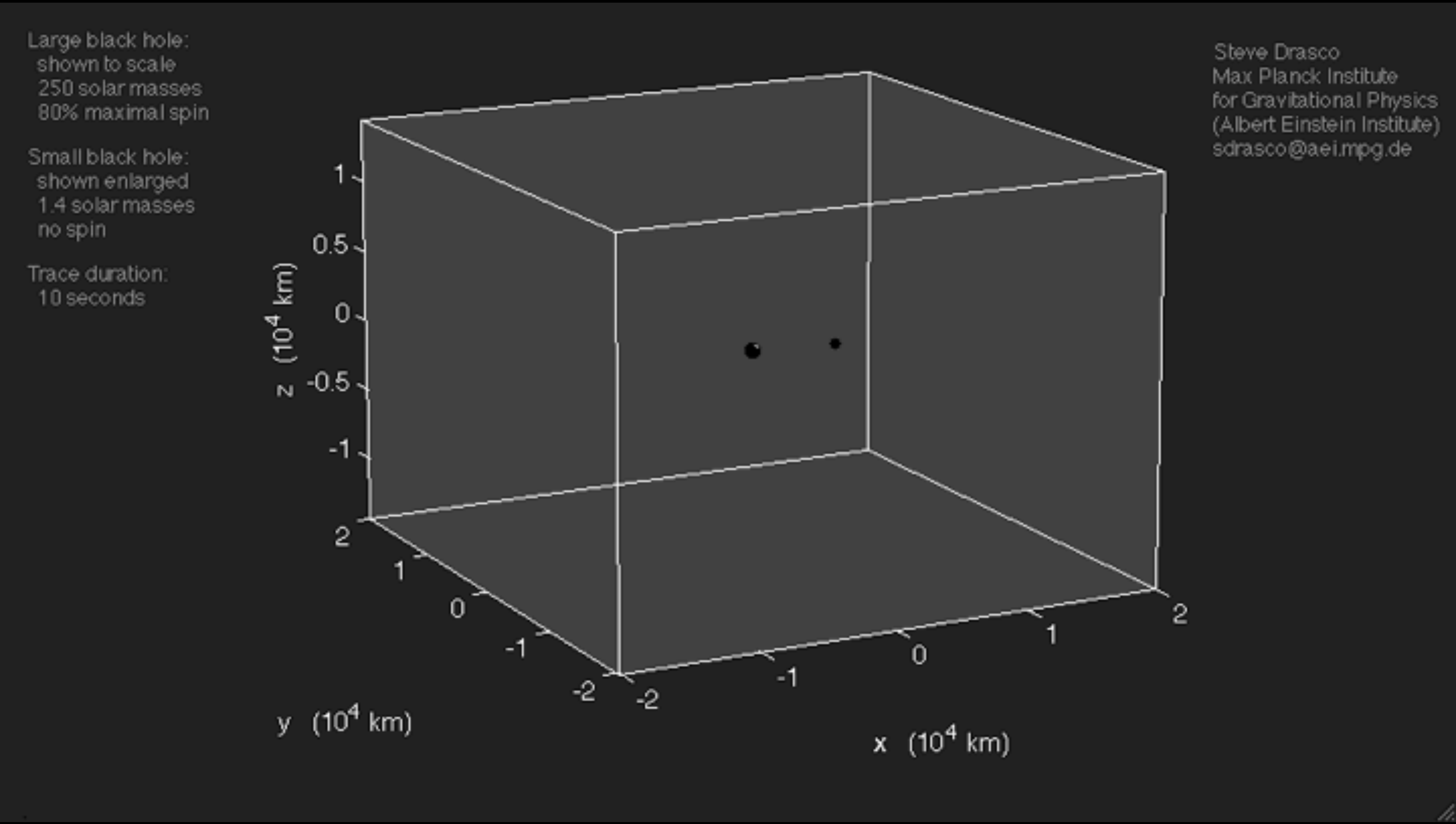
SOME BINARIES WON'T CHIRP APPRECIABLY DURING OBSERVATION



LUMINOSITY IS A STEEP FUNCTION OF FREQUENCY



SMALL BLACK HOLE FALLING INTO A BIG BLACK HOLE

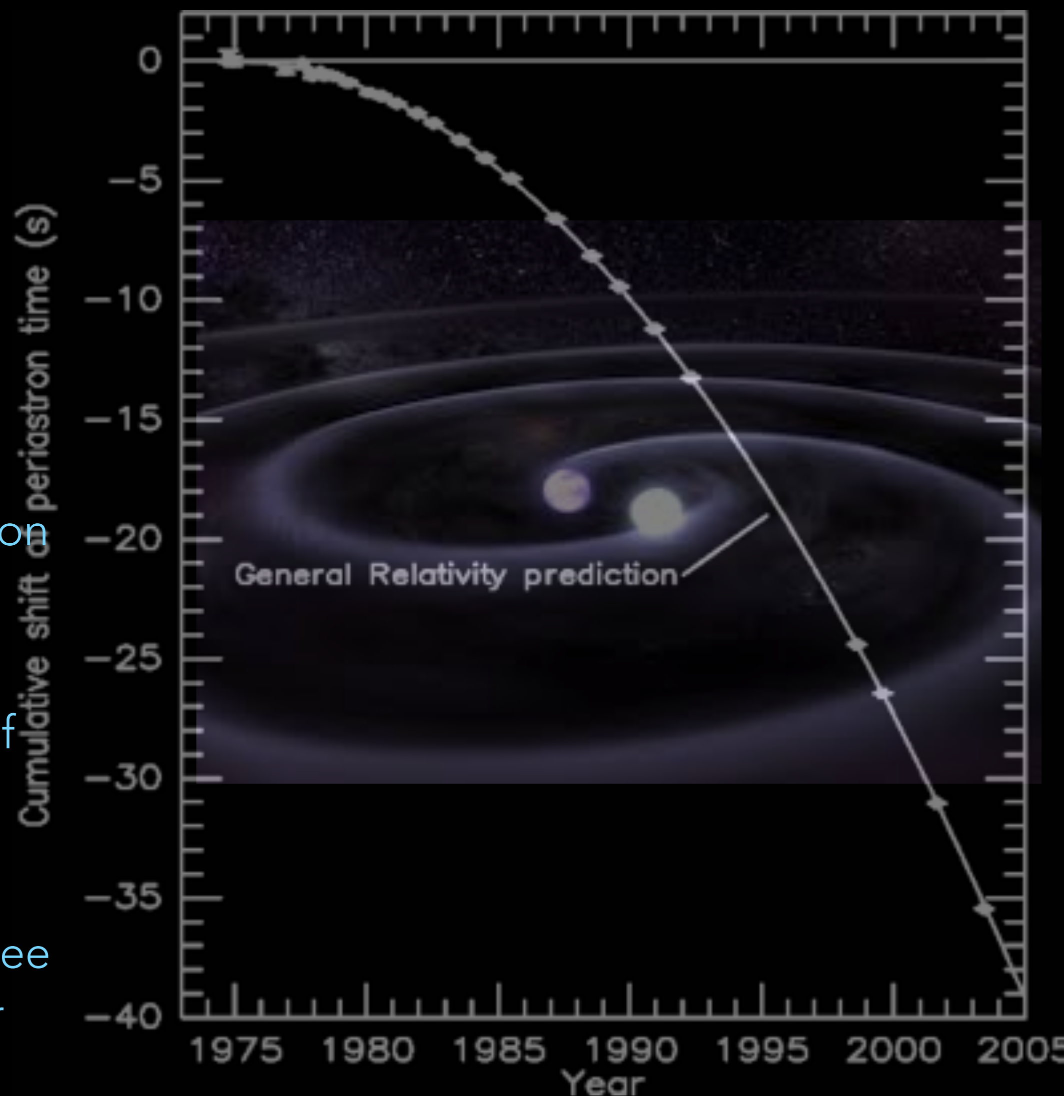


Animation Steve Drasco

BINARY NEUTRON STARS

- probably progenitors of short gamma ray bursts
- **can measure:**
  - chirpmass pretty well but component masses are difficult to constrain
- observations should:
  - constrain models of formation and evolution of compact binaries
  - possibly equation of state of supra-nuclear matter
- **rates highly uncertain**
  - advanced detectors could see between 0.5 to 400 per year

$$\mathcal{M} = \frac{m_1^{3/5} m_2^{3/5}}{(m_1 + m_2)^{1/5}}$$

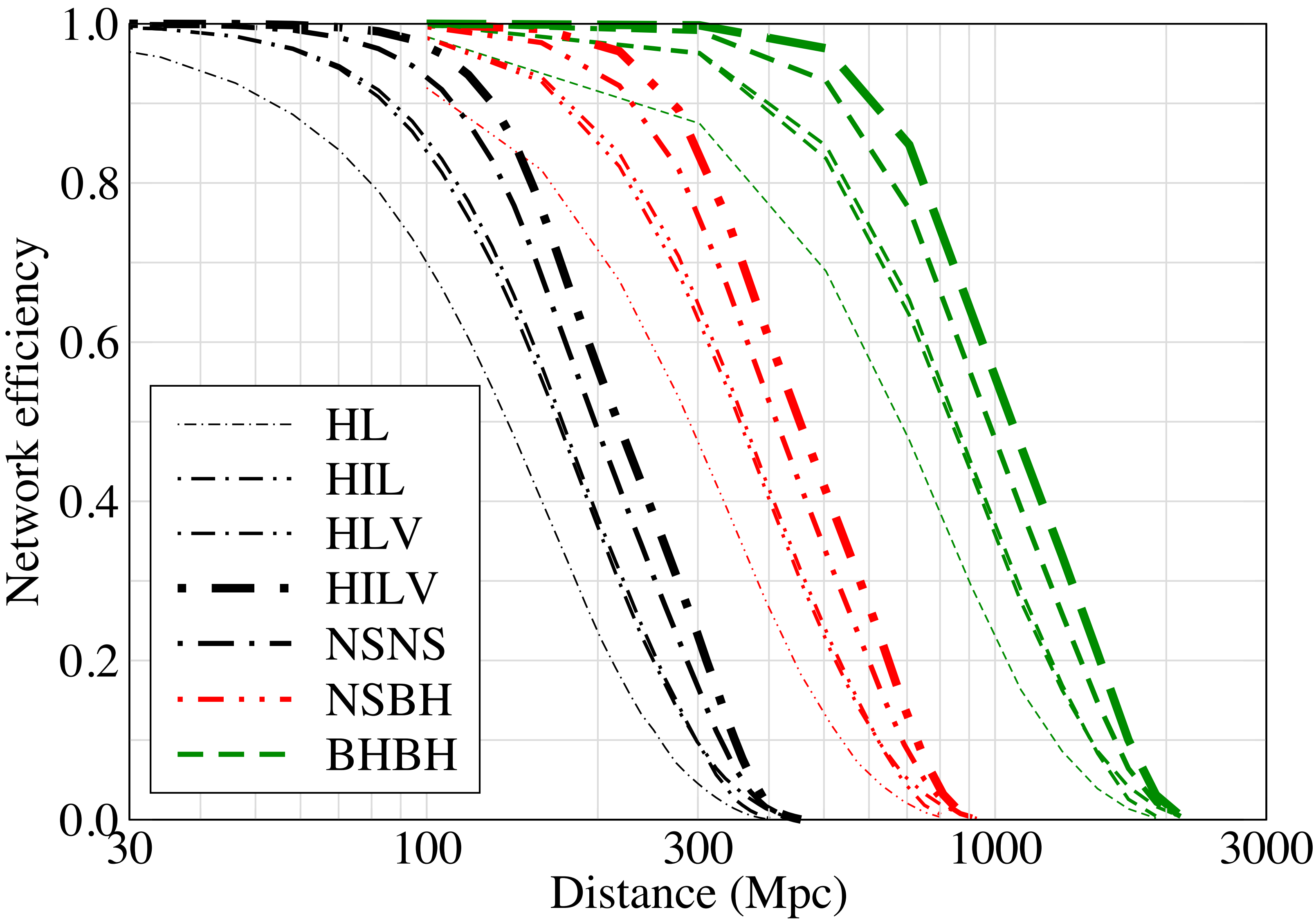


Plot: Weisberg+, Image: NASA

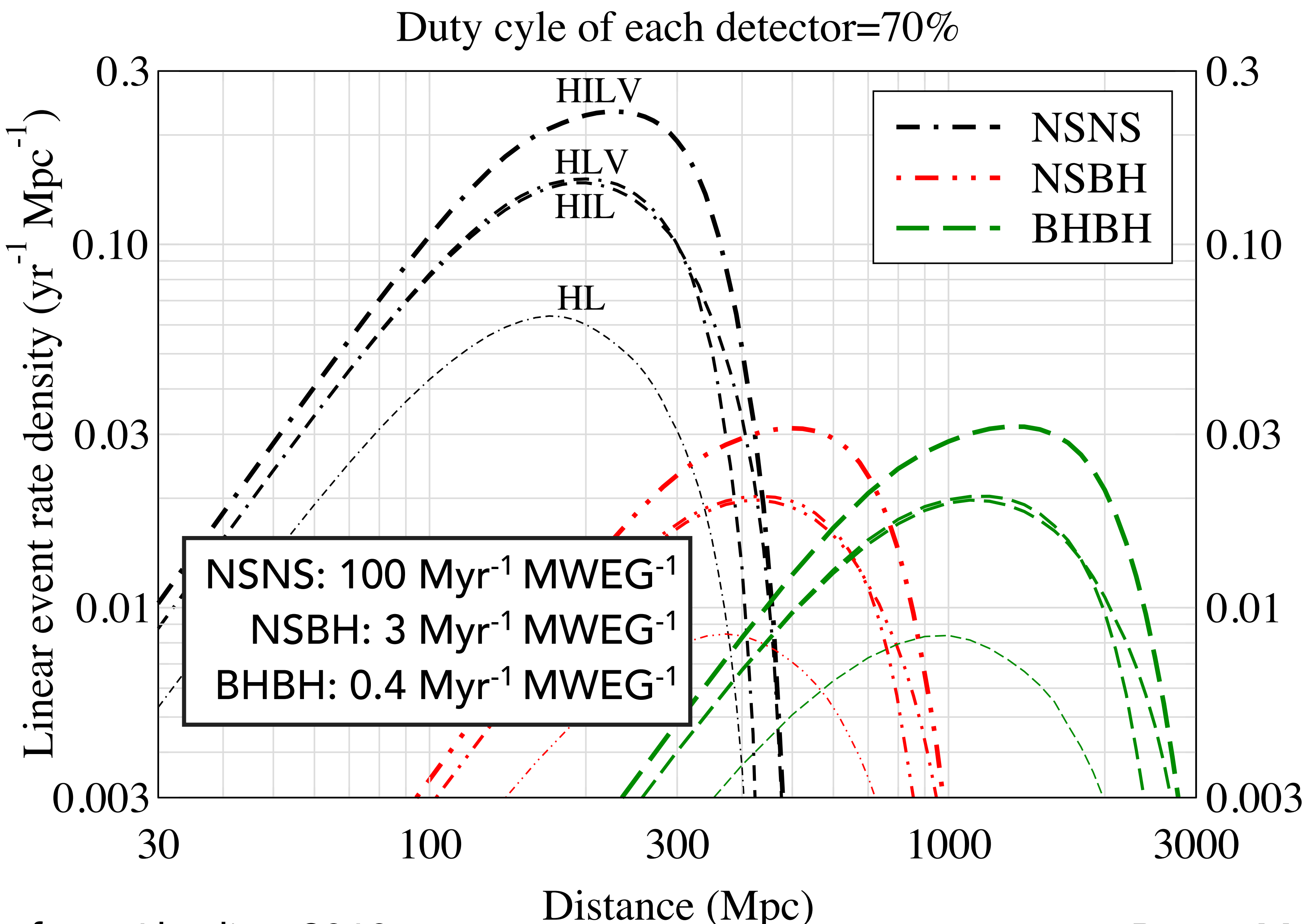
EXPECTED NS-NS MERGER RATES

- observed short GRB rate ~ 10 yr^{-1} Gpc^{-3}
- we won't observe all GRBs because
  - most GRB satellites are not sensitive to the whole sky and gamma emission is not expected to be isotropic
- comoving volume rate depends on the beaming angle
  - smaller the beaming angle, less likely we will observe them and so greater the intrinsic rate
- half beaming angle of 5° gives a comoving volume rate of 2,000 yr^{-1} Gpc^{-3}
  - implies a detection rate of ~ 50 yr^{-1} at LIGO-Virgo design sensitivity
- population synthesis models predict uncertain rates for all populations

COMPLETENESS OF SURVEYS



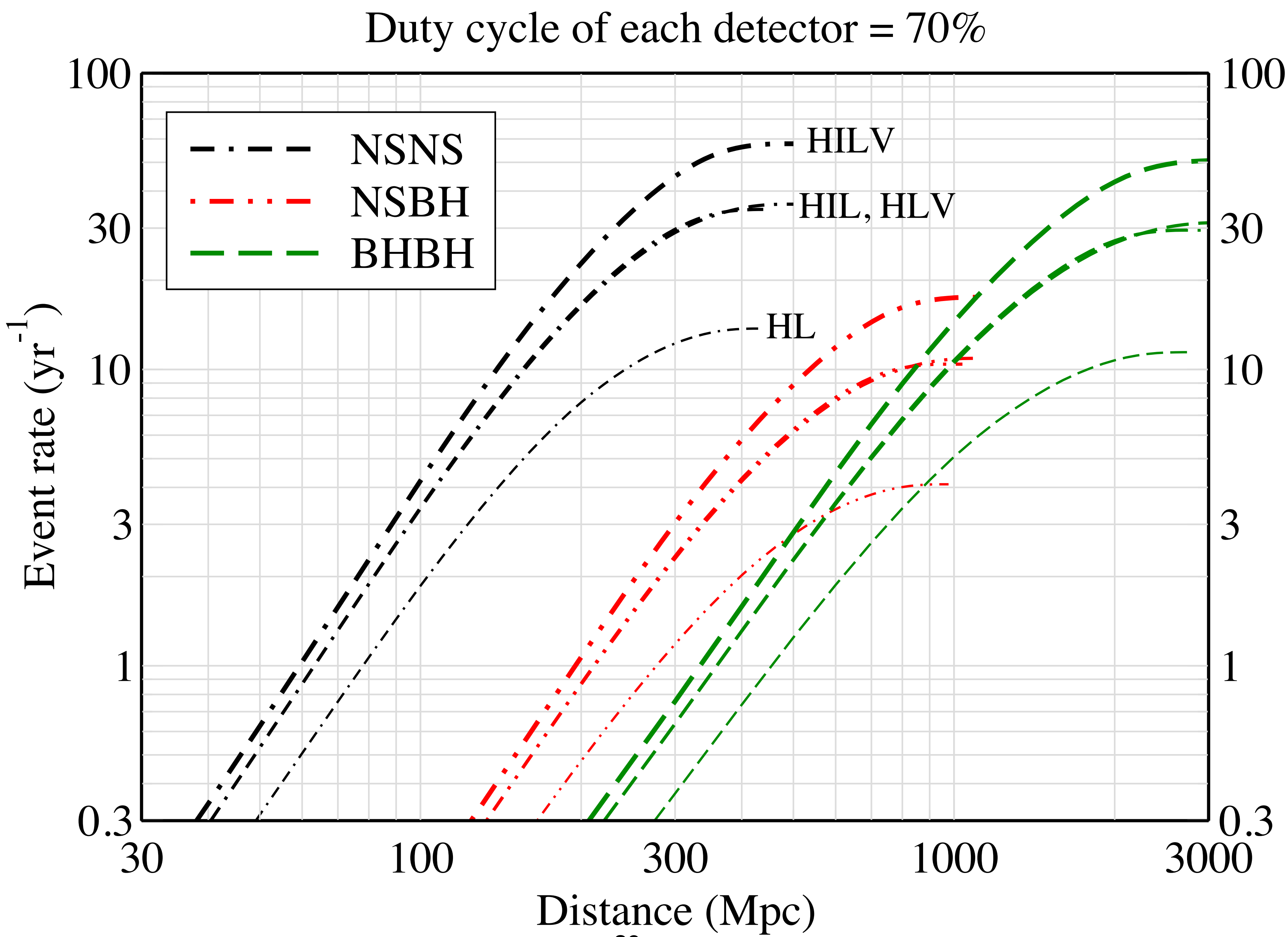
EXPECTED LINEAR RATE DENSITY



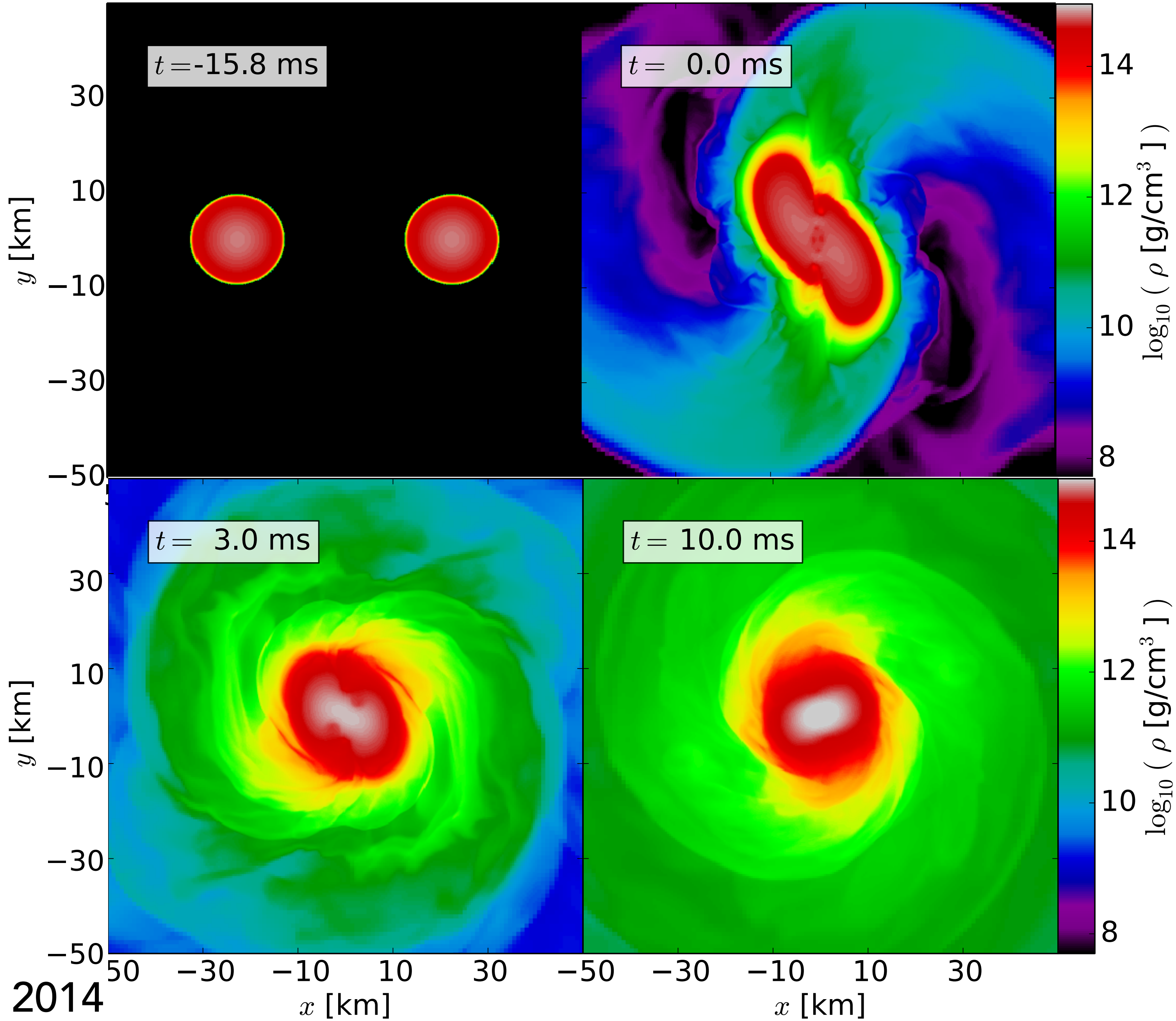
Rates from Abadie+ 2010

Rates: Mon

CUMULATIVE RATE AS A FUNC. OF DIST.

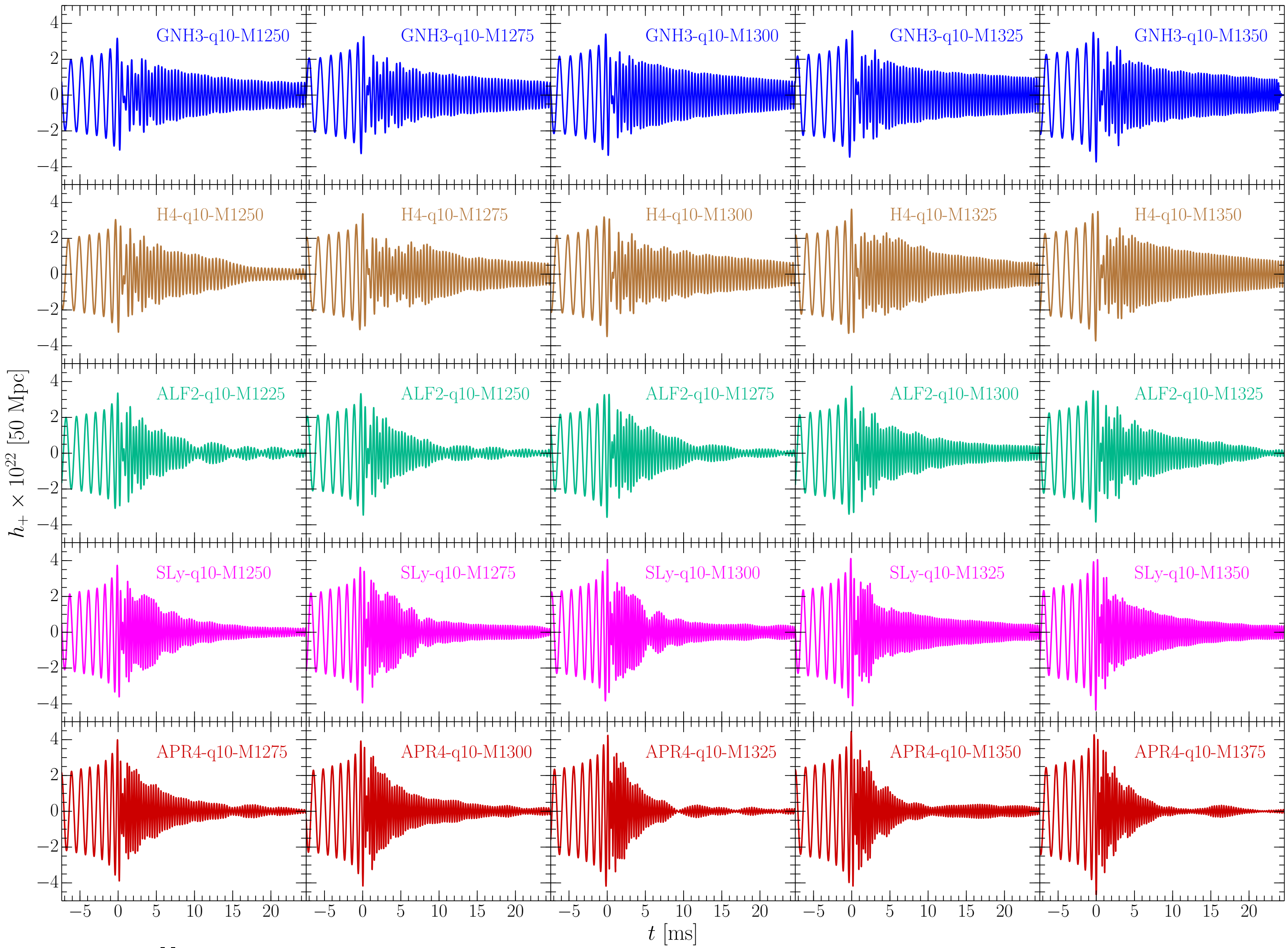


BINARY NEUTRON STAR MERGER



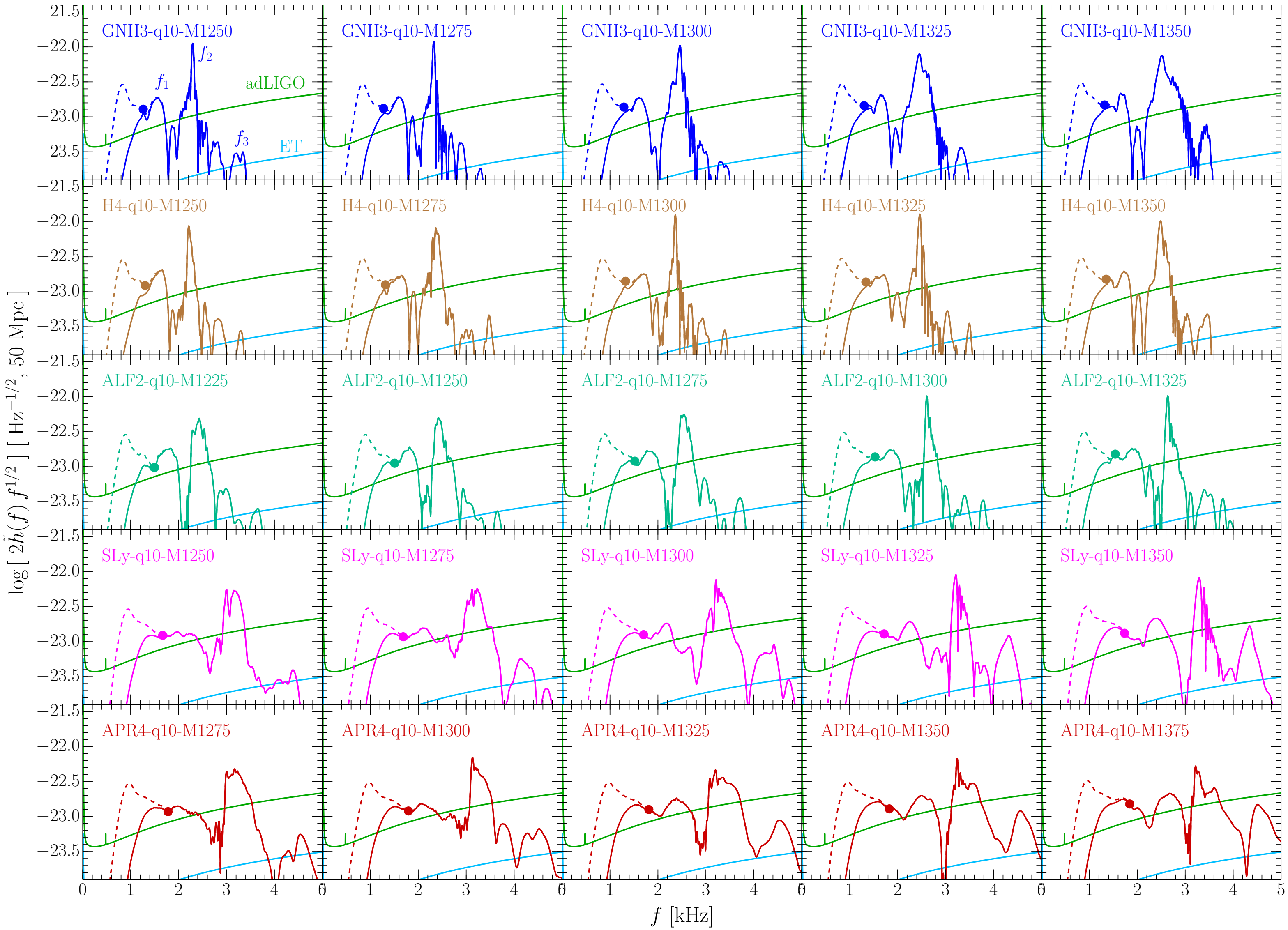
Takami+ 2014

BINARY NEUTRON STARS: POST-MERGER WAVEFORMS



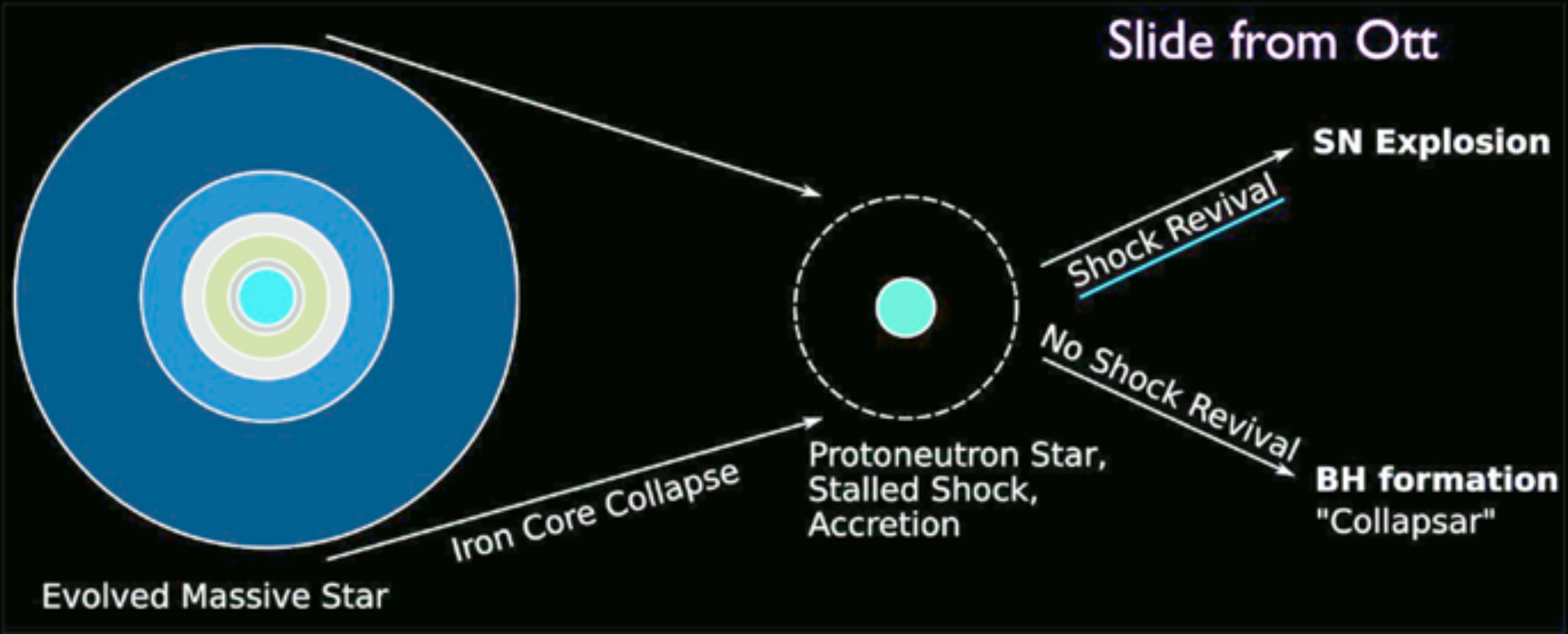
Takami, Rezzolla, Baiotti, 2014

BINARY NEUTRON STARS: POST-MERGER SPECTRUM



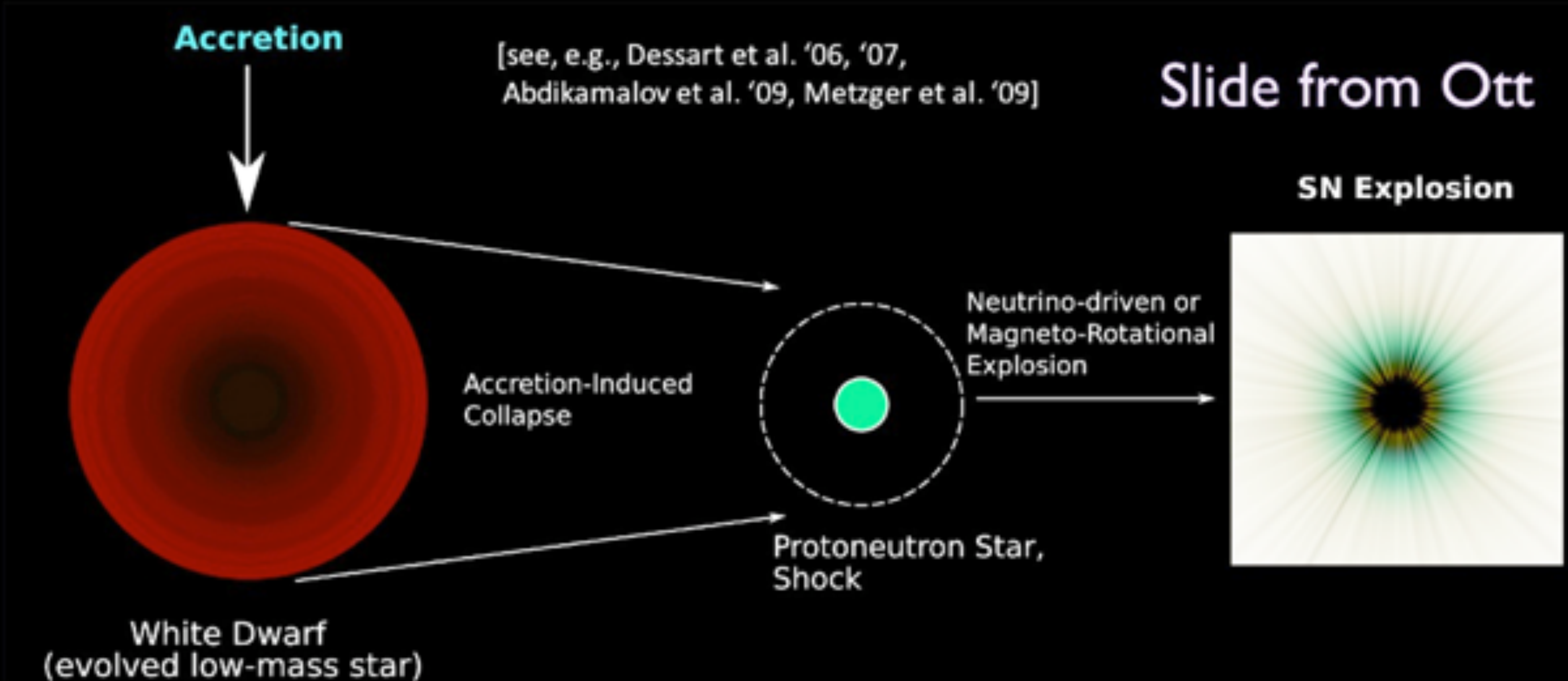
Takami, Rezzolla, Baiotti, 2014

Core Collapse SNe

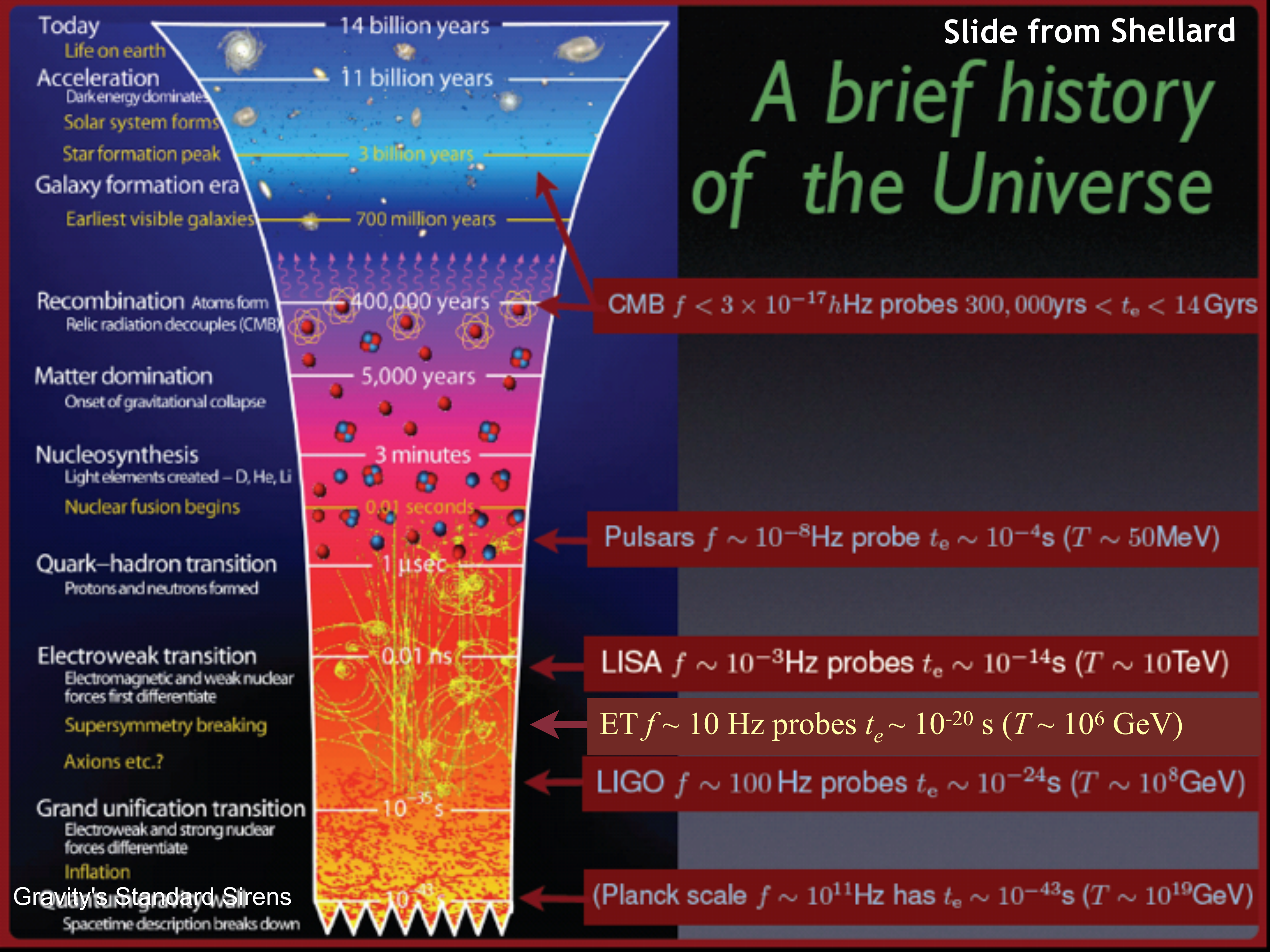


- Energy reservoir
  - few x 10^{53} erg
- Explosion energy
  - 10^{51} erg
- Time frame for explosion
  - 300 - 1500 ms after bounce
- Formation of black hole
  - At baryonic mass > 1.8-2.5 M

Accretion Induced Collapse

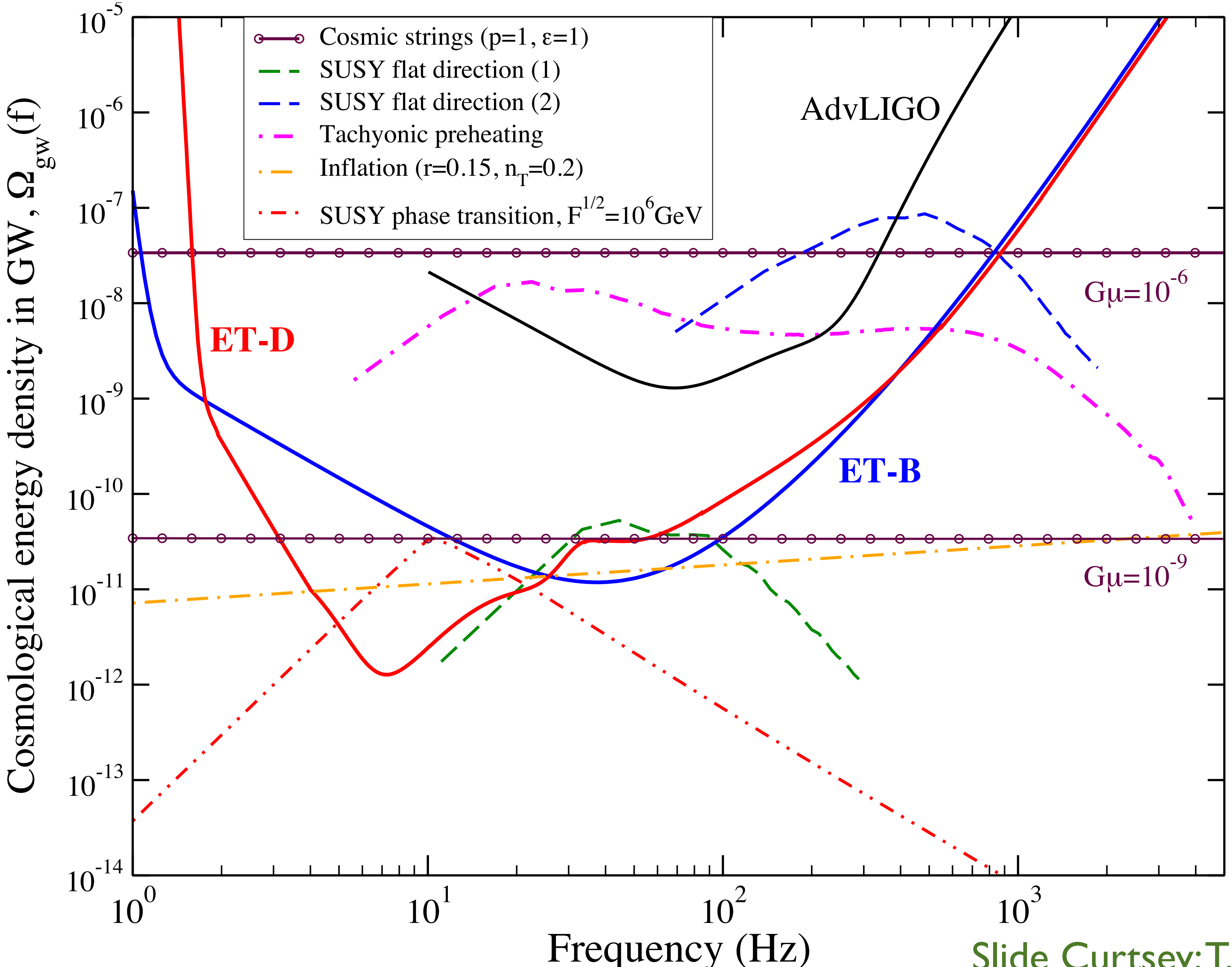


- Collapse of accreting, probably rotating White Dwarfs
  - Neutrino-driven or magneto-rotational explosion
- Explosion probably weak, sub-luminous
  - Might not be seen in optical
- Potential birth site of magnetars - highly (10^{15}- 10^{16} G) magnetized neutron stars



Slide from Shellard
A brief history of the Universe
Today
14 billion years
Life on earth
Acceleration
Dark energy dominates
11 billion years
Solar system forms
Star formation peak
3 billion years
Galaxy formation era
Earliest visible galaxies
700 million years
Recombination Atoms form
Relic radiation decouples (CMB)
400,000 years
Matter domination
Onset of gravitational collapse
5,000 years
Nucleosynthesis
Light elements created – D, He, Li
3 minutes
Nuclear fusion begins
0.01 seconds
Quark–hadron transition
Protons and neutrons formed
1 μsec
Electroweak transition
Electromagnetic and weak nuclear forces first differentiate
0.01 ns
Supersymmetry breaking
Axions etc.?
Grand unification transition
Electroweak and strong nuclear forces differentiate
Inflation
Spacetime description breaks down
CMB $f < 3\times10^{-17}h$Hz probes $300,000$yrs $< t_e < 14$ Gyrs
Pulsars $f\sim10^{-8}$Hz probe $t_e\sim10^{-4}$s ($T\sim50$MeV)
LISA $f\sim10^{-3}$Hz probes $t_e\sim10^{-14}$s ($T\sim10$TeV)
ET $f\sim10$ Hz probes $t_e\sim10^{-20}$ s ($T\sim10^6$ GeV)
LIGO $f\sim100$ Hz probes $t_e\sim10^{-24}$s ($T\sim10^8$GeV)
(Planck scale $f\sim10^{11}$Hz has $t_e\sim10^{-43}$s ($T\sim10^{19}$GeV)
Gravity's Standard Sirens

PRIMORDIAL BACKGROUNDS

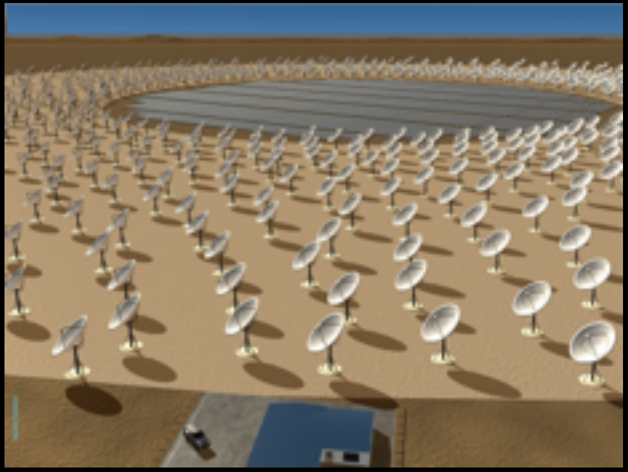


Slide Curtsey: T. Dent

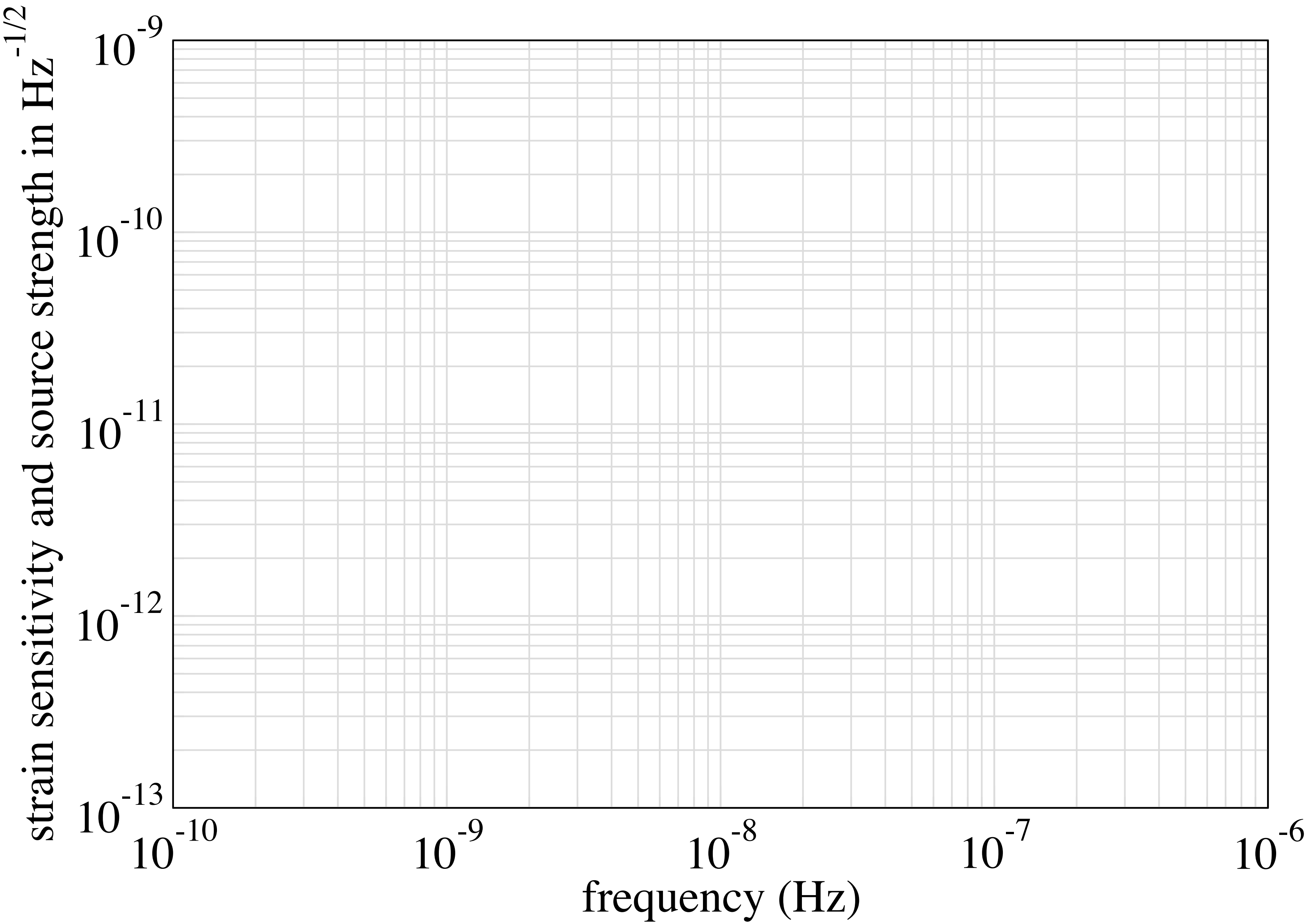
OBSERVING THE SOURCES

PULSAR TIMING ARRAYS

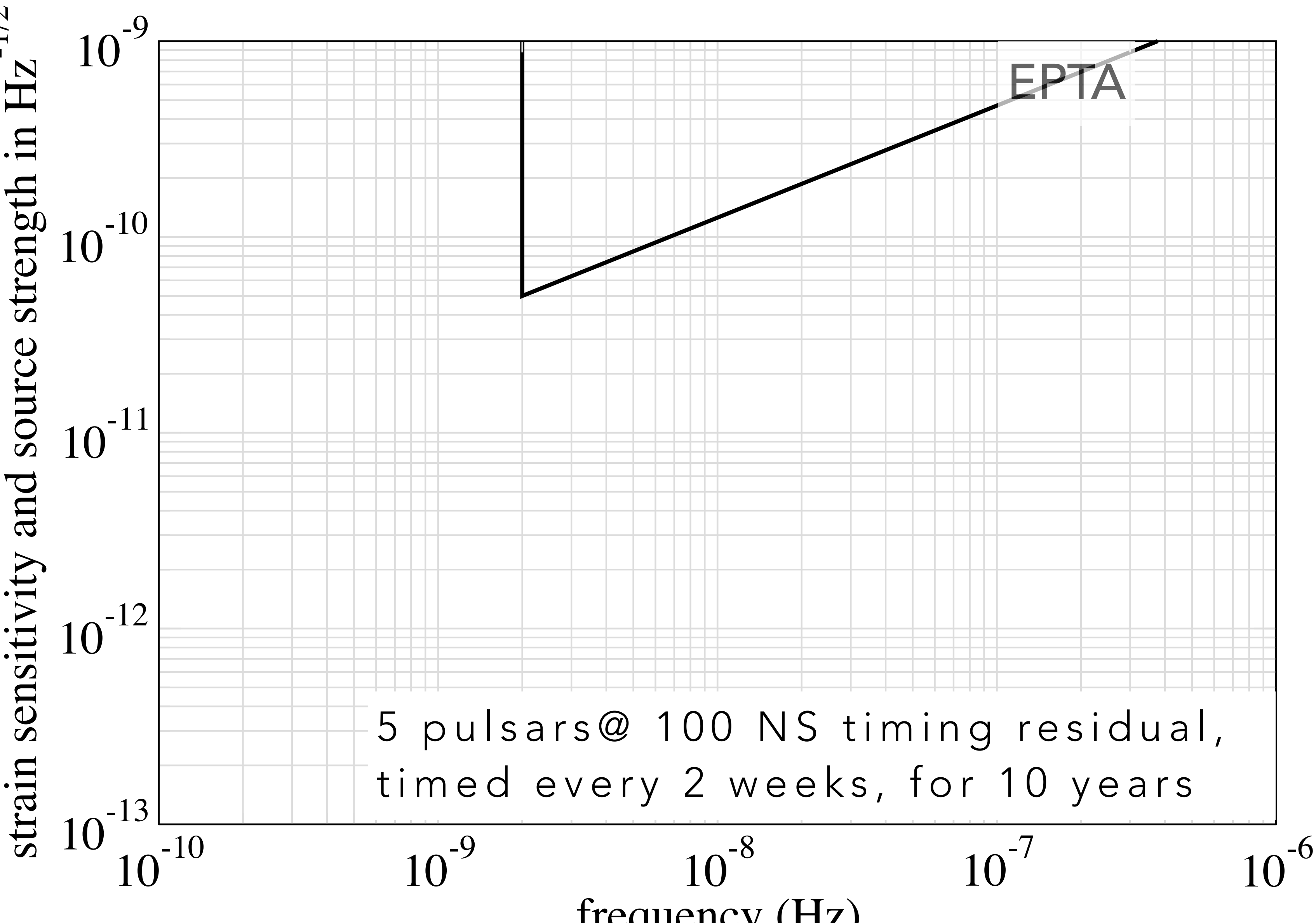
- European Pulsar Timing Array (EPTA)
- Parkes Pulsar Timing Array (PPTA)
- North American Nano-hertz Gravitational Wave Observatory (NanoGrav)
- International Pulsar Timing Array (IPTA)
- Square Kilometre Array (SKA and its predecessors)



FREQUENCY RANGE

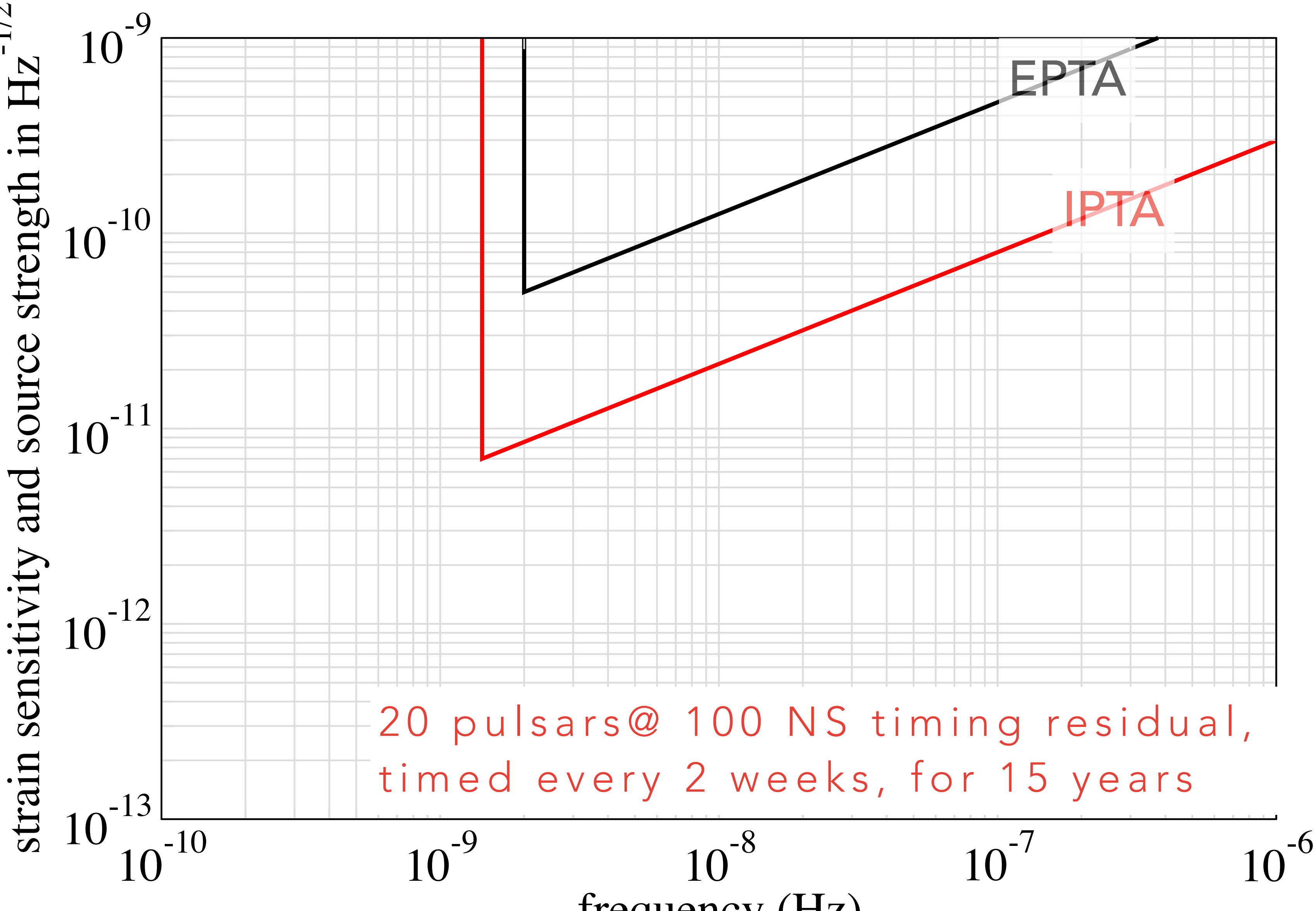


EPTA, PPTA, NANOGRAV



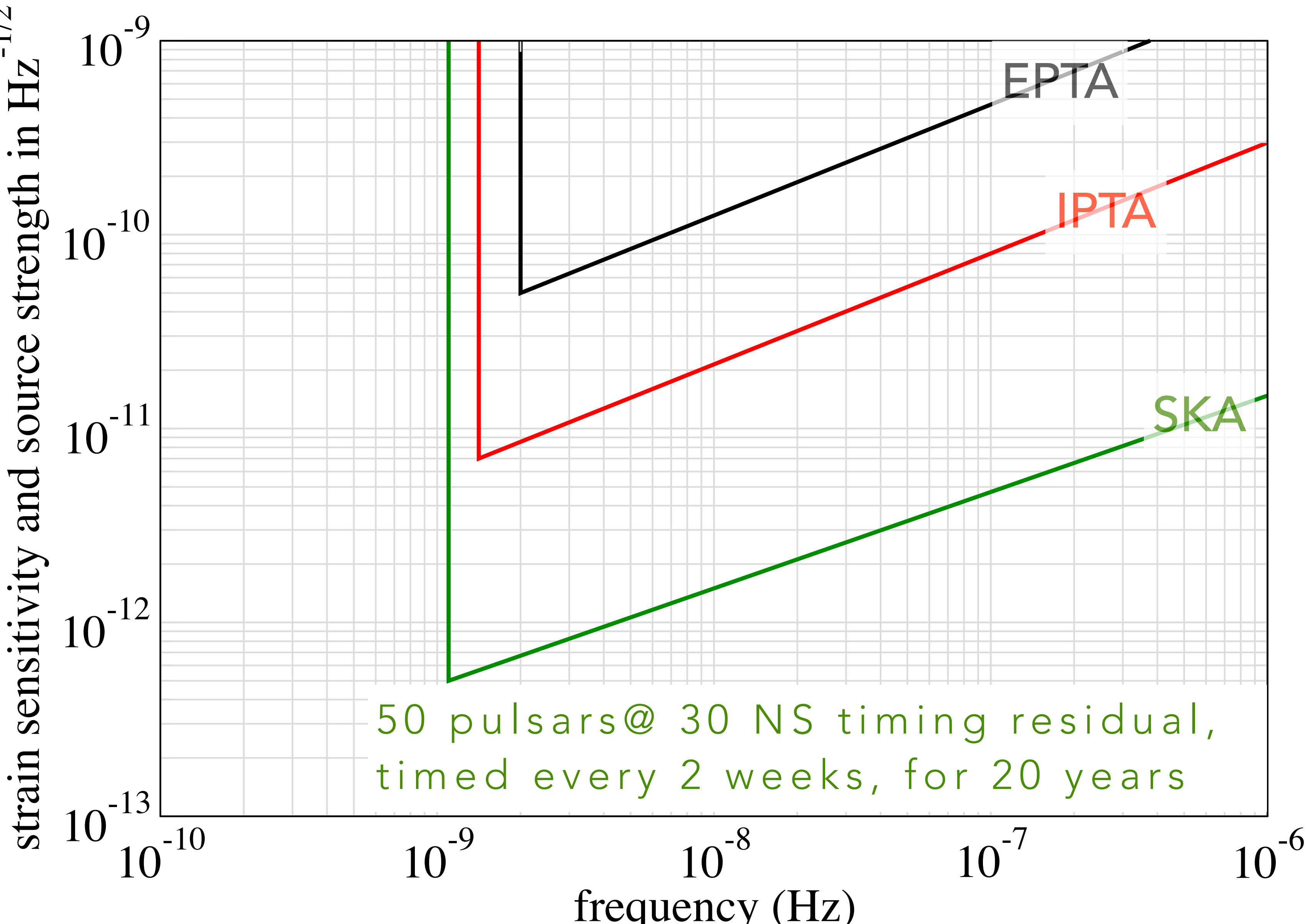
sensitivity data from Moore+ 2014

IPTA



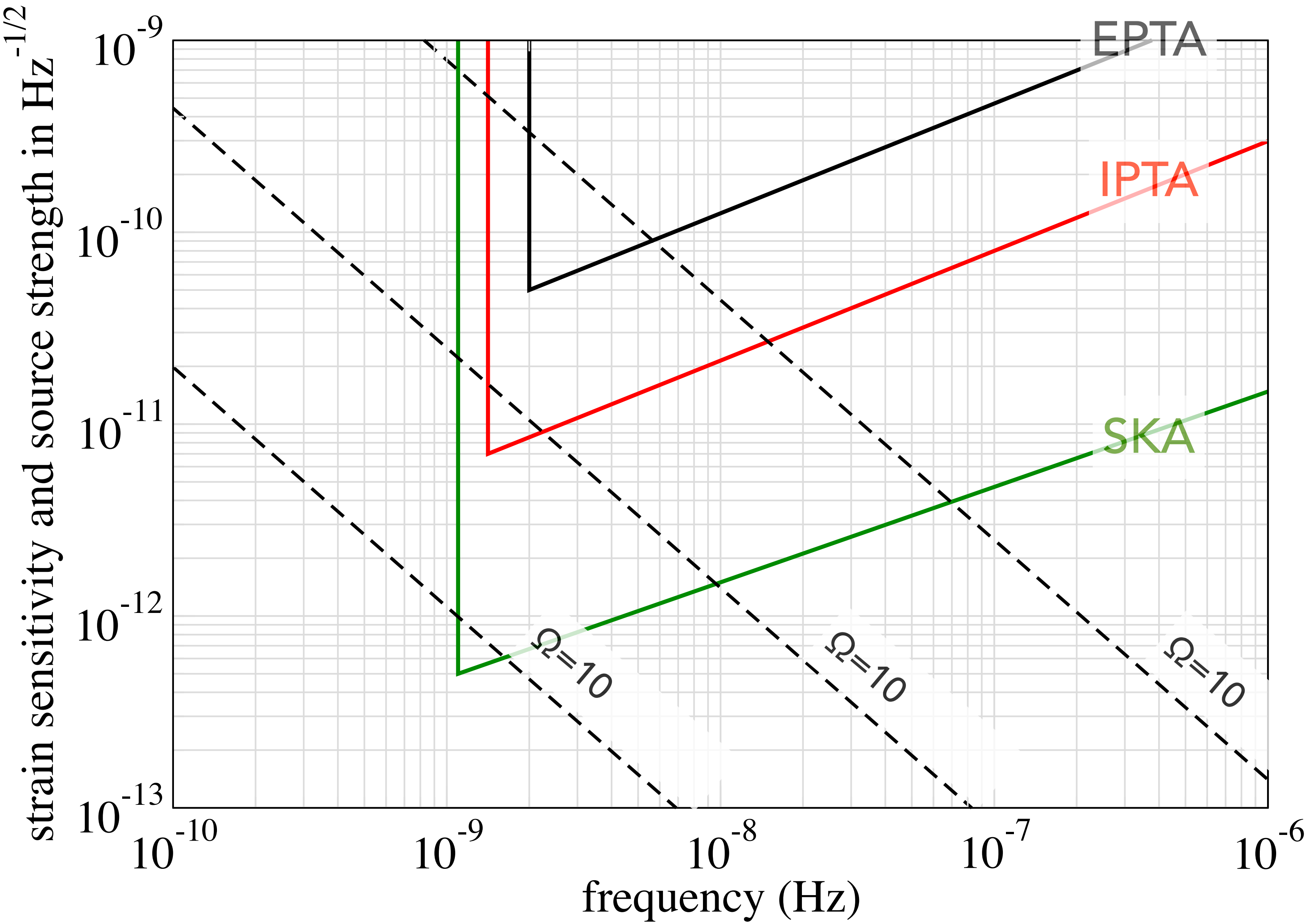
sensitivity data from Moore+ 2014

SKA

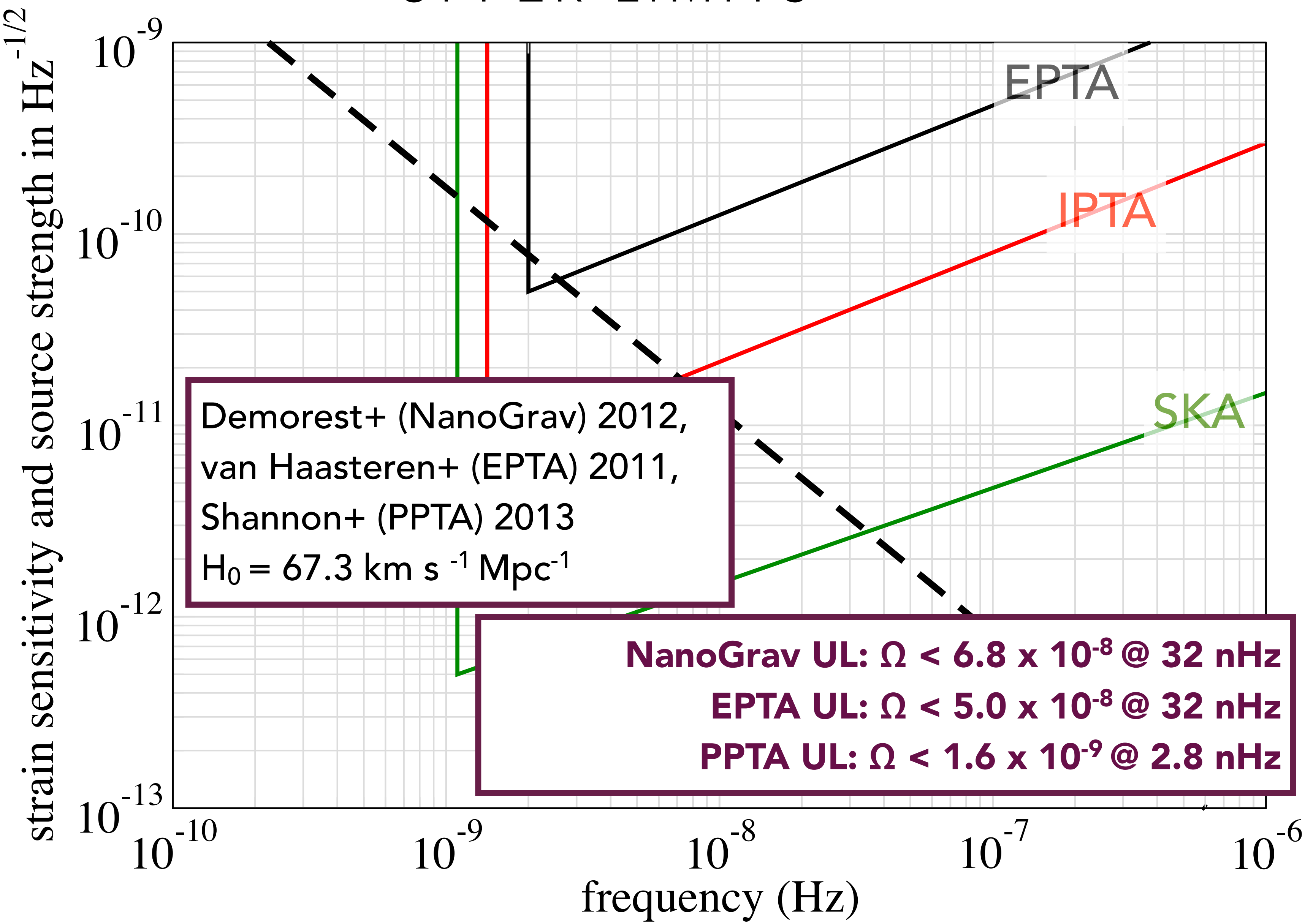


sensitivity data from Moore+ 2014

STOCHASTIC BACKGROUNDS

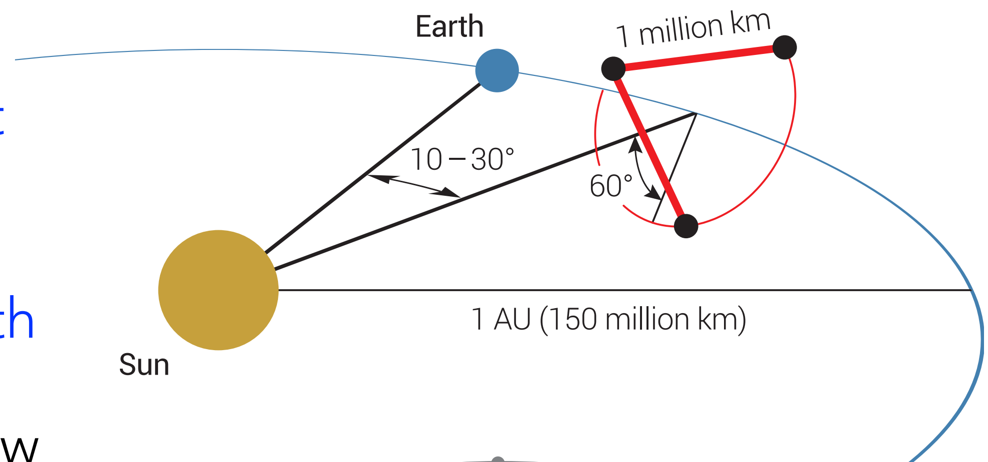


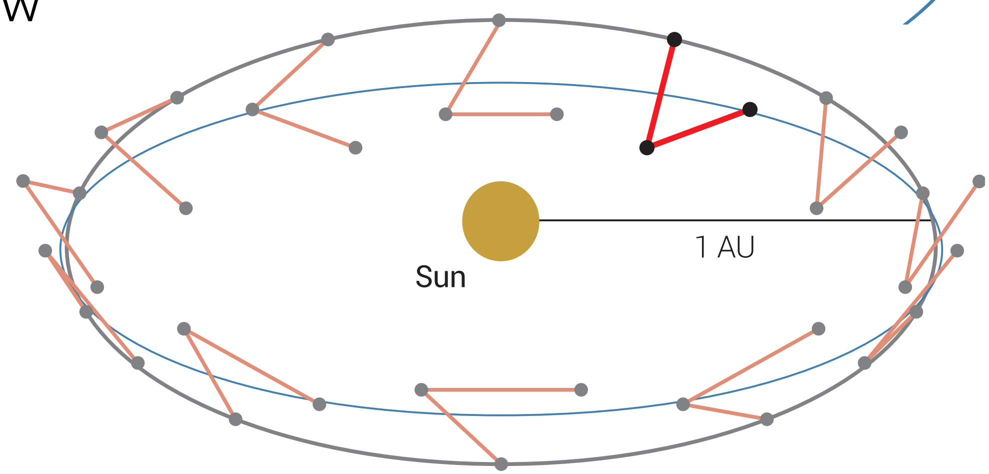
SMBBH BACKGROUND: CURRENT BEST UPPER LIMITS



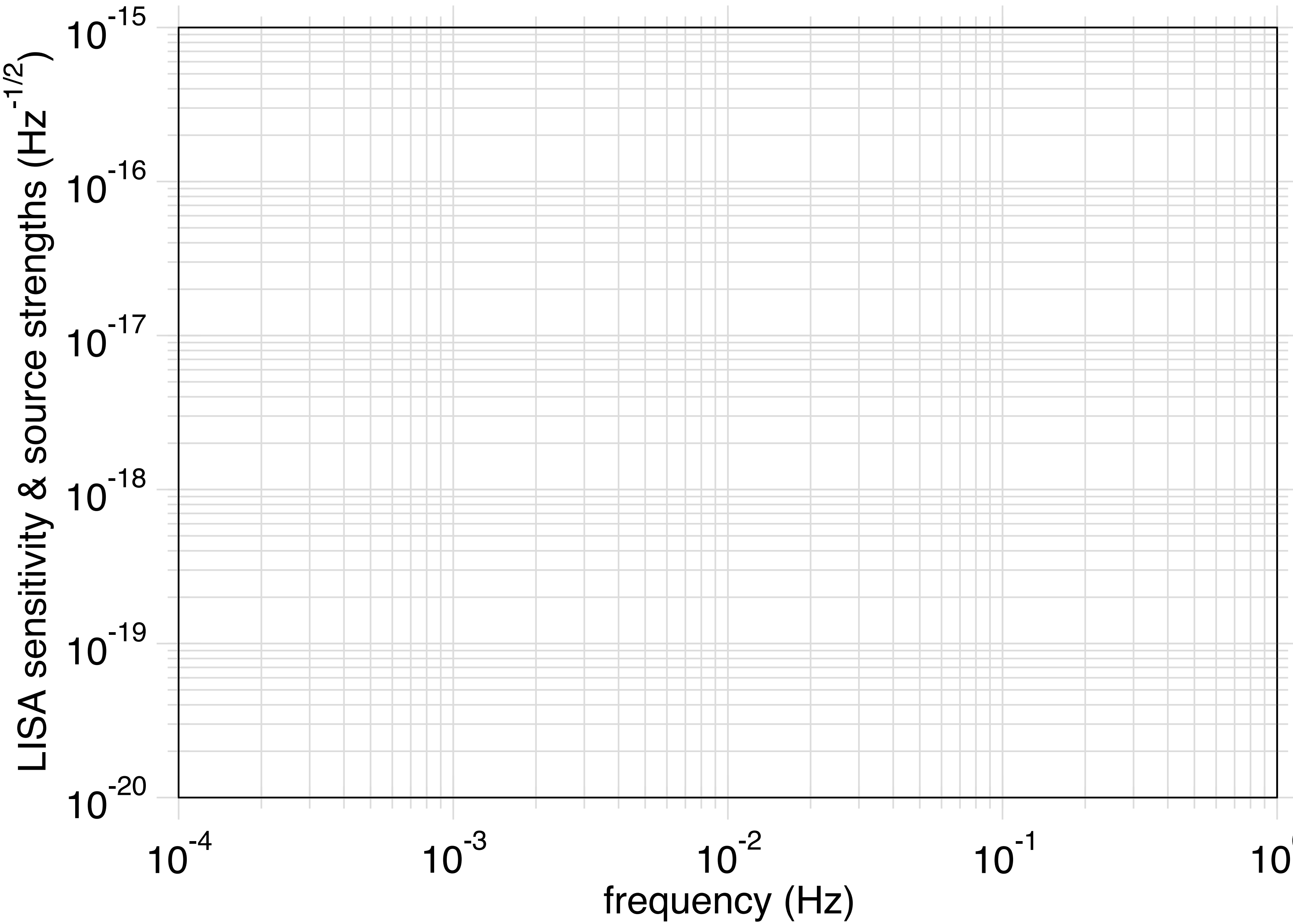
ELISA: L3 MISSION IN ESA'S COSMIC HORIZON PROGRAMME

- Consists of 3 spacecraft in heliocentric orbit
  - Distance between spacecraft ~ 1 million km
  - 10 to 30 degrees behind earth
- The three eLISA spacecraft follow Earth almost as a rigid triangle entirely due to celestial mechanics
  - The triangle rotates like a cartwheel as craft orbit the sun

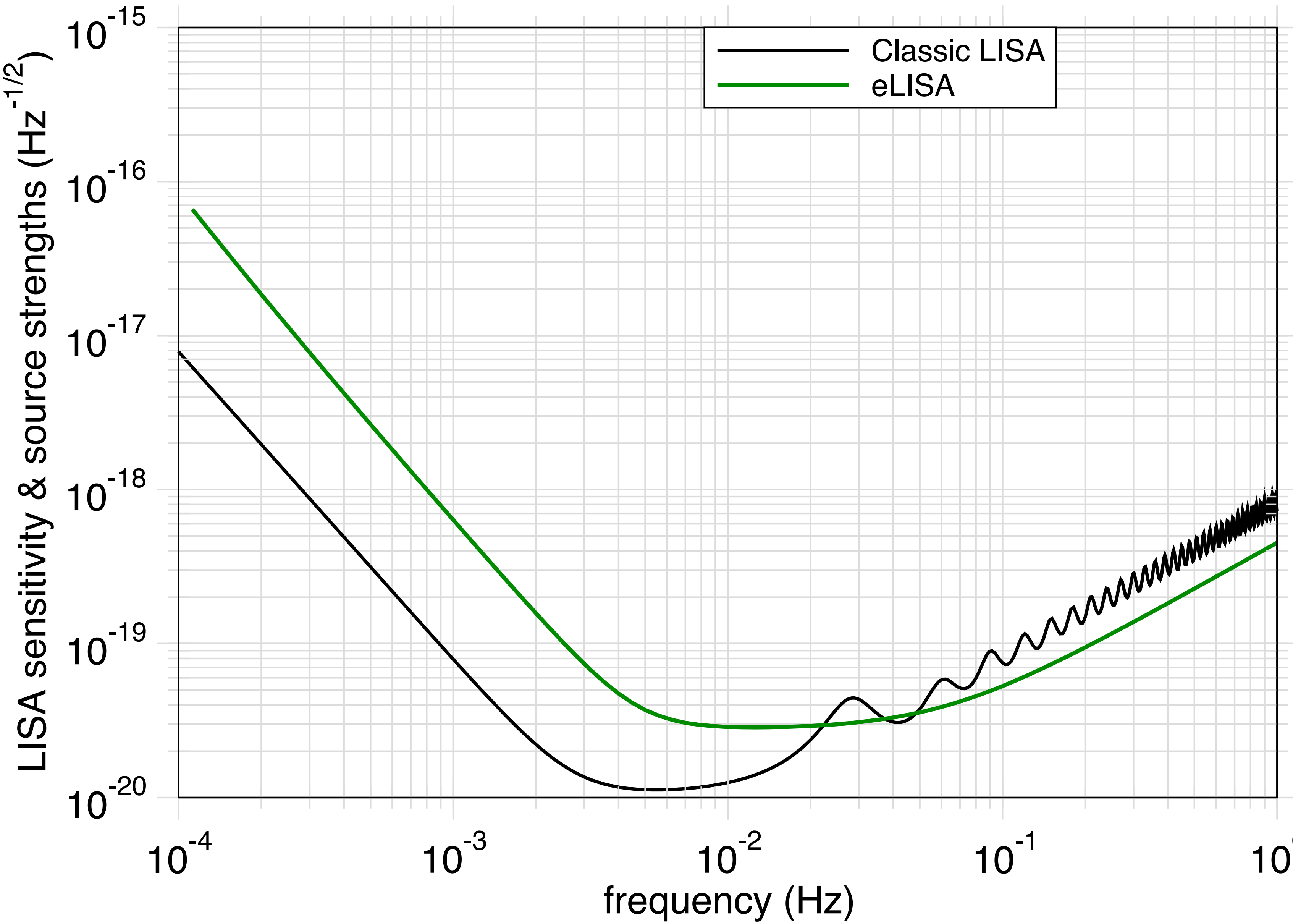




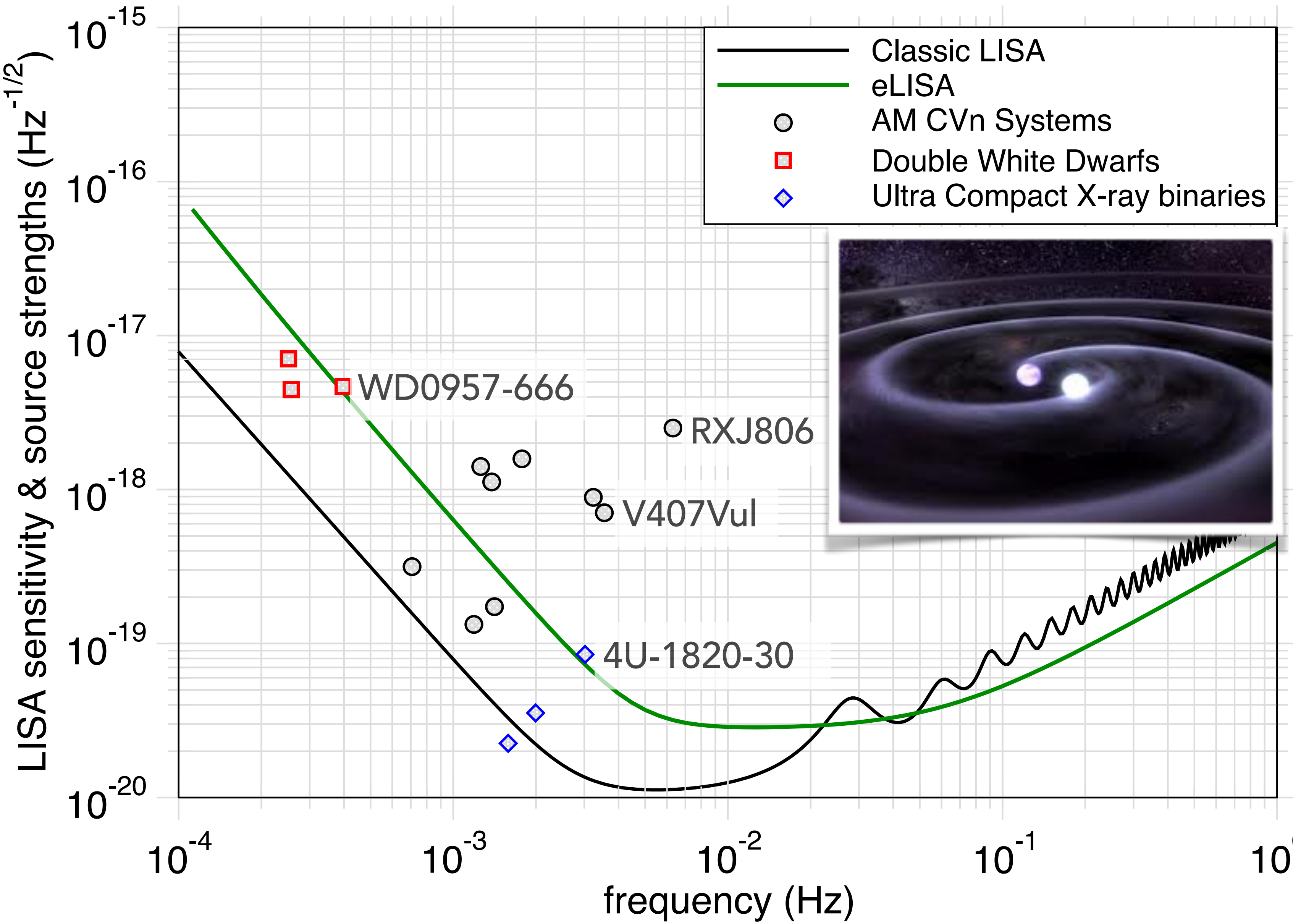
LISA: FREQUENCY RANGE



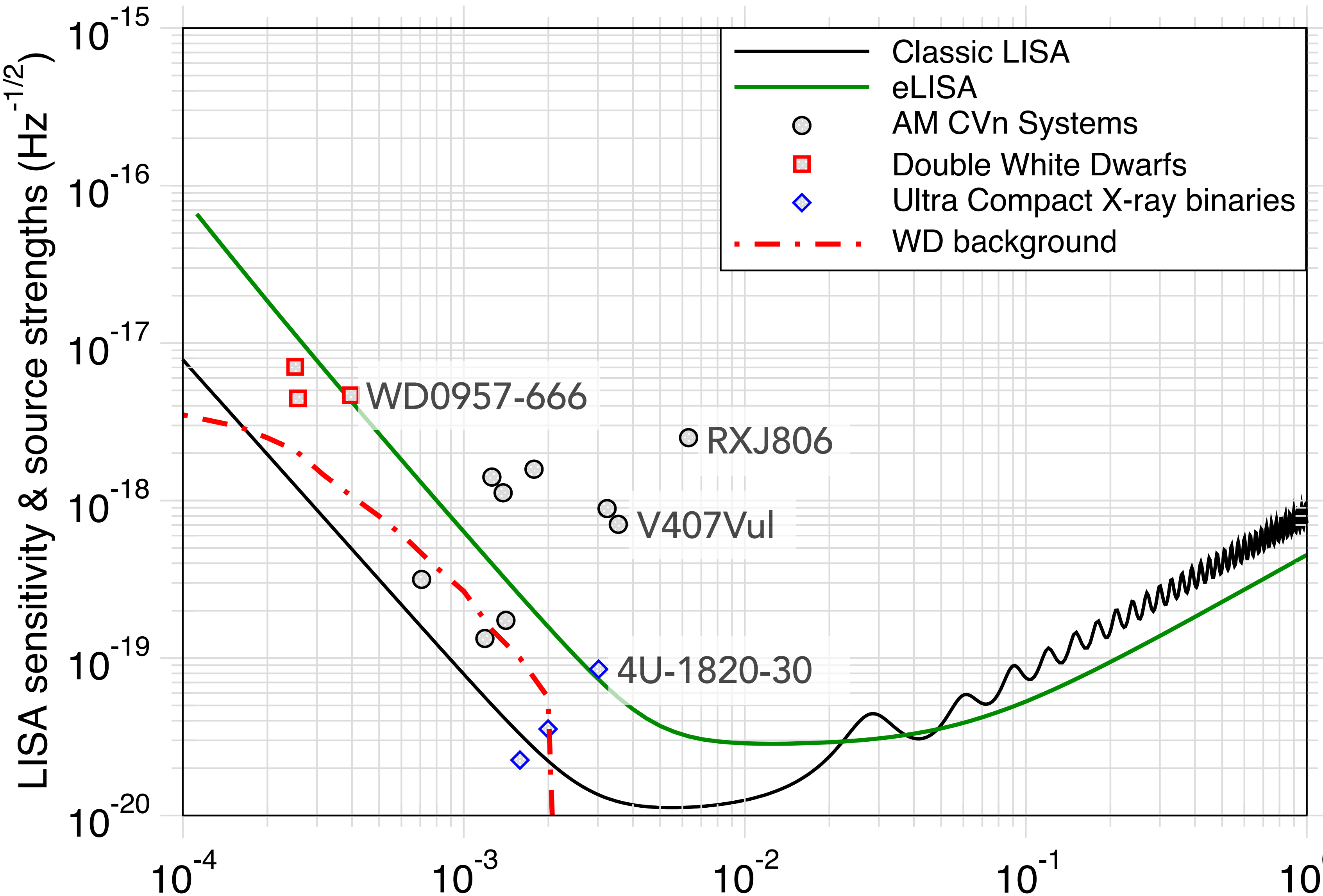
LISA SENSITIVITY



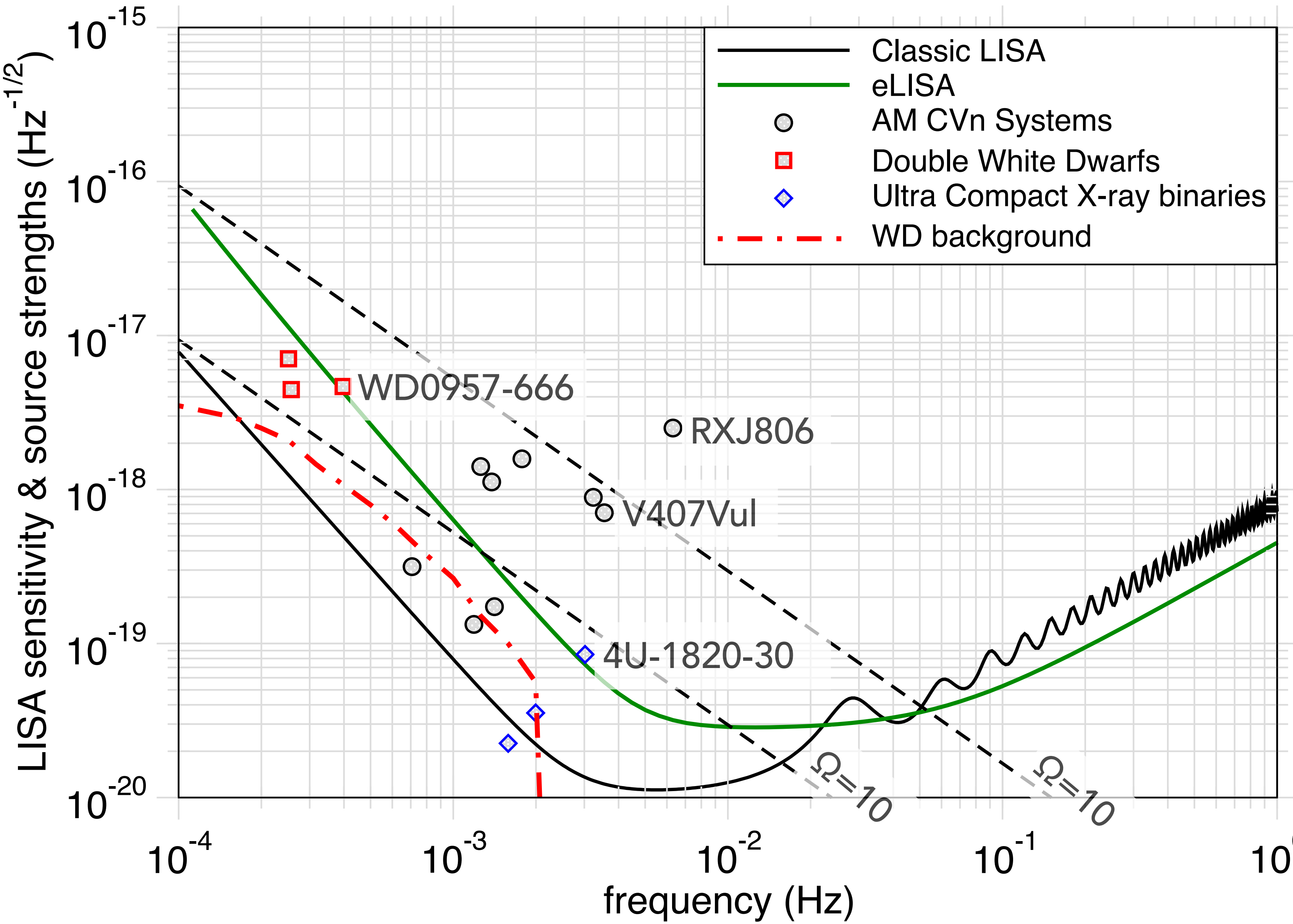
KNOWN SOURCES IN LISA BAND



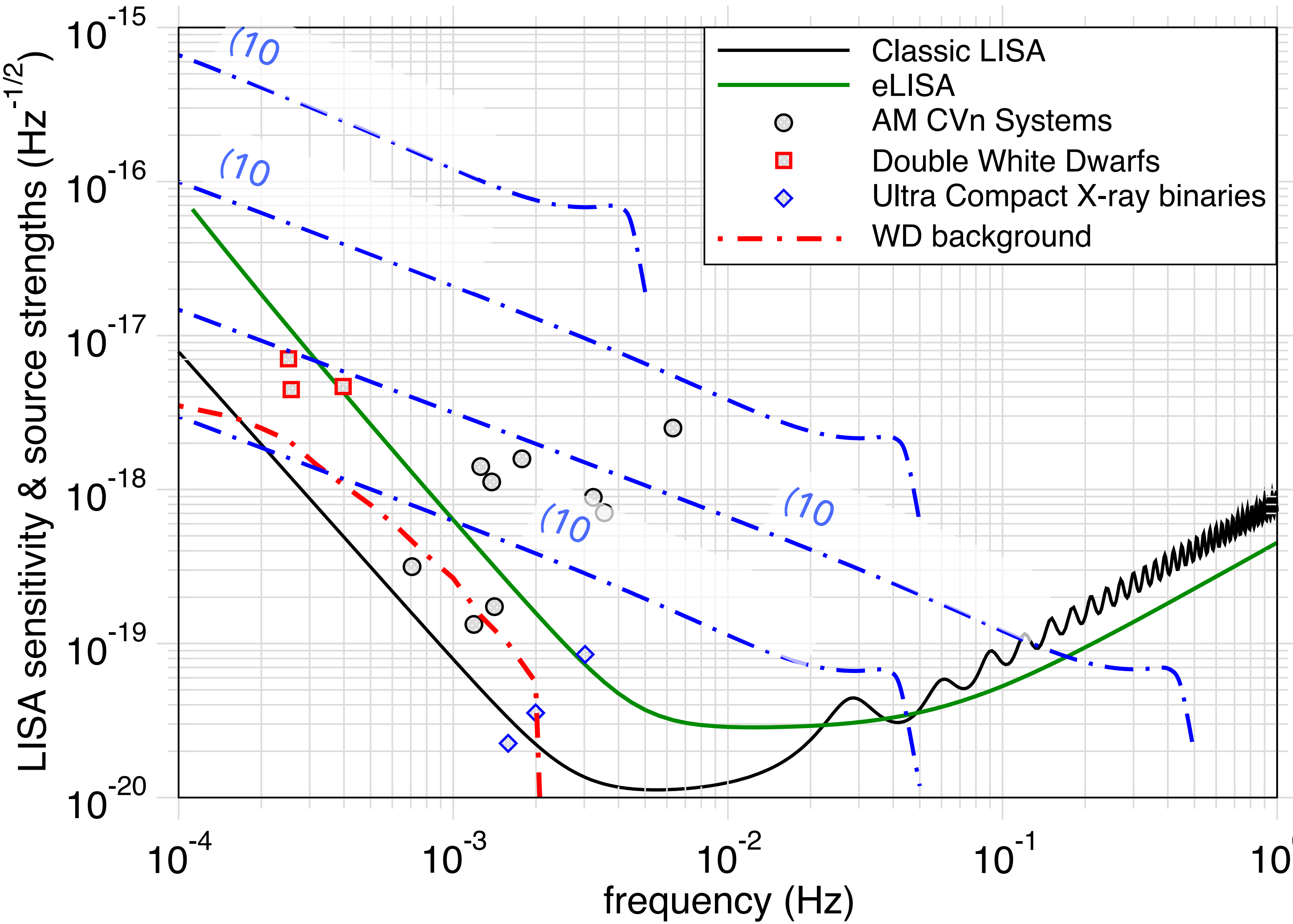
LISA WHITE DWARF BACKGROUND



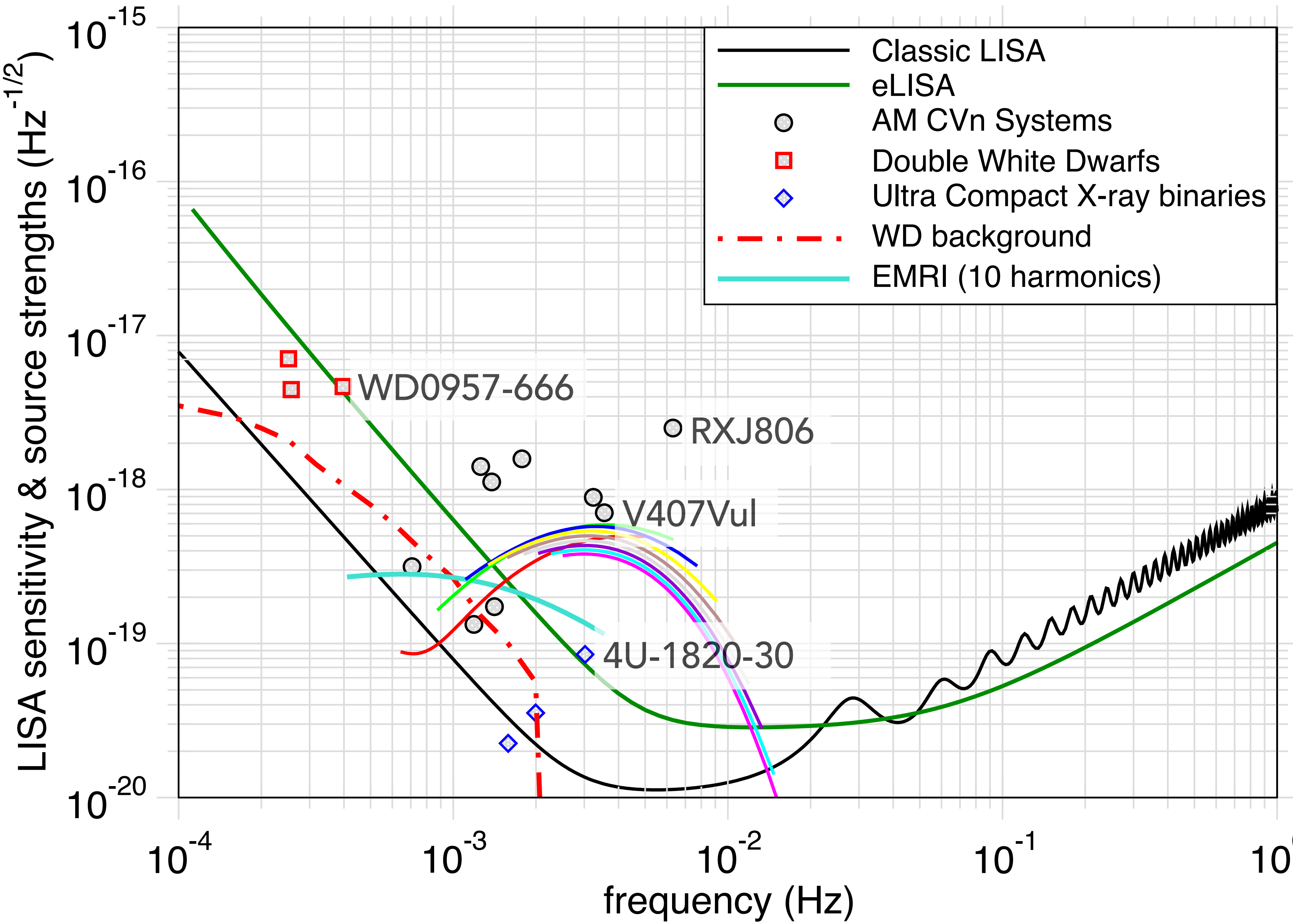
STOCHASTIC BACKGROUNDS



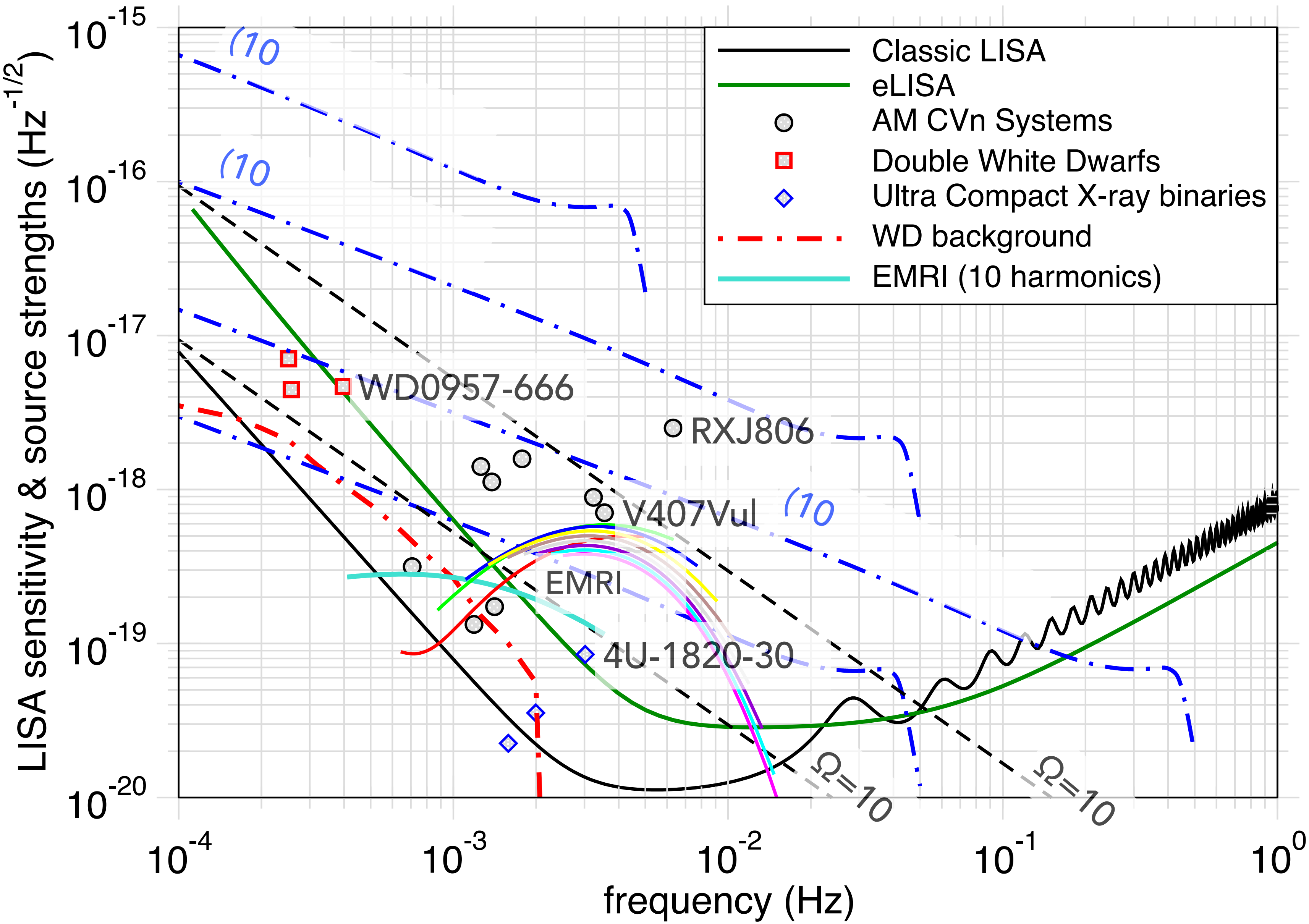
SUPERMASSIVE BLACK HOLES



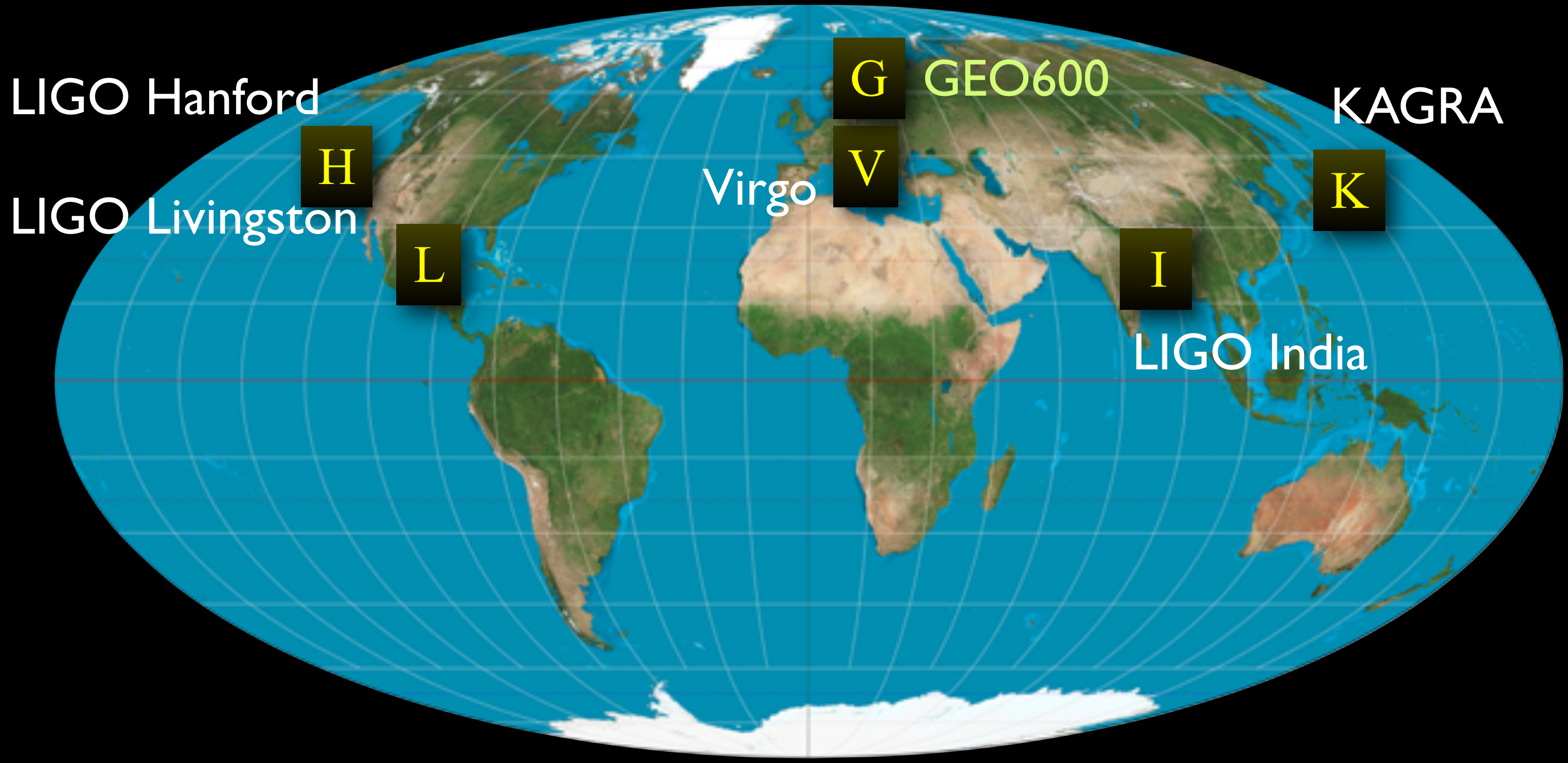
EXTREME MASS RATIO INSPIRALS



LISA SOURCE SUMMARY

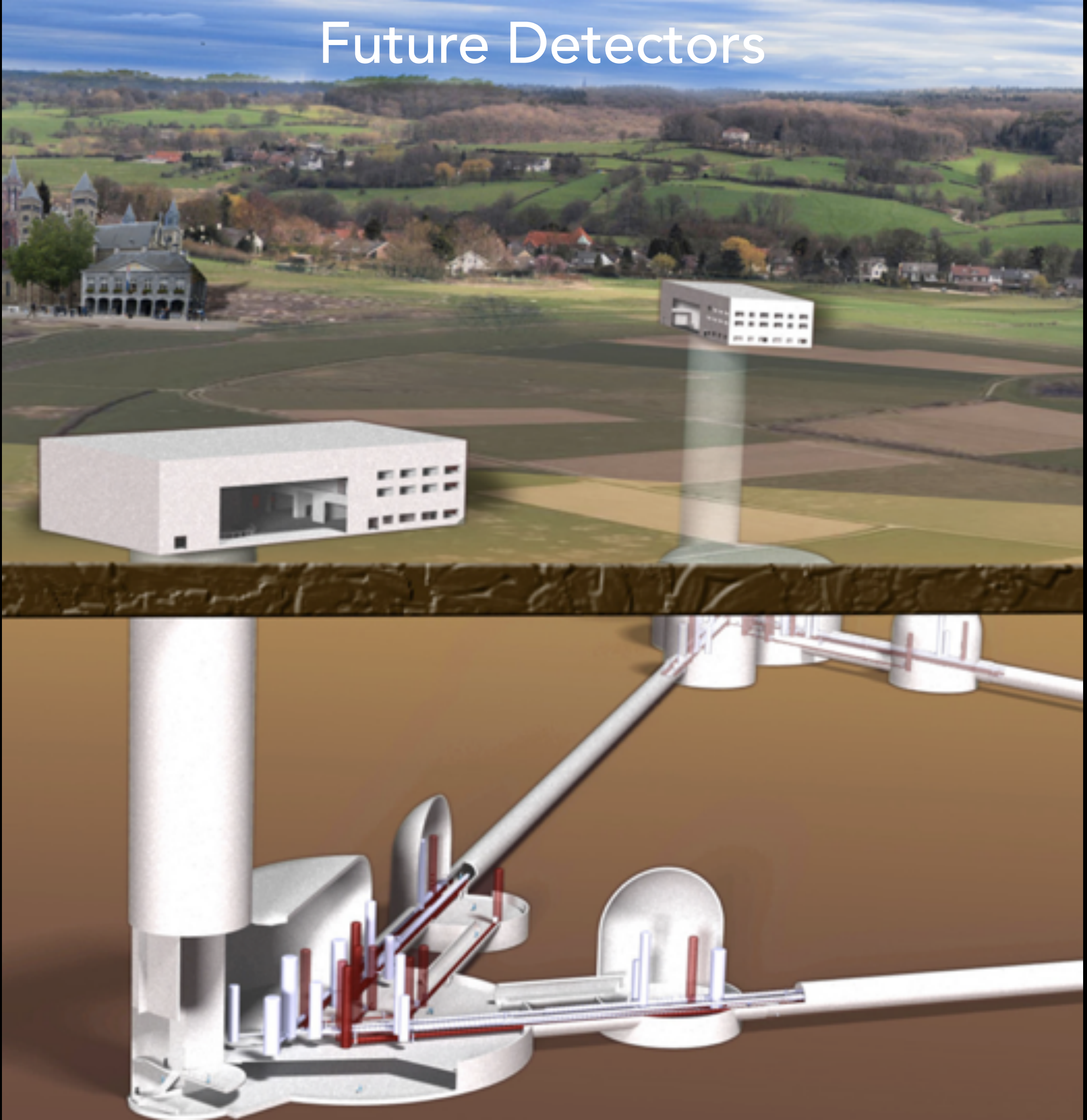


ADVANCED DETECTOR NETWORK



- 2006-2010: detectors took 2 years worth of data at unprecedented sensitivity levels
- 2015-2022: five large detectors will become operational
- Advanced LIGO detectors both installed and locked, commissioning over the next 3 years should see first detections

Future Detectors



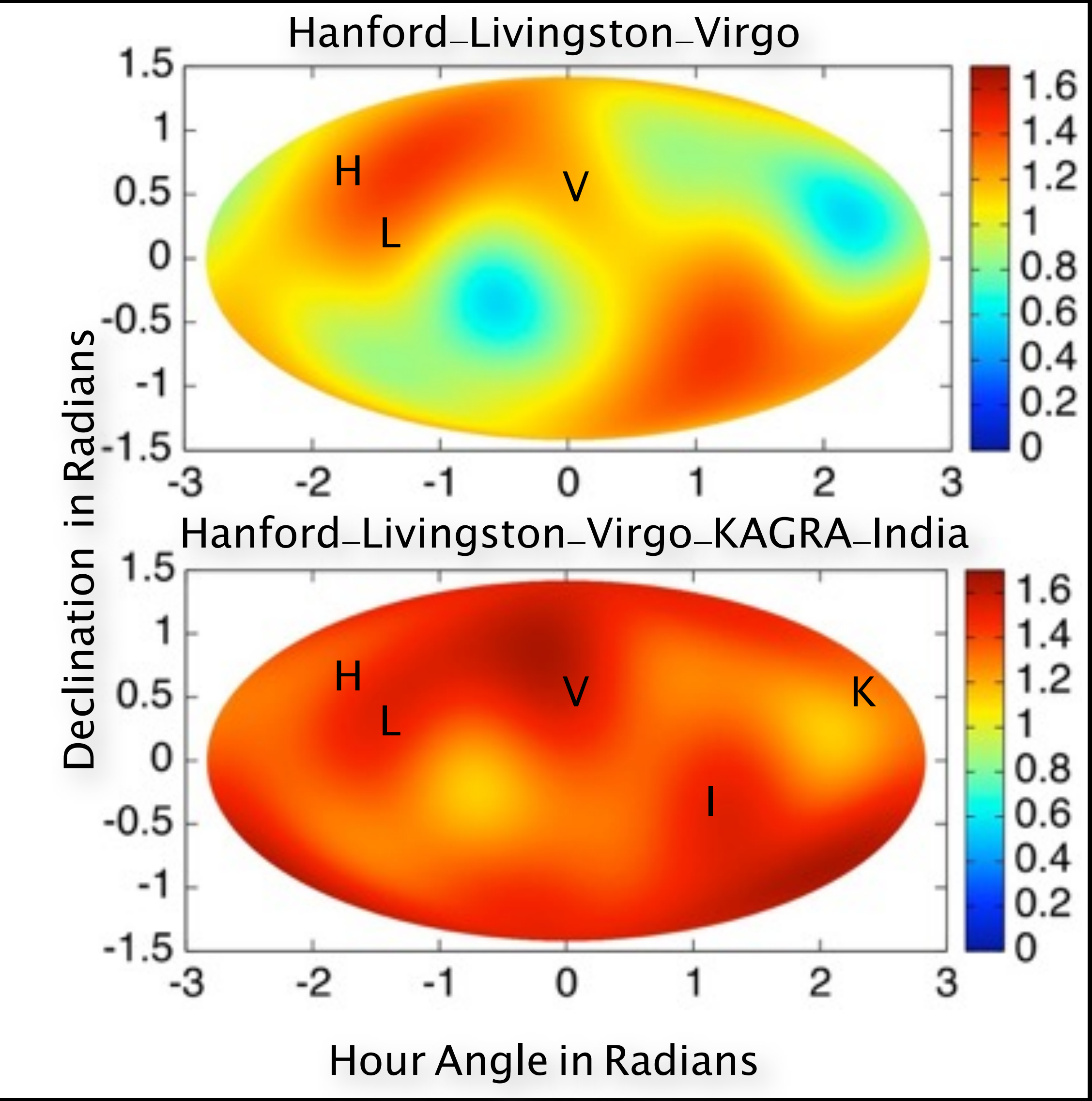
Voyager: x 3 improvement in aLIGO strain sensitivity

Cosmic Explorer: new 40 km arm length interferometer

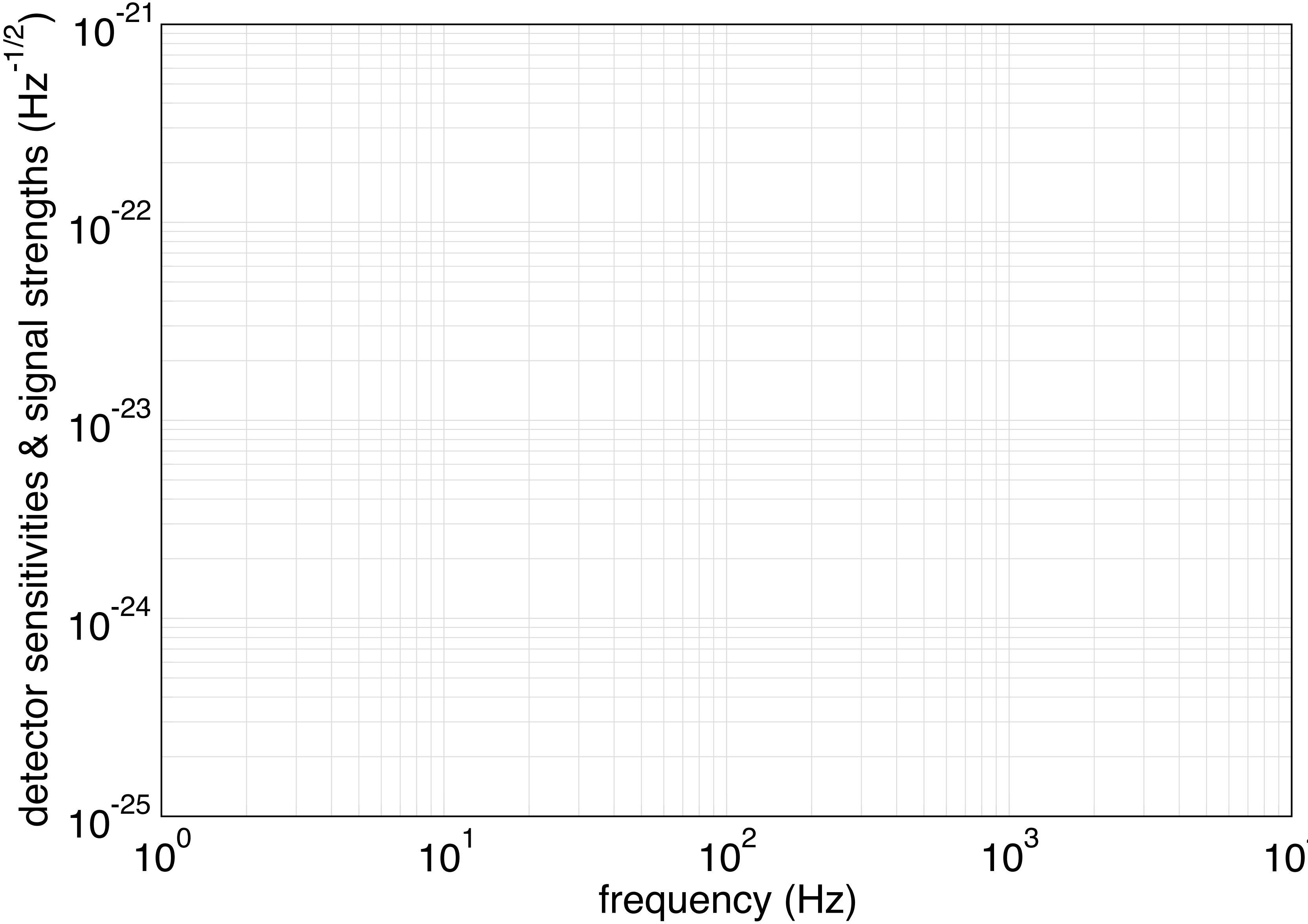
Einstein Telescope: triangular, 10 km arm length, underground, cryogenic detectors

NETWORK SKY SENSITIVITY

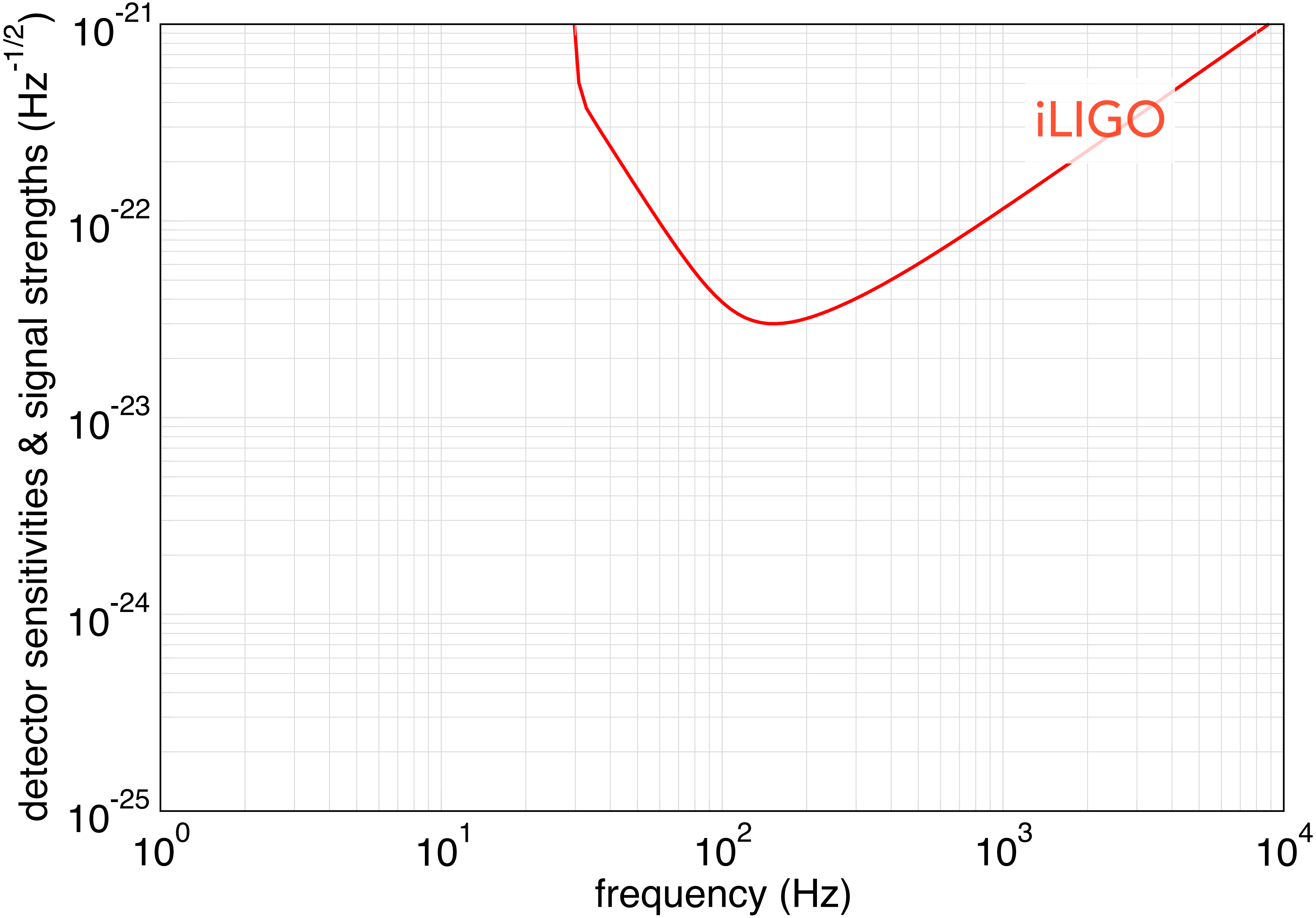
- A *network* of gravitational wave detectors is always on and sensitive to *most* of the sky
- We can integrate and build SNR by coherently tracking signals in phase



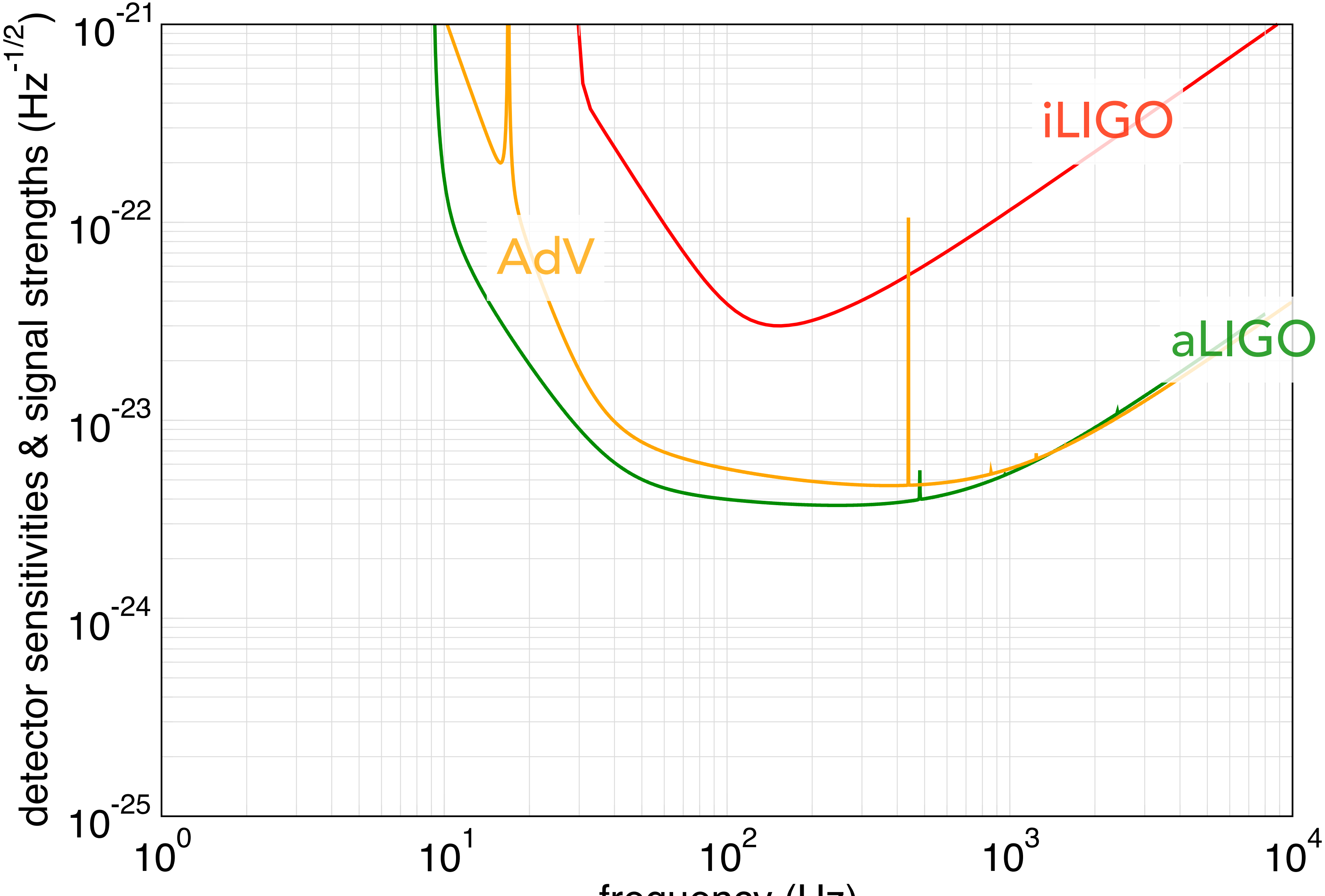
FREQUENCY SENSITIVITY OF GROUND-BASED DETECTORS



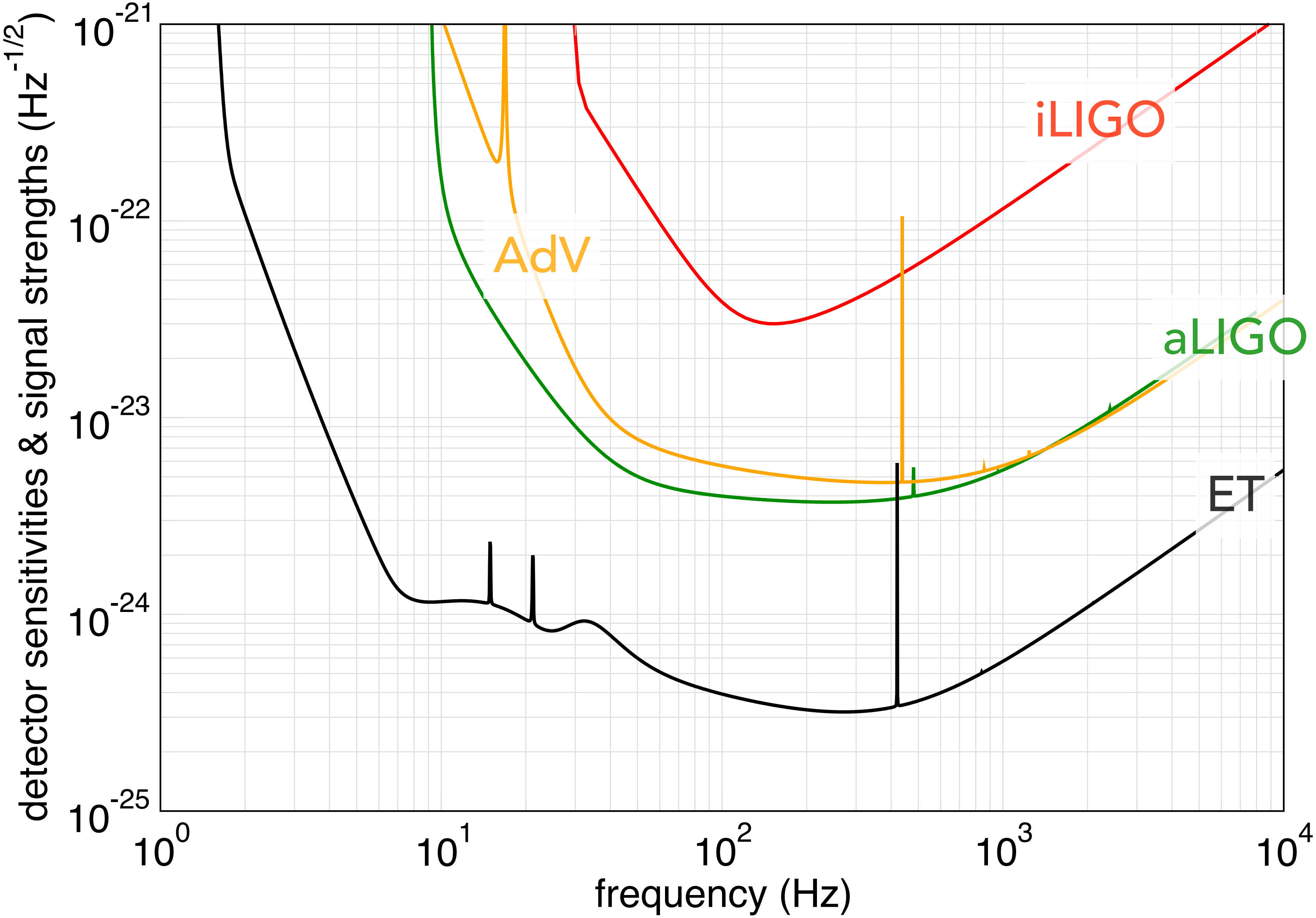
INITIAL DETECTORS



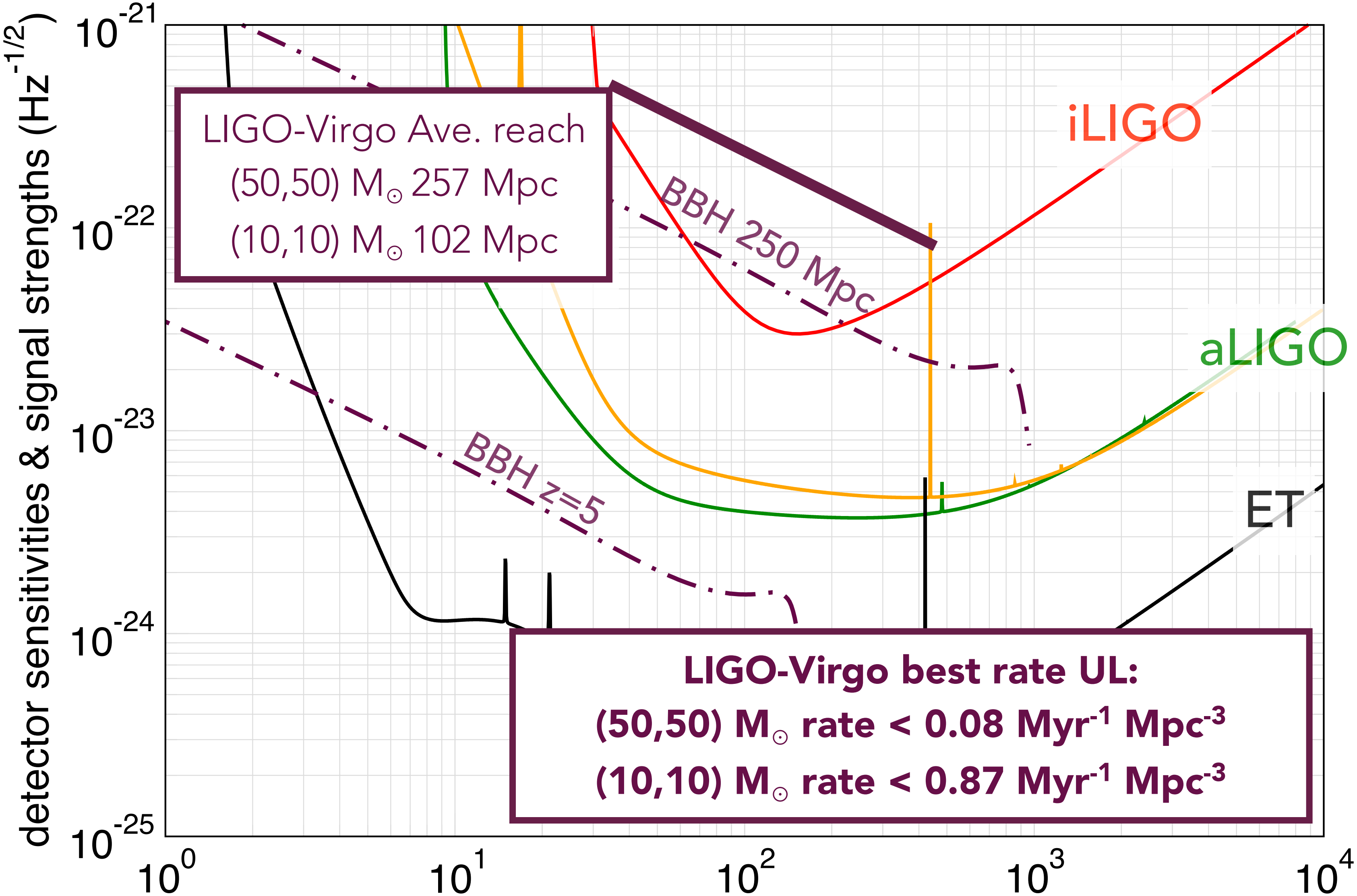
ADVANCED LIGO AND VIRGO



ET SENSITIVITY

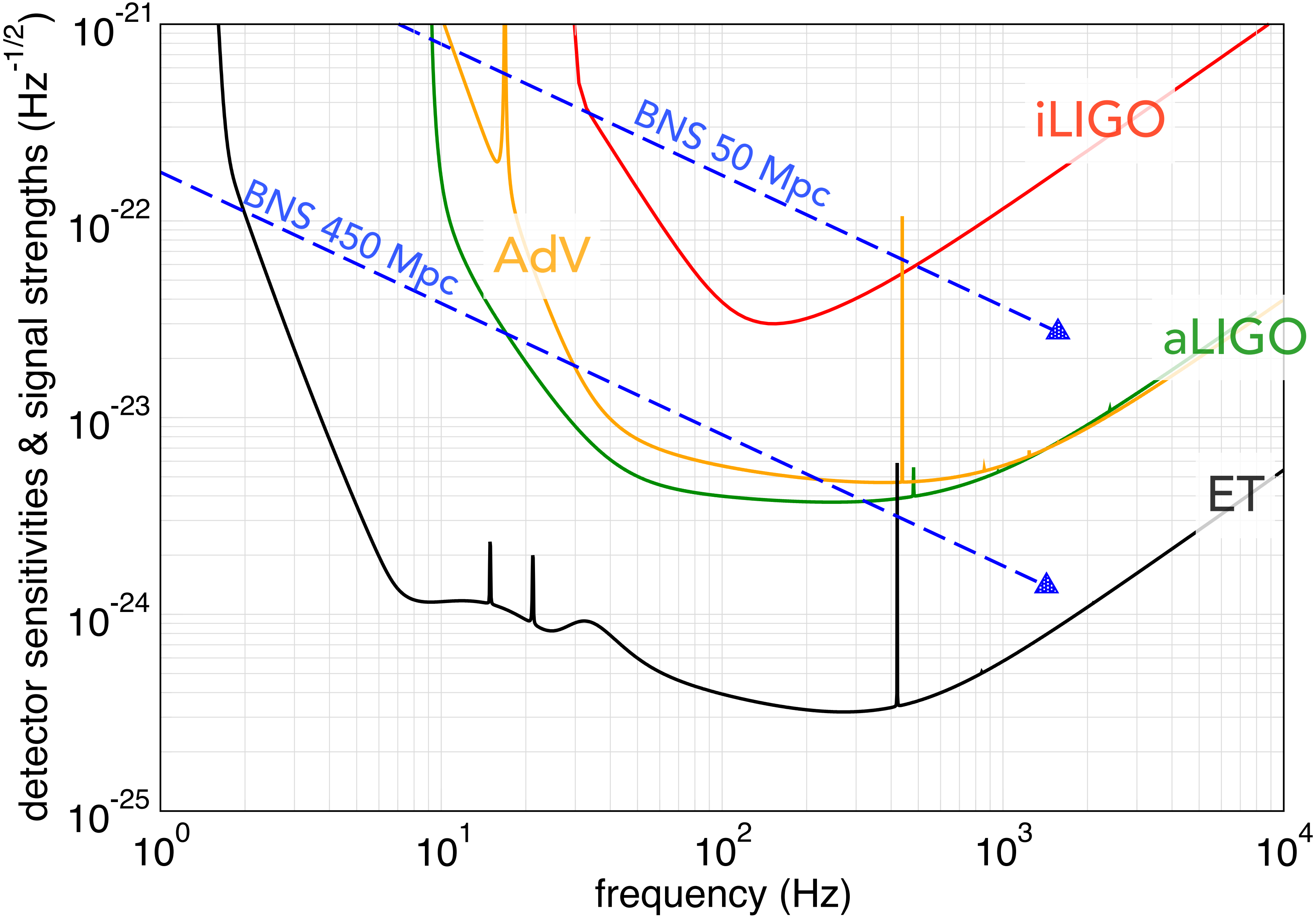


BINARY BLACK HOLES (10+10) $M_\odot$

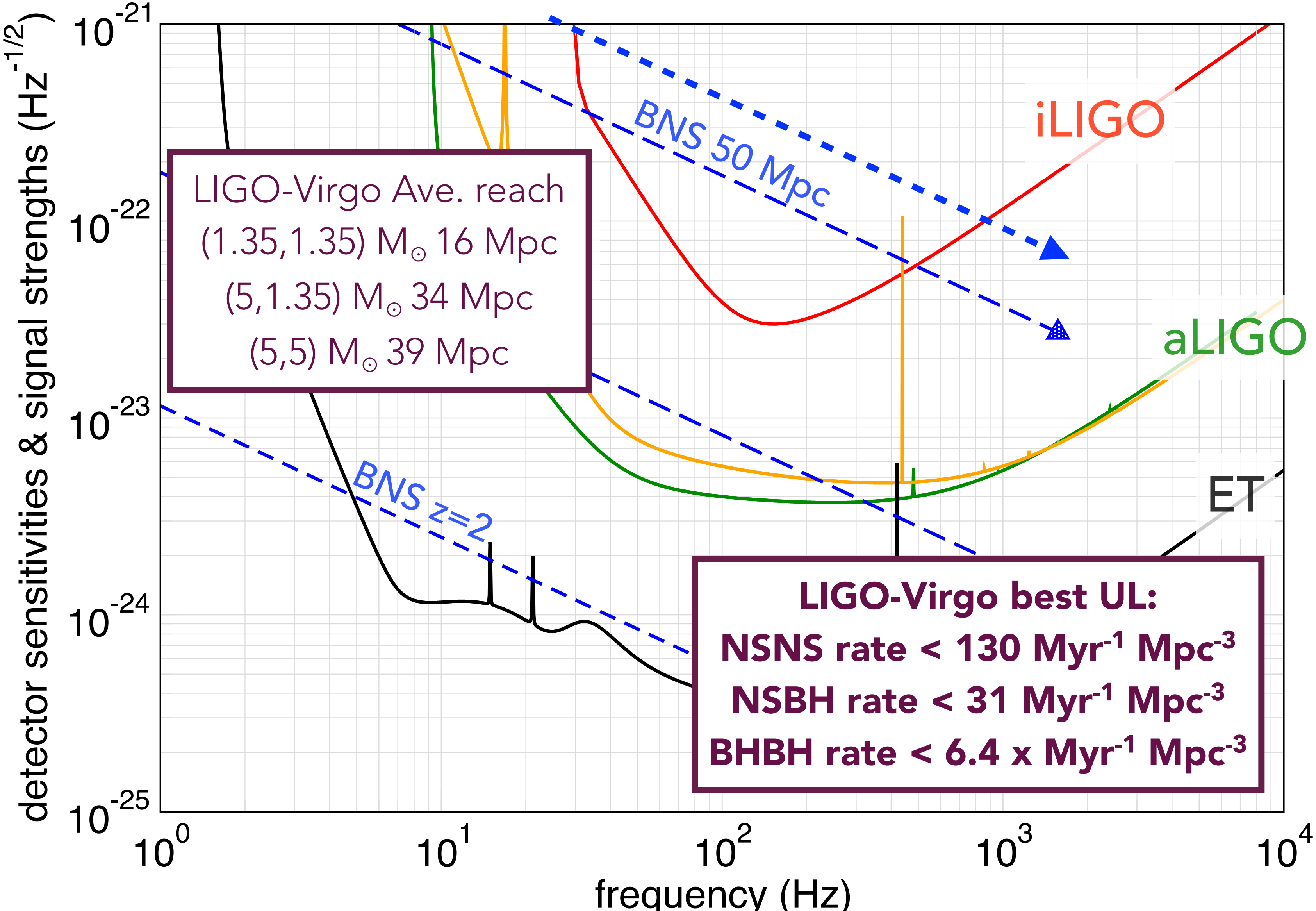


Aasi+ (LVC) 2013

BINARY NEUTRON STARS (1.4+1.4) $M_\odot$

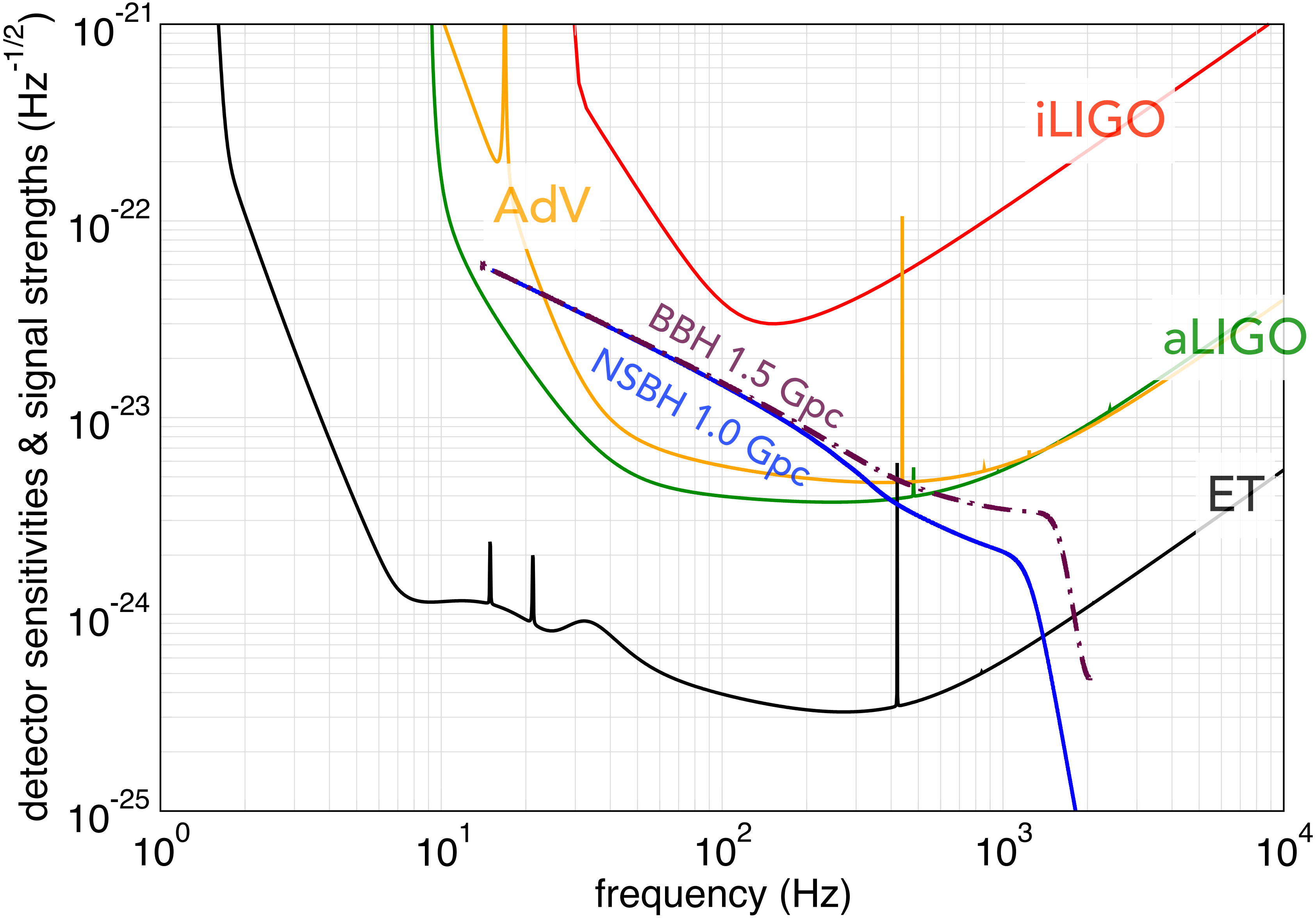


BINARY NEUTRON STARS: EFFECT OF REDSHIFT

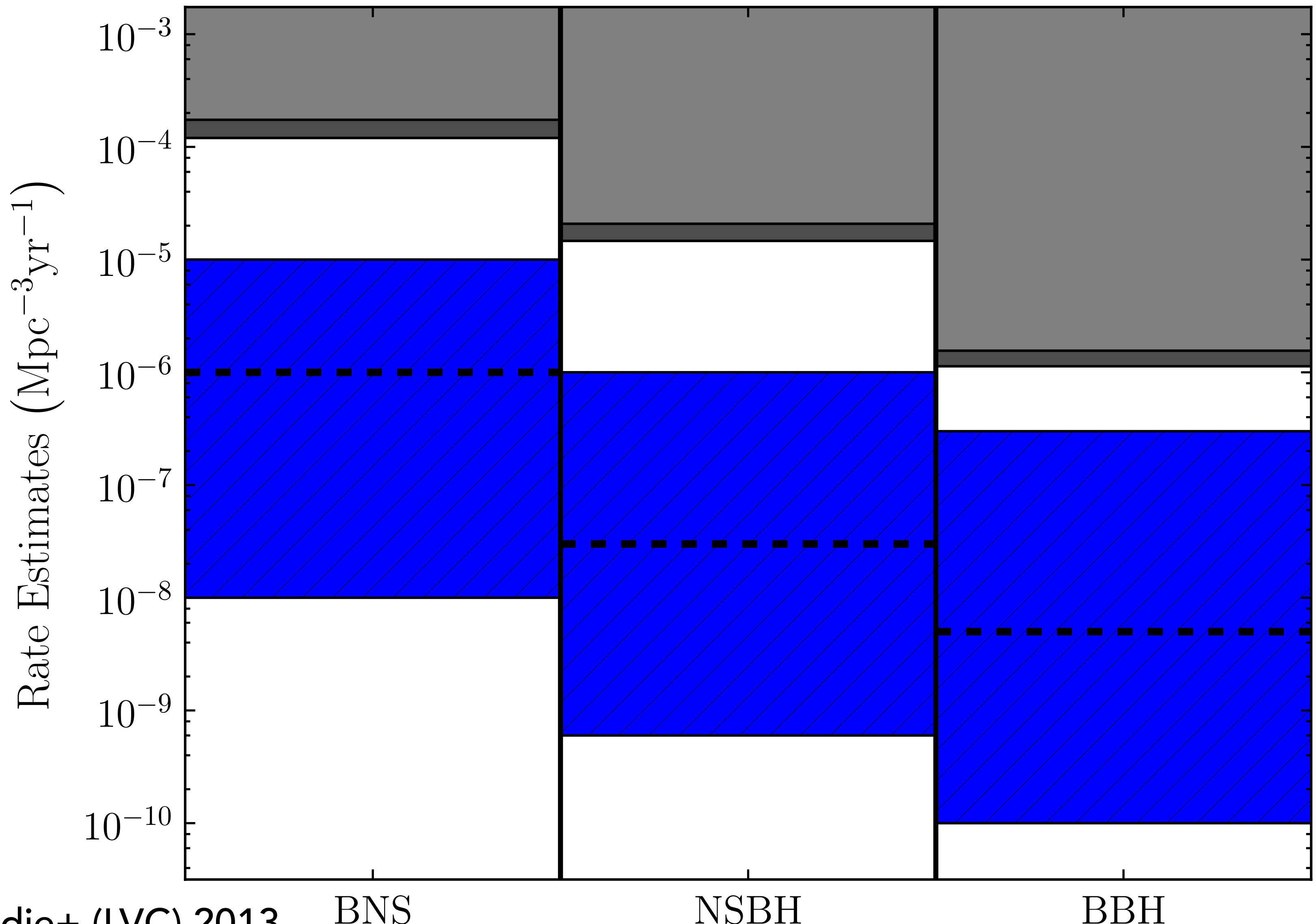


Abadie+ (LVC) 2013

NEUTRON STAR-BLACK HOLE BINARY
$(10+1.4)\ M_\odot$

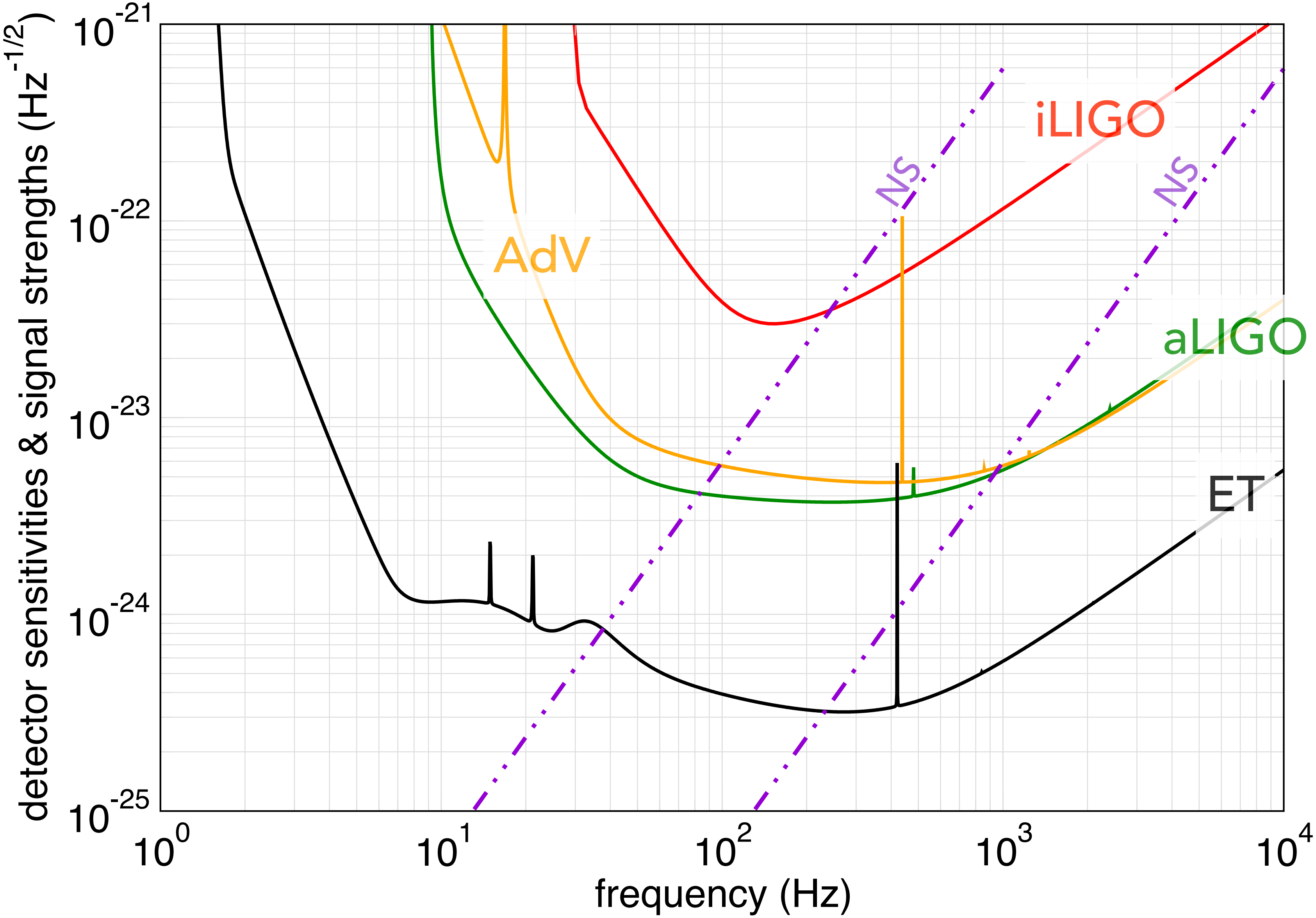


LIGO-VIRGO BEST UPPER LIMITS AND IMPLICATIONS FOR DETECTION

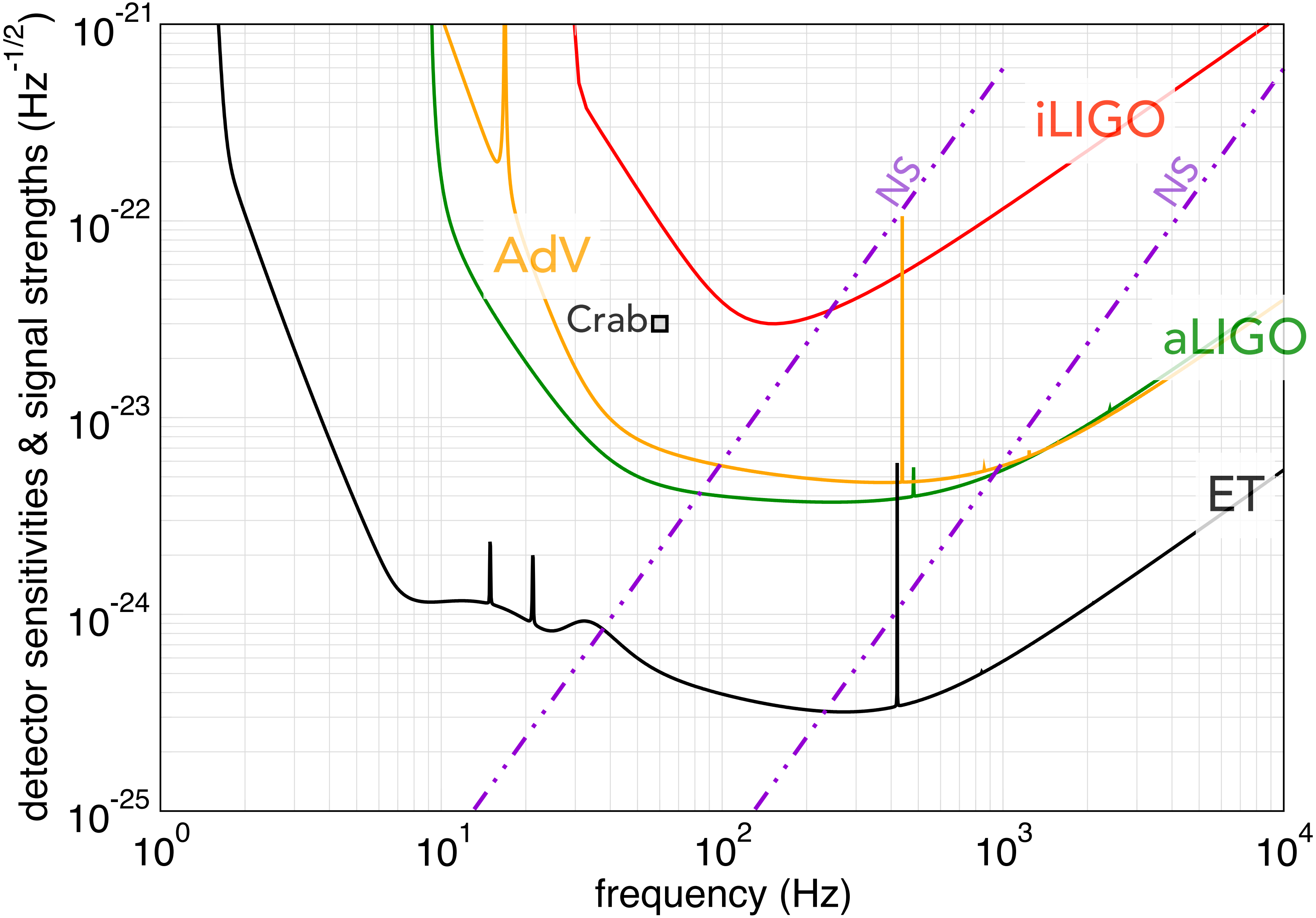


Abadie+ (LVC) 2013

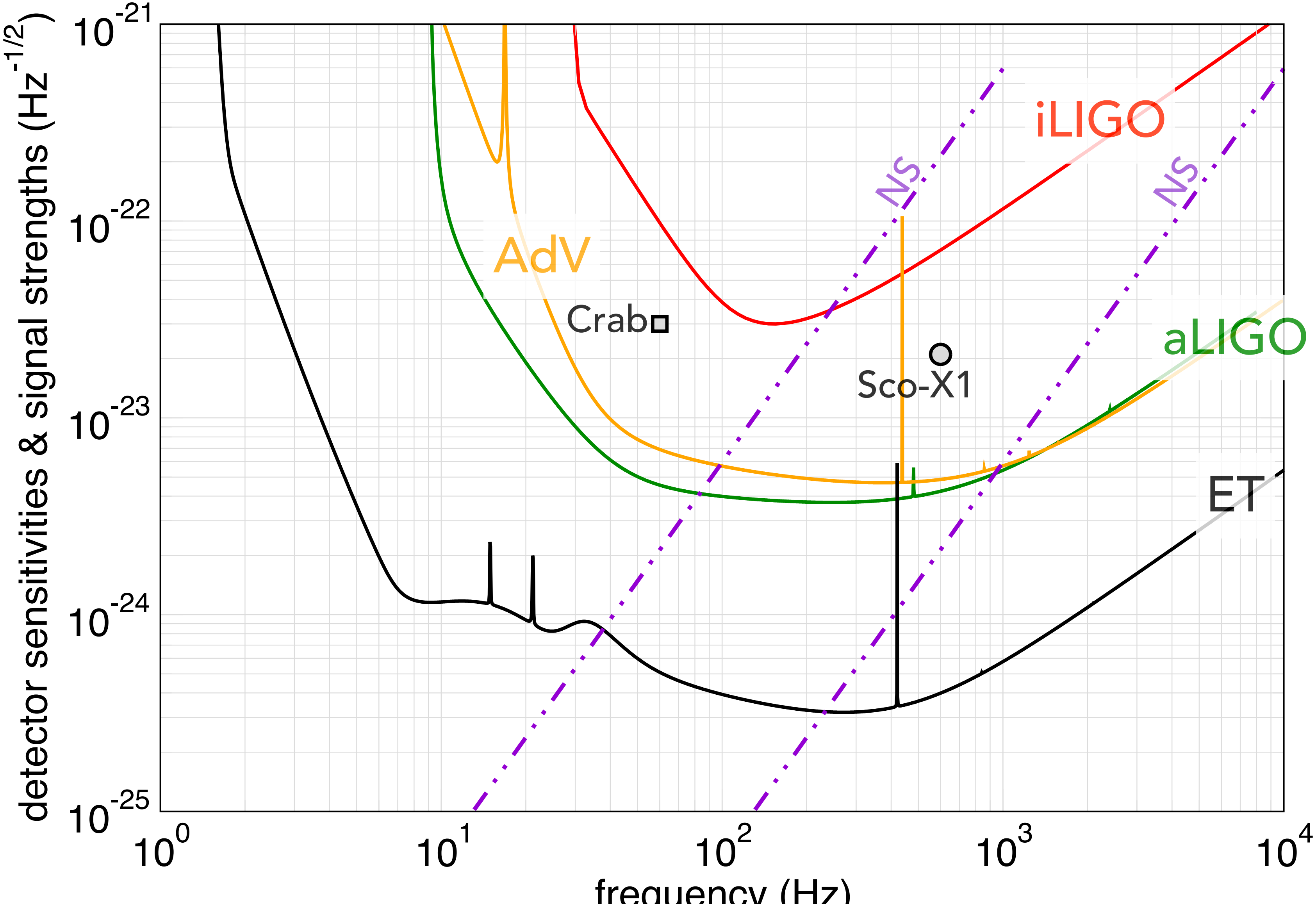
CONTINUOUS WAVES FROM MILLISECOND PULSARS



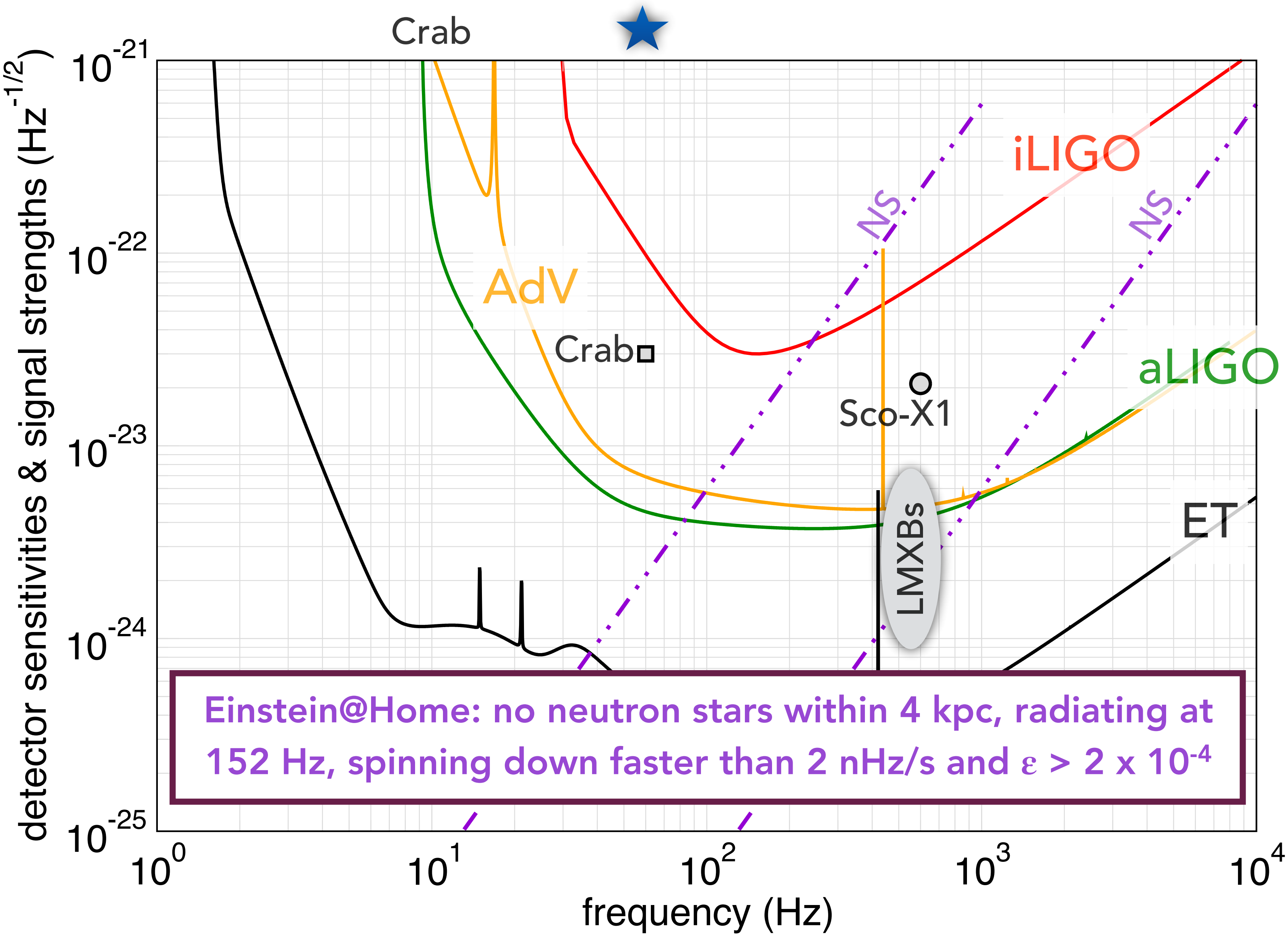
CRAB, VELA AND OTHER ISOLATED PULSARS



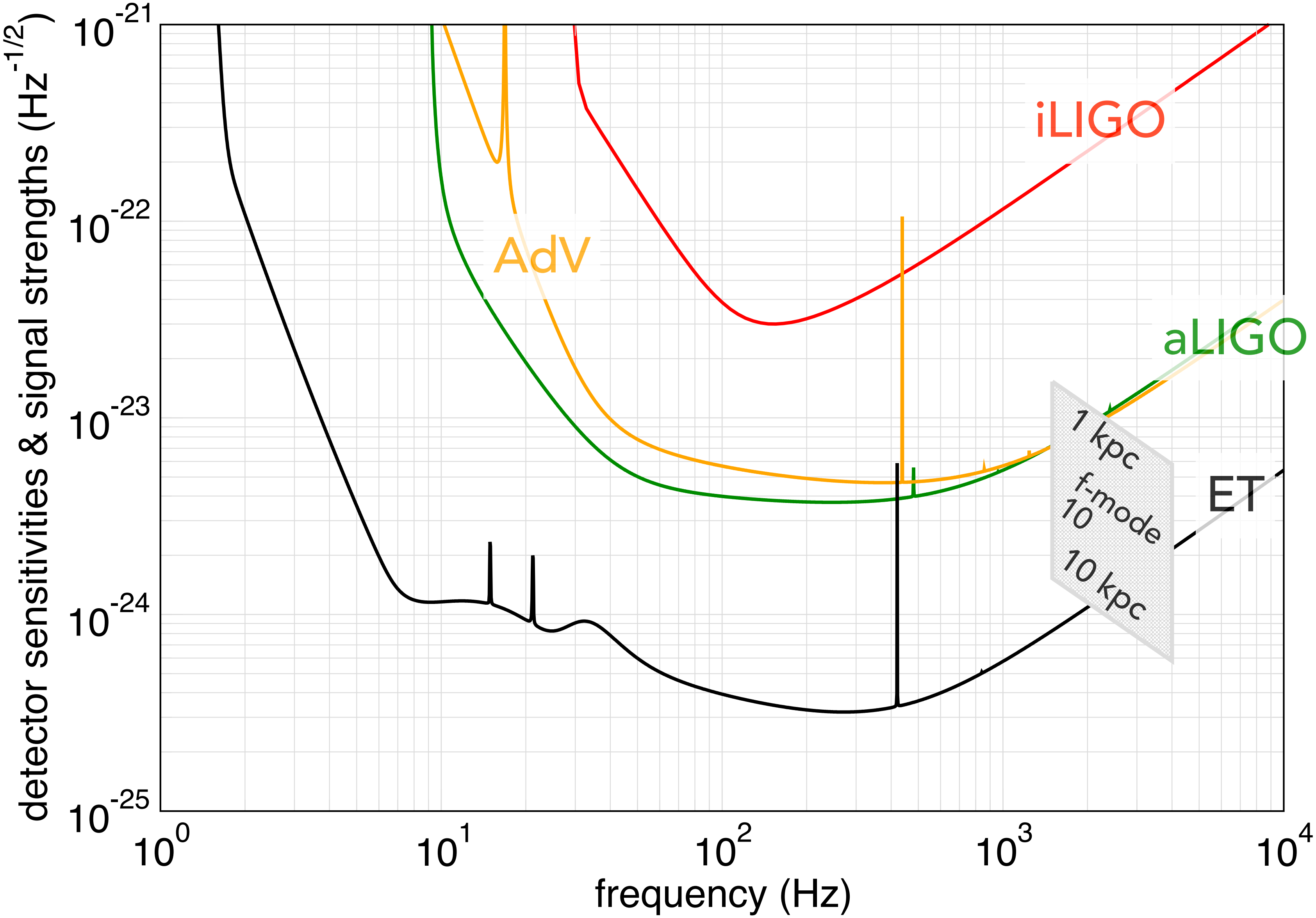
SCO X1


detector sensitivities & signal strengths (Hz^-1/2)
10^-21
10^-22
10^-23
10^-24
10^-25
10^0
10^1
10^2
10^3
10^4
frequency (Hz)
iLIGO
AdV
aLIGO
ET
NS
NS
Crab
Sco-X1

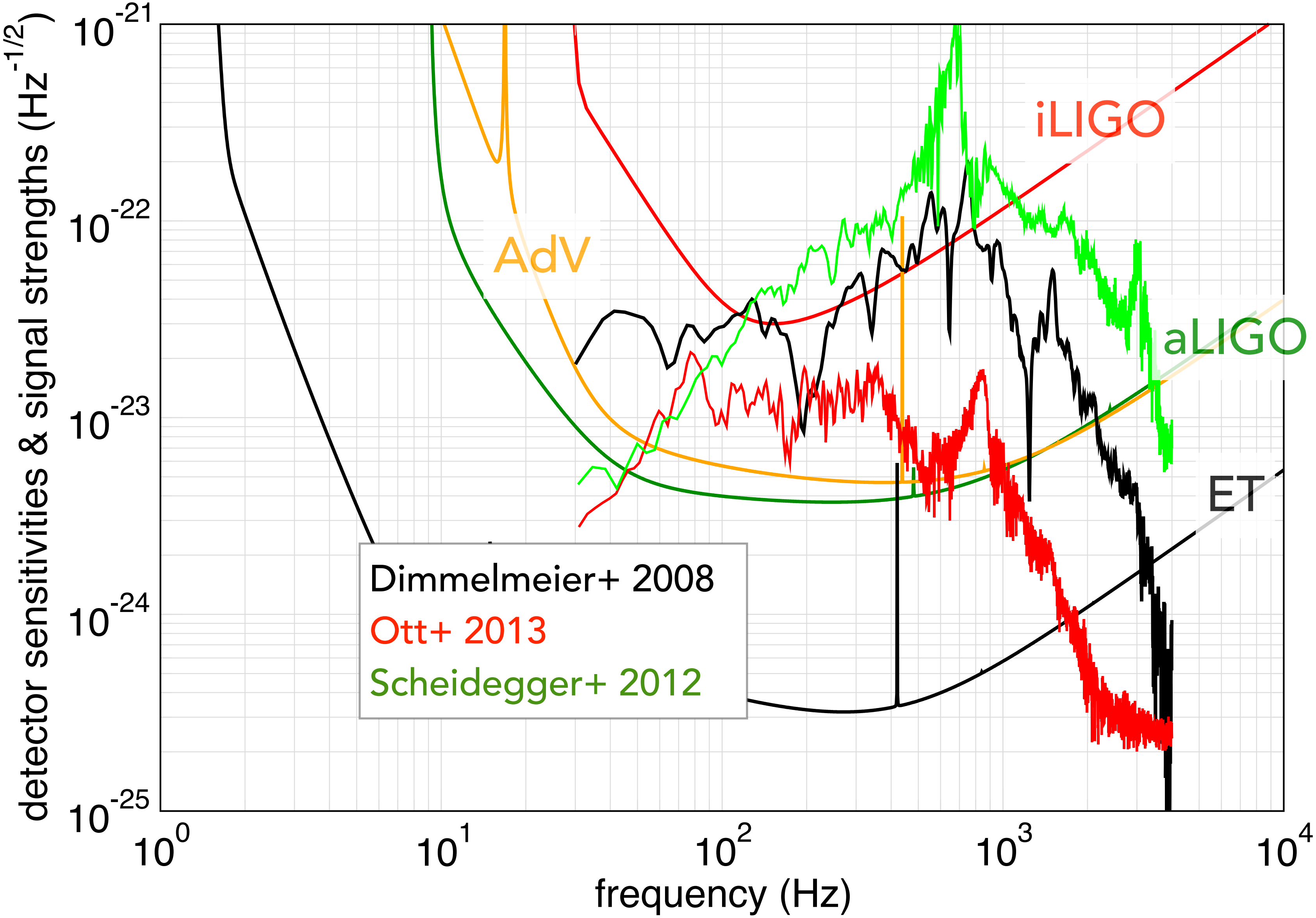
LOW-MASS X-RAY BINARIES



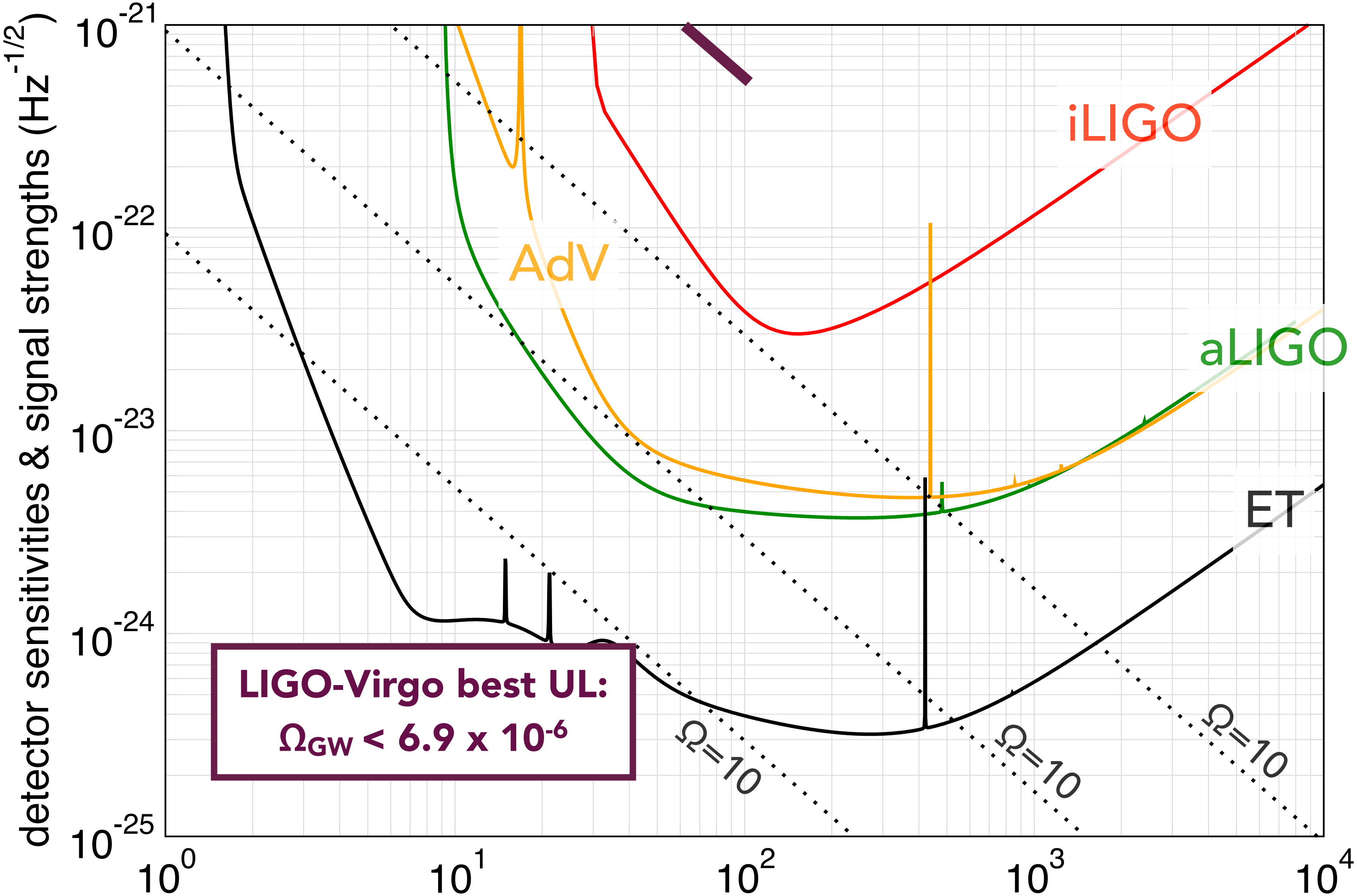
NORMAL MODES OF NEUTRON STARS



GALACTIC SUPERNOVAE SPECTRA

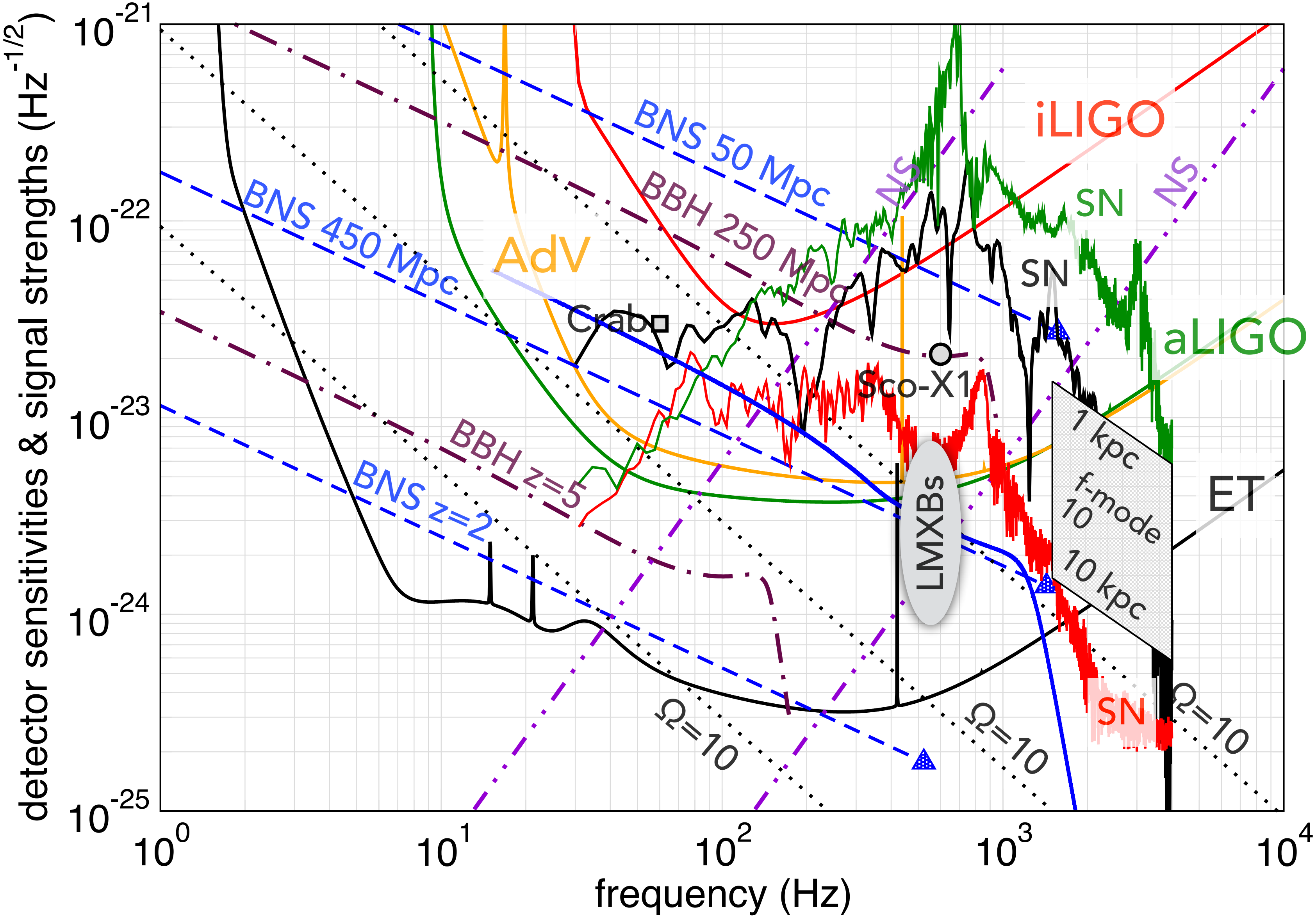


STOCHASTIC BACKGROUND

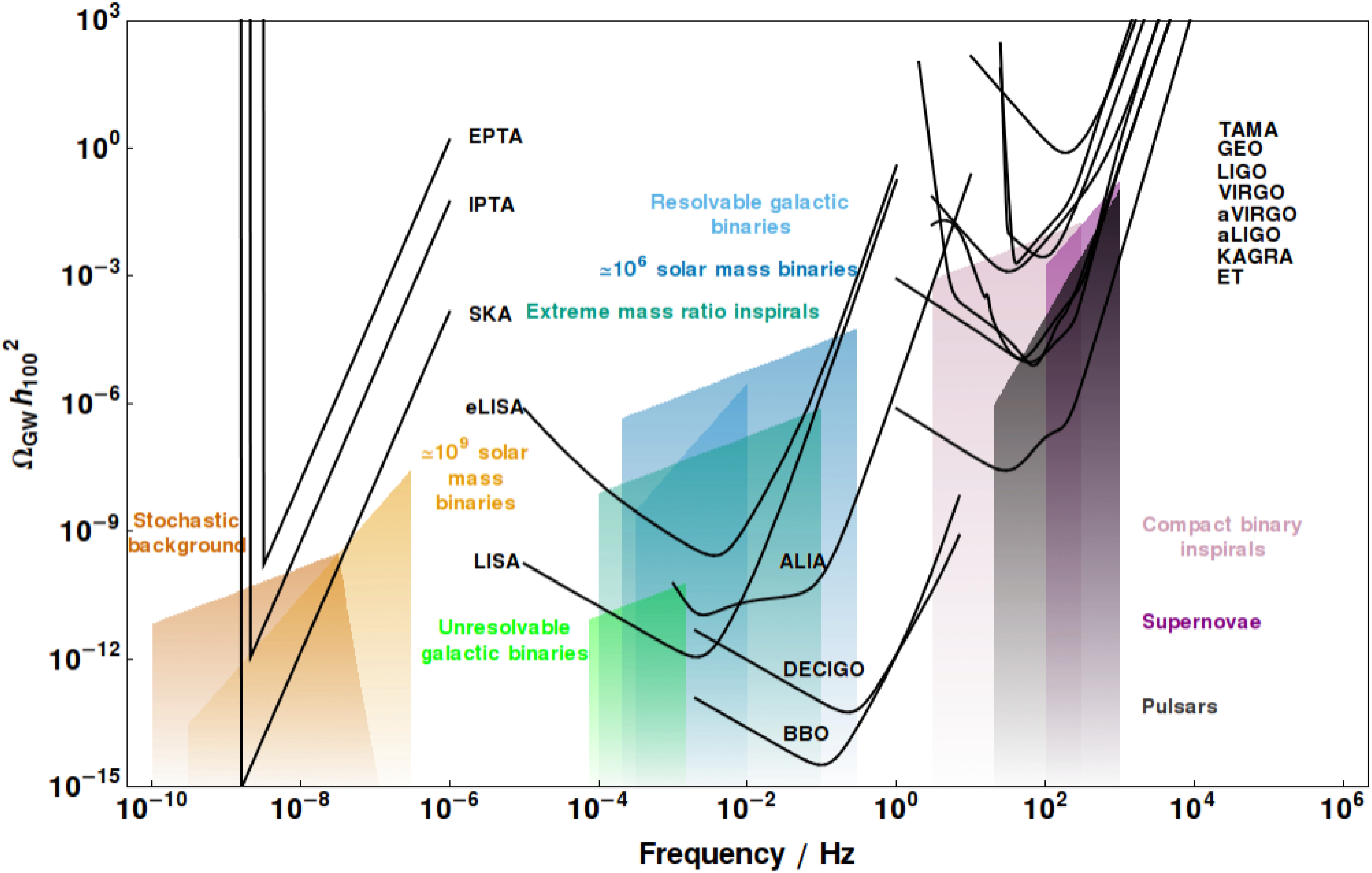


Abbott+ (LVC) 2009

SOURCE SUMMARY: GROUND-BASED DETECTORS



SENSITIVITY IN TERMS OF ENERGY DENSITY IN GRAVITATIONAL WAVES

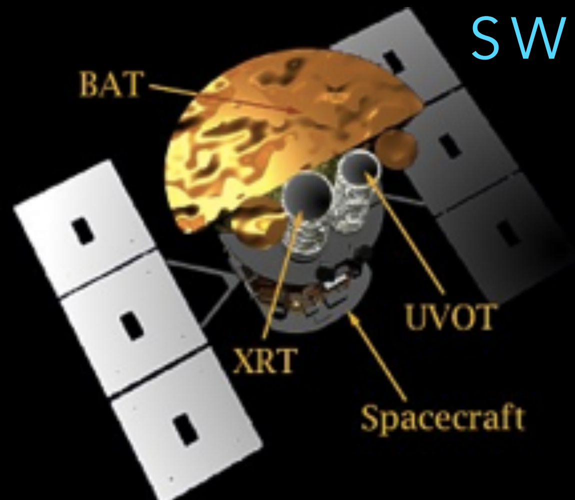


Moore+ 2014

FUNDAMENTAL PHYSICS, ASTROPHYSICS AND COSMOLOGY WITH GRAVITATIONAL WAVES

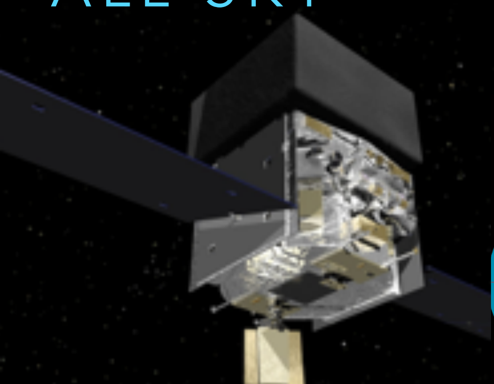
MULTI-MESSENGER ASTROPHYSICS: SYNERGY BETWEEN GW-EM

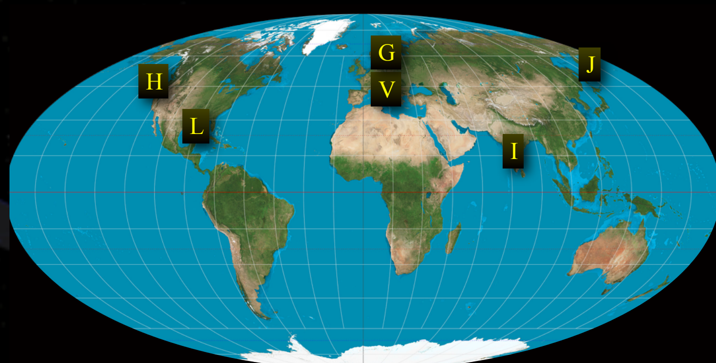
SWIFT - BAT



KECK

FERMI GBM - ALL SKY



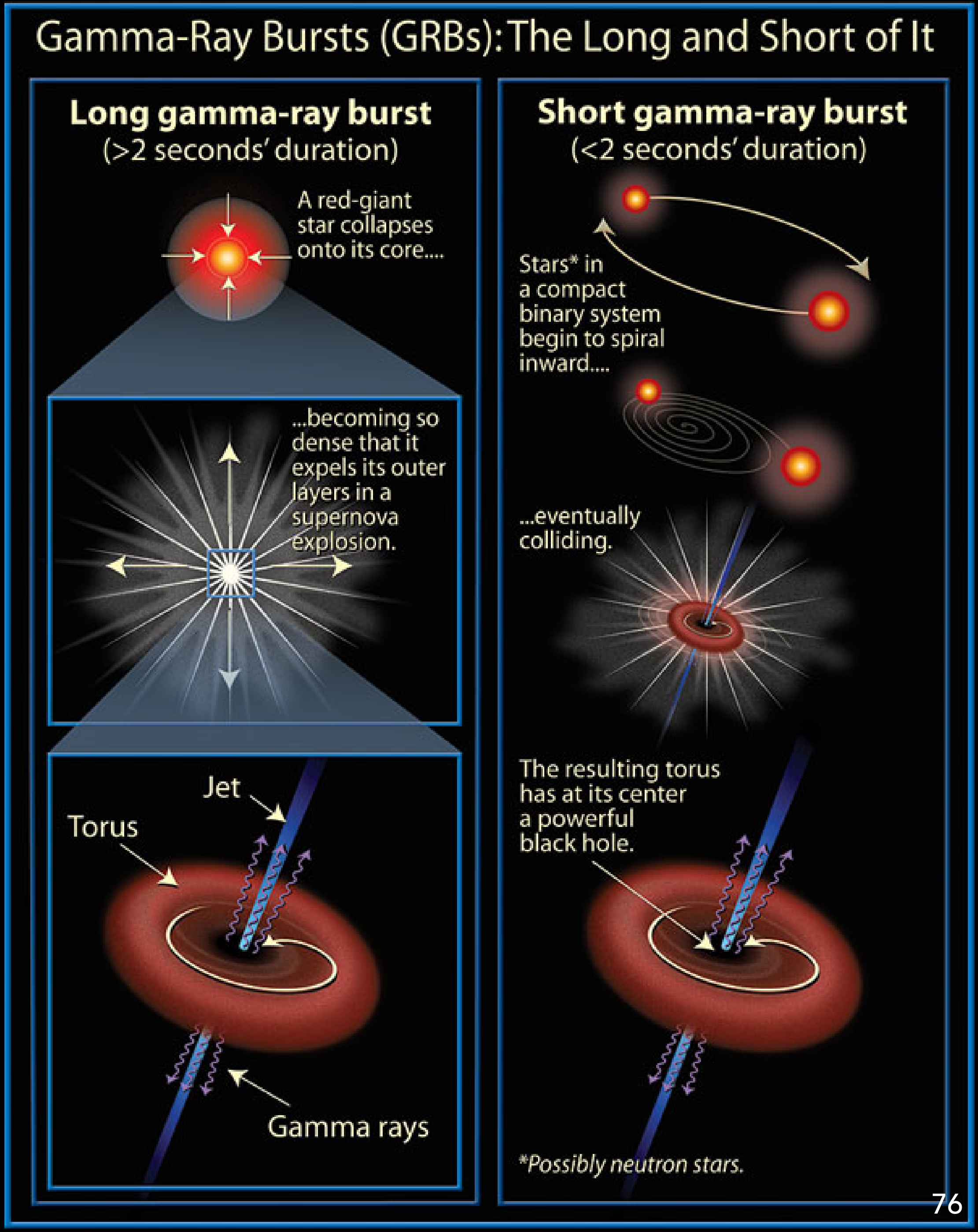


GW NETWORK - ALL SKY

Parkes Radio Telescope

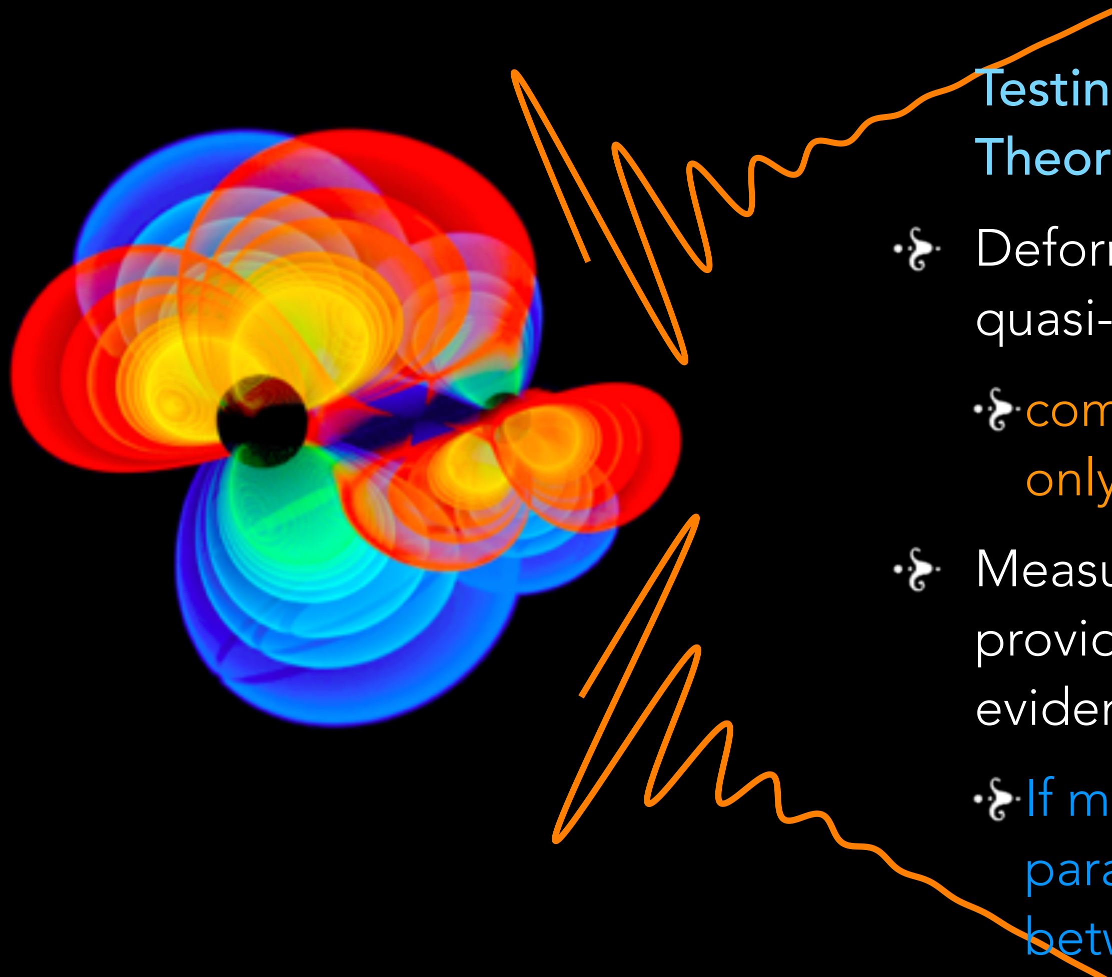
PROGENITORS OF GAMMA-RAY BURSTS

- What causes these giant explosions?
- What are the different classes of GRBs?
- Synergy between EM and GW Astronomy
  - Distances measured with GW
  - Redshift measured with EM
  - Could potentially be very useful for cosmography



BLACK HOLES ARE … MOST PERFECT MACROSCOPIC OBJECTS

S. Chandrasekhar

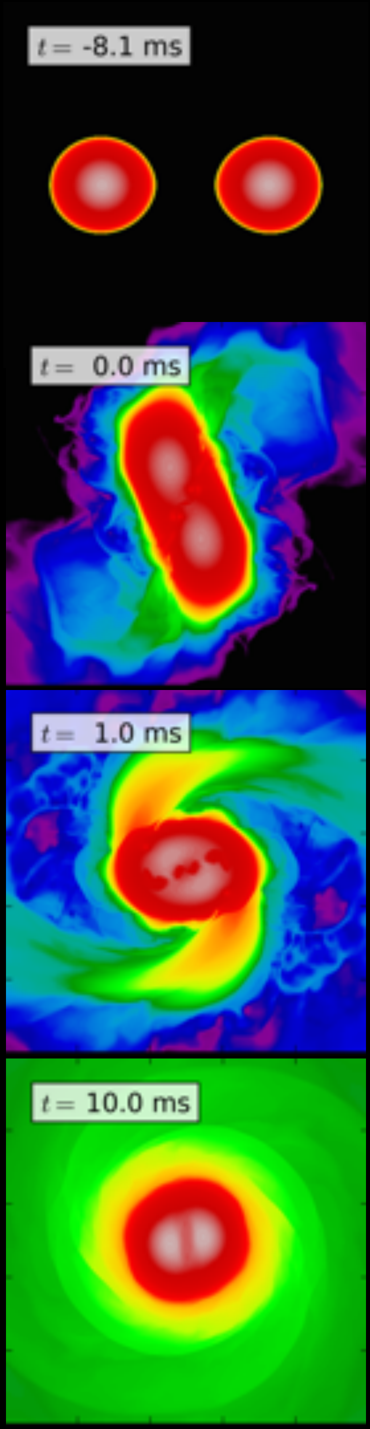


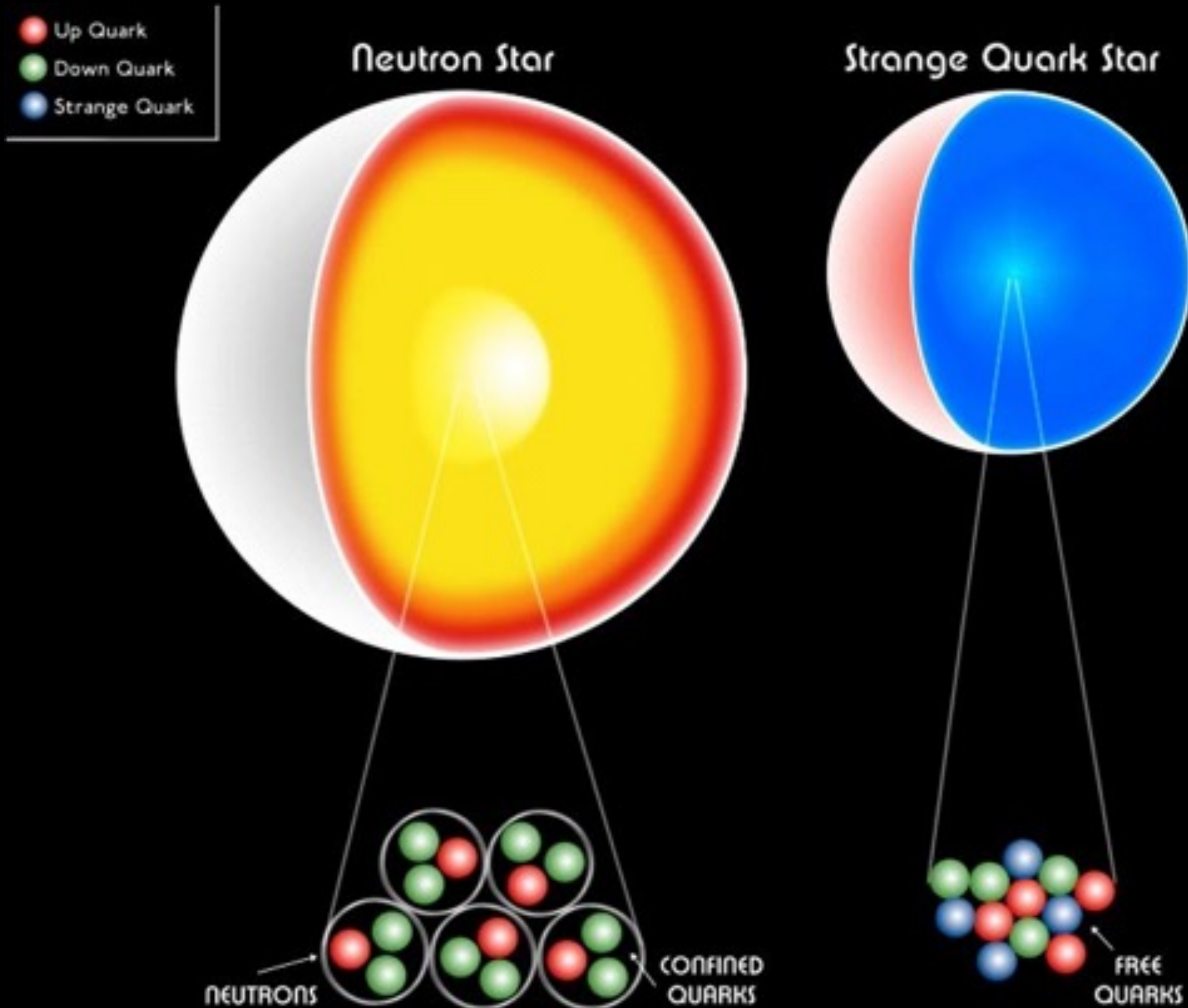
Testing Black Hole No-Hair Theorem

- Deformed black holes emit quasi-normal modes
  - complex frequencies depend only on the mass and spin
- Measuring two or modes would provide a smoking gun evidence of black holes
  - If modes depend on other parameters, consistency between different mode frequencies would fail

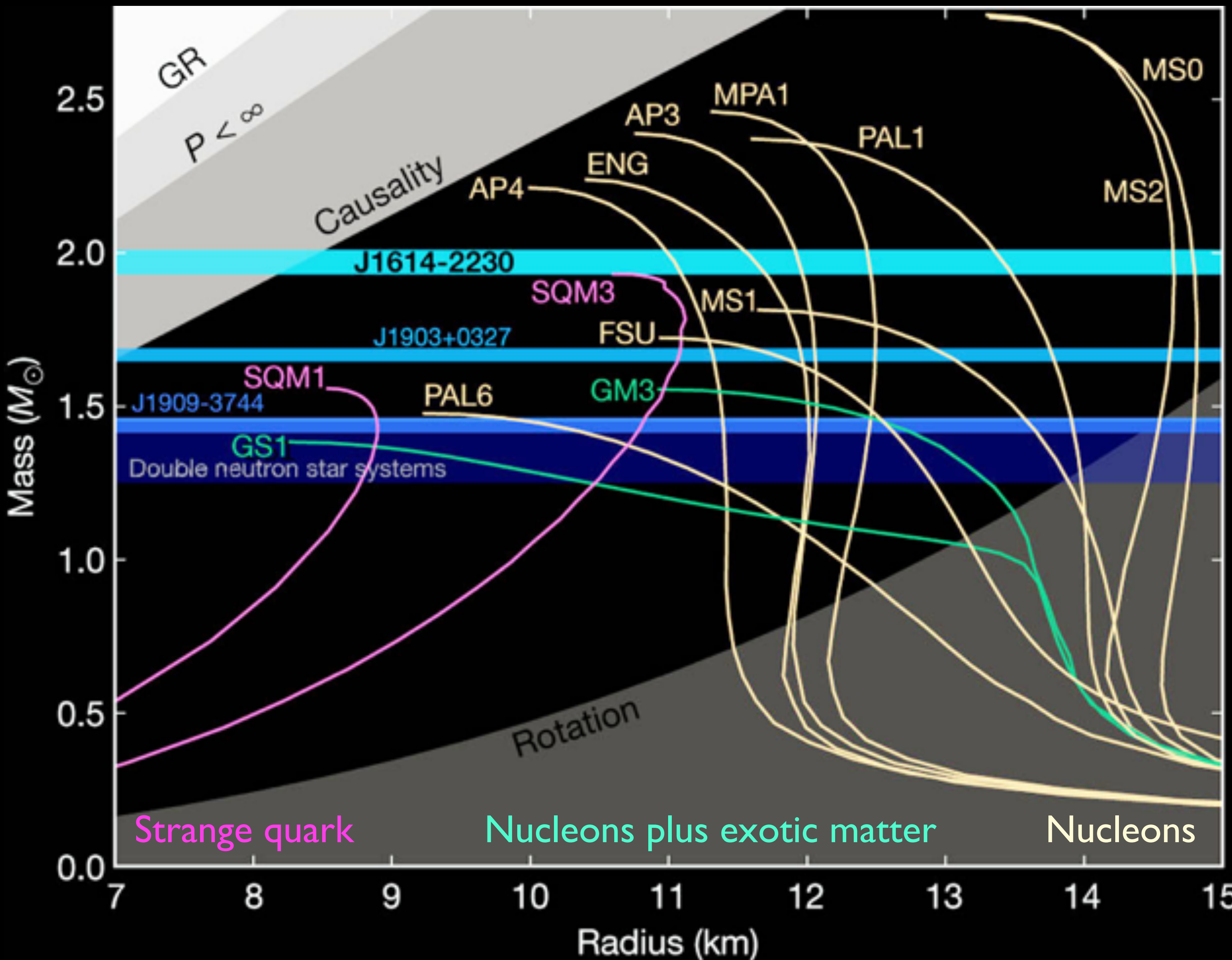
Dreyer+ 2004, Berti+ 2006, Berti+ 2007, Kamaretsos+ 2012, Gossan+2012

Measuring Neutron Star Equation of State





Densities ~ 4 x10^{17} kg/m^3



Demorest+, Nature 2010

Cosmology with Binary Neutron Stars

- Compact binaries are standard sirens; GW observations can measure the luminosity distance
- But can we measure distance and redshift *both* from GW observations alone?
- Tidal interactions between neutron stars have the opposite effect of cosmology; this helps break the mass-redshift degeneracy

Read and Messenger 2012; Messenger+ 2014

SUMMARY

- Detector sensitivity reaching levels where one should expect detections
  - BNS for ground-based detectors and SMBBH background for PTAs
- Many challenges remain
  - Understanding detector noise, timing residuals, etc., is critical
  - Source modelling is mature but the parameter space is huge and in some cases simulations are not able to reproduce what happens in nature
  - Analysis methods have come a long way but improvements in efficient analysis and parameter estimation algorithms is needed
- Future
  - GW Astronomy will kick start in a few years; we need to think "What Next?"
  - Improvements in current facilities and new infrastructure

FUNDAMENTAL PHYSICS

- properties of gravitational waves
  - Testing GR beyond the quadrupole formula
  - How many polarisations are there?
  - Do gravitational waves travel at the speed of light?
- EoS of supra-nuclear matter
  - signature of EoS in GW emitted when neutron stars merge
- black hole no-hair theorem and cosmic censorship
  - are astronomical black hole candidates black holes of general relativity?
- equation-of-state of dark energy
  - compact binaries are standard candles/sirens
- independent constraint/measurement of neutrino mass
  - delay in the arrival times of neutrinos and gravitational waves

ASTROPHYSICS

- formation and evolution of compact binaries and their populations
  - masses, mass ratios, spin distributions, demographics
- unveiling progenitors of short-hard GRBs
  - Understand the demographics and different classes of sh-GRBs
- understanding Supernovae
- finding why neutron stars stall, pulsars glitch and magnetars flare
  - what causes stalling of spin frequencies in LMXBs, sudden excursions in pulsar spin frequencies and what is behind ultra high-energy transients of EM radiation in magnetars?
- ellipticity of neutron stars as small as 1 part in a billion (10μm)
  - Mountains of what size can be supported on neutron stars?
- onset/evolution of relativistic instabilities

COSMOLOGY

- cosmography
  - strengthen existing distance calibrations at high z
  - calibration-free measurements of distance and cosmological parameters, possibly redshift from GW observations alone
- black hole seeds
  - when and where did seed black holes form and how did they grow?
- anisotropic cosmologies
  - in an anisotropic Universe the distribution of H on the sky should show residual quadrupole and higher-order anisotropies
- primordial gravitational waves
  - quantum fluctuations in the early Universe produce a stochastic b/g
- production of GW during early Universe phase transitions
  - phase transitions, pre-heating, re-heating, etc., could produce detectable stochastic GW

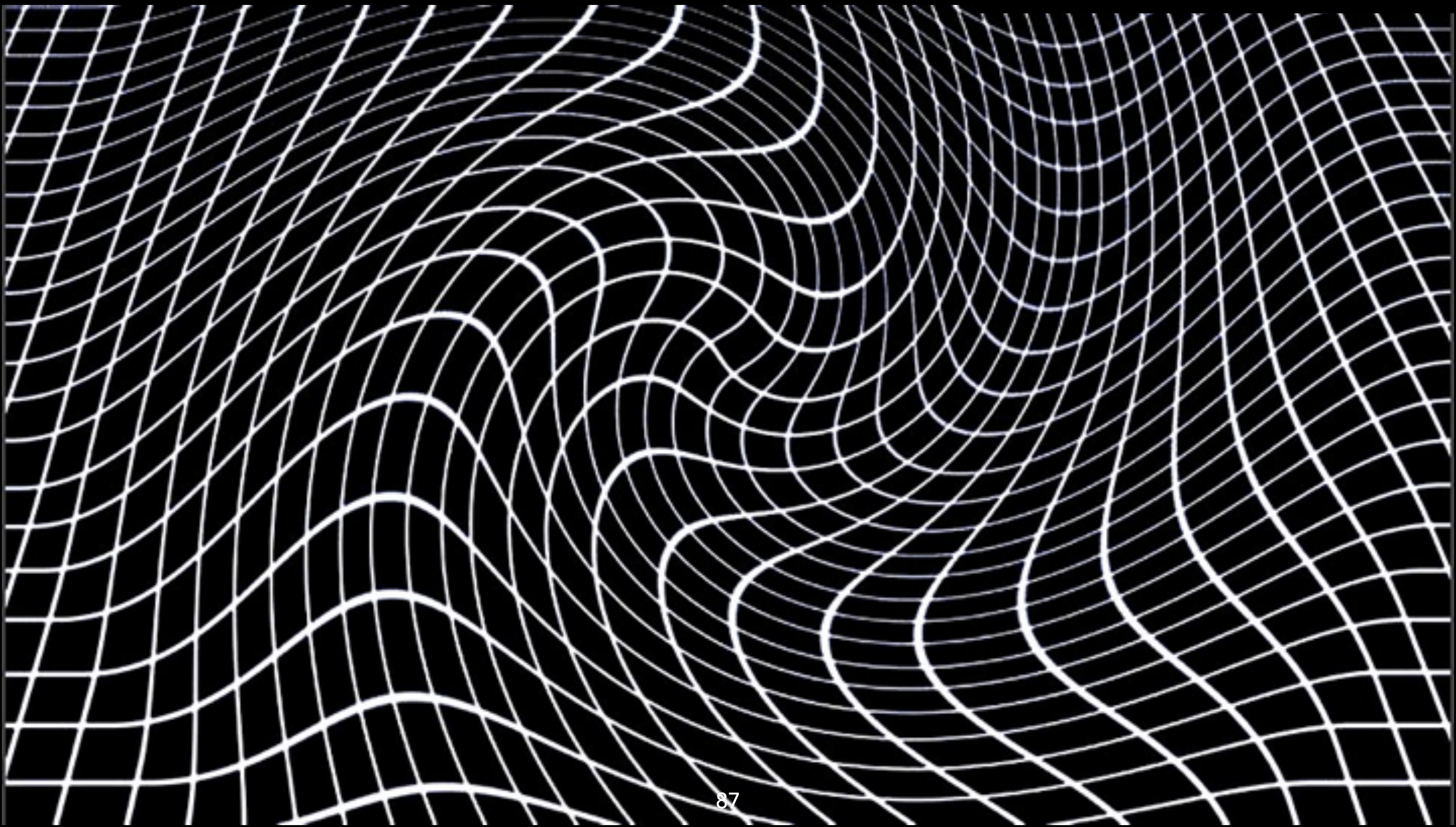
SPARE SLIDES

CHALLENGES

- models and simulations of sources
  - supernova bounce, neutron star cores, corner cases of parameter space in binary systems, GRB afterglows
- rapid parameter estimation of gravitational wave events
  - especially important if we do find high event rate
  - testing GR etc. need to be made computationally efficient
- improved understanding of “detector” noise and false alarm rate

STANDARD LORE: GRAVITATIONAL WAVES STRETCH AND SQUEEZE SPACETIME

Science photo library



HOW RIGID IS SPACETIME?

- In Einstein equations

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

- the coupling constant has dimensions of force

$$G_F = c^4/G \sim 10^{44}\,\mathrm{N}$$

- Under what circumstance can such a force be felt? Consider force on an orbiting body:

$$F = \frac{mv^2}{r} \qquad \frac{m}{r} = \frac{v^2}{G} \qquad F = \frac{v^4}{G} = \frac{c^4}{G}\left(\frac{v}{c}\right)^4$$

- Black holes in a binary can experience G_F

G_F AND GRAVITATIONAL WAVES

- strain produced by a self-gravitating mass

$$h \sim \frac{GM}{c^2 R} \frac{GM}{c^2 D}$$

source's mass

source's characteristic size

distance to source

- so $h \sim \dfrac{GM^2/R^2}{G_F} \dfrac{R}{D}$ or $h \sim \dfrac{v^4/G}{G_F} \dfrac{R}{D}$

- strain can be largest for most compact sources, i.e. black holes and neutron stars

- since GW is defined in the wave zone

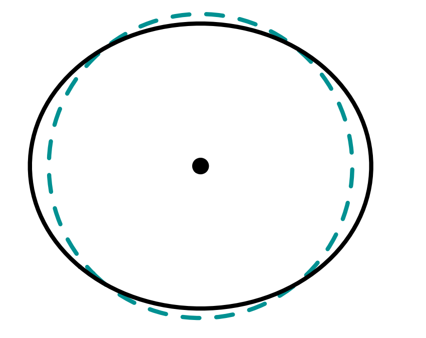
$$D > \lambda_{\mathrm{GW}} \gg R \Rightarrow h < 1$$

NEUTRON STAR EQUATIONS OF STATE

- Size of a neutron star (for nucleonic matter) decreases with increasing mass!
- There are a plethora of equations of state that are consistent with the current observed neutron star masses
- Neutron star radius measurements by X-ray observations have a lot of systematics
  - Current constraints place the radius anywhere between 8 and 20 km - too large a range to determine EOS
- Gravitational wave observations could, in principle, provide a clean measurement of NS EoS

EFFECT OF TIDES ON INSPIRAL DYNAMICS - A FIFTH PN EFFECT

- several authors have suggested using PN Tidal effects to measure EOS of neutron stars
- the most recent studies use a population of BNS events to measure EOS
- quantity of interest is the tidal deformability parameter



$$\lambda = \frac{Q}{\mathcal{E}} = \frac{\text{size of quadrupole deformation}}{\text{strength of external tidal field}}$$

Love number k_2

Radius R

$$\lambda = \frac{2}{3} k_2 R^5 \quad (G = c = 1)$$

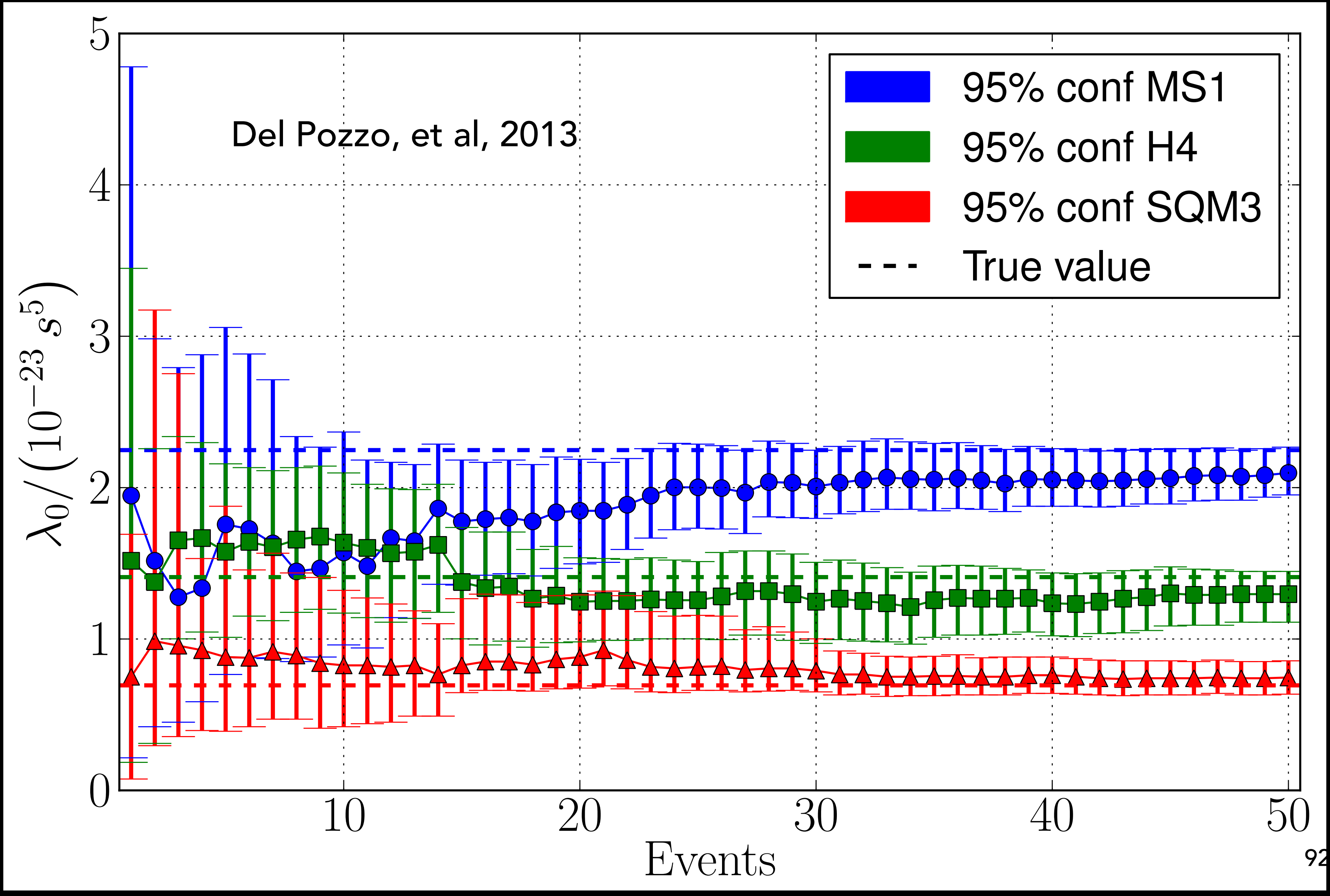
Tidal deformability

$$\Lambda \equiv G\lambda (G m_{\rm NS}/c^2)^{-5}$$

$$\Lambda \in [300, 600]$$

Hinderer 2008, Flanagan and Hinderer 2008, Hinderer+ 2010, Read+ 2009, 2013, Pannarale+ 2011, Damour+ 2012, Lackey+ 2012, Del Pozzo+ 2012, Lackey & Wade 2014; image and equation curtsey J. Read

HOW WELL CAN WE MEASURE EOS IN ADVANCED VIRGOAND LIGO?



BLACK HOLE QUASI NORMAL MODES

- The deformation is radiated away as gravitational waves with a characteristic spectrum called *quasi-normal modes* which are damped sinusoids
- Far away from the source the waveform emitted by a perturbed black hole has the form:

$$h(t) = A\,\frac{M}{r}\,\exp(-t/\tau)\,\cos(\omega t + \varphi_0)$$

† Amplitude A depends on the nature of perturbation

† r is the distance to the black hole

† ω and τ are the mode frequency and damping time

TYPICAL VALUES OF THE DOMINANT MODE

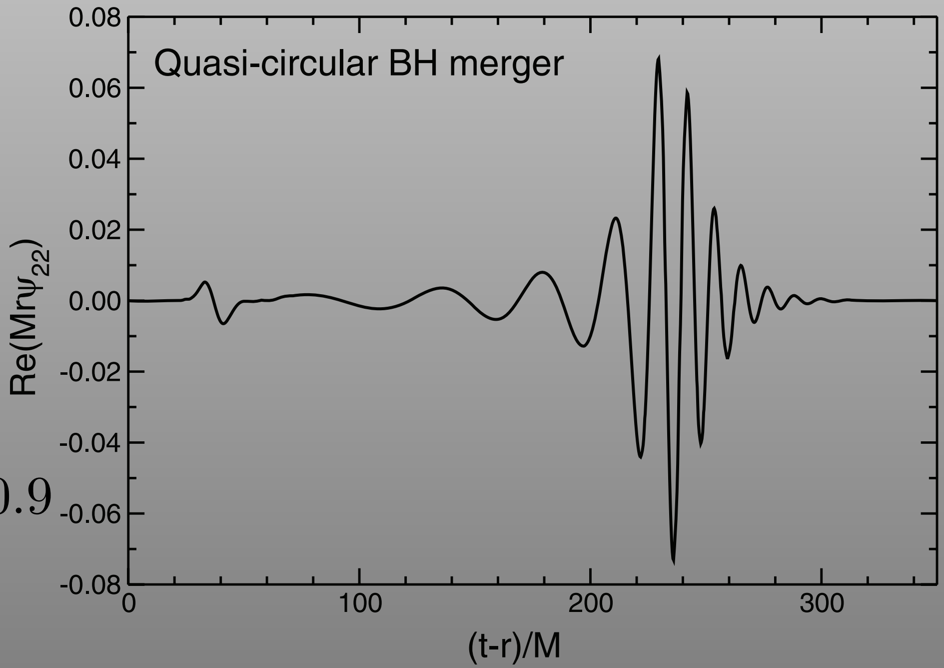
- Gravitational waves being quadrupolar the most dominant mode excited is *l* = 2
- The frequency and the decay time of the 22 mode (i.e. *l*=2, *m*=2) are:

$$f = 1.2\,\mathrm{kHz}\left(\frac{10M_\odot}{M}\right)$$

$$\tau = 0.55\,\mathrm{ms}\left(\frac{M}{10M_\odot}\right)$$

$$Q = \frac{1}{2}\tau\omega \sim 2$$

$$f = 2.0\,\mathrm{kHz} \;\text{ and }\; Q = 5 \text{ for } j = 0.9$$

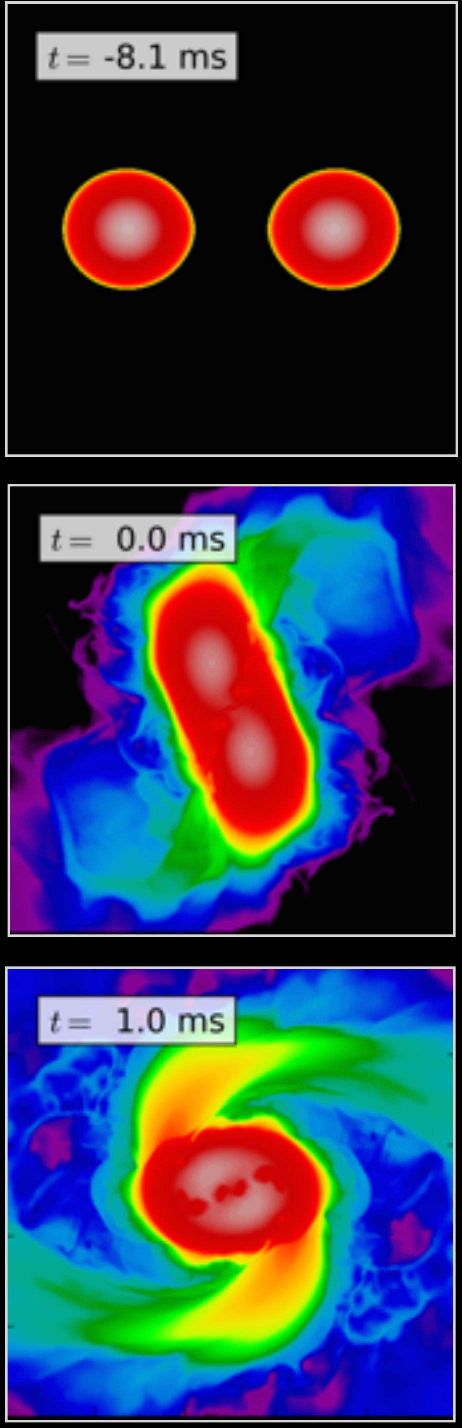


MASS-REDSHIFT DEGENERACY

- Gravitational waveforms from binary black holes of different total mass are identical in shape - there is no mass-scale in General Relativity
- So a binary of total mass M at z=0 looks identical to a binary of total mass M / (1+z) at redshift z
- This is the mass-redshift degeneracy
  - Makes it impossible to measure the redshift to BBH sources by GW observations alone
- This was also thought to be the case for BNS - at least recently

NEUTRON STAR BINARY SPECTROSCOPY: BASIC IDEA

- Inspiral signal is followed by a merger waveform: merger signal depends on the neutron star equation of state
- For most equations of state, heavier neutron stars are smaller and so larger post-merger oscillations
- But here is the tension:
  - cosmological expansion causes the frequency to redshift
  - so the **observed mass** of the binary is larger
  - but larger masses should have greater frequencies
- This tension between cosmology and microphysics helps resolve the mass-redshift degeneracy



BINARY NEUTRON STAR "SPECTROSCOPY"

- Post-Newtonian phasing formula has M and f together

$$\Psi(f) = 2\pi f t_C - \phi_C + \sum_{k=0}^{7} \alpha_k (\pi M f)^{(k-5)/3}$$

- So it is possible to scale away cosmological frequency redshift: $f \to f/(1+z)$ and $M \to M(1+z)$

- The tidal term, on the other hand, cannot be scaled away

$$\Psi_{\rm Tide}(f) = -\frac{1250\, k_2\, \alpha_0}{3} (\pi M f)^{5/3} \left(\frac{R}{M}\right)^5$$

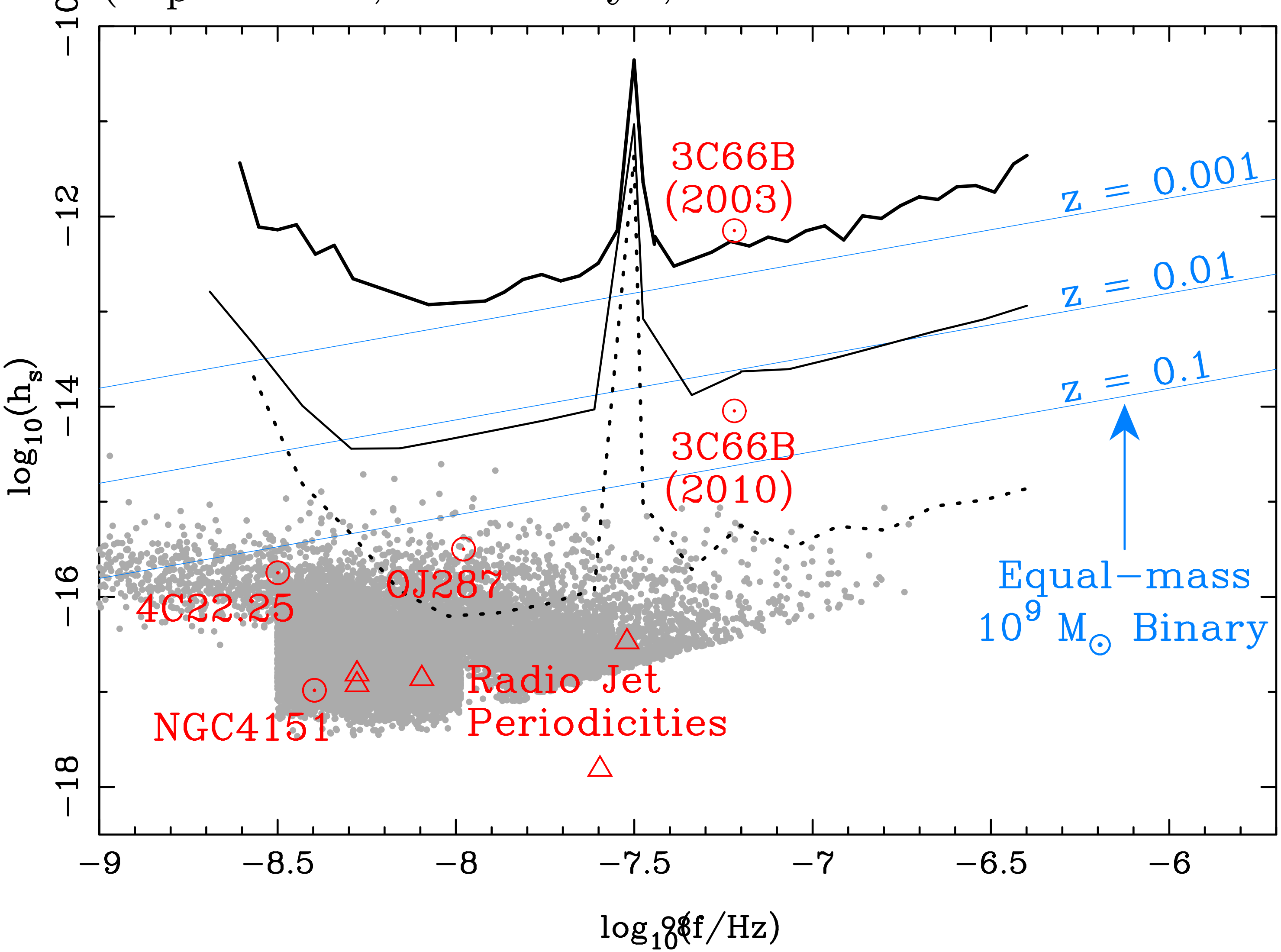
- This helps measure redshift directly from GW observations

Messenger+Read PRL 2012; Messenger+ PRX 2014

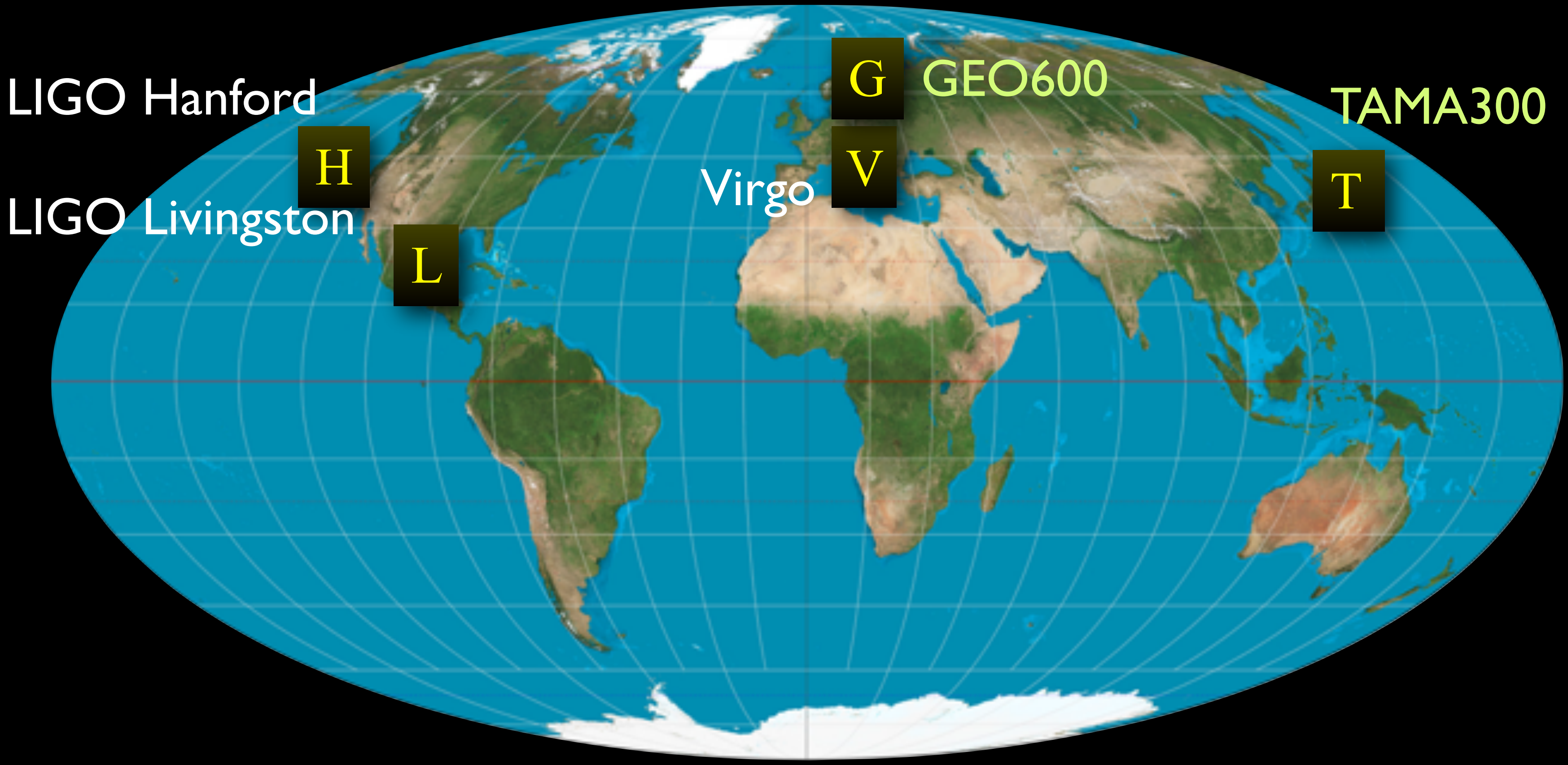
Sensitivities of PTAs to SMBBH

IPTA ($N_p = 40$, $T = 15\,\mathrm{yr}$)

SKA ($N_p = 100$, $T = 10\,\mathrm{yr}$, RMS residuals $= 20\,\mathrm{ns}$).



INITIAL INTERFEROMETER NETWORK



- Between 2006-2010 larger detectors took 2 years worth of data at unprecedented sensitivity levels
- No detections so far but beginning to impact astrophysics