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Honors Thesis

Extracting Gravitational Waves from Noisy Data Using a Maximum Entropy Method Approach

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Abstract

Gravitational waves are virtually undetectable ripples in the fabric of space and time. The LIGO Scientific Collaboration aims to achieve the first direct detection of these waves with the LIGO detectors. LIGO has been updated to Advanced LIGO, which is about ten times more sensitive than before. To prepare for this next generation in gravitational wave detection, we test methods for extracting gravitational wave signals and their parameters from noisy data. We test the MaxEnt method's extraction ability by injecting fake signals into artificially constructed noise and extracting a signal. We then estimate the extracted signal's parameters and evaluate MaxEnt's effectiveness by comparing estimated parameters with the originals.

Introduction

Einstein's general theory of relativity predicts the existence of gravitational waves, ripples in the fabric of spacetime caused by the sudden movement of massive objects. Hulse and Taylor found the first indirect evidence of gravitational waves from a binary pulsar system, winning the 1993 Nobel Prize in Physics [1]. Gravitational waves are thought to move at the speed of light and in rapid succession alternately stretch and compress the dimensions of the space and objects through which they pass. The Laser Interferometer Gravitational Wave Observatory (LIGO) detectors seek to detect gravitational waves by measuring the changes in length these waves produce [2].

There are two LIGO detectors located at sites in Livingston, Louisiana and Hanford Washington. The detectors are Michelson-Morley interferometers, each with two perpendicular arms, each four kilometers long. A beam splitter splits a laser down both vacuum tube arms, where mirrors at the ends reflect the beams back. The reflected beams recombine and produce an interference pattern, registered by a photodetector. Even slight changes in the lengths of the arms produce noticeable changes in the interference pattern. Therefore a gravitational wave traveling through the detector should change the lengths of the arms and produce changes in the interference pattern. The LIGO Scientific Collaboration (LSC) runs these detectors and uses the data they provide in the search for and gravitational waves [2][3].

However, several issues make direct detection of gravitational waves difficult. We expect the signal of even the strongest gravitational waves to be extremely weak, therefore, the sensitivity of the LIGO detectors must be very high. LIGO's sensitivity is such that plausible gravitational wave sources might only change the lengths of the arms by a distance thousands of times smaller than the width of an atomic nucleus [1][4]. With such high sensitivity, even the slightest noise in the signal becomes more than a slight problem. Seismic vibrations, passing trucks, dust particles in the vacuum arms, overhead power lines, and a myriad of other sources all produce unwanted noise. Despite the superior techniques and technologies utilized to isolate the detectors and reduce noise, some level of noise is unavoidable. These combined factors of noise, high sensitivity, and the weak strength of gravitational waves together made the detection of gravitational waves highly improbable with the previous generation of LIGO detectors. However, the current generation of Advanced LIGO detectors (aLIGO) has increased detector sensitivity about tenfold. The LSC expects aLIGO to be sensitive enough to detect at least some gravitational wave sources and events [5]. Therefore, since there is a significant likelihood of direct detections, the LSC needs powerful and effective tools at its disposal to differentiate gravitational wave signals from noise.

The maximum entropy method is a statistical method useful for extracting a meaningful signal from a noisy dataset, with an established history and precedent in astrophysics [6]. The LIGO Scientific Collaboration (LSC) has implemented this method in an algorithm designed to extract gravitational wave signals. The inception of this specific implementation of the MaxEnt method was in 2007, when T. Z. Summerscales et al. proposed using MaxEnt to recover gravitational waveforms produced by core-collapse supernova to gain insight into the waveform parameters of these supernovae, in order to better understand the underlying physical processes. However, MaxEnt can be used for any unknown waveform [4].

We represent the data $\boldsymbol{d}$ by the model $\boldsymbol{d} = \boldsymbol{R}\boldsymbol{h} + \boldsymbol{n}$, where $\boldsymbol{h}$ is the gravitational wave signal, $\boldsymbol{R}$ is the detector response dependent upon the wave's direction of incidence and frequency content, and $\boldsymbol{n}$ is the noise in the detector. To find $\boldsymbol{h}$, we want to maximize the probability $P(\boldsymbol{h}|\boldsymbol{d}, \boldsymbol{R}, I)$, where I

is any other relevant parameters which may include any reasonable *a priori* expectations and noise characterization. MaxEnt finds $\boldsymbol{h}$ by minimizing the following function, which is the same as maximizing P,

$$F = \frac{1}{2}\chi^2 - \alpha S$$

where

$$\chi^2 = (\boldsymbol{Rh} - \boldsymbol{d})^T \boldsymbol{N}^{-1} (\boldsymbol{Rh} - \boldsymbol{d})$$

and $\boldsymbol{N}^{-1}$ is the inverse of the noise covariance matrix. T simply denotes vector or matrix transposition. S is a regularizer equivalent to the Shannon information entropy which helps with smoothness and preventing overfitting. The term α is a Lagrange parameter that helps to balance being true to the data and overfitting noise [4][7][8].

We aim to test the effectiveness of this implementation and to help refine it by injecting false gravitational waveforms into noise, and subsequently testing MaxEnt's ability to extract these signals. Our goals are to understand the extent to which MaxEnt can be effective at extracting these waveforms, to improve its power and efficacy of extraction of such waveforms, and to work out any bugs in its implementation.

Methodology

Our aim is to test the efficacy of the maximum entropy algorithm (MaxEnt) at extracting gravitational wave signals. Since the LSC has not yet directly detected any gravitational waves, we test MaxEnt by creating and injecting fake, idealized gravitational wave signals into noise [5]. We construct a fake waveform and inject it into recolored white noise with a frequency profile that matches models of aLIGO detector noise. We subsequently run MaxEnt upon the combined fake signal and noise, which returns to us an extracted signal. We then compare the signal extracted by MaxEnt to the originally injected signal to see how well MaxEnt performed (It is important to note that MaxEnt works blindly; that is to say, no information about the ideal, injected waveform is passed to MaxEnt as input). We do this by fitting the same kind of curve used for our injected signal to the extracted waveform, and comparing the fitted curve's parameters to the original parameters of the injected curve.

We test MaxEnt's ability to extract a specific type of gravitational wave burst called ringdown waveforms. When a black hole or other massive celestial body consumes mass, the body is perturbed and vibrates [4]. These vibrations produce ringdown waveforms, waveforms that have one consistent frequency, but whose strength decays exponentially over time, much like a bell struck with a hammer produces one tone that fades over time. A ringdown takes the following form of a sinusoidal function modulated by an exponential decay

$$g(t) = A\sin(2\pi f t + \varphi)e^{-t/\tau}$$

where t indicates time, A is the amplitude, φ is the sinusoidal phase constant, f is the frequency, and τ is the decay constant. The specific parameters of comparison we use are the frequency f and the decay constant τ, which characterizes how quickly the signal fades.

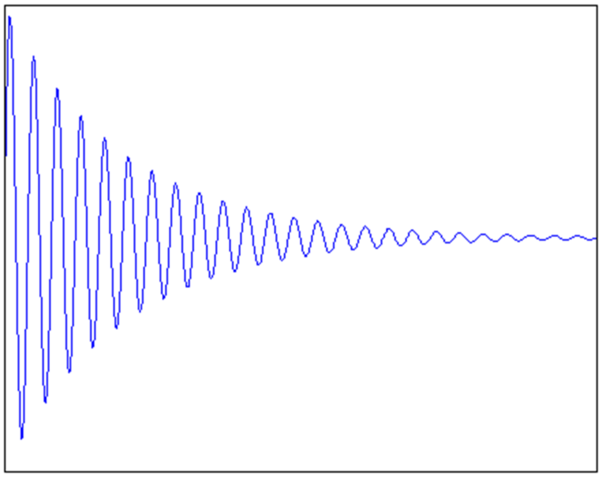

Fig. 1: Example of a ringdown waveform

The specific waveform we use for injection is a ringdown with two polarizations, where both polarizations have $\tau = 0.2$ and $f = 2284.68$ Hz. We perform 247 injections, varying the strength of the signal and the noise.

We determine the frequency of the injections by calculating the power spectrum as a function of frequency and recording the most powerful frequency. To find the decay constant, we calculate the cross-correlation between injections processed by MaxEnt and ideal ringdown functions with decay constants from 0.01 to 0.30, in increments of 0.01. To find the cross-correlation between the two discrete signals x and y, we first normalize x and y individually so that their auto-correlations (cross-correlation of a signal with itself) are equal to one. We then calculate the cross-correlation as a function of the lag between x and y using the formula

$$C_{xy}(m) = \sum_n x_{n+m}\, y_n.$$

The maximum value for C_{xy} reflects the level of similarity between the two signals [9].

Results

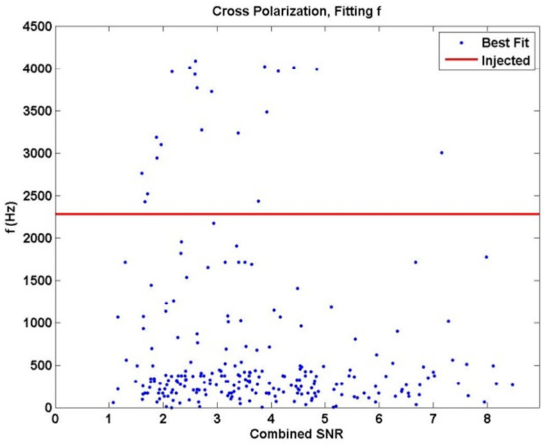

Fig. 2: Best-fit frequencies for cross polarization

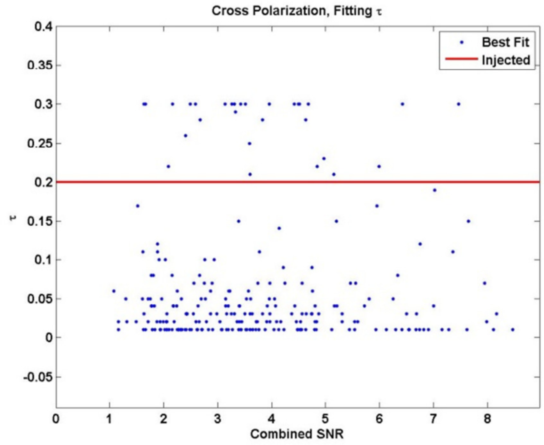

Fig. 3: Best-fit decay constants for cross polarization

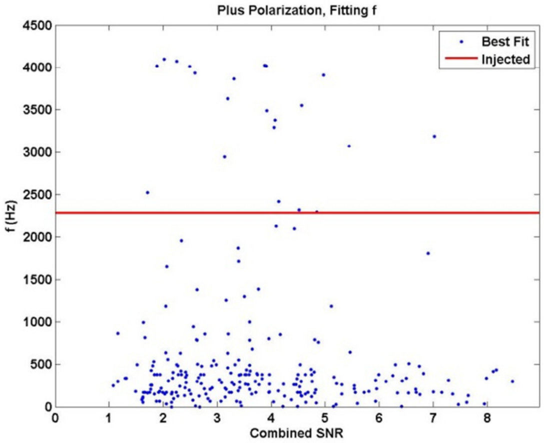

Fig. 4: Best-fit frequencies for plus polarization

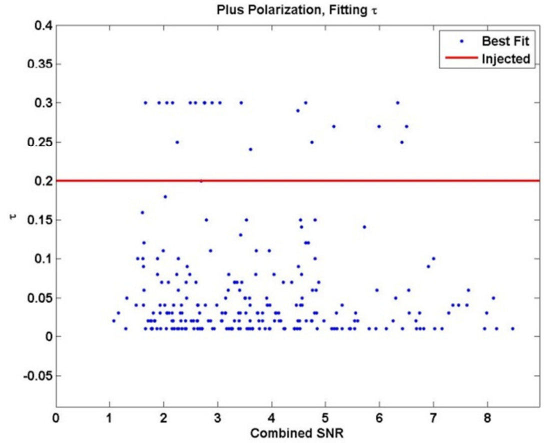

Fig. 5: Best-fit decay constants for plus polarization

For each injection, we plot the extracted parameters for each polarization against the combined signal-to-noise ratio (SNR) which results from simulating noise on the detector network. The red lines indicate the values of the injected parameters, and the blue data sets indicate the various parameters extracted from the injections that were processed by MaxEnt.

Discussion

We expected that MaxEnt would extract the waveforms with varying levels of accuracy, depending upon the strength of the noise relative to the injected signal. If the signal-to-noise ratio is very high, MaxEnt might return a signal looking very much like the one we originally injected, that is, virtually indistinguishable from noise. For lower signal-to-noise ratios, MaxEnt might return a much more noisy or fuzzy looking version of our original injection. If the signal-to-noise ratio is too low, MaxEnt might not find any meaningful signal, and simply return noise. Given these assumptions, as SNR increases, the extracted parameters should approach the injected parameters.

However, the data does not seem to support these expectations, as there does not appear to be any significant correlation between combined SNR and closeness to the original parameter values—that is, closeness to the red lines. The most likely explanation for this outcome is that the SNR values are too low to produce noticeable effects. This is almost certainly the case for frequency, because using the power spectrum to find the frequency of an injection should only fail when the frequencies present in the noise have more power than the frequency of the injection. However, it is more difficult to make such straightforward claims for the decay constants, since we calculate them using cross-correlation: any number of factors such as a high frequency (2285 Hz) relative to the sampling rate (8192 Hz) producing aliasing of the signal, relatively low SNRs, or the fact noise persists while ringdowns decay could all affect the resulting decay constants.

By running tests with various signal-to-noise ratios, we now have a better idea of how much information we can recover based upon the noise levels, and can better understand MaxEnt's strengths and weaknesses, and determine its value as a tool for the LSC. We found our methods of extracting parameters from MaxEnt output to be fairly inaccurate for SNRs of about 8.5 or less. This indicates that to effectively utilize MaxEnt as a signal extraction tool, we either need to improve our methods of parameter extraction, or that MaxEnt is unable to extract much useful signal at low SNRs, or some combination of both.

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