LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

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Tilt effect in a single mode cavity

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1 Introduction

Study the tilt sensitivity of the single mode cavity proposed by the Moscow group.

2 Setup

Symmetric FP cavity, T (power transmittance of ITM) is changed from 1% ~ 10^{-6}. T for ETM = 0.

Mirror shape :

1) Single mode y0=30, α=0.175, β=-0.05

2) 5cm spherical : spherical mirror with beam size of 5.07cm on TMs. This is the beam size of Single mode cavity with the parameter used.

3) 6cm spherical : spherical mirror with beam size of 6cm on TMs.

For each tilt angle, cavity is locked. Fig.1 is a power loss (power with tilt normalized by the power without tilt) in a FP cavity, T(ITM)=0.014, with 5cm spherical mirror as a function of the ETM tilt (0~0.5μ rad). Green is the unlocked case without locking, i.e., lock without tilt and use that length for all tilt angles, and the blue dash is the result with locking for each angle, i.e., the cavity lock length is adjusted at each tilt.

The green is 1 – Loss(θ) / T(ITM) and blue dashed is 1 – Loss(θ). Loss(θ) is the mode mixing of the incoming field and the tilted cavity beam axis, or green solid line and dashed line in Fig.2. And these are consistent with naïve interpretation. The large loss seen without lock is due to the length change induced by the tilt, which is corrected by the locking, hence the rise from solid to dashed. But, however well the length is adjusted, the mode mixing due to the tilt cannot be recovered, hence the small drop by Loss(θ).

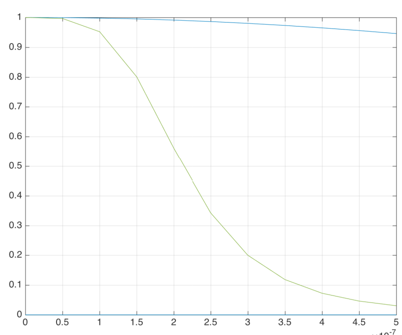

Figure 1 effect of locking

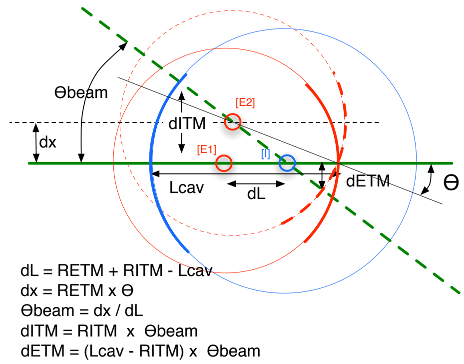

Figure 2 resonating beam in a cavity with ETM tilted

3 Fields in cavities

Fig.3 compares power distribution in three cavities, all T(ITM)=10^{-6}, without and with tilt of 0.1 μrad. This is the power reflected by ETM, so it is cutoff at 0.17m. The red line, 5.07cm case, there is very little power change. 5.07cm is chosen to make the slope of blue and Gaussian case the same. Very loss T(ITM) is chosen to make the cavity field least affected by the incoming source field.

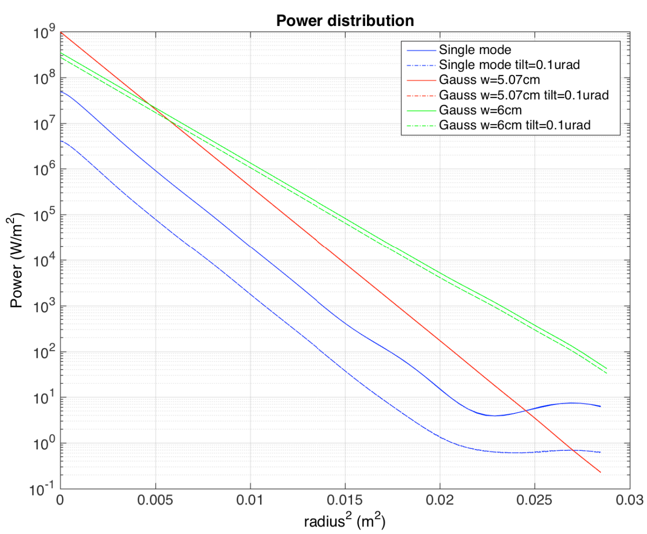

Figure 3 Power vs r^2 in three cavities

The round trip loss for each cavity at different tilt angle is given in Table.1. This is the total power loss per round trip.

	Single mode cavity	5cm spherical	6cm spherical
Tilt = 0	3.3ppm	0.0006ppm	0.44ppm
Tilt = 0.1 μrad	65ppm	0.0013ppm	1.8ppm

4 Definition of signal loss

The fields in a cavity with finite apertures are not possible to be expressed analytically, so the base mode of the field in a given cavity is defined by the field coming out of the ETM without ETM tilt, i.e., the solid lines in Fig.3 are used to define the base mode fields, and the power loss of this mode in the field reflected by ETM is used as the definition of loss. The base field is calculated for each case with different T(ITM).

The gravitational wave signal is proportional to the base mode power, so this is an appropriate quantity.

It can be easily seen that the loss of the single mode cavity is the largest, then 6cm spherical. And the 5cm spherical shows almost no loss.

5 Angle dependence of loss and T(ITM)

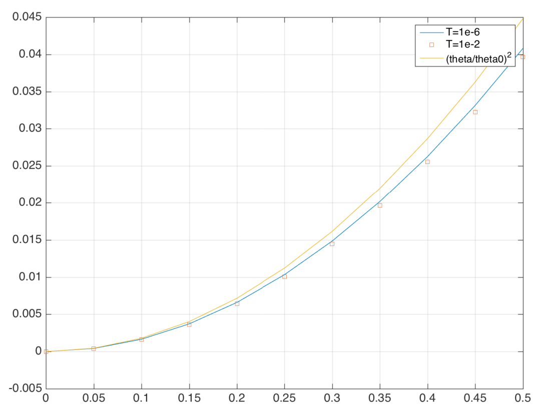

Figure 4 loss vs ETM tilt angle : 5cm spherical cavity

Fig.4 shows the loss of the signal power, defined in the previous section, as a function of the tilt angle. The blue line is the case using T(ITM) = 10^{-6}, red box using T=0.01. There is very small dependence on T or finesse, that is because the loss is essentially the mode coupling of the incoming beam and the resonating field, as is discussed in Sec.1.

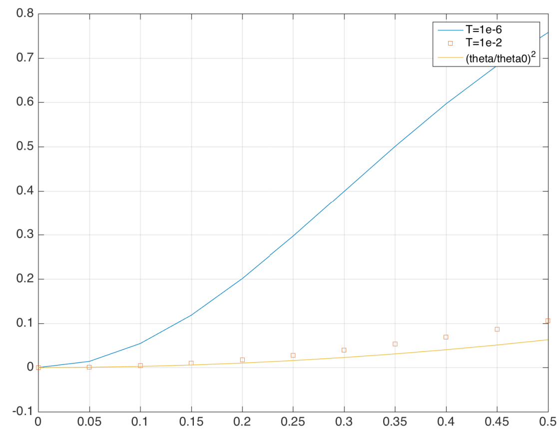

Figure 5 loss vs ETM tilt angle : 6cm spherical cavity

Fig.5 shows the same loss vs tilt angle. Now there is large difference between the losses with different T values, or finesse values. As Table 1 shows, the round trip loss in a 5cm cavity is negligible, even with large tilt, but the loss is not negligible in a 6cm cavity. And this clipping loss cannot be recovered by the locking, so the total loss depends on the finesse or T.

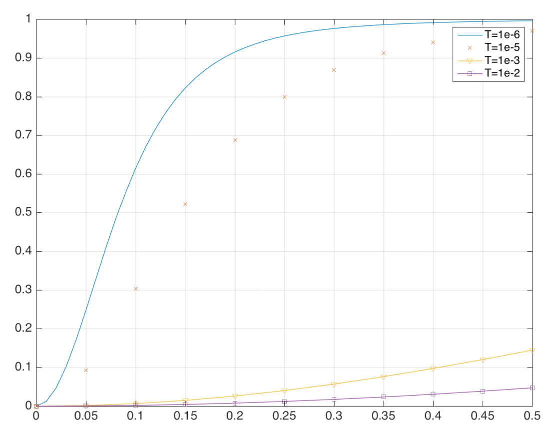

Figure 6 Loss vs ETM tilt anfle : single mode cavity

Fig.6 shows the loss of the single mode cavity, for four different T values. There is a strong dependence, just as was shown for the 6cm cavity case. The reason is due to the round trip loss.

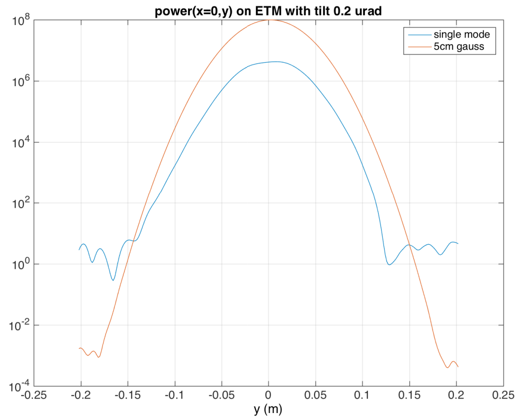

Figure 7 Asymmetry of power distribution and tail

The beam axis tilt, θbeam in Fig.2, and the beam width fit by Gaussian shape in a single mode cavity are almost the same as those in 5cm spherical cavity.

Fig.7 shows the power distribution of the field on ETM coming from ITM, with $T=10^{-6}$ and tilt angle of 0.2 μrad. The blue line is the one in a single mode cavity and the red is the one in a 5cm spherical cavity. The asymmetry is larger and the tail is wider. The power outside of 0.17 is the loss when reflected by ETM.

6 Statement

In the real experiment, there is finite T for ITM, like 1%. The loss of the signal does depend on T due to the clipping loss. The eigen solution approach does not have this T in the formulation. How to reconcile these two?Type equation here.

Other issues to be considered is the mode mismatch loss (squeezing hates loss), and the beam size (thermal noise hates small beam size).

7 Analytic formula of the loss in a spherical mirror cavity

There are two kinds of losses, one is due to the mode coupling and the other is the clipping loss.

The input mode is the one resonating along the solid green line, and the resonating cavity mode is the one along the dashed green line. The power loss of the 00 mode is $(\theta_{beam} / \theta_0)^2$, where θ_{beam} is the angle between the two mode base axis, as shown in Fig.1, and θ_0 is the divergence angle at the crossing where the two axis intersects. This loss can be expressed using the quantities at ETM as

$$L_{mode}(\theta_{ETM}) = \left(\frac{R}{\sqrt{(2R-L)L}}\frac{\theta_{ETM}}{\theta_{div}}\right)^2 = \left(\frac{\theta_{ETM}}{\theta_{mode}}\right)^2$$

$$\theta_{div} = \frac{\lambda}{\pi\, w_{ETM}}$$

θ_{mode} = 3.52 μrad for the 5cm cavity and θ_{mode} = 2.14 μrad for the 6cm cavity. This is independent on the finesse or T(ITM).

Among the input power, 1-L_{mode} goes into the resonating field, whose axis is tilted from the original input beam axis. The signal mode, defined by the cavity without tilt, is along the input beam axis, so only 1-L_{mode} among the resonating power is the original signal mode. So the total loss of the signal is 2 L_{loss}, first due to the mode mismatch from the input field to the resonating field, and another loss due to the mode mismatch of the resonating field to the original signal field.

Another one is the loss due to the finite mirror aperture. Naive estimation of the clipping loss is the integration of the Gaussian distribution out of the mirror radius.

$$L_{clip} = \exp\left(-2\,\frac{{R_m}^2}{w^2}\right)$$

where R_m is the mirror aperture and the w is the Gaussian beam width.

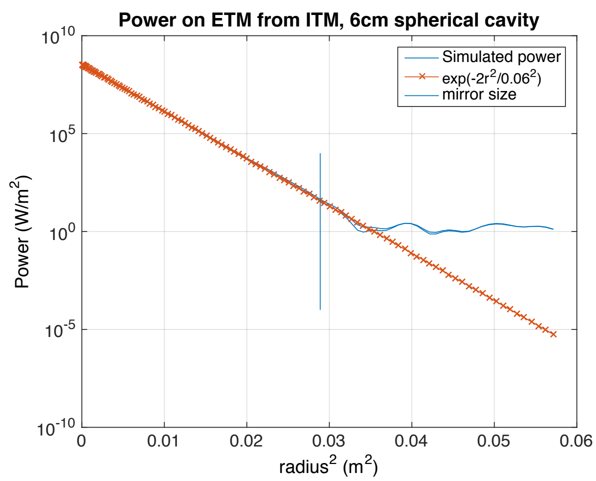

Figure 8 Diffraction tail

Fig.8 compares the stationary field in the 6cm cavity vs a Gaussian distribution. This is the field on ETM coming from ITM. The vertical line is the mirror radius and all power outside of the mirror is lost on the reflection. The blue line shows a long tail out of the mirror aperture, which is caused by the diffraction induced by the clipping. The numerical value in the above estimation is 0.1ppm per mirror, while the numerical modeling gives 0.2ppm per mirror. In the following, the diffractive loss in a tilted cavity is estimated using a Gaussian shape, and the loss is multiplied by 2 based on this argument. It is not quite accurate, but will be good enough to understand the numerical results.

The simplified estimation of the clipping loss is done as follows. When the ETM is tilted by θ_{ETM}, the beam centers on ITM and ETM are shifted by $dx_{ITM} = R^2 / (2R - L)\,\theta_{ETM}$ and $dx_{ETM} = R(L-R) / (2R - L)\,\theta_{ETM}$. The loss is estimated by calculating the power out of the mirror surface with a Gaussian beam off centered by these dx's.

The sum of the loss on ITM and ETM are

$$L_{clip}(\theta) = 4\,\frac{Rm^2}{w^4}\exp\left(-2\frac{Rm^2}{w^2}\right)\frac{R^2(R^2+(L-R)^2)}{(2R-L)^2}\theta^2 = \left(\frac{\theta}{\theta_{clip}}\right)^2$$

θ_{clip} = 843 μrad for 6cm cavity. This does not include the factor 2 by diffractive tail.

The power is lost each time the field is reflected by the mirror. The total loss for this kind of loss can be calculated as follows.

$$E(L) = \frac{Esrc}{1-\sqrt{1-T(ITM)}\sqrt{1-L}} = E(0)(1-\frac{L}{T})$$

In terms of power loss, it is 2L / T.

By combining the two losses, the total loss in a tilted cavity can be expressed as follows.

$$L_{total}(\theta) = \frac{4}{T}\,L_{clip} + 2\,L_{mode} = (\frac{4}{T\,\theta_{clip}{}^2} + \frac{2}{\theta_{mode}{}^2})\theta^2$$

Fig.9 shows the comparison of the simulated loss value and the analytic formula, L_{total}. The clipping loss is very small and there is no T dependence.

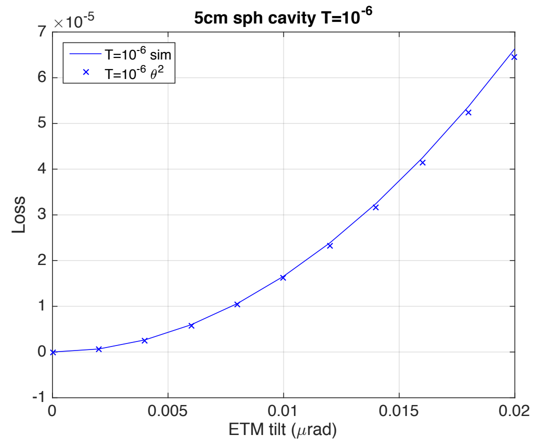

Figure 9 Loss of 5cm spherical cavity

Fig.10 shows the comparison in a 6cm spherical cavity with various T values. The dependence on the finesse is consistent between the simulation and the analytic formula, L_{total}. The analytic formula has one factor based on the untilted cavity, i.e., the effect of the diffractive tail. But Fig.10 compares effects of T values which ranges four orders of magnitude.

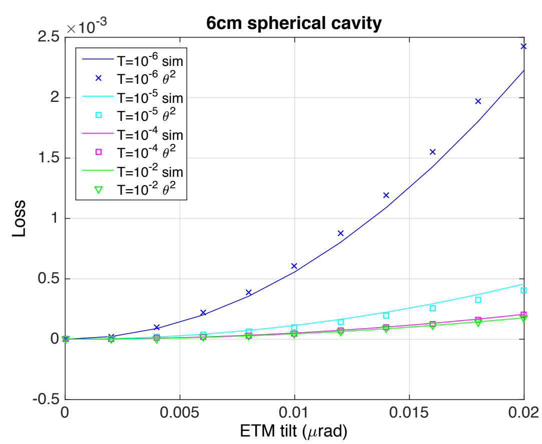

Figure 10 Loss of 6cm spherical cavity

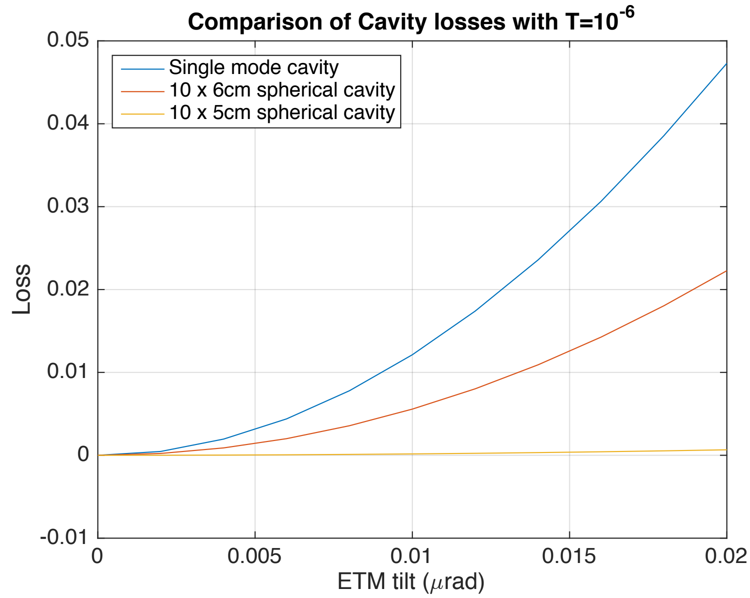

Figure 11 Comparison of losses in different cavities

Fig.11 compares the angle dependence of the loss in three different cavities, all with T(ITM)=10^{-6}. The loss in a single mode cavity is much larger than spherical cavities and losses in 5cm and 6 cm spherical cavity are multiplied by 10.