Quantization Noise in Advanced LIGO Digital Control Systems

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Project Introduction

- Mixed Signal Systems
- Digital Control vs Analog Control
- Quantization Noise: One of the major demerits of Digital Control Systems
- Causes of Quantization Noise

Quantization Noise

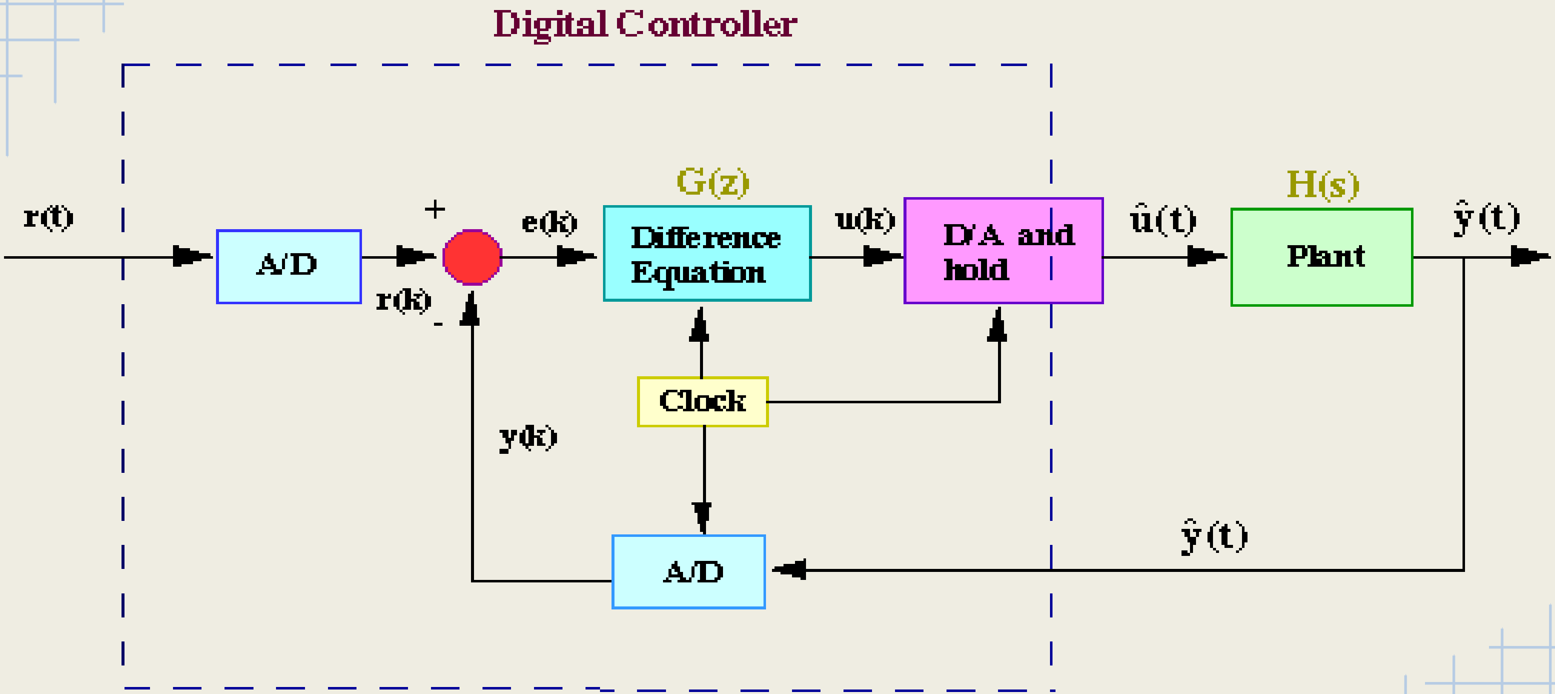
In Analog: 1.25 + 2.34500000199999 = 3.59500000199999

In double precision computer,
(1.25) + (2.34500000199999) = 3.5950000012

Quantization Noise = (approximately) 10^{-12}

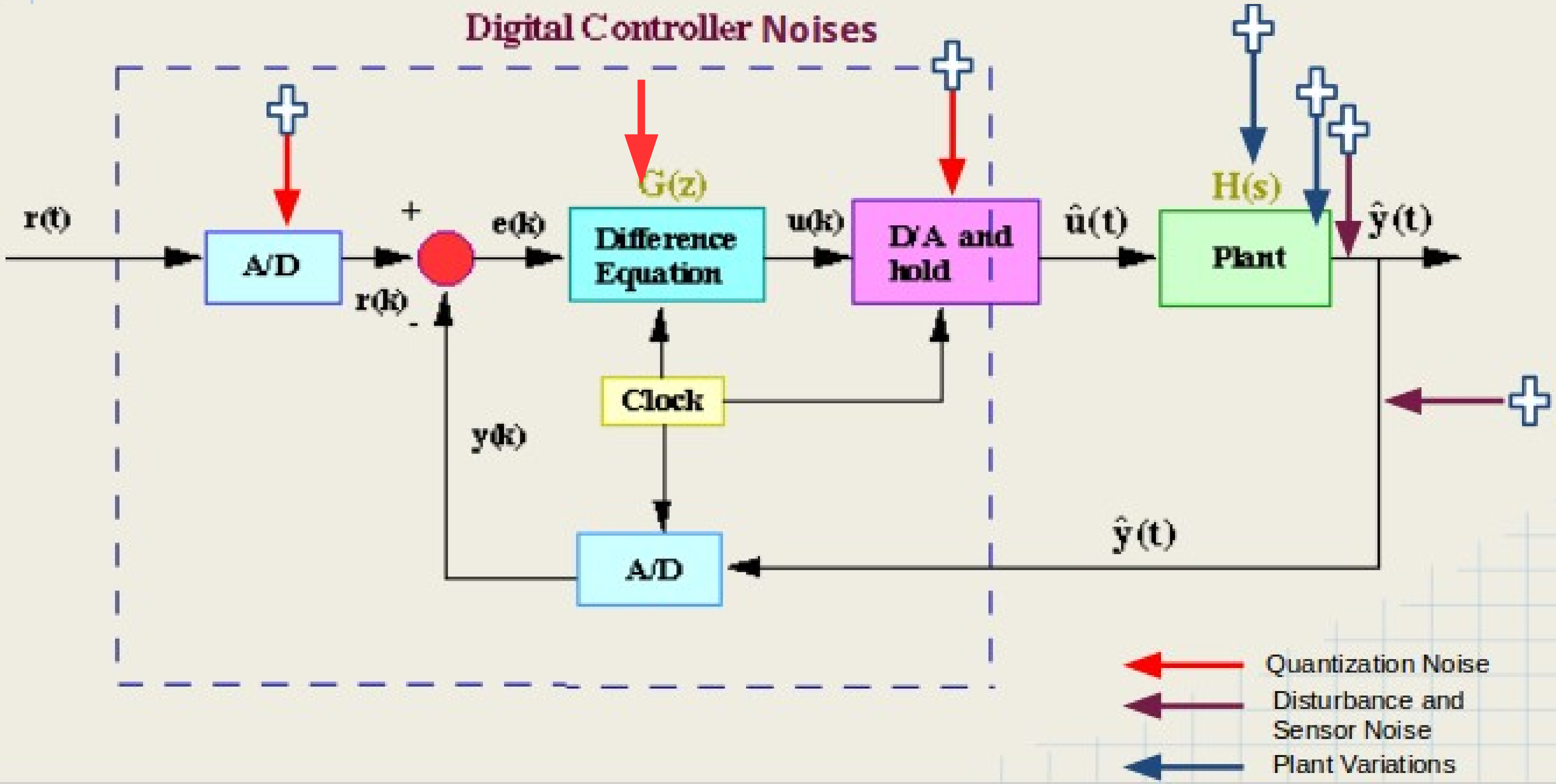
Similarly, two (B+1) bit numbers, on multiplication give a (2B+1) number which then needs to be truncated for a B+1 precision computer

Digital Control System : I



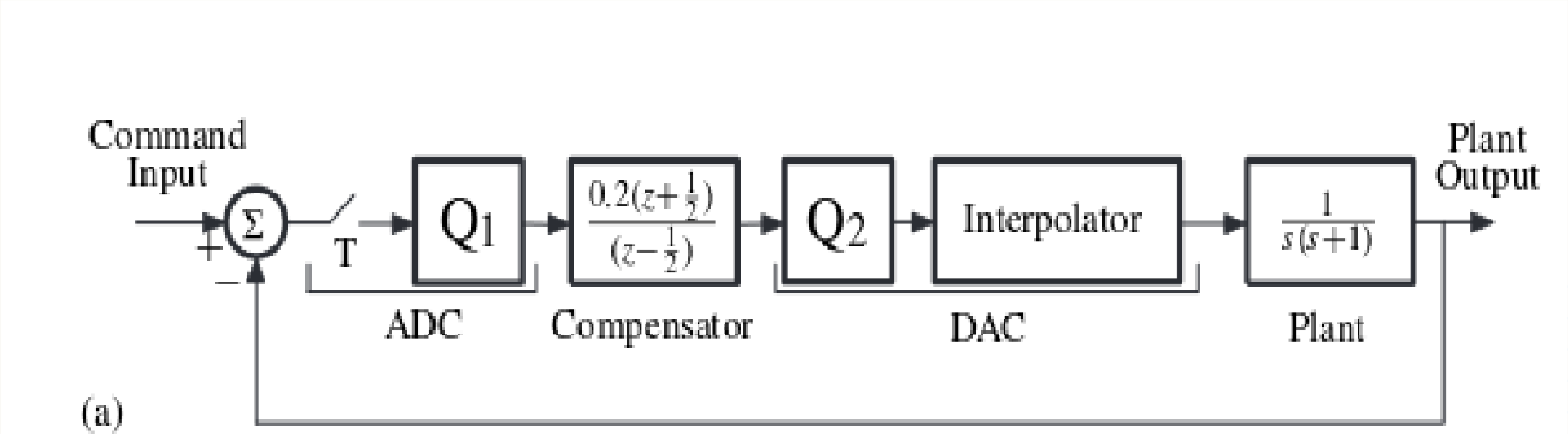
Reference: National Instruments

Digital Control System : II



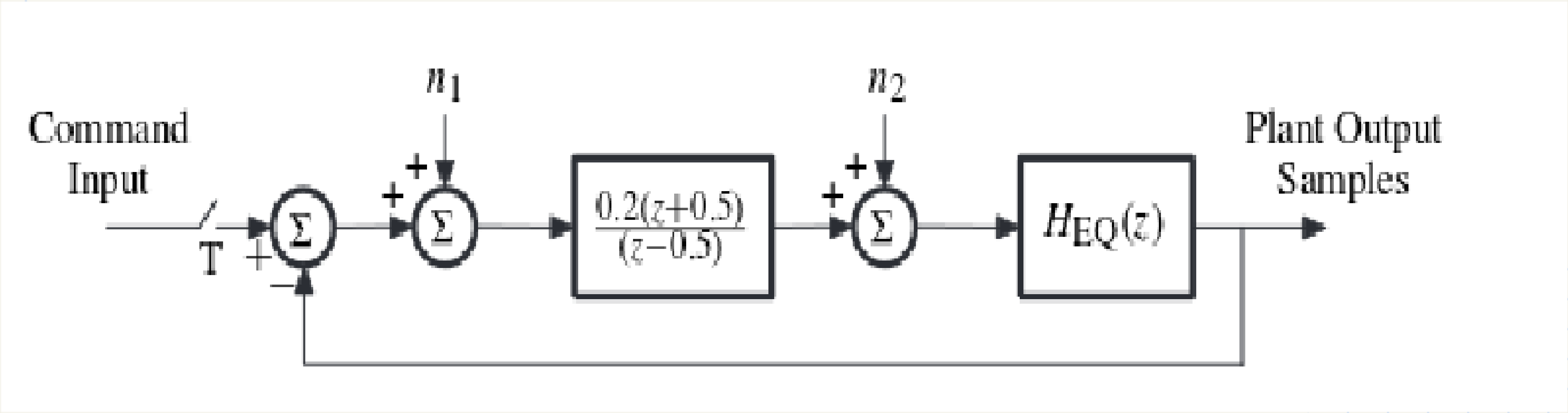
Digital Control System : III

Modeling Quantization Noise in a Digital Control System:



Digital Control System : IV

With Approximate Additive Quantizer Model:



Reference: Widrow and Kollar Book on Quantization

Sources and Measurement

Three primary sources:

- Finite Precision of Digital Computers **(ADC Quantization Noise)**
- Mathematical Calculations **(Digital Filter Quantization Noise)**
- Truncation of Numbers to drive finitely precise DAC **(DAC Quantization Noise)**

Improvements Possible

- To improve digital filter performance:
  - Change filter structure
  - Better the precision
  - Error Feedback
- To improve DAC performance:
  - Use higher precision DAC
  - Noise Shaping
- For ADCs:
  - Change Hardware Implementation and Design (Algorithm)

Quantization Noise Analysis of the Digital Controller

Background

- Filter Structure (Mathematical Operations, Order)
- State Space Representation of Digital Filters
- Low Noise Form (Matts Evans)
- Time Complexity and Performance
- For double precision implementation: (ref. Denis Martynov)
  - Output(double)-Output(single) = Noise(single)
  - Noise(double) = Extrapolation factor * Noise(single)

Improvements in Noise Estimation

- Precise Noise Estimation
- SNR Distribution and Warning System
- Code running time
- SNR Plot

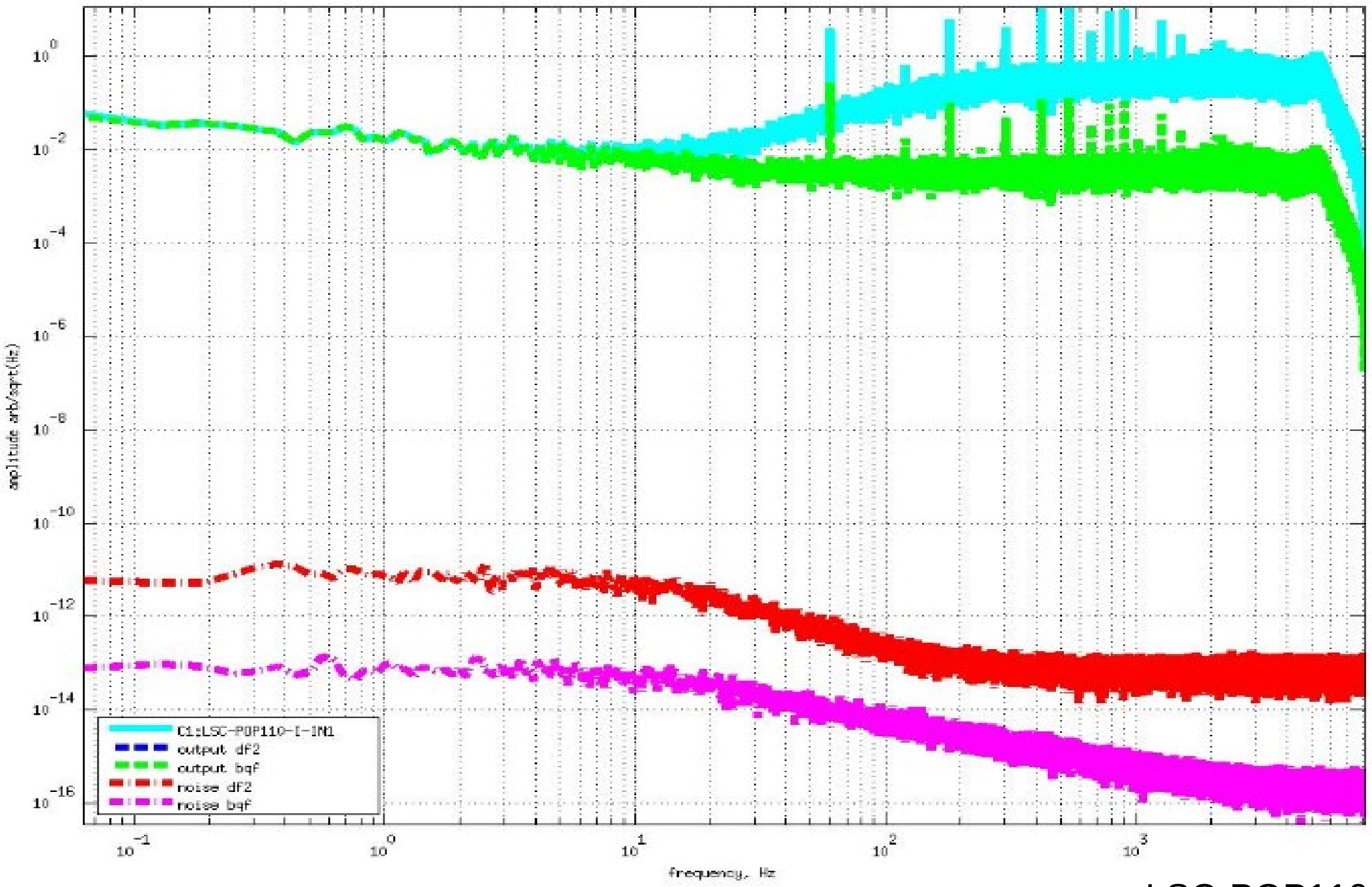
Automatic Digital Controller Checker Tool

A software tool based on MATLAB which performs the following:

- Searches for valid channel names (For sites, only recorded channels)
- Construct channel names from filter modules in Foton file archive (for all files)
- Download Data -> Perform Noise Estimation -> Plot
- Save the Data for future analysis...and repeat.

Testing on 40m Controller

- Caltech’s 40m prototype Interferometer Digital filters were Analyzed



LSC-POP110_I filter

For aLIGO sites

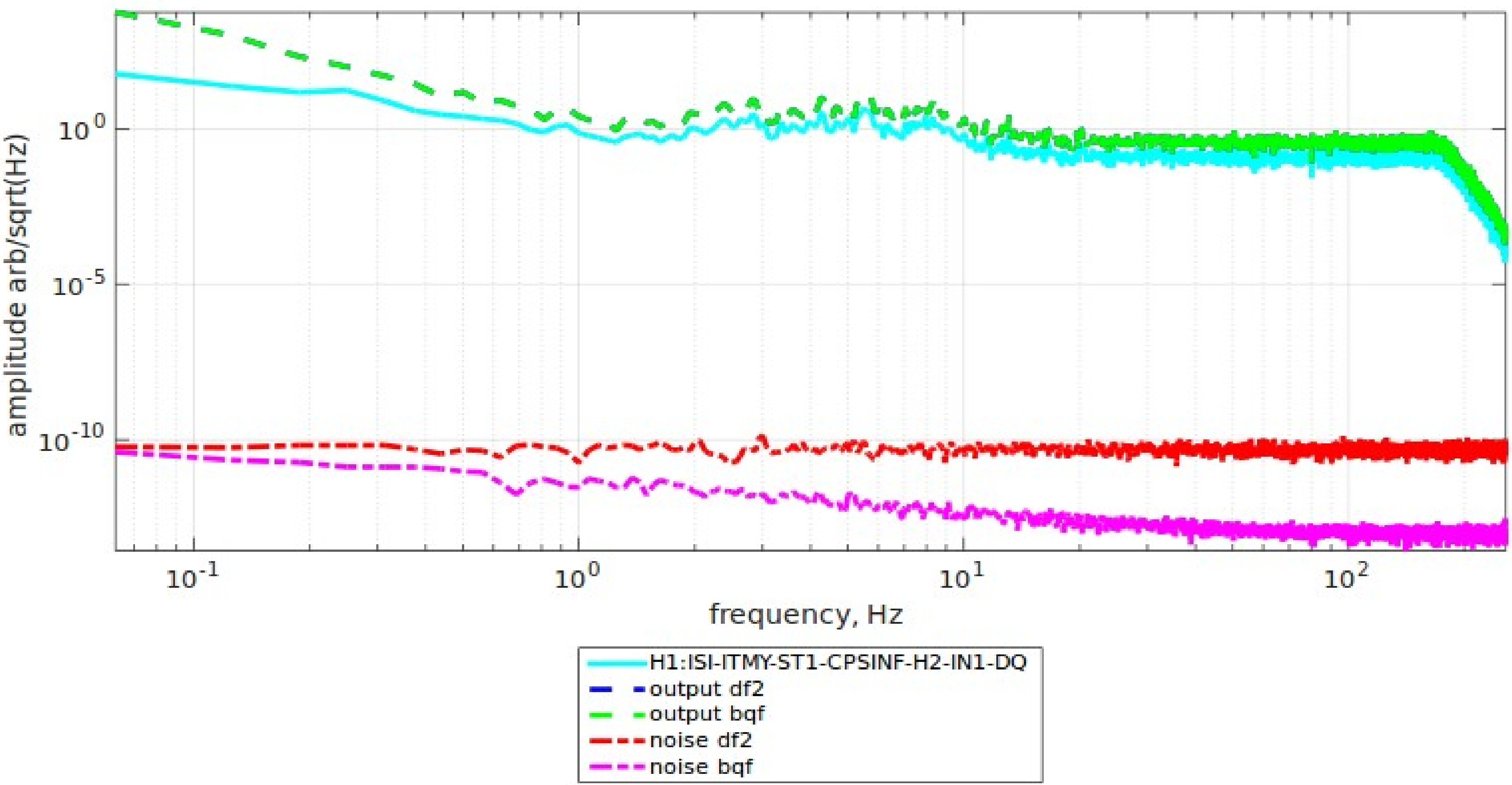
- Remote Access to input/output data for digital filters
- Only channels that are recorded
  - Some output channel (only) recorded filters have been checked by inverting the filter
- Foton file archive checked out of SVN at
  - Hanford: GPS Time: 1117896120 : Jun 9 14:41 UTC
  - Livingston: GPS Time: 1117562416: Jun 5 18:00 UTC
- The complete set of resultant plots is available at :

https://drive.google.com/folderview?id=0BzjRW8WwGjzJfkE3cVFzczJVU0JpSkZUTm1DR0dpWF9BWFlNVTh3VGg3UG93dHRLTURPZWs&usp=sharing

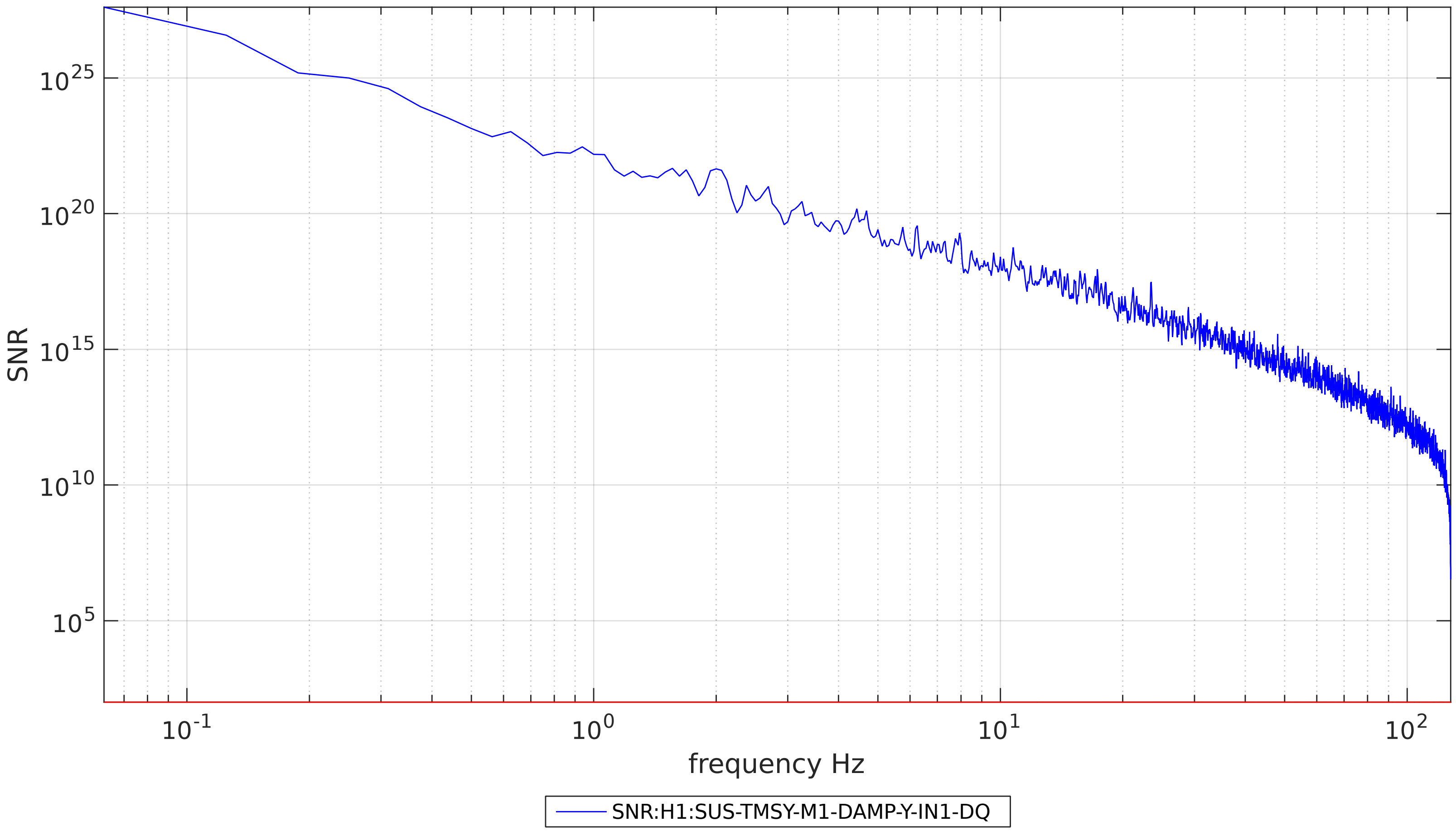
Observations and Inferences

- General Behaviour

--Digital Filter Noise is way below Output spectrum level.

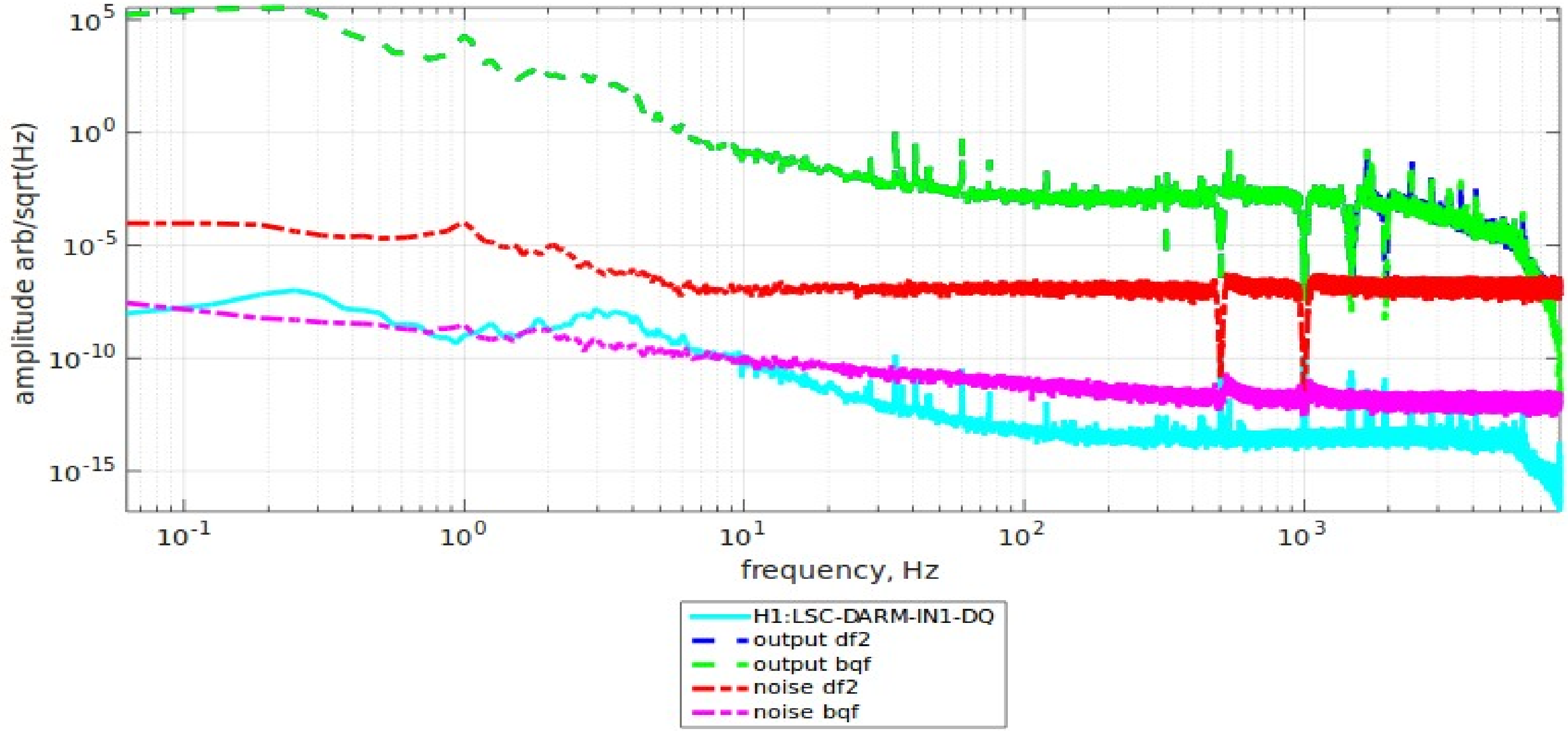


amplitude arb/sqrt(Hz)
10^{0}
10^{-5}
10^{-10}
10^{-1}
10^{0}
10^{1}
10^{2}
frequency, Hz
H1:ISI-ITMY-ST1-CPSINF-H2-IN1-DQ
output df2
output bqf
noise df2
noise bqf

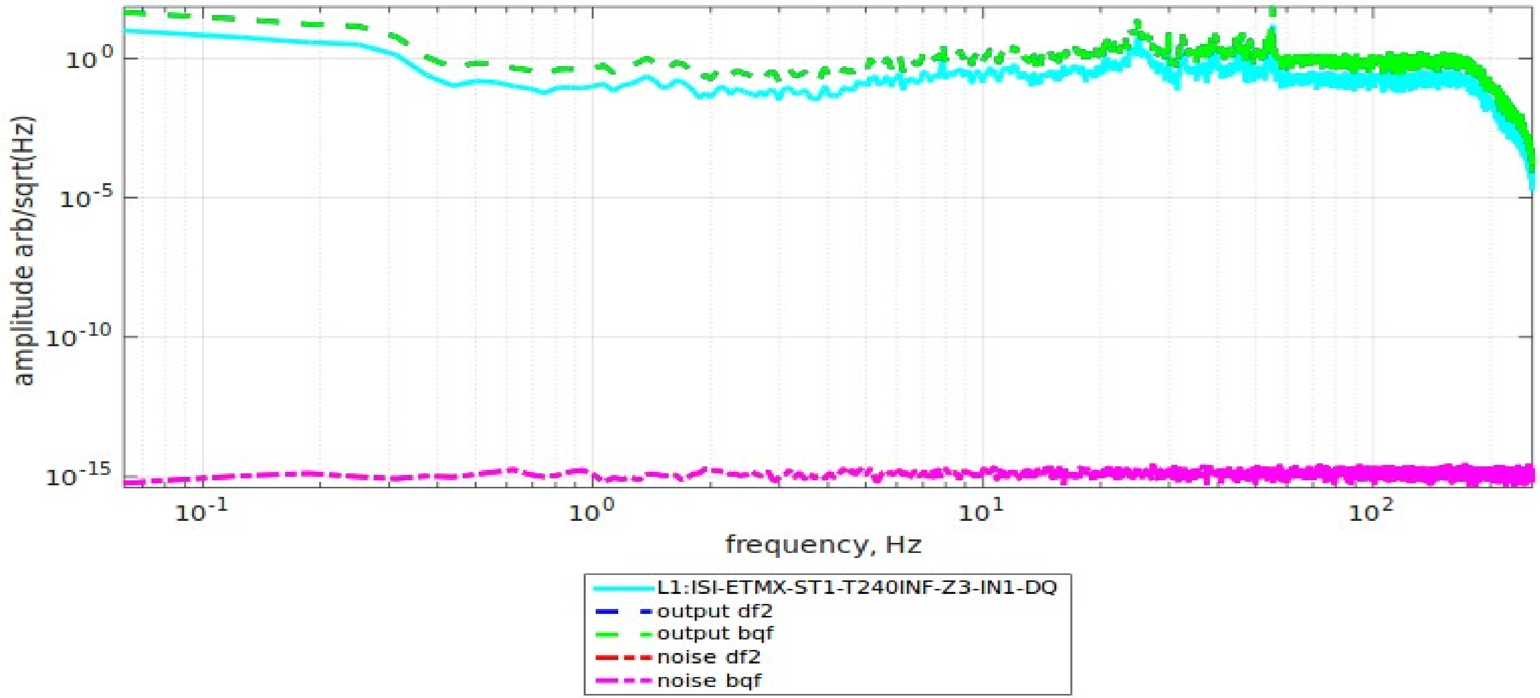


SNR
10^25
10^20
10^15
10^10
10^5
10^-1
10^0
10^1
10^2
frequency Hz
SNR:H1:SUS-TMSY-M1-DAMP-Y-IN1-DQ

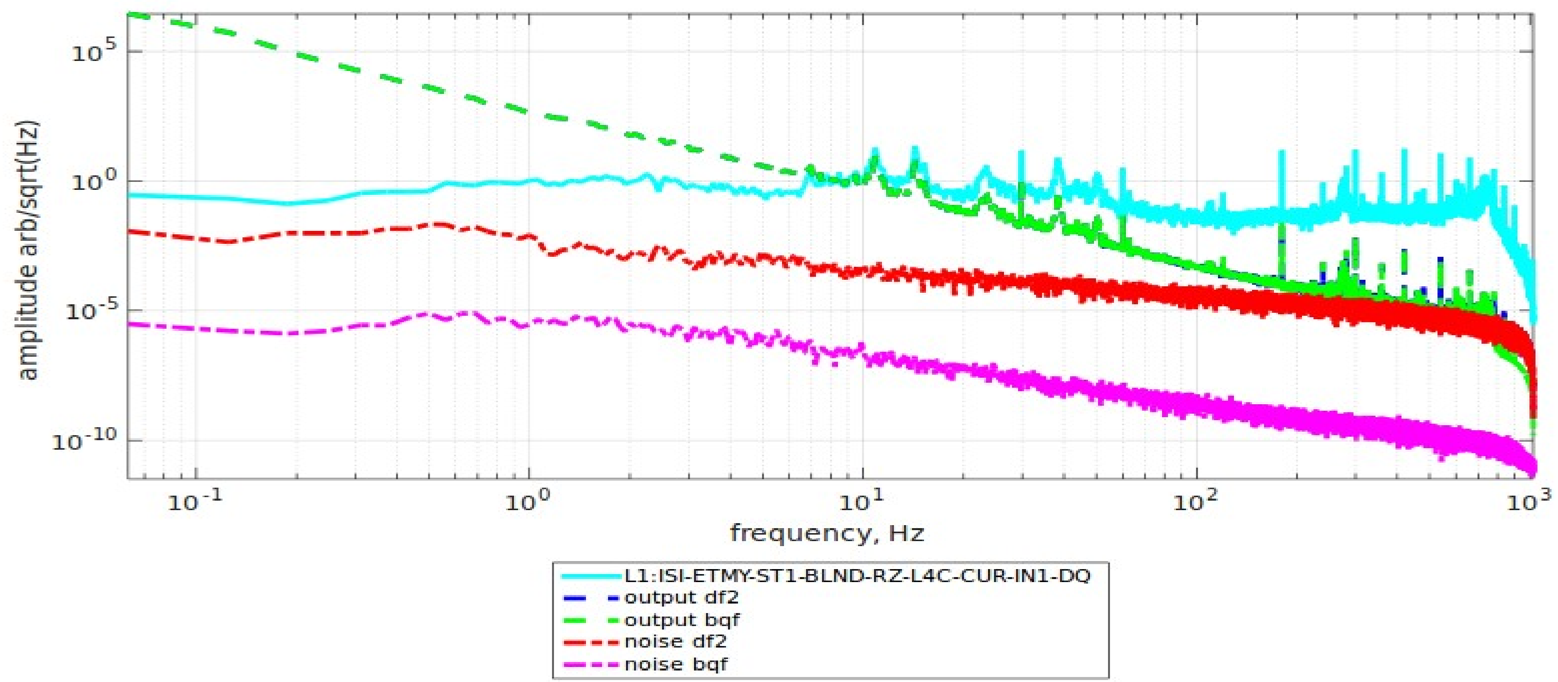
Filters with High Phase Lag (Higher Order filters) --SNR level lower



DF2 performs equally well as LNF
--Gain like filters/filters not performing many calculations



DF2 above output spectrum
--When Input signal is of very low order + High Phase Lag filter (Combined Effect)



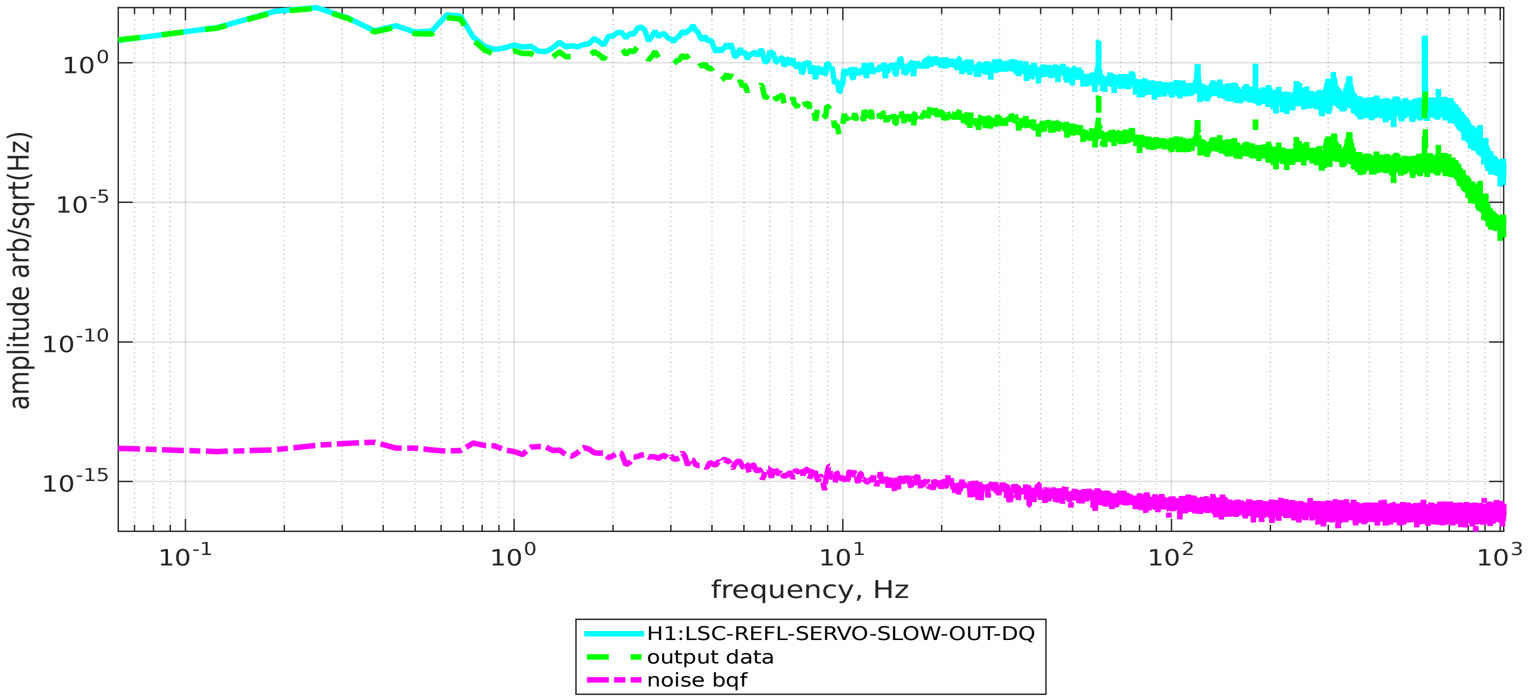
Other Observations and Inferences

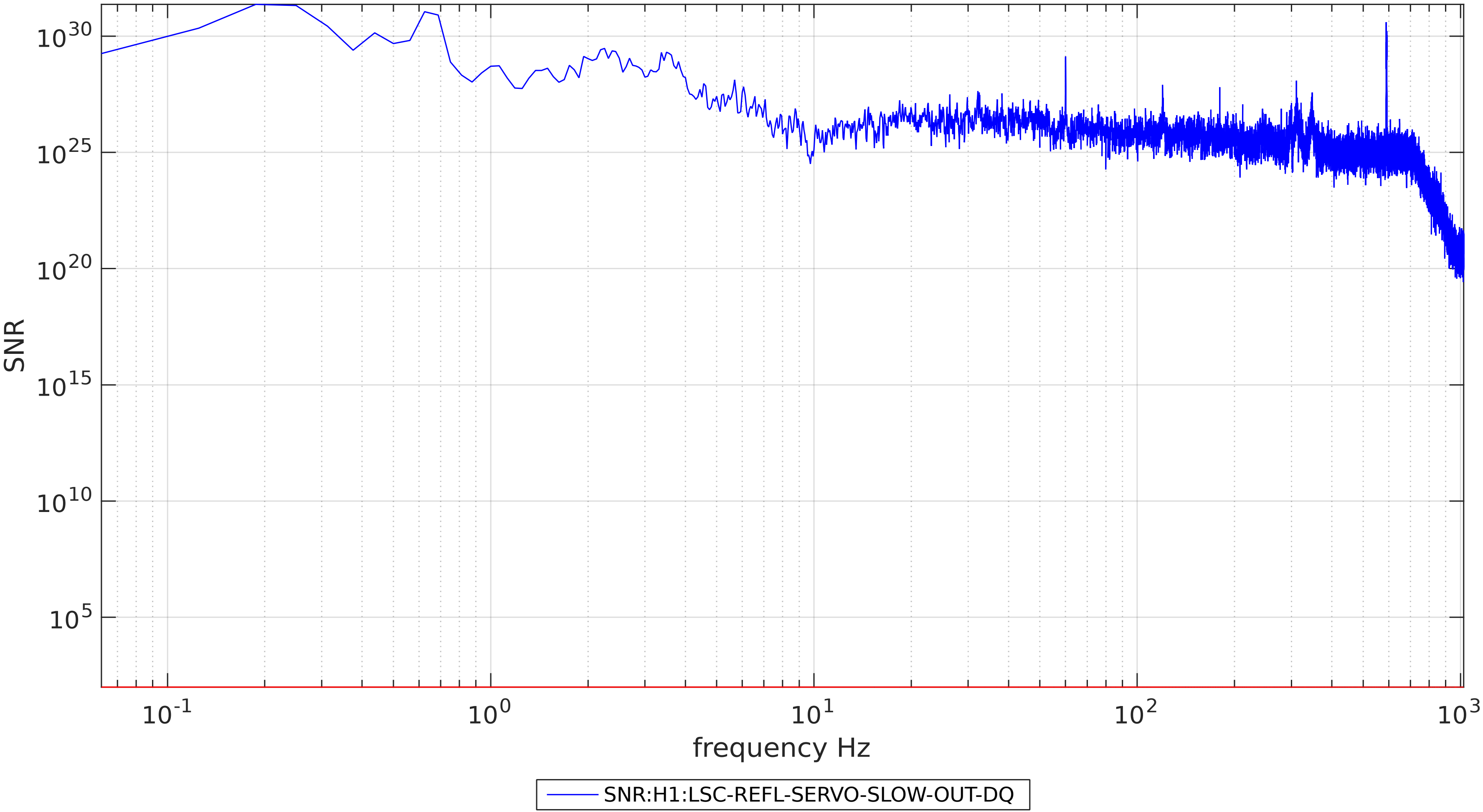
- Dependence and Independence on Input

--Inference: More on the independent side. To an approximation.

- Generally, LNF is better than DF2 by an order of 100 -10,000 SNR

Filter Inversion





SNR
frequency Hz
SNR:H1:LSC-REFL-SERVO-SLOW-OUT-DQ

Limitations and Conclusions

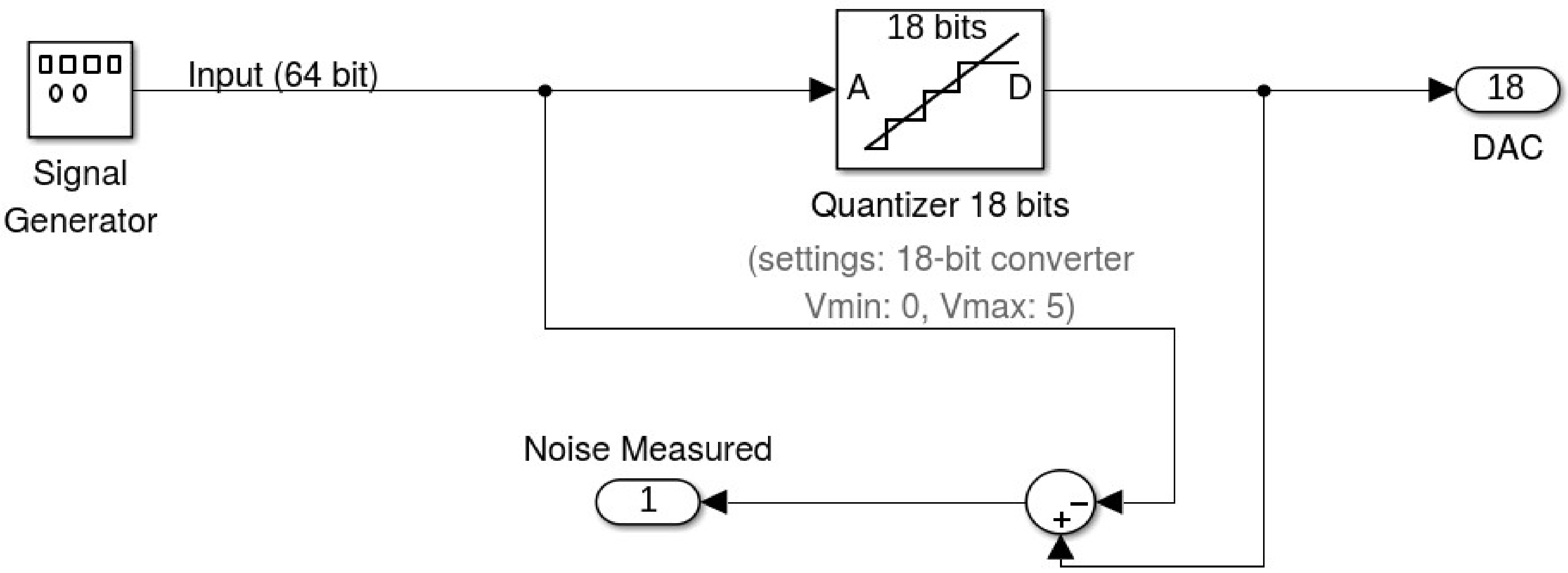
- A major limitation :
  - History of filters: The case when a filter is an integral type or higher order integrals
  - Remedy: Proper Sample time for the filter
- Only recorded channels tested, but there could be problems within the controller
- Major conclusion: LNF filter performs great for most filters (>90%). Even for the other 10%, SNR > 10^2 -10^3
- Not all filters can be inverted (from Output to Input) for analysis

DAC Quantization Noise

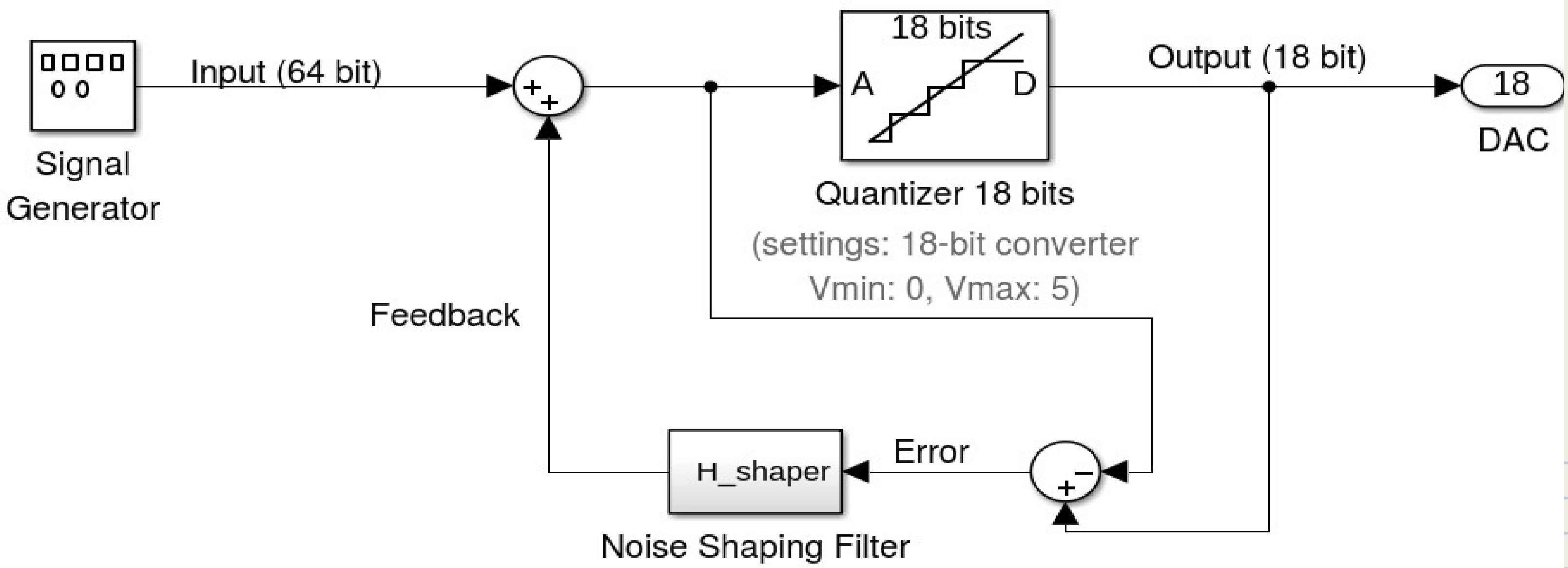
Ways to mitigate DAC Noise

- Using higher precision DACs
  - But, there are hardware limitations
  - Also, processing speed
- DAC architecture improvements
- DAC Noise Shaping
  - Low noise in a particular band of frequencies at the cost of higher overall noise level.

DAC Noise Measurement



DAC Noise Shaping



Background : Noise Shaping

- On simple block diagram analysis,

$$X'(z) = X(z) + E(z)\,(-1 + H\_shaper(z))$$

where, X'(z) is output transfer function in z-domain
and similarly, X(z) is input, E(z) is quantization error
and H_shaper(z) is feedback transfer function

- Since, the noise needs to be fed back after a delay, the above equation is modified to be like:

$$X'(z) = X(z) + E(z)\,(-1 + z^{-1}H\_target(z))$$

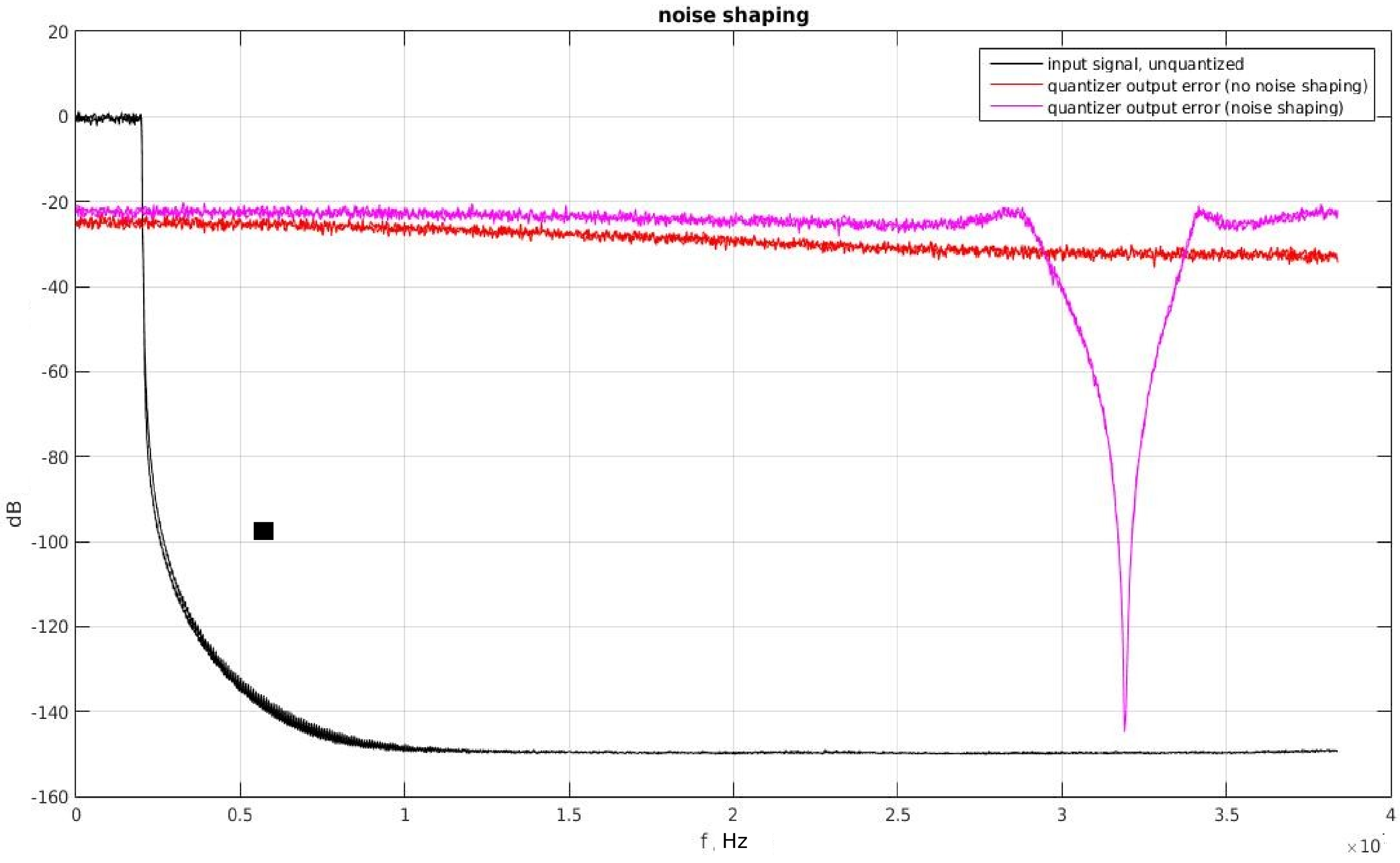
where the delay is accounted for in the code.

- Essentially, noise is now “shaped” or modified according to our own choice.

Customized Noise Shaping for aLIGO DAC

- The robustness of the noise shaping algorithm.
- Suppress any peak (notch) in Quantization noise
- Or, suppress a particular band of frequencies all together.

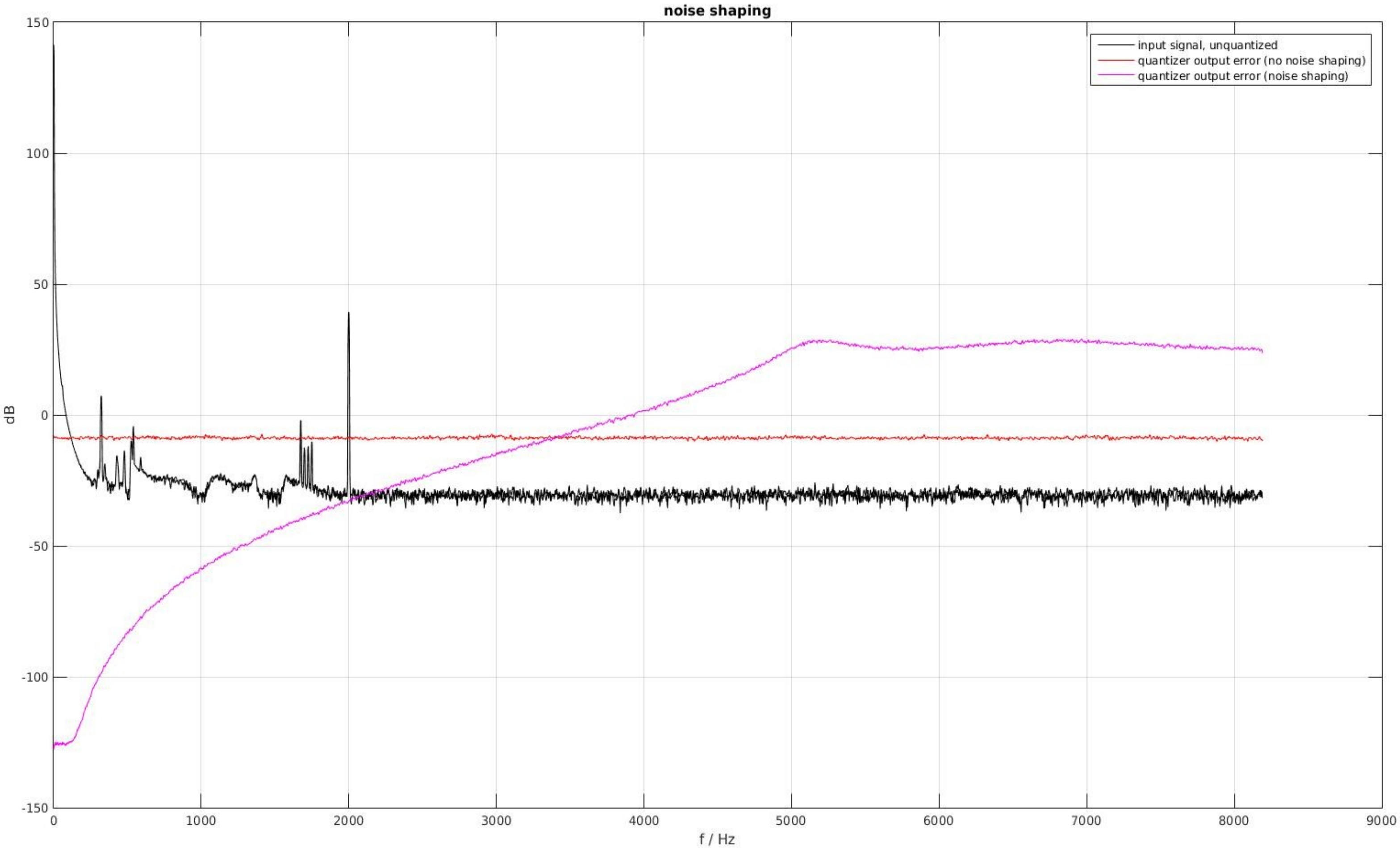
  With a compensation elsewhere.


noise shaping
input signal, unquantized
quantizer output error (no noise shaping)
quantizer output error (noise shaping)
dB
20
0
-20
-40
-60
-80
-100
-120
-140
-160
0
0.5
1
1.5
2
2.5
3
3.5
4
f , Hz
×10

Simulations in MATLAB

- Algorithm implemented in MATLAB gave successful results for any arbitrary noise shape.
- For a high pass shaped noise (which is desirable for GW detection):

Plot : Next Slide



noise shaping
input signal, unquantized
quantizer output error (no noise shaping)
quantizer output error (noise shaping)
150
100
50
0
-50
-100
-150
dB
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
f / Hz

Implementation in C

To enable the frontend code to take advantage of the noise shaping algorithm developed.

- Filtering done using SOS coefficients
- No plotting in this case, hence error debugging with respect to MATLAB simulation results
- Noise shaped data given to the DAC

Project Conclusions and Scope

- There are two major conclusions of the project work and the research done in this project:
  - For most of the filters analyzed, the low noise form performed better than DF2 and also SNR for most of them was acceptable.
    - That being said, an exclusive testing of the controller still remains to be done as signals inside the controller were not tested.

Project Conclusions and Scope

DAC Quantization Noise

A primary concern due to its higher level has been mitigated to a great extent, according to the noise shaping algorithm proposed.

The future scope would be to completely implement it in the system and take advantage of it.

Q & A

Thank You!