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Cosmography and Black Hole Spectroscopy by Coherent Synthesis of the Terrestrial and Space GW Antennae Network: Orbit Optimization

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1 Mission

Our mission is to come up with a new simplier and cheaper LISA-like detector. The detector consists of three spacecraft forming an equilateral triangle with an arm length of around 100 km. The constellation moves on the Earth orbit, trailing 5 degrees behind the Earth with inclination to the ecliptic of 60 degrees. This is an ambitious goal and therefore the project consists of several separate parts which can be considered as full projects themselves. Thus far I have been engaged with orbit determination.

2 Orbit

2.1 Requirements

Since spacecraft orbits have non-zero eccentricity and due to gravitational forces from the planets in the solar system arm lengths, internal angles and relative velocities among the spacecraft vary continuously. In order for the orbit of the spacecraft to be stable it is essential that the arm lengths of the detector do not change significantly. The length change should not exceed 2% of the total arm length. And relative velocities have to be less than 15 m/s so that it would be possible to reduce the Doppler effect. [2]

2.2 Initial parameters

Let us call vertices of the triangle A, B and C.

In order to uniquely identify a specific orbit of a spacecraft it is necessary to know an initial position of the spacecraft (x, y, z) and its initial velocity (vx, vy, vz) or another six parameters called Keplerian elements: $a, e, i, \omega, \Omega, f$, these are a semimajor axis, eccentricity, inclination, argument of periapsis, longitude of the ascending node and true anomaly respectively. The first five parameters determine the exact orbit and the last one determines the position of the spacecraft on this orbit.

The initial steps of choosing the orbits for the spacecraft are the same as for LISA. These two papers have been used for determing initial parameters in this project [2, 3]. The only difference is up to now the arm length.

The Sun is disposed in the origin and the base plane is the ecliptic plane. For the center of mass we have

$$COM = (AU, 0, 0), \tag{1}$$

where AU is astronomical unit.

The direction from the origin to the projection of A on the ecliptic plane is put as x-axis. The initial positions of three spacecraft A, B and C are determined accodring to following formulae.

$$\begin{cases} A = (COM_x + \frac{R}{2}, COM_y, COM_z + \frac{\sqrt{3}}{2}R) \\ B = (COM_x - \frac{r}{2}, COM_y + \frac{l}{2}, COM_z - \frac{\sqrt{3}}{2}r) \\ C = (COM_x - \frac{r}{2}, COM_y - \frac{l}{2}, COM_z - \frac{\sqrt{3}}{2}r) \end{cases} \tag{2}$$

where l is the arm length, the radius of the circumscribed circle $R = \frac{l}{\sqrt{3}}$, and the radius of the inscribed circle $r = \frac{R}{2}$.

The spacecraft A is supposed to be at aphelion at the initial moment. The semimajor axis for the orbit is assumed to be equal to one astronomical unit (AU).

$$a = AU \tag{3}$$

Since apocenter distance

$$r_{ap} = (1+e)a = |r_A| = \sqrt{\left(a + \frac{R}{2}\right)^2 + (\frac{\sqrt{3}}{2}R)^2} = \sqrt{a^2 + aR + R^2}, \tag{4}$$

the eccentricity is equal to

$$e = \frac{r_{ap}}{a} - 1 = \sqrt{1 + \frac{R}{a} + \left(\frac{R}{a}\right)^2} - 1 \tag{5}$$

The inclination can be determined as

$$i = \arcsin \frac{A_z}{AU + A_x} = \arcsin \frac{\frac{\sqrt{3}}{2}R}{AU + \frac{R}{2}} \tag{6}$$

According to the initial position of the spacecraft A being at its aphelion, the argument of periapsis:

$$\omega = \frac{3\pi}{2} \tag{7}$$

These four parameters (a, e, i, w) are the same for all orbits in the formation.

The longitude of the ascending node, Ω, and the mean anomaly, M are different for each orbit and given by:

$$\begin{cases} \text{A: } \Omega, M \\ \text{B: } \Omega + \frac{2\pi}{3}, M - \frac{2\pi}{3} \\ \text{C: } \Omega - \frac{2\pi}{3}, M + \frac{2\pi}{3} \end{cases} \tag{8}$$

Since the vertice A is situated at aphelion at $t = 0$ and the projection of its radius-vector on the ecliptic plane is x-axis, $\Omega = \frac{3\pi}{2}$ and $M = \pi$.

Using these parameters it is possible to calculate initial velocities of A, B and C. To do this it is convenient to convert Keplerian elements to Cartesian coordinates.

A great number of papers on conversion of Keplerian elements to Cartesian elements have been published. For instance this paper is very straightforward [1].

2.3 The orbit calculation without Earth

Having initial positions and velocities of three spacecraft we can calculate the orbits taking into account only the attraction force from the Sun by solving a system of differential equations:

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{z} = v_z \\ \dot{v_x} = -\frac{GM_s x}{r^3} \\ \dot{v_y} = -\frac{GM_s y}{r^3} \\ \dot{v_z} = -\frac{GM_s z}{r^3}, \end{cases} \tag{9}$$

where M_s is a mass of the Sun and r is the distance of a spacecraft from the Sun. A numerical solution of this one-body problem is obtained with Dormand Prince 5 method which is a member of the RungeKutta family of Ordinary Differential Equations (ODE) solvers and calculatee fifth-order accurate solutions.

2.4 The orbit calculation with Earth

The initial parameters are the same as if it was with only the Sun but the system of ODE is slightly different:

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{z} = v_z \\ \dot{v_x} = -\frac{GM_s x}{r^3} - \frac{GM_e(x-x_e)}{r_e^3} \\ \dot{v_y} = -\frac{GM_s y}{r^3} - \frac{GM_e(y-y_e)}{r_e^3} \\ \dot{v_z} = -\frac{GM_s z}{r^3} - \frac{GM_e z}{r_e^3}, \end{cases}, \tag{10}$$

where M_e is a mass of Earth, r_e is the distance of a spacecraft from Earth and x_e and y_e are coordinates of Earth at given moment of time.

For simplicity the orbit of Earth is considered to be circular.

3 Results

3.1 The obrits without Earth

Examples of results are represented in the pictures below. The number of steps taken is 1024.

The arm length change and relative velocities meet the requirements.

The arm length change for the arm AB is much larger than for the arm BC since vertices B and C are symmetric with respect to the x-axis, i.e. they are 'symmetrically closer' to each other than to A. In order to know how the formation would behave if this ideal case did not take place it is necessary to break this symmetry which will be done in later work.

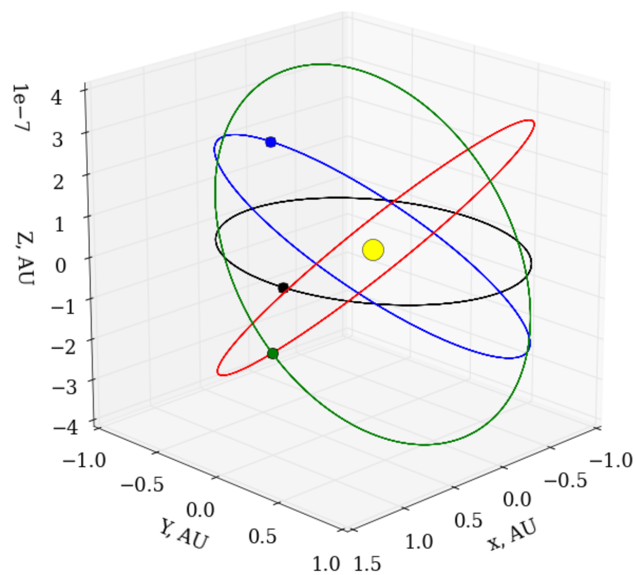

Figure 1: Orbits of spacecraft

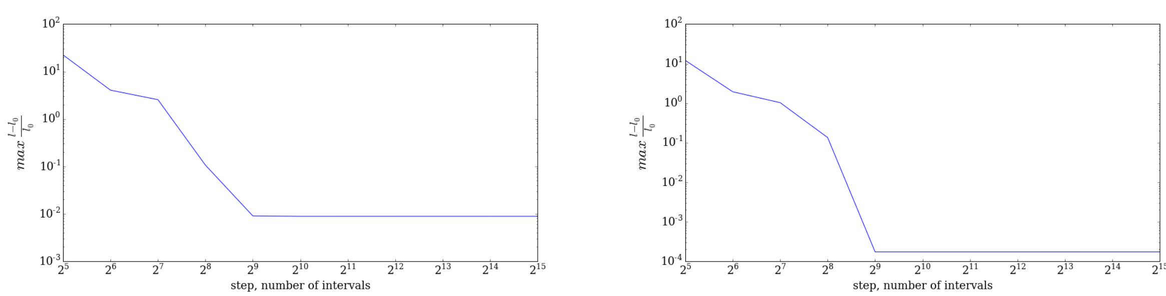

Figure 2: Convergence of Dormand Prince 5 method for the arms AB and BC respectively

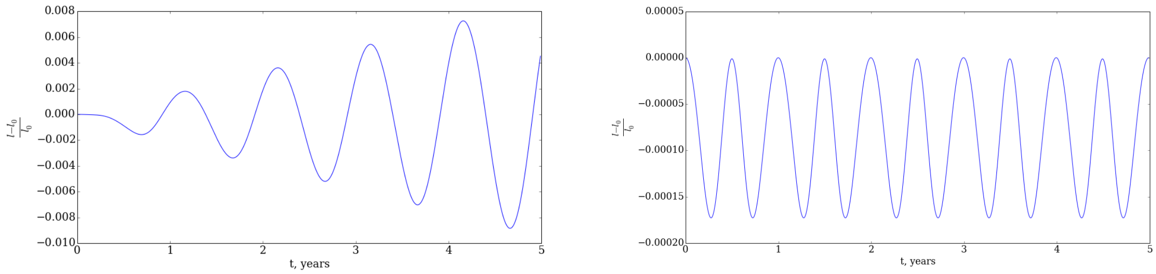

Figure 3: The change of arm length for the arms AB and BC respectively

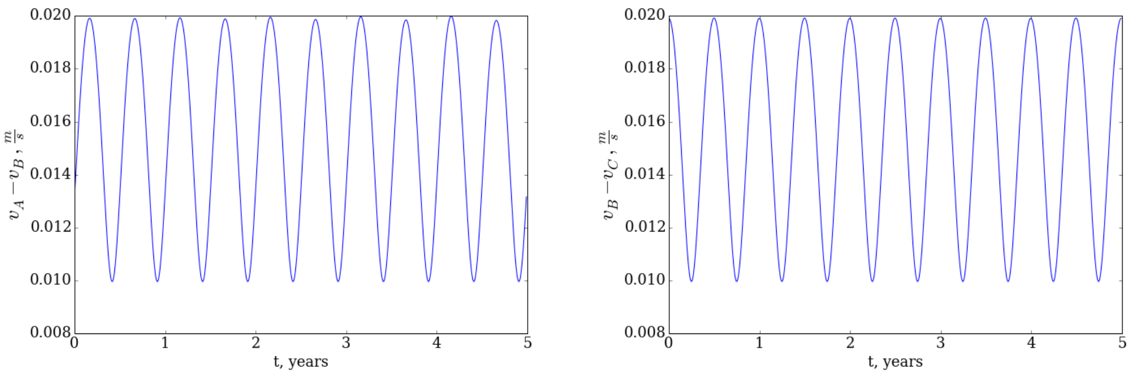

Figure 4: Relative velocities between A and B and between B and C respectively

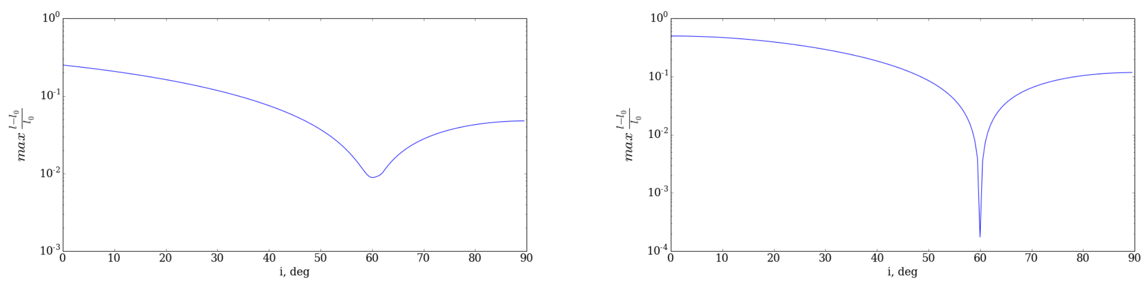

Figure 5: The dependence of maximum arm length change on inclination for AB and BC

3.2 The obrits with Earth

The results including Earth are presented in the pictures below. The number of steps taken is .

The trailing angle is 20 degrees. The method does not work for 5 degrees yet. This case is planned to be investigated now.

The velocities still meet the requirement but the maximum arm length change for the arm AB is greater than 2%. The inclination and initial position of the detector can be changed to decrease this arm length change.

Also the initial arm length can effect the maximum arm length change. Its influence is to be studied.

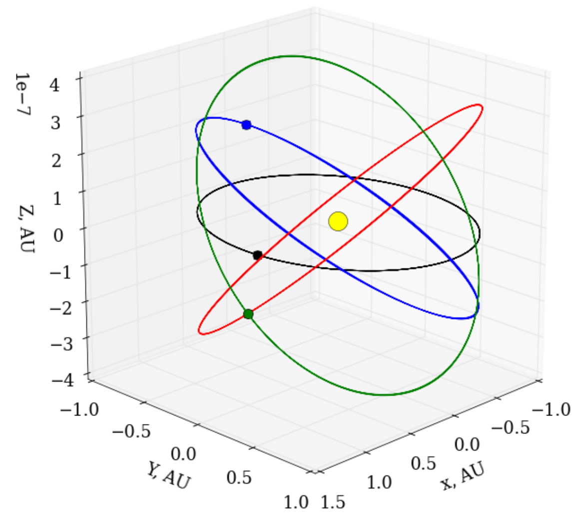

Figure 6: Orbits of spacecraft

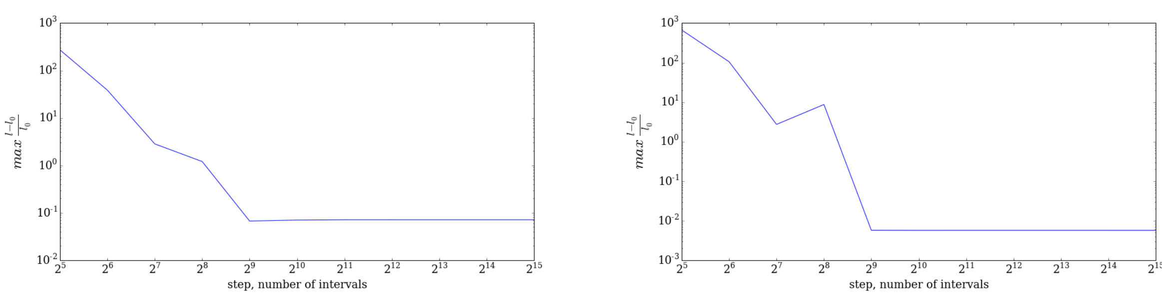

Figure 7: Convergence of Dormand Prince 5 method for the arms AB and BC respectively

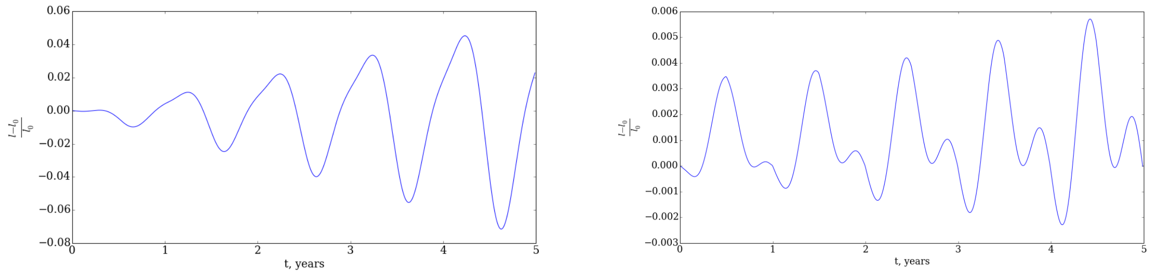

Figure 8: The change of arm length for the arms AB and BC respectively

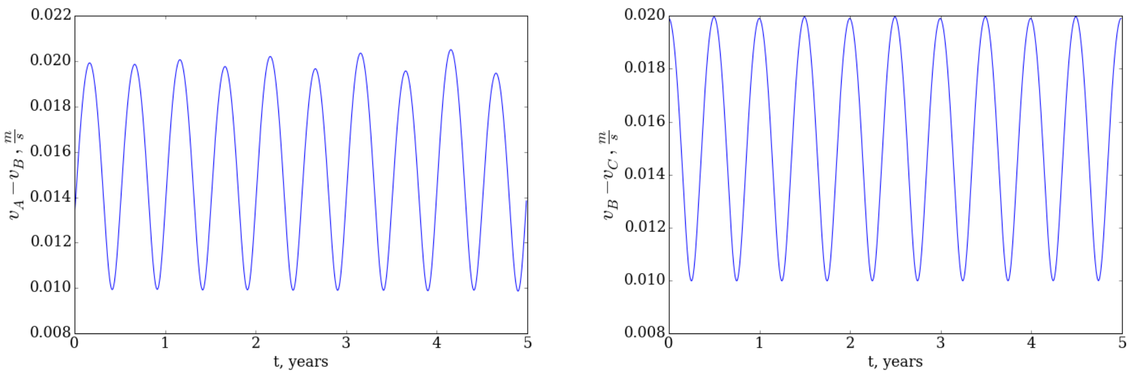

Figure 9: Relative velocities between A and B and between B and C respectively

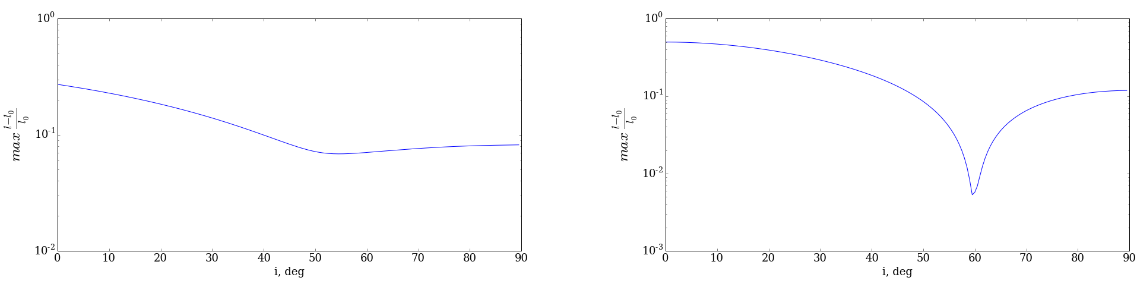

Figure 10: The dependence of maximum arm length change on inclination for AB and BC

References

[1] M Eng and René Schwarz. Keplerian orbit elements – cartesian state vectors. 2014.

[2] Steven P Hughes. Preliminary optimal orbit design for the laser interferometer space antenna(lisa). *Advances in the Astronautical Sciences*, 111(1):61–78, 2002.

[3] Guangyu Li, Zhaohua Yi, Gerhard Heinzel, Albrecht Rüdiger, Oliver Jennrich, Li Wang, Yan Xia, Fei Zeng, and Haibin Zhao. Methods for orbit optimization for the lisa gravitational wave observatory. *International Journal of Modern Physics D*, 17(07):1021–1042, 2008.