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**Effects of scattered light by AERM**

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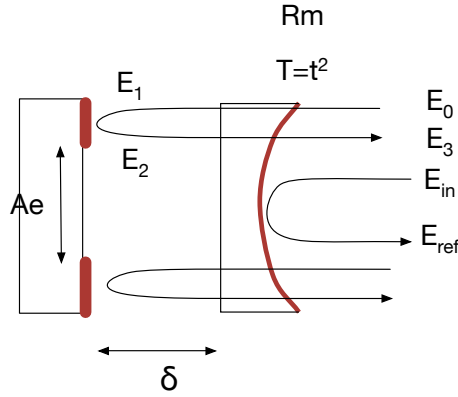
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## 1 Introduction

The noise induced by scattered lights on the surface of AERM is analyzed, and it is shown that the noise is much smaller than the noise induced by the scattering by the ESD coating on AERM.

A section is added for the estimation of the effect using the new geometry for the substrate LF5.

## 2 Reflection by ESD



**Figure 1 Reflection by ESD**

The amplitude of the TEM00 mode of the field reflected by the ESD,  $E_3$ , is given by

$$\begin{aligned}
 A_{00} &= A \exp(-i\phi) \\
 A &= E_0 T \exp\left(-2 \frac{r_0^2}{w^2}\right) / \sqrt{1 + \left(k \frac{n}{2R_m} w^2\right)^2} \\
 \phi &= 2k\delta
 \end{aligned} \tag{1}$$

$r_0$  is the inner radius of the ESD,  $w$  is the beam size on ETM,  $R_m$  is the RoC of ETM,  $T$  is the power transmittance of ETM, and  $\delta$  is the relative motion of ERM and ETM. Time independent phase is neglected which does not contribute to the noise. The derivation is given in appendix, ESD reflection.

The TEM00 mode reflected by ETM is  $E_{ref} + E_3$ , i.e.,  $E_0(1 + A \exp(-i\phi)) \approx E_0(1 - i2Ak\delta)$ . This is to be compared with the signal with strain  $h$ ,  $E_0 \times 2ikhL$ , where  $L$  is the arm cavity length. The noise by the reflection by ESD is derived as  $h(\text{noise by ESD}) = A/L \times \delta$ . Numerically,  $A \sim 10^{-9}$ , and  $h(\text{ESD}) \sim 10^{-9} / 4000 \times \delta$ . The longitudinal noise requirement for the suspended test mass is  $10^{-19} \text{ m}/\sqrt{\text{Hz}}$  at 10Hz, and this noise can be neglected even when the relative motion between ETM and ERM is somehow enhanced more than the suspended motion itself.

### 3 Reflection by Bevel

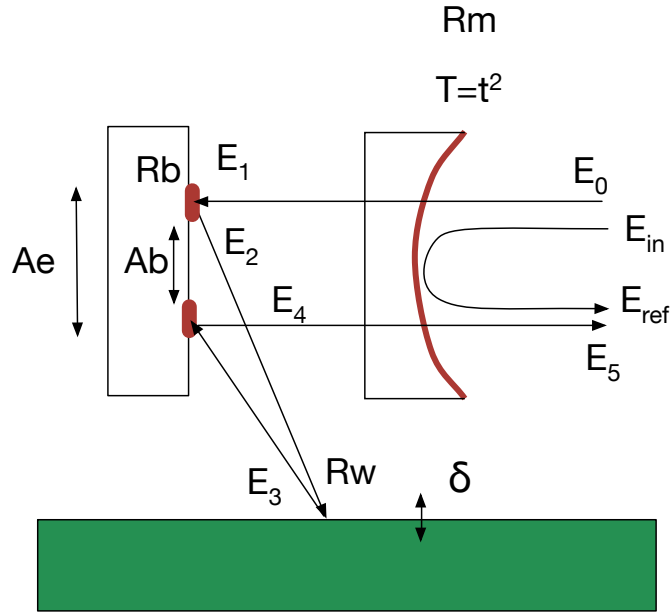


Figure 2 Transmission

The noise considered comes from the reflection by the bevel,  $E_1$  to  $E_2$ , reflected by the wall,  $E_2$  to  $E_3$ , then reflected by the bevel again,  $E_3$  to  $E_4$ , and comes back to the arm,  $E_5$ . The full derivation is given in the Appendix, Bevel reflection. Exact calculation is too complicated, and over estimation is done by the following simplification.

First, the gold coating for the ESD is neglected. The area of the ESD,  $S(ESD)$ , is  $\pi^2(A_{TM}^2 - A_e^2)/4$ , and that of the bevel,  $S(b)$ , is  $\pi^2(A_e^2 - A_a^2)/4$ . So the ratio  $S(ESD)/S(b) = (A_{TM}^2 - A_e^2) / (A_e^2 - A_a^2) \sim 14$ . When the power distribution of the Gaussian beam is included, the power hitting ESD coating to that hitting bevel is less than this. The surface of the ESD coating is as smooth as the test mass surface, so the large angle scattering is small, and let us assume that the total reflection is 10ppm. The reflection by the bevel is assumed to be 0.04, and  $S(ESD) \times 10^{-5} / S(b) \times 0.04 = 0.003$ , so the contribution from the reflection by the ESD is order of magnitude smaller than that by bevel. For the reverse process, from the wall to bevel or EDS to the arm, the same argument can be applied to estimate that the reflection by ESD coating can be neglected.

The field reflected by the bevel hits the wall. The reflection by the wall to the mirror is not a normal reflection, but rather a back scattering with finite incident angle, like  $E_2$  to  $E_3$  reflection. It is assumed that 10% of the energy is scattered to larger angle with uniform distribution, i.e., power is proportional to the solid angle of the target. Among the field reflected by the wall, the power captured by the bevel area is calculated and all power is assumed to reflect back into the arm. The field from the arm to bevel to wall to bevel to arm is assumed to propagate coherently, and the direction at the end in the arm is assumed to be parallel to the cavity axis.

Then the amplitude of  $E_5$  in Fig.2 is given by the same expression as eq.(1), only with an extra factor which comes from the solid angle of the bevel.

$$\begin{aligned}
A_{00} &= A \exp(-i\phi) \\
A &= E_0 T \exp\left(-2 \frac{r_0^2}{w^2}\right) / \sqrt{1 + \left(k \frac{n}{2R_m} w^2\right)^2} \cdot C \\
\phi &= 2k\delta \\
C &= R_b \sqrt{R_w} A_b (A_e - A_b) / 4B^2
\end{aligned} \tag{2}$$

$R_b$  is the power reflectance of AERM,  $R_w$  is the reflectance of the wall, and  $B$  is a typical distance between the wall and bevel,  $\sim 1$ m. The numerical value of  $C$  comes out to be  $7 \times 10^{-6}$ . This extra factor mainly come from two factors, one is the reflectance of the ERM babel surface and the small solid angle of the bevel. The ESD coating area, which has larger solid angle, does not contribute much because of the low scattering toward the cavity.

The numerical value of the amplitude  $A$  comes out to be  $\sim 10^{-14}$ , and the strain noise is  $10^{-14} / 4000 \times \delta$ , where  $\delta$  is the motion of the wall. The motion of the ground is  $10^{-9}$  m/ $\sqrt{\text{Hz}}$  at 10Hz, so  $h(\text{bevel}) \sim 10^{-26}$  m/ $\sqrt{\text{Hz}}$ , which is well below the signal level.

There are many simplifications done to carry out this calculation, and each simplification is based on the argument that large safety margin is added in the estimation. The argument can be applied to any scattering source on the AERM, anywhere not covered by the ESD coating. So the conclusion is that the upper limit of the noise induced by lights scattered by AERM surface is negligible.

#### 4 Revision for the new substrate material LF5

The geometry of the AERM is

“ESD Gold starts at 226 mm aperture (223 was SF2 design for the hole), and this area is polished and the power reflectance is 0.05.

Bevel starts at 190 and ends at 192 mm (assume only 1mm bevel for now)”

Using the same formula as (1), the reflection by the flat area between the ESD inner aperture and the hole, increase the TEM00 mode by 30% compared to the estimation given above. The reflection by the bevel reduces by factor of 5, based on the formula (2).

These results show that the noise induced by the reflections by various parts of the newly proposed AERM is essentially the same as the original AERM design and are fully acceptable.