Phenomenological fit of the peak luminosity from non-precessing binary-black-hole coalescences

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LIGO-T1600018-v4

dated: February 11, 2016

Abstract

This technical document describes a fitting formula for the peak luminosity of non-precessing quasicircular binary-black-hole coalescences, calibrated to numerical-relativity simulations.

1 Introduction

For the coalescence of binary black holes within general relativity, the initial masses and spins of a quasicircular binary uniquely determine the mass and spin of the final black hole, as well as the energetics of the coalescence. For the final mass, final spin and total radiated energy, fitting formulas have been calibrated to numerical relativity simulations, with several such fits published recently. [1, 2, 3]

However, to our knowledge no such results were available yet in the literature for the peak intrinsic gravitational-wave luminosity that the binary reaches close to its merger. In this document, we report a peak-luminosity fit calibrated to 89 quasicircular non-precessing NR simulations, each characterized by the binary's mass ratio and each BH's dimensionless spin component χ_1, χ_2 in the direction of the total angular momentum of the binary. These NR waveforms were produced by the SXS consortium ([4], SpEC code [5]) or with the BAM code [6, 7, 3], and are listed in Table 2. Note that, in contrast to the total radiated energy, the peak luminosity is independent of the total mass of the system. For the fit below, we have considered the strongest spherical harmonic modes, $22, 21, 33, 32, 44, 43$.

The algebraic form of the fit was constructed by sequentially considering reduced one-dimensional data sets of non-spinning, as well as equal-mass and equal-spin simulations, where the parameter space is particularly well covered by the NR data and physics is simpler. The final fit over the full data set is parametrized by the symmetric mass ratio η, [1] a dominant effective spin parameter $\chi_{\text{eff}} = (m_1\chi_1 + m_2\chi_2)/(m_1 + m_2)$ and the subdominant spin difference $\Delta\chi = \chi_1 - \chi_2$. This fit is constrained by consistency to the reduced subsets, and in expanding to higher orders we include only terms that improve the quality of the fit as measured by the Akaike Information Criterion (AIC, [8]) and Schwarz's Bayesian Information Criterion (BIC, [9]).

2 Fitting formula

Including the constraints, the final ansatz for the non-precessing peak-luminosity fit is

$$\begin{aligned} P_{\max}\left[\frac{c^5}{G}\mathrm{W}\right] \approx\ & \left(0.0128513 + f_{21}\chi_{\text{eff}} + f_{22}\chi_{\text{eff}}^2 + f_{23}\chi_{\text{eff}}^3 + f_{24}\chi_{\text{eff}}^4 + f_{25}\chi_{\text{eff}}^5\right)\eta^2 \\ & - 16\left(-0.00355112 + (-0.00771306 + f_{21})\chi_{\text{eff}} + (-0.00387767 + f_{22})\chi_{\text{eff}}^2\right. \\ & \left. + (-0.00112333 + f_{23})\chi_{\text{eff}}^3 + (-0.00211423 + f_{24})\chi_{\text{eff}}^4 + (-0.00235491 + f_{25})\chi_{\text{eff}}^5\right)\eta^4 \\ & + a_{10}\sqrt{1-4\eta}^{\,a_{12}}\,\eta^{a_{11}}\,\Delta\chi + a_{20}\sqrt{1-4\eta}^{\,a_{22}}\,\eta^{a_{21}}\,\Delta\chi^2\,. \end{aligned} \tag{1}$$

[1] We use the convention that $m_1 > m_2$, so that $\eta = q/(1+q)^2$ with $q = m_1/m_2$.

Inserting the fitted coefficients from Table 1, the best-fit formula becomes

$$\begin{aligned} P_{\max}\left[\frac{c^5}{G}\mathrm{W}\right] \approx {} & \left(0.012851 + 0.007822\chi_{\mathrm{eff}} + 0.010222\chi_{\mathrm{eff}}^2 + 0.015806\chi_{\mathrm{eff}}^3 + 0.001136\chi_{\mathrm{eff}}^4 - 0.009868\chi_{\mathrm{eff}}^5\right)\eta^2 \\ & + \left(0.056818 - 0.001747\chi_{\mathrm{eff}} - 0.101507\chi_{\mathrm{eff}}^2 - 0.234915\chi_{\mathrm{eff}}^3 + 0.015658\chi_{\mathrm{eff}}^4 + 0.195569\chi_{\mathrm{eff}}^5\right)\eta^4 \\ & + 0.026161\,(1-4\eta)^{0.541826}\,\eta^{3.162958}\Delta\chi + 0.000777\,(1-4\eta)^{0.449915}\,\eta^{1.780035}\Delta\chi^2\,. \end{aligned} \tag{2}$$

	Estimate	Standard Error	t-Statistic	P-Value
a10	0.0261613	0.0298177	0.877375	0.382978
a11	3.16296	0.589473	5.36574	$8.022358872914045 \cdot 10^{-7}$
a12	1.08365	0.23737	4.56525	0.0000183299
a20	0.000777103	0.00216148	0.359523	0.720175
a21	1.78003	1.35331	1.31532	0.192256
a22	0.89983	0.702625	1.28067	0.204105
f21	0.00782227	0.000857354	9.12373	$6.210370847884475 \cdot 10^{-14}$
f22	0.0102219	0.00109013	9.37669	$2.009584658411308 \cdot 10^{-14}$
f23	0.0158055	0.00494573	3.19579	0.00201378
f24	0.00113562	0.00208548	0.544537	0.587625
f25	−0.00986815	0.00619512	−1.59289	0.115229

Table 1: Coefficients for the full direct fit in η, χ_{eff}, $\Delta\chi$, together with their statistical determinants.

Note that some of the coefficients in Table 1 are only weakly constrained, but including them in the ansatz is indeed preferred by the AIC, BIC and total residual error. Additional work on further improving the physics model of the fit and its statistical significance is in progress.

References

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q	χ_1	χ_2	$L_{\max}$	tag	code
1.00	-0.95	-0.95	0.000716972	d15_q1_sA_0_0_-0.95_sB_0_0_-0.95	SXS
1.00	-0.90	-0.90	0.000743501	d15_q1_sA_0_0_-0.9_sB_0_0_-0.9	SXS
1.00	-0.85	0.00	0.0008708	q1_-85_0_-0.425_T_80_440	BAM
1.00	-0.80	-0.80	0.00075736	d15_q1_sA_0_0_-0.8_sB_0_0_-0.8	SXS
1.00	-0.60	-0.60	0.000815473	d15_q1_sA_0_0_-0.6_sB_0_0_-0.6	SXS
1.00	-0.50	0.00	0.00092796	d19.0_q1.0_s0_0_-0.5_s0_0_0	SXS
1.00	-0.44	-0.44	0.00085812	d15.3_q1.00_sA_0.000_0.000_-0.438_sB_0.000_-0.000_-0.438	SXS
1.00	-0.20	-0.20	0.000941915	d15_q1_sA_0_0_-0.2_sB_0_0_-0.2	SXS
1.00	0.00	0.00	0.00102267	BBH_CFMS_d18_q1_sA_0_0_0_sB_0_0_0	SXS
1.00	0.00	0.50	0.00116985	d18.0_q1.0_s0_0_0.5_s0_0_0	SXS
1.00	0.20	0.20	0.00113277	d15_q1_sA_0_0_0.2_sB_0_0_0.2	SXS
1.00	0.44	0.44	0.00129673	d13_q1_sA_0_0_0.44_sB_0_0_0.44	SXS
1.00	0.50	-0.50	0.00104364	q1_-50_50_0_T_80_400	BAM
1.00	0.60	0.60	0.00144723	d15_q1_sA_0_0_0.6_sB_0_0_0.6	SXS
1.00	0.75	0.75	0.00162813	BBH_SKS_d15.4_q1_sA_0_0_0.750_sB_0_0_0.750	SXS
1.00	0.80	0.80	0.00170496	d15_q1_sA_0_0_0.8_sB_0_0_0.8	SXS
1.00	0.85	-0.85	0.00104765	q1_-85_85_0_T_96_480	BAM
1.00	0.85	0.85	0.0017872	d15_q1_sA_0_0_0.85_sB_0_0_0.85	SXS
1.00	0.90	0.90	0.00187794	d15_q1_sA_0_0_0.9_sB_0_0_0.9	SXS
1.00	0.95	0.95	0.0019825	d15_q1_sA_0_0_0.95_sB_0_0_0.95	SXS
1.00	0.96	0.96	0.00200366	BBH_SKS_d15.4_q1_sA_0_0_0.960_sB_0_0_0.960	SXS
1.00	0.98	0.98	0.00205288	d15.4_q1.00_sA_0.000_0.000_0.980_sB_-0.000_-0.000_0.980	SXS
1.00	0.99	0.99	0.00208938	BBH_SKS_d15.4_q1_sA_0_0_0.995_sB_0_0_0.995_v2	SXS
1.20	0.00	0.00	0.00100585	BBH_SKS_d16_q1.20_sA_0_0_0_sB_0_0_0	SXS
1.50	-0.50	0.00	0.000798153	d17.0_q1.5_s0_0_-0.5_s0_0_0	SXS
1.50	-0.50	0.50	0.000855272	d16.0_q1.5_s0_0_-0.5_s0_0_0.5	SXS
1.50	0.00	0.00	0.000930387	d18.0_q1.5_s0_0_0_s0_0_0	SXS
1.50	0.50	-0.50	0.00104086	d16.0_q1.5_s0_0_0.5_s0_0_-0.5	SXS
1.50	0.50	0.00	0.0011168	d14.0_q1.5_s0_0_0.5_s0_0_0	SXS
2.00	-0.85	0.85	0.000618361	q2_85_-85_-0.283333_T_80_440	BAM
2.00	-0.50	0.50	0.000676021	q2_50_-50_-0.166667_T_80_400	BAM
2.00	0.00	-0.85	0.000728687	q2_-85_0_-0.283333_T_80_400	BAM
2.00	0.00	0.00	0.000773095	d13_q2_sA_0_0_0_sB_0_0_0	SXS
2.00	0.00	0.85	0.000850273	q2_85_0_0.283333_T_80_400	BAM
2.00	0.50	-0.50	0.00092679	q2_-50_50_0.166667_T_80_400	BAM
2.00	0.50	0.50	0.0010603	q2_0.5_0.5_96_460	BAM
2.00	0.60	0.00	0.00103979	d13.9_q2.00_sA_0.000_0.000_0.600_sB_0.000_0.000_0.000	SXS
2.00	0.75	0.75	0.00132149	q2._0.75_0.75_D11.11	BAM
2.00	0.85	0.00	0.00122211	q2_0_85_0.566667_T_80_360	BAM
2.32	0.00	0.00	0.000682528	BBH_SKS_d15_q2.32_sA_0_0_0_sB_0_0_0	SXS
2.51	0.00	0.00	0.000629181	BBH_SKS_d15.5_q2.51_sA_0_0_0_sB_0_0_0	SXS
3.00	-0.85	0.00	0.000371196	q3_0_-85_-0.6375_T_80_480	BAM
3.00	-0.50	-0.50	0.000411786	d14.0_q3.0_s0_0_-0.5_s0_0_-0.5	SXS
3.00	-0.50	-0.50	0.000416532	q3_-0.5_-0.5_96_400	BAM
3.00	-0.50	0.00	0.000420085	d14.0_q3.0_s0_0_-0.5_s0_0_0	SXS
3.00	-0.50	0.50	0.000430804	q3_50_-50_-0.25_T_80_460	BAM
3.00	0.00	0.00	0.000520935	BBH_CFMS_d13.2_q3_sA_0_0_0_sB_0_0_0	SXS
3.00	0.50	-0.50	0.000693338	q3_-50_50_0.25_T_80_400	BAM
3.00	0.50	-0.50	0.000693561	d14.0_q3.0_s0_0_0.5_s0_0_-0.5	SXS
3.00	0.50	0.00	0.000711534	d14.0_q3.0_s0_0_0.5_s0_0_0	SXS
3.00	0.50	0.50	0.000734943	d14.0_q3.0_s0_0_0.5_s0_0_0.5	SXS
3.27	0.00	0.00	0.000472826	BBH_SKS_d14.3_q3.27_sA_0_0_0_sB_0_0_0	SXS
3.50	0.00	0.00	0.000433676	BBH_SKS_d14_q3.50_sA_0_0_0_sB_0_0_0	SXS
4.00	-0.85	0.85	0.000261648	q4_85_-85_-0.51_T_96_552	BAM
4.00	-0.75	-0.75	0.000262425	q4aM075_T_112_448	BAM
4.00	-0.50	-0.50	0.000287548	q4aM05_T_96_384	BAM
4.00	-0.50	0.50	0.000293052	q4_50_-50_-0.3_T_80_400	BAM
4.00	-0.25	-0.25	0.000323052	q4aM025_T_112_448	BAM
4.00	0.00	0.00	0.000366212	d13_q4_sA_0_0_0_sB_0_0_0	SXS
4.00	0.00	0.00	0.000371026	q4a0_T_112_448	BAM
4.00	0.00	0.85	0.000375831	q4_85_0_0.17_T_80_400	BAM
4.00	0.25	0.25	0.000438529	q4a025_T_96_384	BAM

q	χ_1	χ_2	$L_{\max}$	tag	code
4.00	0.50	0.50	0.000541471	q4a05__T__96__384	BAM
4.00	0.75	0.75	0.000705258	q4a075__T__112__448	BAM
4.50	0.00	0.00	0.000312999	BBH__SKS__d13.5__q4.50__sA__0__0__0__sB__0__0__0	SXS
5.00	-0.50	0.00	0.00021029	d13.0__q5.0__s0__0__-0.5__s0Ecc1em3	SXS
5.00	0.50	0.00	0.00038938	d13.0__q5.0__s0__0__0.5__s0Ecc4em4	SXS
5.04	0.00	0.00	0.000266286	BBH__SKS__d13__q5.04__sA__0__0__0__sB__0__0__0	SXS
5.52	0.00	0.00	0.000233385	BBH__SKS__d13__q5.52__sA__0__0__0__sB__0__0__0	SXS
6.00	0.00	0.00	0.000205162	d13__q6__sA__0__0__0__sB__0__0__0	SXS
6.58	0.00	0.00	0.000178372	BBH__SKS__d12.7__q6.58__sA__0__0__0__sB__0__0__0	SXS
7.19	0.00	0.00	0.000154084	BBH__SKS__d12.7__q7.19__sA__0__0__0__sB__0__0__0	SXS
7.76	0.00	0.00	0.000137884	BBH__SKS__d12.4__q7.76__sA__0__0__0__sB__0__0__0	SXS
8.00	-0.85	-0.85	0.0000872657	q8am085v6D10__96c025	BAM
8.00	-0.50	0.00	0.0000999707	d13.0__q8.0__s0__0__-0.5__s0	SXS
8.00	0.00	0.00	0.000129976	d13.0__q8.0__s0__s0	SXS
8.00	0.50	0.00	0.00019735	d13.0__q8.0__s0__0__0.5__s0	SXS
8.00	0.80	0.00	0.000292206	q8a0a08__c0.25__100	BAM
8.00	0.85	0.85	0.000314259	q8++0.85__T__80__200__-4pc	BAM
8.27	0.00	0.00	0.00012308	BBH__CFMS__d12.5__q8.27__sA__-0.000__0.000__-0.000__sB__0.000__-0.000__0.000	SXS
8.73	0.00	0.00	0.000112447	BBH__SKS__d12.2__q8.73__sA__0__0__0__sB__0__0__0	SXS
9.17	0.00	0.00	0.000103859	BBH__SKS__d12.5__q9.17__sA__0__0__0__sB__0__0__0	SXS
9.66	0.00	0.00	0.0000957664	BBH__SKS__d12__q9.66__sA__0__0__0__sB__0__0__0	SXS
9.99	0.00	0.00	0.0000905358	BBH__SKS__d12.2__q9.99__sA__0__0__0__sB__0__0__0	SXS
10.00	0.00	0.00	0.0000878912	q10c25e__T__112__448	BAM
18.00	-0.80	0.00	0.0000218193	q18a0aM08c025__96__fine	BAM
18.00	-0.40	0.00	0.0000254528	q18a0aM04c025__96__fine	BAM
18.00	0.00	0.00	0.0000323058	q18a0a0c025__144	BAM
18.00	0.40	0.00	0.0000451323	q18a0a04c025v4__eta1__T__120__SH	BAM

Table 2: Numerical-relativity waveforms used in the calibration of our fits with their relevant parameters and their peak luminosity, as well as a tag that uniquely identifies the simulation in its source data set, and the name of the code used to generate it.