Analysis of Scattering in Lungo Arm Cavities

This documents treats the problem of scattered light noise in Lungo arm cavities. The ultimate goal is to understand what should be the diameter of the beam tube to keep scattered light noise below the sensitivity curve.

Diameter of LIGO tubes is 120 cm. Projection of beam tube baffles on the radial plane is 10cm. At the same time beam path is off centered from the tube axis by 20cm. Closest distance between the beam path and edge of baffles is 30cm.

Diameter of Kagra beam tube is 80cm, projection of arm baffles to the radial plane covers 4cm. Effective distance from the beam axis to the edge of baffles is 36cm.

Necessary diameter of Lungo beam tube is estimated based on two effects: noise from scattered light and amount of clipping loss on the baffles in the beam tube. These two problems are addressed in this report.

In analysis of scattering, formalism from Kip-T940063 is used. Equations for DARM displacement noise and surface aberrations of test masses are derived in reports T070089 and T1300354 by Peter, Mike and Hiro.

1 Overview of the method

Scattering noise arises when a small fraction of light is scattered out from the main beam, hits the wall and recombines into the main beam as shown in Fig 1. Motion of the wall couples through the phase modulation of the main beam and radiation pressure on the test mass.

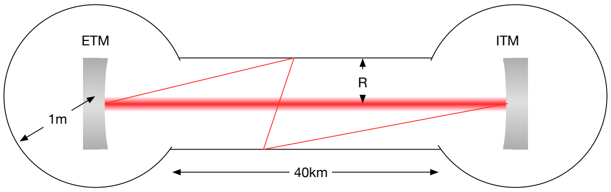

Figure 1: Scheme of back and forward scattering in the arm cavity. Light scatters out from the main beam and hits the wall. Some light scatters back into the main beam. Another part of light bounces multiple times between the walls and recombines with the main beam on the opposite test mass.

Amount of light scattered out from the main beam depends of BRDF of the mirror B_{mir}. Once scattered light hits the scattering object, it partially scatters back into the main beam. In this report, we assume that scattering object is a baffle or a chamber wall. The probability to scatter light into unit solid angle equals to

$$B_{sc} = 0.02\ St^{-1}$$

Light should be scattered into particular solid angle to reach the main beam. Cross section σ of this process is given by equation

$$\sigma = \lambda^2 B_{mir}$$

and total optical power that recombines with the main beam from the annulus of width $d\theta$ is

$$\begin{aligned} \frac{dP}{P_0} &= 2\pi sin\theta d\theta B_{mir} B_{sc} \frac{\lambda^2}{r^2} B_{mir} \\ &= \lambda^2 B_{sc} dK \end{aligned}$$

where θ is angle between the main beam path and scattered light, $r = R/sin\theta$ is distance from the optic to the scattering object; K combines factors that depend on θ; P_0 is total power in the main beam.

Differential arm noise x_{darm} due to motion of the scattering object x_{sc} is given by equation

$$x_{darm}^2 = \frac{1}{2} \frac{\int dP}{P_0} x_{sc}^2$$

Taking into account radiation pressure noise we get

$$x_{darm}^2 = \frac{1}{2} \left(\lambda^2 + \left(\frac{8\Gamma P_0}{cM\pi f^2} \right)^2 \right) B_{sc} x_{sc}^2 K \qquad (1)$$

where M is mass of test masses, f is frequency, Γ is signal gain. $\Gamma = 15.7$ for SRM transmission of 0.35, $\Gamma = 4$ for SRM transmission of 0.1, $\Gamma = 1.5$ for SRM transmission of 0.03.

The goal of this work is to find K $[m^{-2}]$ for wide and narrow angle scattering. $K = \int dK$ represents an integral over scattering objects.

2 Optical loss

First we consider scattering out from the main beam. Probability of a cavity photon to be scattered out from the main beam at angle θ to unit solid angle is given by equations

$$\begin{aligned} B_{mir,wide} &= \frac{Z_{wide}}{\pi} cos\theta, \ \pi/2 > \theta > 0.1 \\ B_{mir,nar} &= \frac{Z_{nar}}{\theta^n}, \ 0.1 > \theta > 0 \end{aligned} \qquad (2)$$

where Z_{wide} and Z_{nar} are determined by total power loss from test masses in wide and narrow angles.

2.1 Wide angle

For advanced LIGO mirrors Z_{wide} is expected to be 10ppm. However, for some of test masses it is measured on the level of 30ppm. For Lungo, we expect that wide angle scattering will increase proportionally to the beam area

$$1064nm: \ Z^{lungo}_{wide} = \frac{w^2_{lungo}}{w^2_{ligo}} Z^{ligo}_{wide} \approx 35ppm$$

$$2100nm: \ Z^{lungo}_{wide} = \frac{w^2_{lungo}}{w^2_{ligo}} \left(\frac{1064}{2100}\right)^2 Z^{ligo}_{wide} \approx 18ppm$$

where w_{lungo} and $w_{ligo} = 6.2cm$ are beam sizes on test masses in Lungo and aLIGO interferometers. Assuming g-factor of arm cavities close to zero for Lungo, we get $w_{lungo} = 11.65cm$ for 1064nm light and $w_{lingo} = 16.38cm$ for 2100nm light.

2.2 Narrow angle

Exponent n is determined by the coating profile. If there is aberration on the coating with wavelength λ_{ab} then scattering cone has half angle of $\theta = \lambda/\lambda_{ab}$ where λ is optical wavelength. Optical power scattered at angle θ is a function of aberration amplitude. Equation for narrow angle scattering was simulated by Hiro based on the power spectrum of LIGO coating profile. Parameters from equation 2 are

$$1064nm: \ Z_{nar} = 3.2 \cdot 10^{-8}, \ n = 2.45$$

$$2100nm: \ Z_{nar} = 1.1 \cdot 10^{-8}, \ n = 2.45$$

In order to find optical loss from small angles we integrate $B_{mir,nar}$ over solid angles determined by $0.1 > \theta > R_c/L$, where R_c is coating radius. Total loss in the narrow angles is given by equation

$$L_{nar} = \frac{2\pi}{n-2} \frac{Z_{nar}}{(R_c/L)^{n-2}}$$

2.3 Clipping loss

When beam passes through the baffles in the arm cavities or reflects from the optic, part of the beam is clipped and lost. Fig 2 shows amount of optical loss due to baffles of different sizes. Test mass radius in aLIGO is 17cm but coating quality at the edge of the mirror is worse than at the center. Beam decentering on test masses also effectively reduces R_c. For this reason, we assume coating radius is 2cm smaller compared to test mass radius.

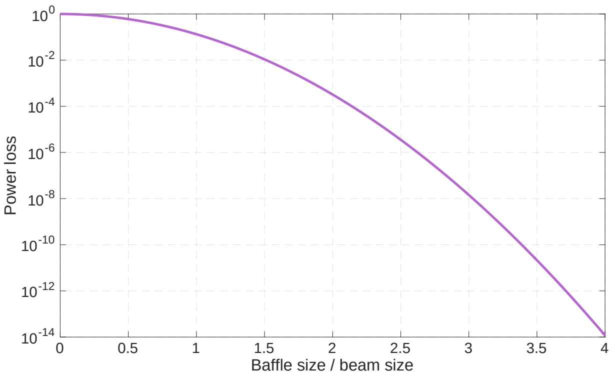

Figure 2: Power loss of the Gaussian beam due to baffles.

2.4 Summary of losses

Table 1 summarizes scattering loss for advanced LIGO and Lungo interferometers. Actual loss in advanced LIGO cavities is smaller compared to the model since we consider pessimistic loss scenario.

	w, cm	R_c, cm	Wide, ppm	Narrow, ppm	Clip, ppm	Total, ppm
aLIGO	6.2	15	10	47	6	63
Lungo 1064nm	11.7	28	35	100	10	145
Lungo 2100nm	16.4	40	18	27	7	52

Table 1: Scattering from one test mass in aLIGO and Lungo.

Narrow angle scattering can be divided into two categories. Scattered light from test mass can hit the beam tube or reach opposite chamber. Distribution of scattered light between these two categories depend on tube radius R. In case of large beam tube radius $R \ll R_c$, most scattered light will reach the opposite chamber and can be damped using suspended beam baffle. In the other extreme case $R = R_c$ all narrow angle scattered light hits the beam tube.

Beam tube radius should be large enough to avoid significant power loss on the baffles. Amount of light clipped by every baffle should be less than $\sim 10^{-8}$. According to Fig 2, this condition implies that radius of the tube should be

$$R > 3w.$$

For cryogenic Lungo operating at 2100nm, minimal radius of beam should be $R = 49.2cm$ plus radial baffle size of $\approx 5cm$. This gives total beam tube diameter of 110cm.

3 Calculation of K

In this section we estimate parameter K from equation 1 for wide and narrow angle scattering. We find K assuming that noises from scattering elements sum incoherently.

3.1 Wide angle

We assume that radius of the chamber is 1m. At scattering angle $\theta < 0.3$ scattered light gets into the tube, at larger angles light hits chamber walls. Fig 3 shows that most significant contribution to DARM in wide angle scattering model comes from $\theta \approx 0.6rad$. Total integral over the chamber and tube elements gives

$$1064nm: \ K_{wide} = 2.5 \cdot 10^{-10} \ [m^{-2}]$$

$$2100nm: \ K_{wide} = 6.8 \cdot 10^{-11} \ [m^{-2}]$$

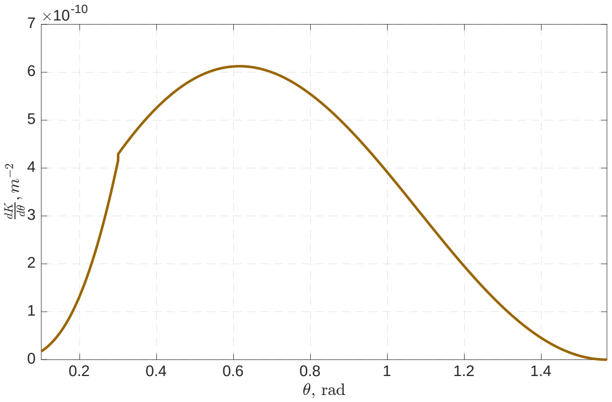

Figure 3: Contribution to backscattered power from different θ assuming wide angle scattering model for 1064nm light.

3.2 Back scatter in beam tube

Scattered light hits the beam tube if $\theta < R/L$. Some light scattered back into the main beam through the same test mass. Summing together tube segments (baffles) we get

$$1064nm: \; K_{tube} = 8.9 \cdot 10^{-10} \; [m^{-2}]$$
$$2100nm: \; K_{tube} = 8.8 \cdot 10^{-11} \; [m^{-2}]$$

assuming beam tube radius $R = 50cm$.

Fig 4 shows dependence of K on beam radius R. Along the y-axis we plot $\sqrt{K}$ since DARM noise is proportional to this number. Scattered light noise from the beam tube increases by a factor of 1.7 if we choose $R = 0.4m$ compared to $R = 0.6m$.

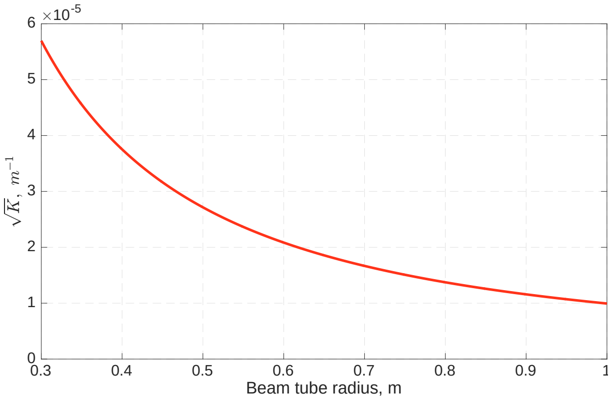

Figure 4: Scattering noise level in DARM for different beam tube radii R for 1064nm light.

3.3 Forward scatter in beam tube

Scattered light noise from multiple bounces is expected to be smaller compared to noise that comes from straight backscattering mechanism. It we consider scenario when light scattered from the test mass hits the baffle, then another baffle and then scatters forward to another test mass, we get

$$dK = \frac{1}{(L-r)^2} Y B_{mir}(\theta) B_{mir}(\tilde{\theta}) 2\pi sin\theta d\theta$$

where $Y \approx 2\pi B_{sc}$ denotes a fraction of light scattered from the first baffle to the place of secondary scattering, $\tilde{\theta} = 1/(L/R - 1/\theta)$ is angle of incidence of scattered light on the opposite optic.

In estimation of K for forward scattering in the beam tube we assume radius of beam tube $R = 0.5m$. Summation over baffles gives

$$1064nm: \; K_{forward} = 1.8 \cdot 10^{-10} \; [m^{-2}]$$
$$2100nm: \; K_{forward} = 1.8 \cdot 10^{-11} \; [m^{-2}]$$

3.4 Clipping in the beam tube

In the case if size of the tube is not significantly larger than size of the beam, then a fraction of light scatters back from baffle towards the optic according to equation

$$\triangle K_{clip} = \frac{1}{r^2} X_{clip} B_{mir} \qquad (3)$$

where r is distance from the baffle to test mass, X_{clip} is the fraction of the beam clipped by the baffle. For $R = 3w$ we get $X_{clip} = 10^{-8}$

Assuming $R = 0.5m$ and summing over baffles we get

$$1064nm: \; K_{clip} = 1.1 \cdot 10^{-18} \; [m^{-2}]$$
$$2100nm: \; K_{clip} = 4.9 \cdot 10^{-11} \; [m^{-2}]$$

3.5 Distribution of scattering

In this section we analyze from with part of the tube we get most contribution to K. Distribution of scattered light in the tube significantly depends on n. Apart from the main mirror BRDF model with $n = 2.45$, we consider another two:

$$Z_{nar} = 1.1 \cdot 10^{-10}, \; n = 3$$
$$Z_{nar} = 3.5 \cdot 10^{-7}, \; n = 2.2$$

where Z_{nar} is normalized for mirror scattering of $100ppm$ in the narrow angle.

Fig. 5 shows dK/dL depending on the position of the scattering object in the tube. For $n = 2.5$ contribution from all segments is almost equal, for $n > 2.5$ most scattered light noise comes from the far end of the tube relative to the scattering mirror. In case of $n < 2.5$ significant scattered light noise comes from the closer end of the beam tube relative to the scattering mirror.

3.6 Opposite chamber

Scattered light noise from narrow angles that does not hit the beam tube, hits the chamber of the opposite test mass. In this case $R/L > \theta > R_c/L$. Assuming $R = 0.5m$ and

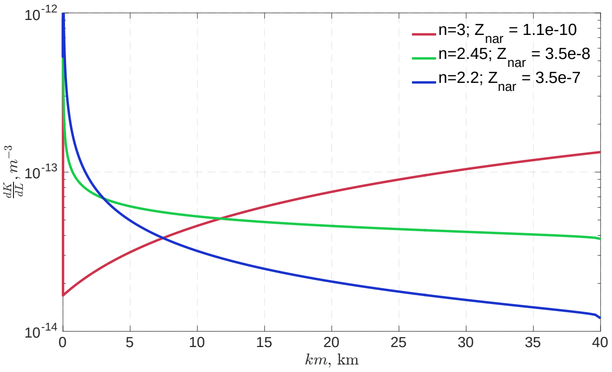

Figure 5: Contribution to K from different tube segments.

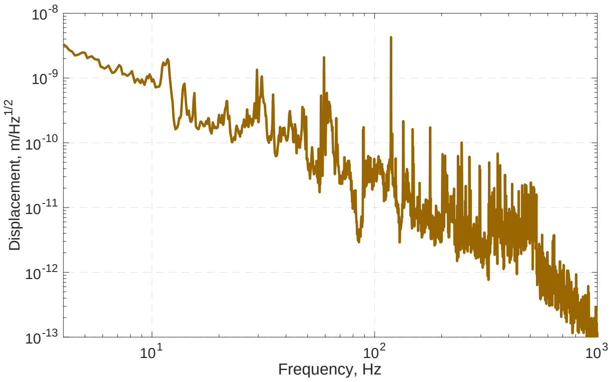

Figure 6: Motion of the beam tube at LLO.

$R_c = 0.28m$ for 1064nm Lungo and $R_c = 0.4m$ for 2100nm Lungo, we sum over annulus and get

$$1064nm: \ K_{chamber} = 1.2 \cdot 10^{-9} \ [m^{-2}]$$
$$2100nm: \ K_{chamber} = 2.6 \cdot 10^{-11} \ [m^{-2}]$$

4 Scattered light noise

In this section we estimate contribution of scattered light noise to DARM. We discuss contribution of phase noise and radiation pressure noise, show motion of the beam tube and compute projections to DARM for 1064nm Lungo and 2100nm Lungo.

4.1 Phase noise vs radiation pressure

Total DARM noise is the sum of phase noise and radiation pressure noise. Expected total arm power for 1064nm Lungo is 1.1MW and for 2100nm Lungo is 3MW. In both cases SRM transmission of 0.03 and $\Gamma = 1.5$. Radiation pressure actuates on test masses and scales as $1/f^2$ compared to phase noise. Crossover frequencies are

$$1064nm: \ f = 8.1Hz$$
$$2100nm: \ f = 8.5Hz$$

These frequencies imply that phase noise becomes the dominant scattered light noise in the whole interferometer sensitivity band. For comparison, aLIGO crossover frequency is $f = 50Hz$ for intracavity power of $770kW$, $\Gamma = 15.7$ and mass of test mass is 40kg.

4.2 Tube motion

In order to estimate DARM noise due to scattered light noise according to equation 1, we need to know motion of the chamber and tube. In this work we use motion of the tube measured at LLO and shown in Fig 6

4.3 Noise projection

Fig 7 shows contribution of scattered light noise to DARM assuming beam radius of $R = 50cm$ for 1064nm Lungo and 2100nm Lungo. Scattered light noise is incoherent sum from four test masses.

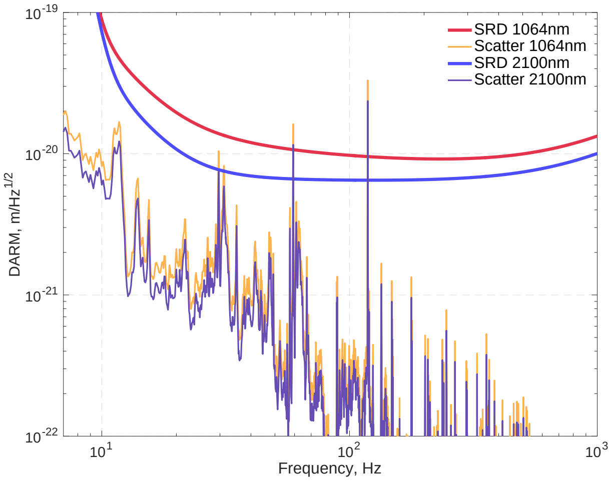

Figure 7: Contribution of scattered light noise to Lungo sensitivity from four test masses.

Fig 7 shows that scattered light noise is projected below the sensitivity curve for $R = 0.5m$. In this estimation we assume that noise from the opposite chamber is reduced by a factor of 100 at 10Hz by suspending the baffle.

5 Conclusions

Scattered light noise from the beam tube projects below sensitivity noise limit if $R > 3w$. In this regime noise from beam clipping is comparable with noise that arises due to scattering on the test mass. Scattered light noise grows exponentially as smaller R. For this reason, beam tube radius should be larger than

$$R > 3w + H_{baffle} + H_{tol}$$

where H_{baffle} is height of baffle projection on the radial plane of the tube, H_{tol} is tolerance of the position of tube axis and beam axis.

For cryogenic interferometer operating at 2100nm, beam tube diameter should be $> 120cm$ assuming $H_{tol} = 5cm$.