Calibrating optical springs

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September 14, 2016

In a signal-recycled interferometer, the transfer function of DARM fluctuation ΔL to dark-port power fluctuation ΔP is determined by

- the ITM transmissivity T_{i},
- the SRM transmissivity T_{s},
- the microscopic one-way SRC phase ϕ,
- the homodyne angle ζ, and
- the optical gain g, which depends on the above four quantities as well as the beamsplitter power P_{bs}.

We also define the arm pole $f_{\mathrm{a}} = cT_{\mathrm{i}}/8\pi L$, and the SRM amplitude reflectivity $r_{\mathrm{s}} = \sqrt{1 - T_{\mathrm{s}}}$.

For O1, we aimed for *resonant sideband extraction*: $\phi = \zeta = \pi/2$. The DARM optical plant has a particularly simple form in this case:

$$\frac{\Delta P}{\Delta L} = \frac{g}{1 + \mathrm{i}f/p}, \qquad (1)$$

with

$$p = f_{\mathrm{a}} \frac{1 + r_{\mathrm{s}}}{1 - r_{\mathrm{s}}}. \qquad (2)$$

If $\phi \neq \pi/2$ or $\zeta \neq \pi/2$, the DARM optical plant is more complicated:

$$\frac{\Delta P}{\Delta L} \propto \frac{t_s e^{i\beta}\left[\left(1 - r_s e^{2i\beta}\right)\cos\phi\cos\zeta - \left(1 + r_s e^{2i\beta}\right)\sin\phi\sin\zeta\right]}{1 + r_s^2 e^{4i\beta} - 2 r_s e^{2i\beta}\left[\cos 2\phi + (\mathcal{K}/2)\sin 2\phi\right]}\sqrt{\frac{2P_{bs}\omega_0^2}{\omega_a^2 + \omega^2}}, \tag{3}$$

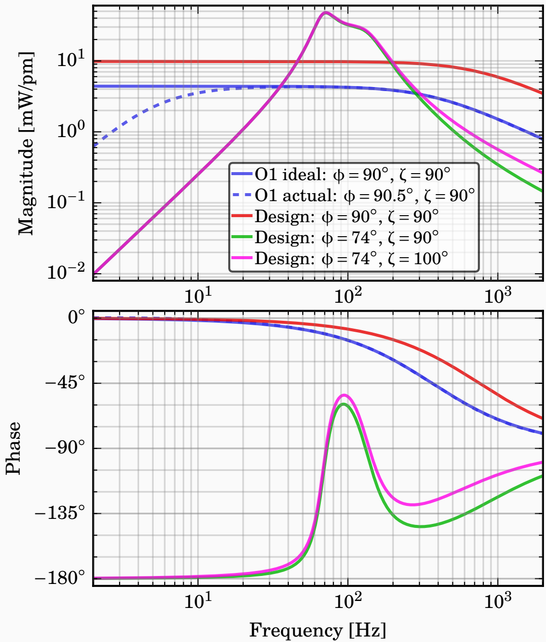
where $\beta = -\arctan f/f_a = -\arctan \omega/\omega_a$,

$$\mathcal{K} = \frac{8P_{bs}}{ML^2}\frac{\omega_0}{\omega^2(\omega_a^2 + \omega^2)}, \tag{4}$$

and ω_0 is the angular frequency of the laser.

(Buonanno and Chen 2001, also Rob Ward's thesis)

DETUNED RSE: CURRENT AND FUTURE OPTICAL PLANTS



We can recast this transfer function into a form that is (nearly) a zpk representation:

$$\frac{\Delta P}{\Delta L} = g \times \frac{1 + \mathrm{i}f/z}{\left(1 + \frac{\mathrm{i}f}{|p|Q_p} - \frac{f^2}{|p|^2}\right) - \frac{\xi^2}{f^2}}, \tag{5}$$

with parameters g, $|p|$, Q_p, z and ξ.

The RSE pole p is now complex:

$$p = f_{\mathrm{a}} \times \frac{1 - r_{\mathrm{s}} e^{2i\phi}}{1 + r_{\mathrm{s}} e^{2i\phi}}. \tag{6}$$

The Q of the pole is

$$Q_p = \frac{|p|}{2 \operatorname{Re} p}. \tag{7}$$

As $\phi \to \pi/2$, the pole p becomes real, and so Q_p attains its minimum value of 1/2.

The RSE zero z is always real:

$$z = f_a \times \frac{\cos(\phi + \zeta) - r_s \cos(\phi - \zeta)}{\cos(\phi + \zeta) + r_s \cos(\phi - \zeta)} \tag{8}$$

As $\phi \to \pi/2$, $z \to p$ regardless of the value of ζ.

The square of the spring frequency is

$$\xi^2 = f_\mathrm{a}^2 \times \frac{2\alpha r_\mathrm{s} \sin 2\phi}{1 - 2r_\mathrm{s}\cos 2\phi + r_\mathrm{s}^2}, \tag{9}$$

where

$$\alpha = \frac{4P_\mathrm{bs}\omega_0}{(2\pi f_\mathrm{a})^4 M L^2}. \tag{10}$$

If $\xi^2 > 0$, the spring feature is a (somewhat) sharp resonance with an associated phase gain in the transfer function. If $\xi^2 < 0$, the spring feature is broad and has only small a effect on the transfer function phase. In either case, $\xi^2 \to 0$ as $\phi \to \pi/2$.

If $|\xi^2|^{\frac{1}{2}} \ll |p|$, the denominator of the optical plant can be approximately factorized:

$$\frac{\Delta P}{\Delta L} \simeq g \times \frac{-\operatorname{sgn}(\xi^2) \times f^2 \times (1 + \mathrm{i}f/z)}{\left(1 + \frac{\mathrm{i}f}{|p|Q_p} - \frac{f^2}{|p|^2}\right)\left(1 - \frac{\mathrm{i}f}{|\xi^2|^{\frac{1}{2}} Q_\xi} - \frac{f^2}{\xi^2}\right)}, \tag{11}$$

with $Q_\xi = Q_p \times |p|/|\xi^2|^{\frac{1}{2}}$. This shows that when $\xi^2 > 0$, the spring poles are complex and right-handed in the s-domain. When $\xi^2 < 0$, the spring poles are real, with one right-handed and the other left-handed.