Structure of black holes in theories beyond general relativity

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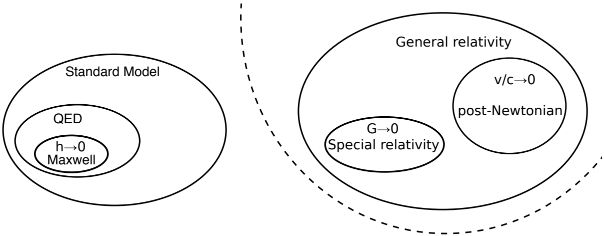
Why corrections to GR?

- No known experiment has been directly inconsistent with GR
- GR + QFT is not renormalizable at high energies
- The full gravity theory has an effective field theory (EFT) with GR as its zeroth order term



Tests of GR

Two ways to study theories of gravity

- Theory independent - framework for calculations
- Theory dependent - statistical tests for detections

Why numerics?

- In principle, LIGO and descendants can detect deviations.
- We need waveform templates for matched filtering,
  - generated from new types of numerical simulations of gravity.

Perturbations of GR

EFT $\implies$ alternative theories of gravity are *corrections* to GR.

- We can do perturbation theory!

We develop a formalism that can represent

- corrections to GR
- matter near the horizons of black holes
  (i.e. "bumpy black holes")

Note that the spacetime does not have to be Ricci-flat.

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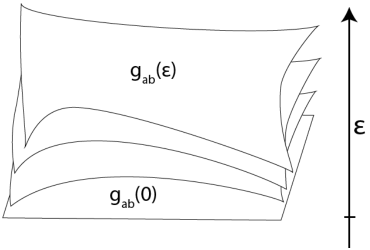
Perturbation Theory

We have a 1-parameter family of geometries, described by the metric g_{ab} with the parameter ϵ. Expanding around $\epsilon = 0$,

$$g_{ab}(\epsilon) = {g_{ab}}^{(\epsilon=0)} + \epsilon \underbrace{\left.\frac{dg_{ab}}{d\epsilon}\right|_{\epsilon=0}}_{h_{ab}^{(1)}} + \mathcal{O}(\epsilon^2)$$

Let ${g_{ab}}^{(\epsilon=0)}$ be a Ricci-flat geometry like Schwarzschild or Kerr.



Theory independent Lagrangian

(Now to the physics)
The action for a general interacting scalar θ

$$I = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \partial_a \theta \partial^a \theta + \epsilon \mathcal{L}_{\text{int}} \right]$$

Theory independent Equations of Motion

After making various calculations and arguments, the gist is that

$$G_{ab} = T_{ab}^{\text{eff}}$$

where $T_{ab}^{\text{eff}} = \mathcal{O}(\epsilon^2)$ and $\nabla^a T_{ab}^{\text{eff}} = 0$

$$\square\theta = S$$

and $S = \mathcal{O}(\epsilon^1)$

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Black hole spacetimes

In this project,

- Conservation of angular momentum $\implies$ axisymmetric
- Gravitation radiation + late times $\implies$ stationary

We can always* put the stationary, axisymmetric metrics into the Weyl-Lewis-Papapetrou (WLP) form:

$$ds^2 = -V(dt - wd\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda}dz^2)\right)$$

Unlike Kerr, this metric doesn't have to be Ricci-flat.

Weyl-Lewis-Papapetrou (WLP) Gauge

Spin-2 gauge in gravity = choice of coordinates
We see manifestly four metric degrees of freedom $\vec{v}$
In perturbation, we have

$$\begin{pmatrix} V(\rho,z) \\ w(\rho,z) \\ \gamma(\rho,z) \\ \lambda(\rho,z) \end{pmatrix} = \begin{pmatrix} V_0 \\ w_0 \\ \gamma_0 \\ \lambda_0 \end{pmatrix} + \frac{1}{2}\epsilon^2 \underbrace{\begin{pmatrix} \delta V \\ \delta w \\ \delta\gamma \\ \delta\lambda \end{pmatrix}}_{\vec{v}}$$

WLP Gauge

The WLP metric:

$$ds^2 = -V(dt - wd\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda}dz^2)\right)$$

One can always fix $\lambda = 0$ in the Ricci-flat case.

For the non-Ricci-flat spacetimes, $z \mapsto f(z)$ keeps the WLP gauge, i.e. $\lambda \mapsto \lambda + \partial_z \log f$.

λ is ambiguous

z gauge fixing

I proved that the gauge freedom exactly corresponds to a constant of integration $C(z)$

$$\partial_z \delta\lambda(\rho, z) = \int_\infty^\rho \rho' \partial_z \left(T^{\text{eff}}_{\rho\rho}(\rho', z) - T^{\text{eff}}_{zz}(\rho', z) \right) d\rho' + C(z)$$

And therefore we can choose $C(z) = 0$ to fully fix our gauge.

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Linearized equations

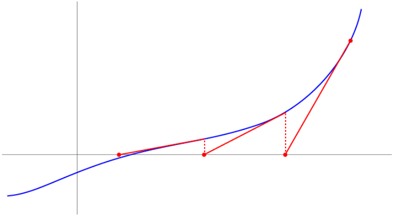
Despite the redundancies in $G_{ab} = T^{\text{eff}}_{ab}$, we can now cast the linearized equations into the form

$$\Delta\vec{v} \equiv \Delta_0 \begin{pmatrix} \delta V \\ \delta w \\ \delta\gamma \\ \delta\lambda \end{pmatrix} + \text{lower order terms} = \text{source}$$

where Δ_0 is the induced 3-Laplacian of the Kerr background

Newton-Raphson method

Given this form, we use an iterative scheme to solve for these four metric variables in $\Delta\vec{v} = \vec{S}$



initial guess: $\Delta_0 \vec{v}_0 = \vec{S}_0$

$$\Delta(\vec{v}_0 + \delta\vec{v}) = \vec{S}$$

$$\Delta\vec{v}_0 + \Delta_0 \delta\vec{v} \approx \vec{S}$$

$$\implies \Delta_0 \delta\vec{v} \approx \vec{S} - \Delta\vec{v}_0$$

iteratively solve: $\boxed{\delta\vec{v} \approx \Delta_0^{-1}\left(\vec{S} - \Delta\vec{v}_0\right)}$

Current goals

- Simulating over a Kerr background with dynamical Chern-Simons
    - Source is in rational polynomial Boyer-Lindquist coordinates
- Implementing the formalism directly (in SpEC)
- Physically interesting quantities of these BHs
    - thermodynamic entropy
    - ISCO
    - orbital frequencies
    - locations of the new horizons

Long Term Goals

- Non-stationary perturbations
  - Quasi-normal modes of these black holes
- Binary black holes
- Gravitational waveforms at infinity
- Non-perturbative solutions(1)
- Extremal-Kerr solutions and relate to the Weak Gravity Conjecture in string theory

Summary

- Tests of GR are important
- Insight into developing theory-independent frameworks
- Choosing gauge is important for numerical calculations
- Numerical simulations $\implies$ LIGO waveforms

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