Structure of black holes in theories beyond general relativity

Weiming Wayne Zhao

LIGO SURF Project
Caltech TAPIR

August 18, 2016

Acknowledgments

Leo C. Stein

Vijay Varma

Mark Scheel

Kevin Barkett and other people in TAPIR

Alan and other people in LIGO

SURF, TAPIR, LSC, Caltech, NSF

Table of Contents

Why corrections to GR?

Why corrections to GR?

- No known experiment has been directly inconsistent with GR

Why corrections to GR?

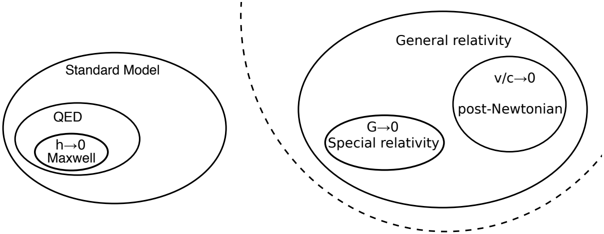
- No known experiment has been directly inconsistent with GR
- GR + QFT is not renormalizable at high energies

Why corrections to GR?

- No known experiment has been directly inconsistent with GR
- GR + QFT is not renormalizable at high energies
- The full gravity theory has an effective field theory (EFT) with GR as its zeroth order term

Why corrections to GR?

- No known experiment has been directly inconsistent with GR
- GR + QFT is not renormalizable at high energies
- The full gravity theory has an effective field theory (EFT) with GR as its zeroth order term



Tests of GR

Tests of GR

Two ways to study theories of gravity

Tests of GR

Two ways to study theories of gravity

- Theory independent - framework for calculations

Tests of GR

Two ways to study theories of gravity

- Theory independent - framework for calculations
- Theory dependent - statistical tests for detections

Why numerics?

Why numerics?

- In principle, LIGO and descendants can detect deviations.

Why numerics?

- In principle, LIGO and descendants can detect deviations.
- We need waveform templates for matched filtering,

Why numerics?

- In principle, LIGO and descendants can detect deviations.
- We need waveform templates for matched filtering,
  - generated from new types of numerical simulations of gravity.

Perturbations of GR

Perturbations of GR

EFT $\implies$ alternative theories of gravity are *corrections* to GR.

Perturbations of GR

EFT $\implies$ alternative theories of gravity are *corrections* to GR.

- We can do perturbation theory!

Perturbations of GR

EFT $\implies$ alternative theories of gravity are *corrections* to GR.

- We can do perturbation theory!

We develop a formalism that can represent

Perturbations of GR

EFT $\implies$ alternative theories of gravity are *corrections* to GR.

- We can do perturbation theory!

We develop a formalism that can represent

- corrections to GR

Perturbations of GR

EFT $\implies$ alternative theories of gravity are *corrections* to GR.

- We can do perturbation theory!

We develop a formalism that can represent

- corrections to GR
- matter near the horizons of black holes
  (i.e. "bumpy black holes")

Perturbations of GR

EFT $\implies$ alternative theories of gravity are *corrections* to GR.

- We can do perturbation theory!

We develop a formalism that can represent

- corrections to GR
- matter near the horizons of black holes (i.e. "bumpy black holes")

Note that the spacetime does not have to be Ricci-flat.

Table of Contents

Perturbation Theory

Perturbation Theory

We have a 1-parameter family of geometries, described by the metric g_{ab} with the parameter ϵ. Expanding around $\epsilon = 0$,

Perturbation Theory

We have a 1-parameter family of geometries, described by the metric g_{ab} with the parameter ϵ. Expanding around $\epsilon = 0$,

$$g_{ab}(\epsilon) = {g_{ab}}^{(\epsilon=0)} + \epsilon \underbrace{\left.\frac{dg_{ab}}{d\epsilon}\right|_{\epsilon=0}}_{h^{(1)}_{ab}} + \mathcal{O}(\epsilon^2)$$

Let ${g_{ab}}^{(\epsilon=0)}$ be a Ricci-flat geometry like Schwarzschild or Kerr.

Perturbation Theory

We have a 1-parameter family of geometries, described by the metric g_{ab} with the parameter ϵ. Expanding around $\epsilon = 0$,

$$g_{ab}(\epsilon) = {g_{ab}}^{(\epsilon=0)} + \epsilon \underbrace{\left.\frac{dg_{ab}}{d\epsilon}\right|_{\epsilon=0}}_{h_{ab}^{(1)}} + \mathcal{O}(\epsilon^2)$$

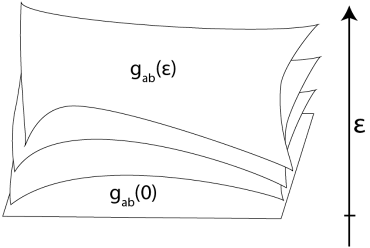
Let ${g_{ab}}^{(\epsilon=0)}$ be a Ricci-flat geometry like Schwarzschild or Kerr.

Perturbation Theory

We have a 1-parameter family of geometries, described by the metric g_{ab} with the parameter ϵ. Expanding around $\epsilon = 0$,

$$g_{ab}(\epsilon) = {g_{ab}}^{(\epsilon=0)} + \epsilon \underbrace{\left.\frac{dg_{ab}}{d\epsilon}\right|_{\epsilon=0}}_{h_{ab}^{(1)}} + \mathcal{O}(\epsilon^2)$$

Let ${g_{ab}}^{(\epsilon=0)}$ be a Ricci-flat geometry like Schwarzschild or Kerr.



Theory independent Lagrangian

(Now to the physics)

Theory independent Lagrangian

(Now to the physics)
The action for a general interacting scalar θ

$$I = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \partial_a \theta \partial^a \theta + \epsilon \mathcal{L}_{\text{int}} \right]$$

Theory independent Lagrangian

(Now to the physics)
The action for a general interacting scalar θ

$$I = \int d^4x \sqrt{-g} \left[R - \frac{1}{2}\partial_a\theta\partial^a\theta + \epsilon\mathcal{L}_{\text{int}} \right]$$

Theory independent Equations of Motion

Theory independent Equations of Motion

After making various calculations and arguments, the gist is that

Theory independent Equations of Motion

After making various calculations and arguments, the gist is that

$$G_{ab} = T_{ab}^{\text{eff}}$$

Theory independent Equations of Motion

After making various calculations and arguments, the gist is that

$$G_{ab} = T^{\text{eff}}_{ab}$$

where $T^{\text{eff}}_{ab} = \mathcal{O}(\epsilon^2)$ and $\nabla^a T^{\text{eff}}_{ab} = 0$

Theory independent Equations of Motion

After making various calculations and arguments, the gist is that

$$G_{ab} = T_{ab}^{\text{eff}}$$

where $T_{ab}^{\text{eff}} = \mathcal{O}(\epsilon^2)$ and $\nabla^a T_{ab}^{\text{eff}} = 0$

$$\Box\theta = S$$

Theory independent Equations of Motion

After making various calculations and arguments, the gist is that

$$G_{ab} = T_{ab}^{\text{eff}}$$

where $T_{ab}^{\text{eff}} = \mathcal{O}(\epsilon^2)$ and $\nabla^a T_{ab}^{\text{eff}} = 0$

$$\Box\theta = S$$

and $S = \mathcal{O}(\epsilon^1)$

Table of Contents

Black hole spacetimes

Black hole spacetimes

In this project,

Black hole spacetimes

In this project,

- Conservation of angular momentum $\implies$ axisymmetric

Black hole spacetimes

In this project,

- Conservation of angular momentum $\implies$ axisymmetric
- Gravitation radiation + late times $\implies$ stationary

Black hole spacetimes

In this project,

- Conservation of angular momentum $\implies$ axisymmetric
- Gravitation radiation + late times $\implies$ stationary

We can always* put the stationary, axisymmetric metrics into the Weyl-Lewis-Papapetrou (WLP) form:

$$ds^2 = -V(dt - w d\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda} dz^2)\right)$$

Black hole spacetimes

In this project,

- Conservation of angular momentum $\implies$ axisymmetric
- Gravitation radiation + late times $\implies$ stationary

We can always* put the stationary, axisymmetric metrics into the Weyl-Lewis-Papapetrou (WLP) form:

$$ds^2 = -V(dt - wd\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda}dz^2)\right)$$

Black hole spacetimes

In this project,

- Conservation of angular momentum $\implies$ axisymmetric
- Gravitation radiation + late times $\implies$ stationary

We can always* put the stationary, axisymmetric metrics into the Weyl-Lewis-Papapetrou (WLP) form:

$$ds^2 = -V(dt - w\,d\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda}dz^2)\right)$$

Unlike Kerr, this metric doesn't have to be Ricci-flat.

Weyl-Lewis-Papapetrou (WLP) Gauge

Weyl-Lewis-Papapetrou (WLP) Gauge

Spin-2 gauge in gravity = choice of coordinates

Weyl-Lewis-Papapetrou (WLP) Gauge

Spin-2 gauge in gravity = choice of coordinates
We see manifestly four metric degrees of freedom $\vec{v}$
In perturbation, we have

$$\begin{pmatrix} V(\rho, z) \\ w(\rho, z) \\ \gamma(\rho, z) \\ \lambda(\rho, z) \end{pmatrix}$$

Weyl-Lewis-Papapetrou (WLP) Gauge

Spin-2 gauge in gravity = choice of coordinates
We see manifestly four metric degrees of freedom $\vec{v}$
In perturbation, we have

$$\begin{pmatrix} V(\rho, z) \\ w(\rho, z) \\ \gamma(\rho, z) \\ \lambda(\rho, z) \end{pmatrix} = \begin{pmatrix} V_0 \\ w_0 \\ \gamma_0 \\ \lambda_0 \end{pmatrix}$$

Weyl-Lewis-Papapetrou (WLP) Gauge

Spin-2 gauge in gravity = choice of coordinates
We see manifestly four metric degrees of freedom $\vec{v}$
In perturbation, we have

$$\begin{pmatrix} V(\rho, z) \\ w(\rho, z) \\ \gamma(\rho, z) \\ \lambda(\rho, z) \end{pmatrix} = \begin{pmatrix} V_0 \\ w_0 \\ \gamma_0 \\ \lambda_0 \end{pmatrix} + \frac{1}{2}\epsilon^2 \begin{pmatrix} \delta V \\ \delta w \\ \delta\gamma \\ \delta\lambda \end{pmatrix}$$

Weyl-Lewis-Papapetrou (WLP) Gauge

Spin-2 gauge in gravity = choice of coordinates
We see manifestly four metric degrees of freedom $\vec{v}$
In perturbation, we have

$$\begin{pmatrix} V(\rho, z) \\ w(\rho, z) \\ \gamma(\rho, z) \\ \lambda(\rho, z) \end{pmatrix} = \begin{pmatrix} V_0 \\ w_0 \\ \gamma_0 \\ \lambda_0 \end{pmatrix} + \frac{1}{2}\epsilon^2 \underbrace{\begin{pmatrix} \delta V \\ \delta w \\ \delta\gamma \\ \delta\lambda \end{pmatrix}}_{\vec{v}}$$

WLP Gauge

The WLP metric:

WLP Gauge

The WLP metric:

$$ds^2 = -V(dt - wd\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda}dz^2)\right)$$

WLP Gauge

The WLP metric:

$$ds^2 = -V(dt - wd\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda}dz^2)\right)$$

One can always fix $\lambda = 0$ in the Ricci-flat case.

WLP Gauge

The WLP metric:

$$ds^2 = -V(dt - wd\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda}dz^2)\right)$$

One can always fix $\lambda = 0$ in the Ricci-flat case.

For the non-Ricci-flat spacetimes, $z \mapsto f(z)$ keeps the WLP gauge,

WLP Gauge

The WLP metric:

$$ds^2 = -V(dt - wd\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda}dz^2)\right)$$

One can always fix $\lambda = 0$ in the Ricci-flat case.

For the non-Ricci-flat spacetimes, $z \mapsto f(z)$ keeps the WLP gauge, i.e. $\lambda \mapsto \lambda + \partial_z \log f$.

WLP Gauge

The WLP metric:

$$ds^2 = -V(dt - wd\phi)^2 + V^{-1}\left(\rho^2 d\phi^2 + e^{2\gamma}(d\rho^2 + e^{2\lambda}dz^2)\right)$$

One can always fix $\lambda = 0$ in the Ricci-flat case.

For the non-Ricci-flat spacetimes, $z \mapsto f(z)$ keeps the WLP gauge, i.e. $\lambda \mapsto \lambda + \partial_z \log f$.

λ is ambiguous

z gauge fixing

z gauge fixing

I proved that the gauge freedom exactly corresponds to a constant of integration $C(z)$

z gauge fixing

I proved that the gauge freedom exactly corresponds to a constant of integration $C(z)$

$$\partial_z \delta\lambda(\rho, z) = \int_\infty^\rho \rho' \partial_z \left(T^{\text{eff}}_{\rho\rho}(\rho', z) - T^{\text{eff}}_{zz}(\rho', z) \right) d\rho' + C(z)$$

z gauge fixing

I proved that the gauge freedom exactly corresponds to a constant of integration $C(z)$

$$\partial_z \delta\lambda(\rho, z) = \int_{\infty}^{\rho} \rho' \partial_z \left(T^{\text{eff}}_{\rho\rho}(\rho', z) - T^{\text{eff}}_{zz}(\rho', z) \right) d\rho' + C(z)$$

And therefore we can choose $C(z) = 0$ to fully fix our gauge.

Table of Contents

Linearized equations

Linearized equations

Despite the redundancies in $G_{ab} = T^{\text{eff}}_{ab}$, we can now cast the linearized equations into the form

Linearized equations

Despite the redundancies in $G_{ab} = T_{ab}^{\text{eff}}$, we can now cast the linearized equations into the form

$$\Delta \vec{v} \equiv \Delta_0 \begin{pmatrix} \delta V \\ \delta w \\ \delta \gamma \\ \delta \lambda \end{pmatrix} + \text{lower order terms} = \text{source}$$

Linearized equations

Despite the redundancies in $G_{ab} = T^{\text{eff}}_{ab}$, we can now cast the linearized equations into the form

$$\Delta\vec{v} \equiv \Delta_0 \begin{pmatrix} \delta V \\ \delta w \\ \delta\gamma \\ \delta\lambda \end{pmatrix} + \text{lower order terms} = \text{source}$$

where Δ_0 is the induced 3-Laplacian of the Kerr background

Newton-Raphson method

Newton-Raphson method

Given this form, we use an iterative scheme to solve for these four metric variables in $\Delta\vec{v} = \vec{S}$

Newton-Raphson method

Given this form, we use an iterative scheme to solve for these four metric variables in $\Delta\vec{v} = \vec{S}$

initial guess: $\Delta_0\vec{v}_0 = \vec{S}_0$

Newton-Raphson method

Given this form, we use an iterative scheme to solve for these four metric variables in $\Delta\vec{v} = \vec{S}$

$$\text{initial guess: } \Delta_0\vec{v}_0 = \vec{S}_0$$

$$\Delta(\vec{v}_0 + \delta\vec{v}) = \vec{S}$$

Newton-Raphson method

Given this form, we use an iterative scheme to solve for these four metric variables in $\Delta \vec{v} = \vec{S}$

$$\text{initial guess: } \Delta_0 \vec{v}_0 = \vec{S}_0$$
$$\Delta(\vec{v}_0 + \delta \vec{v}) = \vec{S}$$
$$\Delta \vec{v}_0 + \Delta_0 \delta \vec{v} \approx \vec{S}$$

Newton-Raphson method

Given this form, we use an iterative scheme to solve for these four metric variables in $\Delta\vec{v} = \vec{S}$

$$
\begin{aligned}
\text{initial guess: } \Delta_0\vec{v}_0 &= \vec{S}_0 \\
\Delta(\vec{v}_0 + \delta\vec{v}) &= \vec{S} \\
\Delta\vec{v}_0 + \Delta_0\delta\vec{v} &\approx \vec{S} \\
\implies \Delta_0\delta\vec{v} &\approx \vec{S} - \Delta\vec{v}_0
\end{aligned}
$$

Newton-Raphson method

Given this form, we use an iterative scheme to solve for these four metric variables in $\Delta\vec{v} = \vec{S}$

$$\text{initial guess: } \Delta_0\vec{v}_0 = \vec{S}_0$$

$$\Delta(\vec{v}_0 + \delta\vec{v}) = \vec{S}$$

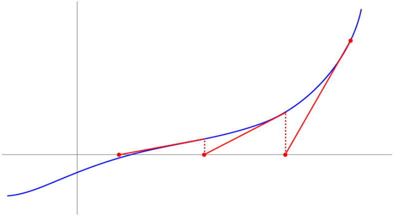
$$\Delta\vec{v}_0 + \Delta_0\delta\vec{v} \approx \vec{S}$$

$$\implies \Delta_0\delta\vec{v} \approx \vec{S} - \Delta\vec{v}_0$$

$$\text{iteratively solve: } \boxed{\delta\vec{v} \approx \Delta_0^{-1}\left(\vec{S} - \Delta\vec{v}_0\right)}$$

Newton-Raphson method

Given this form, we use an iterative scheme to solve for these four metric variables in $\Delta\vec{v} = \vec{S}$



initial guess: $\Delta_0\vec{v}_0 = \vec{S}_0$

$$\Delta(\vec{v}_0 + \delta\vec{v}) = \vec{S}$$

$$\Delta\vec{v}_0 + \Delta_0\delta\vec{v} \approx \vec{S}$$

$$\implies \Delta_0\delta\vec{v} \approx \vec{S} - \Delta\vec{v}_0$$

iteratively solve: $\boxed{\delta\vec{v} \approx \Delta_0^{-1}\left(\vec{S} - \Delta\vec{v}_0\right)}$

Current goals

Current goals

- Simulating over a Kerr background with dynamical Chern-Simons

Current goals

- Simulating over a Kerr background with dynamical Chern-Simons
  - Source is in rational polynomial Boyer-Lindquist coordinates

Current goals

- Simulating over a Kerr background with dynamical Chern-Simons
    - Source is in rational polynomial Boyer-Lindquist coordinates
- Implementing the formalism directly (in SpEC)

Current goals

- Simulating over a Kerr background with dynamical Chern-Simons
  - Source is in rational polynomial Boyer-Lindquist coordinates
- Implementing the formalism directly (in SpEC)
- Physically interesting quantities of these BHs

Current goals

- Simulating over a Kerr background with dynamical Chern-Simons
  - Source is in rational polynomial Boyer-Lindquist coordinates
- Implementing the formalism directly (in SpEC)
- Physically interesting quantities of these BHs
  - thermodynamic entropy

Current goals

- Simulating over a Kerr background with dynamical Chern-Simons
  - Source is in rational polynomial Boyer-Lindquist coordinates
- Implementing the formalism directly (in SpEC)
- Physically interesting quantities of these BHs
  - thermodynamic entropy
  - ISCO

Current goals

- Simulating over a Kerr background with dynamical Chern-Simons
  - Source is in rational polynomial Boyer-Lindquist coordinates
- Implementing the formalism directly (in SpEC)
- Physically interesting quantities of these BHs
  - thermodynamic entropy
  - ISCO
  - orbital frequencies

Current goals

- Simulating over a Kerr background with dynamical Chern-Simons
    - Source is in rational polynomial Boyer-Lindquist coordinates
- Implementing the formalism directly (in SpEC)
- Physically interesting quantities of these BHs
    - thermodynamic entropy
    - ISCO
    - orbital frequencies
    - locations of the new horizons

Long Term Goals

Long Term Goals

- Non-stationary perturbations

Long Term Goals

- Non-stationary perturbations
  - Quasi-normal modes of these black holes

Long Term Goals

- Non-stationary perturbations
  - Quasi-normal modes of these black holes
- Binary black holes

Long Term Goals

- Non-stationary perturbations
  - Quasi-normal modes of these black holes
- Binary black holes
- Gravitational waveforms at infinity

Long Term Goals

- Non-stationary perturbations
  - Quasi-normal modes of these black holes
- Binary black holes
- Gravitational waveforms at infinity
- Non-perturbative solutions(1)

Long Term Goals

- Non-stationary perturbations
  - Quasi-normal modes of these black holes
- Binary black holes
- Gravitational waveforms at infinity
- Non-perturbative solutions(1)
- Extremal-Kerr solutions and relate to the Weak Gravity Conjecture in string theory

Summary

Summary

- Tests of GR are important

Summary

- Tests of GR are important
- Insight into developing theory-independent frameworks

Summary

- Tests of GR are important
- Insight into developing theory-independent frameworks
- Choosing gauge is important for numerical calculations

Summary

- Tests of GR are important
- Insight into developing theory-independent frameworks
- Choosing gauge is important for numerical calculations
- Numerical simulations $\Longrightarrow$ LIGO waveforms

Summary

- Tests of GR are important
- Insight into developing theory-independent frameworks
- Choosing gauge is important for numerical calculations
- Numerical simulations $\implies$ LIGO waveforms

Works cited

A. I. Harte and J. Vines, "Generating exact solutions to Einstein's equation using linearized approximations," `arXiv:1608.04359 [gr-qc]`.

D. Hilditch, "An Introduction to Well-posedness and Free-evolution," *Int. J. Mod. Phys.* **A28** (2013) 1340015, `arXiv:1309.2012 [gr-qc]`.

L. C. Stein, "Rapidly rotating black holes in dynamical Chern-Simons gravity: Decoupling limit solutions and breakdown," *Phys. Rev.* **D90** no. 4, (2014) 044061, `arXiv:1407.2350 [gr-qc]`.

A. Maselli, P. Pani, L. Gualtieri, and V. Ferrari, "Rotating black holes in Einstein-Dilaton-Gauss-Bonnet gravity with finite coupling," *Phys. Rev.* **D92** no. 8, (2015) 083014, `arXiv:1507.00680 [gr-qc]`.

K. Yagi and L. C. Stein, "Black Hole Based Tests of General Relativity," *Class. Quant. Grav.* **33** no. 5, (2016) 054001, `arXiv:1602.02413 [gr-qc]`.

M. Visser, "The Kerr spacetime: A Brief introduction," in *Kerr Fest: Black Holes in Astrophysics, General Relativity and Quantum Gravity*.
2007.
`arXiv:0706.0622 [gr-qc]`.

J. Boyd, *Chebyshev and Fourier Spectral Methods: Second Revised Edition*.
Dover Books on Mathematics. Dover Publications, 2013.
`https://books.google.com/books?id=b4TCAgAAQBAJ`.

G. B. Cook and M. A. Scheel, "Well-behaved harmonic time slices of a charged, rotating, boosted black hole," *Phys. Rev. D* **56** (Oct, 1997) 4775–4781. `http://link.aps.org/doi/10.1103/PhysRevD.56.4775`.