Structure of black holes in theories beyond general relativity LIGO Project Proposal

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1 Introduction

So far in scientific history there has not been a single definitive experiment that shows deviations from the predictions of GR. However we know from quantum field theory that GR as it stands is not renormalizable, so it must not be the full story. LIGO is our first shot at probing gravity in the strong field regime. So we should have alternative hypotheses that we can use the results of LIGO to compare GR against other theories or to parameterize a family of possible theories of gravity. With the former, we need some more insight from numercial merger simulations to develop such prescriptions.

Before embarking on merger simulations, we focus on stationary and isolated, black holes, where no matter is unmodeled. Since astrophysical black holes probably neutralize their electric charge rather quickly, we first consider GR in the Kerr solution for a black hole space time, the unique solution space-time that is stationary, axisymetric, charge neutral, regular, and asymptotically-flat.

The Kerr solution will be the background for a family of solutions to non-minimal coupling terms as a perturbation to the Kerr solution under the Einstein-Hilbert action. The perturbation parameter ϵ will parameterize the solutions of these "bumpy black holes". The non-minimal coupling terms can be from any theory, but we consider concrete examples from anomaly cancellations and Kaluza-Klein reduction in string theory, such as dynamical Chern-Simons (dCS) theory and Einstein-dilaton-Gauss-Bonnet (EdGB). In the perturbation scheme we introduce the linearized Einstein tensor G that is an operator that acts on 2-tensors to give 2-tensors fields over space-time. G satisfies a set of partial differential equations that to first order perturbations, give the contribution of non-minimal interaction and the modified stress-energy tensor.

We say the set of differential equations and boundary conditions is *well-posed* they if give a well-defined spectrum in the solution space. When G and the corresponding boundary conditions is *elliptic*, we deem our question well-posed. Looking at the second-order variation of the metric, G is self-adjoint with respect to an appropriate inner product. This observation motivate two gauge conditions: the solution has to be completely orthogonal, with respect to a nice inner product, to the unperturbed Kerr solution and to the first order variation of the metric.

In our analysis, perturbation theory should only a subset of the possible solutions, we expect perturbation theory to break down when ϵ is of order unity. Furthermore, since GR and the family of alternatives are not UV-complete, we expect effective field theory to break down at least when the energy is on the order of the Planck energy. For the scope of the initial part of the project, we will stay in the weak-coupling regime and will not be focused on studing how the theories behave at or past these breakdowns.

2 Objectives and Approach

(a) Metric Solution

We first aim to solve for some metric solutions for the linearized Einstein tensor to first order in perturbation theory with a Kerr solution as the background. We can first try it with dCS and then EdGB. I potentially can start with the easy case of gravity in $2+1$ dimensions as an exercise to learn my way around the tools I will be needing for the rest of the project. But ultimately, I will be working with the full $3+1$ dimensional theory of gravity for the entire project. This all should be done analytically by pen and paper at this step, probably with the help of Mathematica and the xAct/xPert packages.

An important assumption to check is whether or not G is an elliptic operator. We want to show that boundary value problem is well-posed in this setting.

(b) Numerics

After the basic analytics are complete or it becomes apparent that it is intractable, in either case, we proceed to build code to invert the elliptic operator. We first attempt a method besides directly finding Green's function. We first will perform consistency checks that the vectors in the kernel are annihilated by our numerical operator. Then we try direct inversion and find spectrum of G assuming our formulation allows for it and there exists an orthonormal basis under some well-defined inner product that we construct. Then, we can solve for smoothest eigenfunctions that have the greatest power contribution. We prototype this scheme for finding eigenfunctions in Mathematica at first. If this is not fast enough then we can try implement it in `C++` in the SpEC codebase. After all the numerics check out, we cam build a toolbox for the spectral methods. I can initially recreate some completed code but then graduate to work on unique code for our specific setting.

If the numerics do not converge or are otherwise not well-behaved, we can go back to reformulate our theoretical setup. We consider making more gauge conditions or different ones altogether, as we might have not completely fixed the gauge. Through this iterative process, we can confirm that our theoretic and numerical results agree and the PDE is a well-posed (and solvable) boundary value problem.

(c) Physical implications

Once the theoretical and numerical setups are correct, we try to extract the physics out of black holes in this setting. We hope to understand the explicit spectrum and eigenfunctions of the linearized Einstein operator in this perturbation setting. Physically, we can hope to find the dynamics of merging black holes in the theory-agnostic formalism we've developed. We might be able to calculate corrections to black hole thermodynamics in the various alternative theories to GR.

References

[1] David Hilditch. "An Introduction to Well-posedness and Free-evolution". In: (2013). DOI: 10.1142/S0217751X13400150. eprint: arXiv:1309.2012.

[2] Andrea Maselli et al. "Rotating black holes in Einstein-Dilaton-Gauss-Bonnet gravity with finite coupling". In: (2015). DOI: 10.1103/PhysRevD.92.083014. eprint: arXiv:1507.00680.

[3] Leo C. Stein. "Rapidly rotating black holes in dynamical Chern-Simons gravity: Decoupling limit solutions and breakdown". In: (2014). DOI: 10.1103/PhysRevD.90.044061. eprint: arXiv:1407.2350.

[4] Kent Yagi and Leo C. Stein. "Black Hole Based Tests of General Relativity". In: (2016). DOI: 10.1088/0264-9381/33/5/054001. eprint: arXiv:1602.02413.