Measuring Kerrness in Binary Black Hole Simulation Ringdowns

Progress Report 1

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After a single black hole forms from the merger of a black hole binary, it enters the ringdown phase, radiating away energy in gravitational waves until it settles into a stationary Kerr black hole. However, how soon after the merger does this black hole become close to a Kerr spacetime? Relatedly, how close to the merger phase in a gravitational waveform can LIGO apply data analysis techniques that assume that the remnant black hole is Kerr (or a perturbation thereof)? In this paper, we review several measures of Kerrness, and apply them to binary black hole simulation ringdowns with the Spectral Einstein Code (SpEC).

NUMERICAL RELATIVITY AND KERRNESS BACKGROUND

The first detection of gravitational waves originating from a binary black hole merger [1] has provided a significant confirmation of the predictions of general relativity and has opened the door to many more such tests and observations of black holes. Numerical relativity is the field which concerns the simulation and evolution of spacetimes by solving Einstein's equations. Its role in LIGO includes correlating observed gravitational waveforms with the local properties of their sources [8].

Numerical studies of binary black hole mergers frequently assume a perturbed Kerr spacetime during ringdown [6]. Although simulations have been run which show that the remnant spacetime resulting from a binary black hole merger is a Kerr black hole at the final moment of the simulation [7] [5], the point at which the assumption of a perturbed Kerr spacetime becomes valid has not been established in previous analysis.

The project concerns the evaluation of various measures of “Kerrness”, or similarity to the Kerr spacetime, including established measures such as speciality indices [3] and newly proposed measures such as those proposed by Gómez-Lobo [4] and Bäckdahl and Valiente Kroon [2]. The goal is both to validate that they are appropriate measures of Kerrness through simulations and to use them to more quantitatively define when a remnant spacetime can be considered adequately close to Kerr.

In implementing and analyzing these invariant measurements, the first step is to verify that these measures of Kerrness behave sensibly on a single black hole simulation, both to validate the metrics themselves and as a proof of concept that they have been implemented correctly in the simulation code. Once this verification has occurred, the next step is to compute these measures of Kerrness on binary black hole ringdown simulations.

PROGRESS EVALUATION

Work on the project has centered around investigating the speciality index defined in equation (1) and the $\mathcal{L}$ quantity proposed by Gómez-Lobo (see equation (2)) on single blackhole simulations and binary blackhole ringdowns. In the case of the speciality index, the necessary compute items were already coded and so this quantity was run as a first exercise in single blackhole simulations, and more recently on binary blackhole volume data.

A compute item for the $\mathcal{L}$ invariant does not yet exist. Because this quantity is positive definite, it is possible to consider each term in isolation, and so development of the compute items has proceeded term by term, beginning with the first three due to relatively simpler dependencies.

Single black hole simulations at various resolutions have been performed for S and for

$$S = \frac{27J^2}{I^3} \tag{1}$$

FIG. 1: Definition for the speciality index [3]. This quantity is 1 for a Kerr spacetime.

$$\mathcal{L} \equiv \frac{(\mathfrak{r}(A)+\mathfrak{r}(B))^2 + (\mathfrak{j}(A)_i + \mathfrak{j}(B)_i)(\mathfrak{j}(A)^i + \mathfrak{j}(B)^i) + (\mathfrak{t}(A)_{ij} + \mathfrak{t}(B)_{ij})(\mathfrak{t}(A)^{ij} + \mathfrak{t}(B)^{ij})}{\sigma^{14}} + \frac{\mathfrak{a}_{ij}\mathfrak{a}^{ij} + \mathfrak{b}_{ij}\mathfrak{b}^{ij}}{\sigma^4} + \frac{\left((1-3\lambda^2)\beta + \lambda(3-\lambda^2)\alpha\right)^2}{\sigma^2} + \frac{(\mathfrak{B}_{ij}\mathfrak{B}^{ij})^3}{\sigma^4} + \frac{(\mathfrak{C}_{ij}\mathfrak{C}^{ij})^3}{\sigma^7} + \frac{\Omega}{\sigma^2} \tag{2}$$

FIG. 2: Positive Kerrness invariant proposed by Gómez-Lobo [4]. This quantity vanishes for Kerr initial data, and is being investigated as a measurement of the Kerrness of a spacetime. Compute items for the first three terms have been created and computed on single black hole simulations.

the second and third term of $\mathcal{L}$, with the quantities computed and dumped along various spherical shells around the apparent horizon. In these simulations, numerical convergence properties with respect to resolution level and time step are verified (see FIG. 3 for such a plot for S).

The speciality index (S) was computed on binary blackhole ringdown data from a simulation of GW150914 and preliminary analysis indicates that it is behaving as expected on several shells.

Progress on the project is largely on schedule according to the tentative roadmap provided in the project plan.

Code Development

The SpEC codebase was cloned and a feature branch for the Kerrness project was established.

Compute items have been written for a subset of the Kerrness invariant proposed by Gómez-Lobo shown in equation (2) (see FIG. 4 for the abstract information flow within SpEC).

The development of these compute items involves testing for compiler and runtime errors, followed by computations for the items on a single black hole simulation to verify that they behave according to expectations.

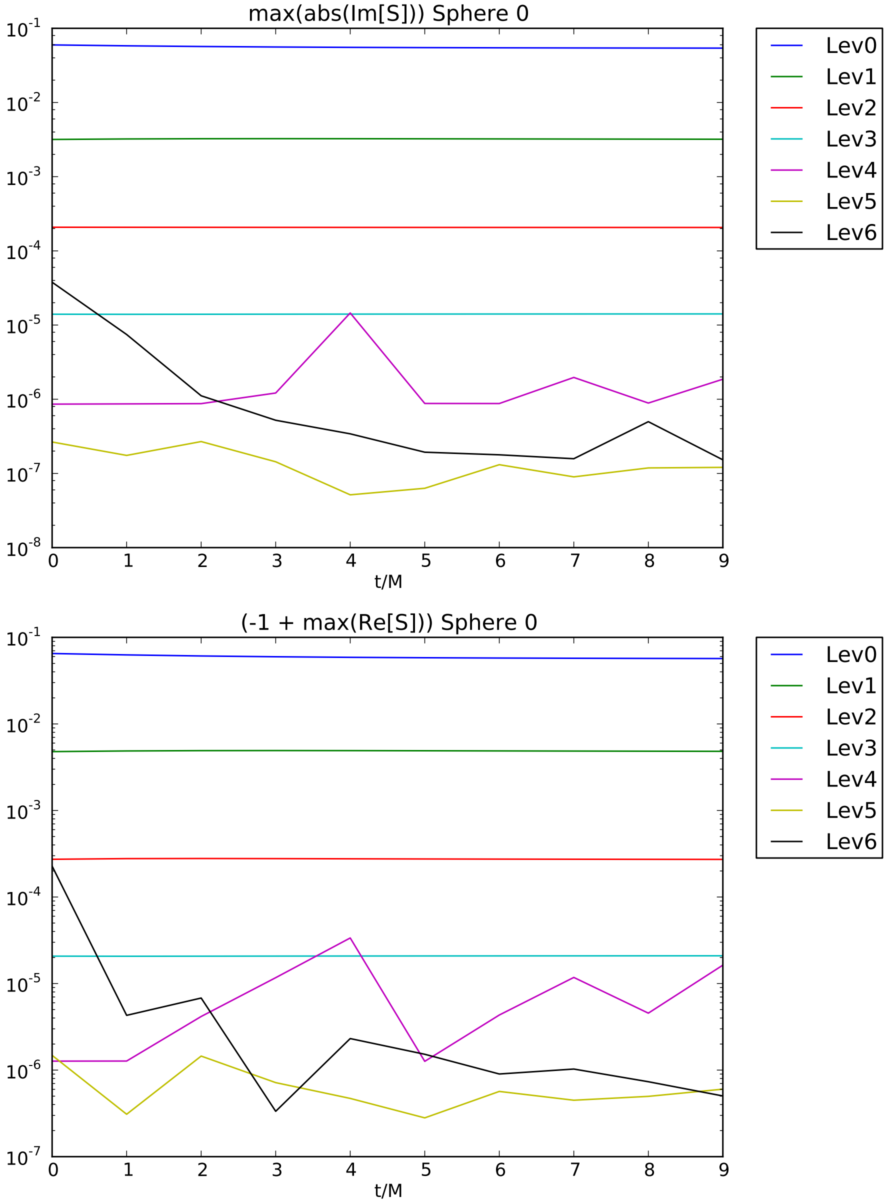

FIG. 3: Logarithmic plots of convergence of the L-infinity norm of the real and imaginary parts of the S invariant from equation (1) on the "sphere 0" shell from a single black hole simulation at various resolution levels (larger number level is higher resolution). The real part of this quantity is expected to converge to 1 and the imaginary part to 0. In addition, this "error" should not increase with time. As can be seen from the plots, the norms are relatively constant with time and appear to hit a noise floor at Lev4 and higher.

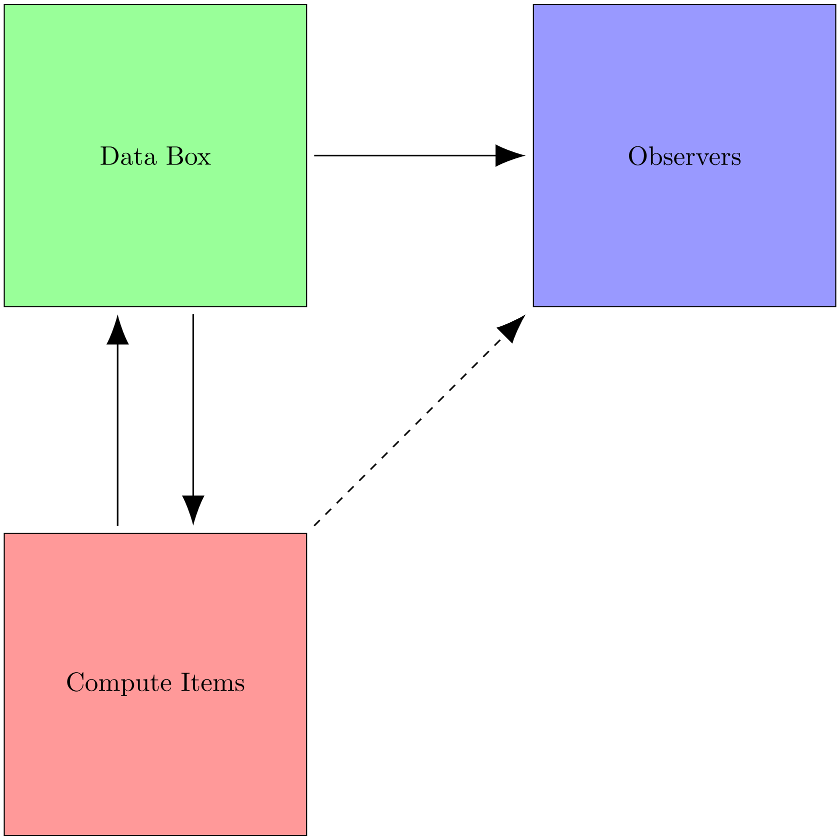

FIG. 4: High level simplified flow of information when working in SpEC. ComputeItems are C++ classes which provide the instructions to compute (and recompute) tensor and scalar quantities. The DataBox stores items which have been computed and transfers these to the Observers when requested and to the ComputeItems when needed in other computations. If a quantity needs to be observed and is not yet in the DataBox, the quantity is computed and then transferred to the DataBox at which point it can be observed.

CHALLENGES

Although a significant portion of the invariant scalar quantity $\mathcal{L}$ (from eq. (2)) has been coded, there remain some issues with convergence and behavior. In particular, the quantity σ in the denominator of (2) must be positive and there are some regions where this is not the case. This could be due to region related issues, or numerics or other coding mistakes, but more investigation is necessary to identify the cause.

Because each quantity has been coded as its own compute item (see FIG. 5 for an overview of these quantities), it is possible to quickly identify which items are the source of problems, and then in principle to correct these issues. Once it has been validated that the code produces sensible results, the code may be refactored into fewer compute items to simplify the design and abstract over details which need not be exposed externally.

Moving forward, new challenges will arise from the binary black hole simulations. The remainder of the project will concern either the simulation of binary black hole ringdowns or computing quantities on volume data from previously run simulations. Both of these require

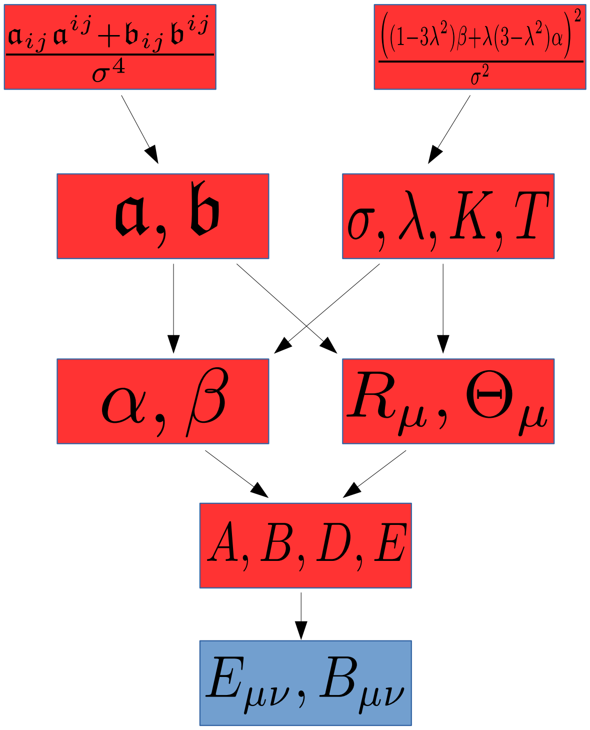

FIG. 5: Simplified dependency chart of quantities referenced by the second and third terms in equation (2). To aid in testing, a compute item was created for each quantity and tested individually. The orthogonal splitting of the Weyl tensor into the electric ($E_{\mu\nu}$) and magnetic ($B_{\mu\nu}$) components is used to compute these quantities on the time slice for use in SpEC.

considerably more time to run than the single black hole simulations used for validation and testing, and thus mistakes will be more expensive. In addition, because configuration is more complex for computations involving binary blackhole mergers, more time will be necessary to set up input files for the runs.

SHORT TERM GOALS

Short term goals for the next month center on further analysis of simulations and computations which have been run, and on the debugging of the currently coded terms of the $\mathcal{L}$ quantity. A decomposition of S computed for the binary blackhole ringdown data along sphereical harmonics will be done to determine if numerical noise in high modes is causing any problems with the results.

The first term of the $\mathcal{L}$ quantity is currently under active development and will be followed by the remaining terms after it has been validated by tests. The terms which have been coded of $\mathcal{L}$ can be run on the binary blackhole ringdown volume data very soon. Once the remaining terms are coded, $\mathcal{L}$ in its entirety can be run on the ringdown data.

Another goal for the next month is to organize what has been coded by moving certain quantities, which are currently independent compute items but which do not need to be exposed externally, into other compute items, and by improving comments.

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