LIGO

Modeling Loss and Thermal Noise in Silicon Gravitational Wave Detector Suspensions

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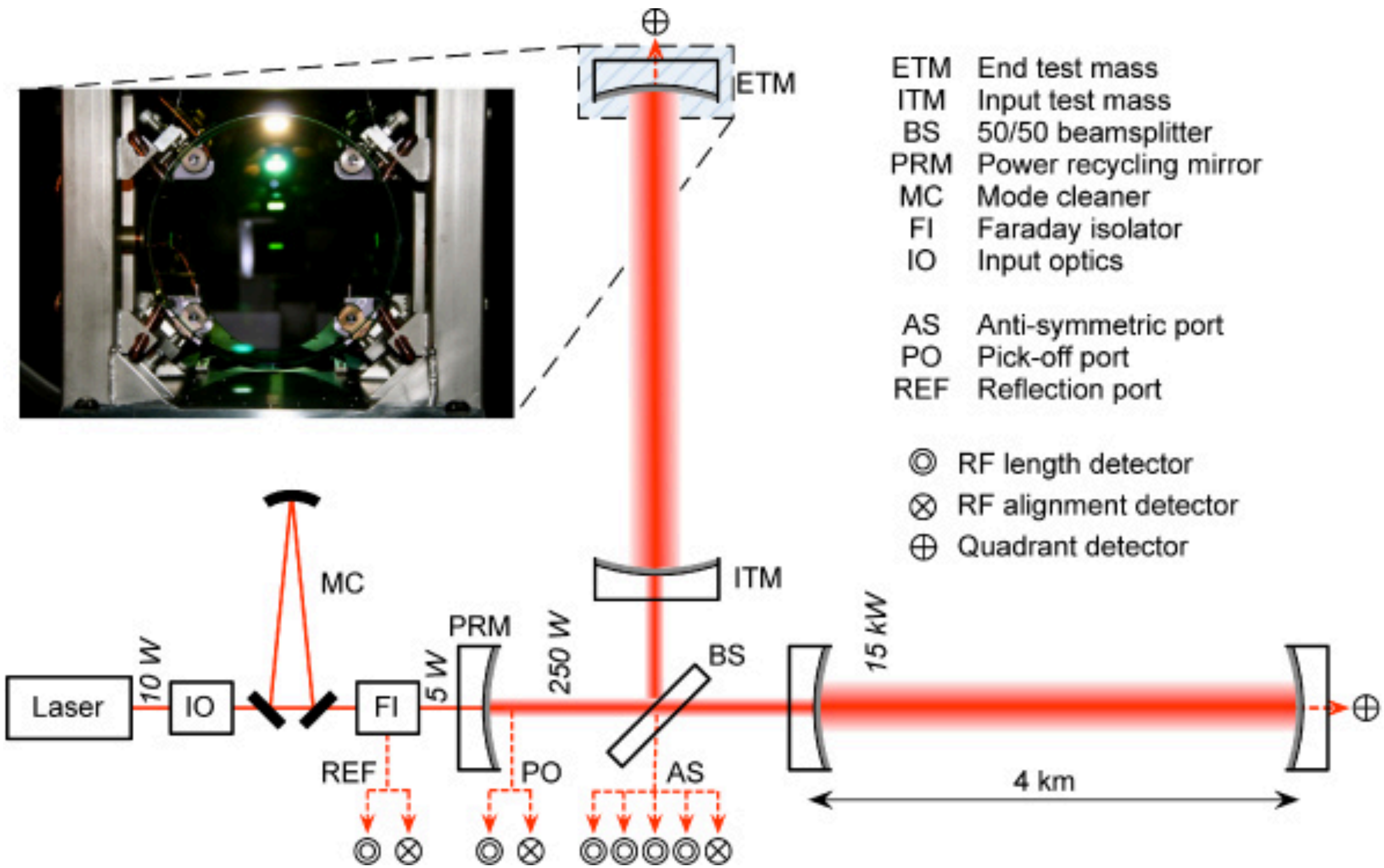
LIGO-G09xxxxx-v1

Overview

- **Advanced LIGO Detectors (2nd Generation)**
- 3rd Generation LIGO Detectors
- Thermal Noise
- Levin's Approach
- Finite Element Analysis
- Conclusions
- Acknowledgements

The Advanced LIGO Detectors



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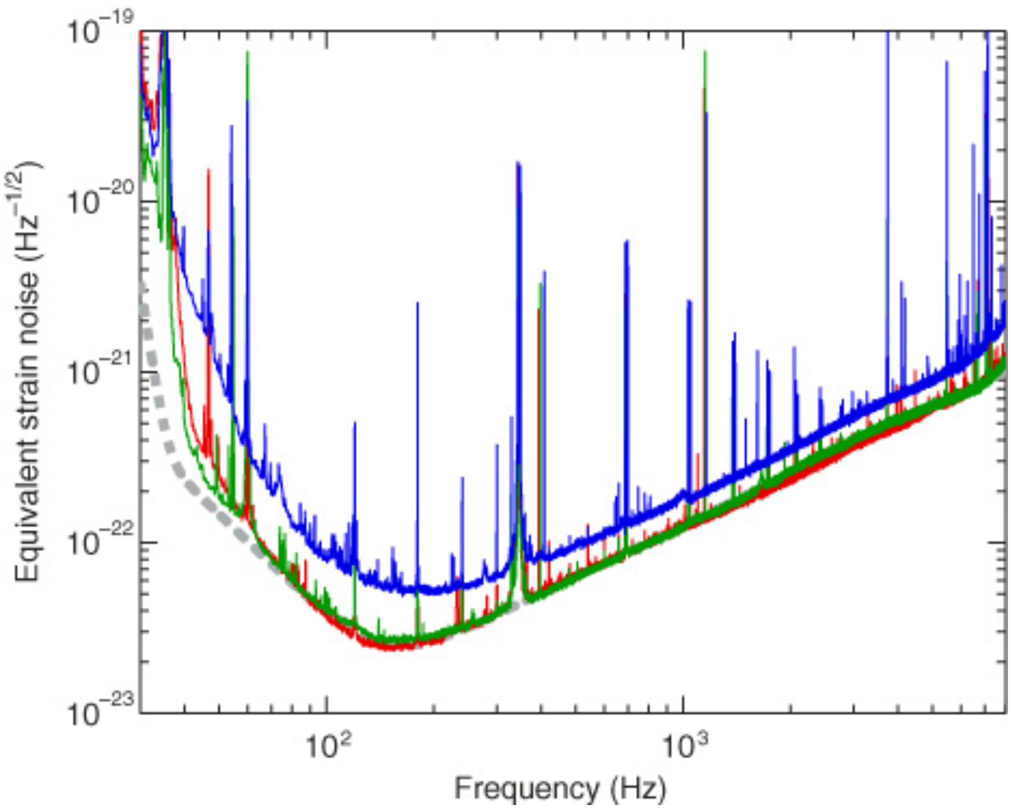
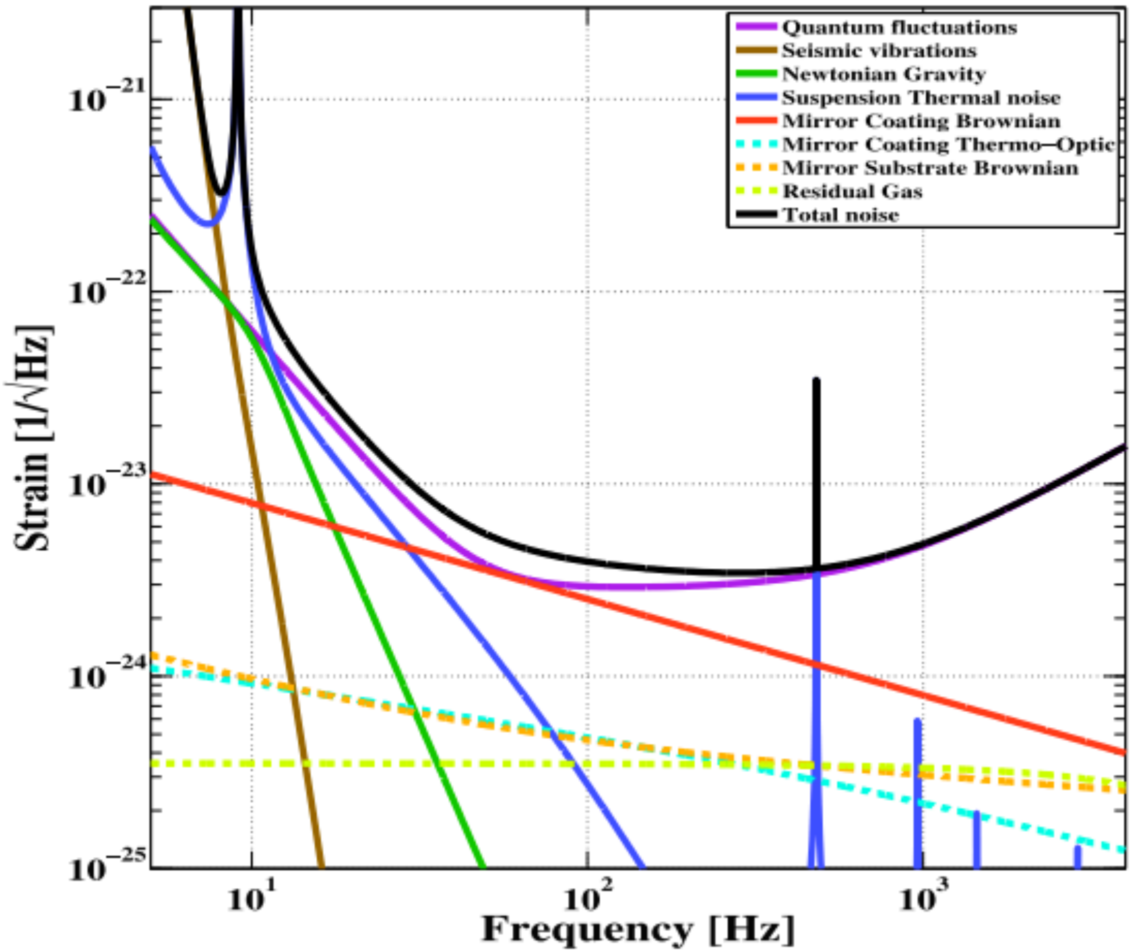
The Advanced LIGO Detectors

- So, what are the issues?

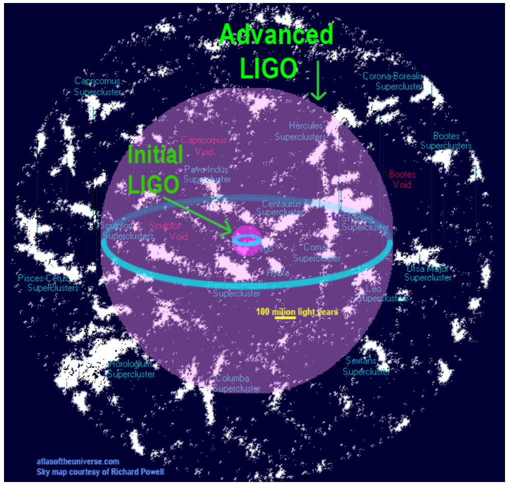


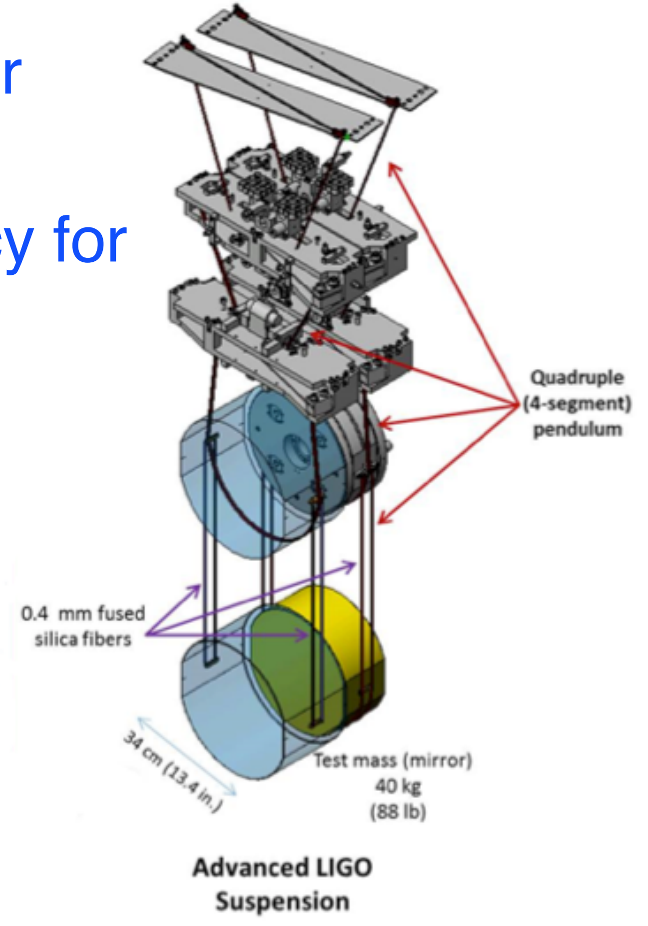
The Advanced LIGO Detectors



The Advanced LIGO Detectors

- Fused silica: low loss = high Q factor
- Quadruple pendulum: $1/f^2$ noise reduction above pendulum frequency for each stage
- Huge upgrade from iLIGO





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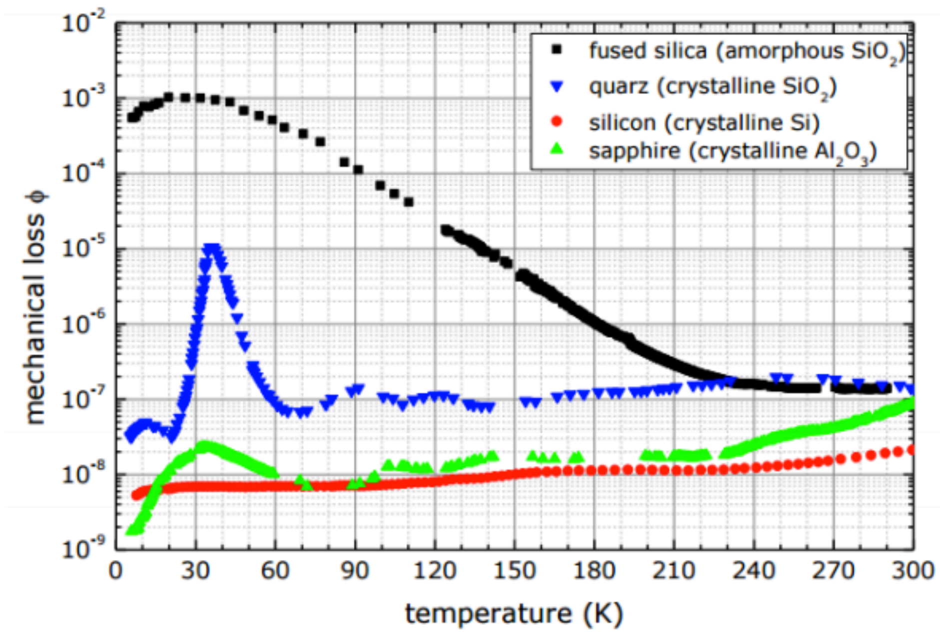
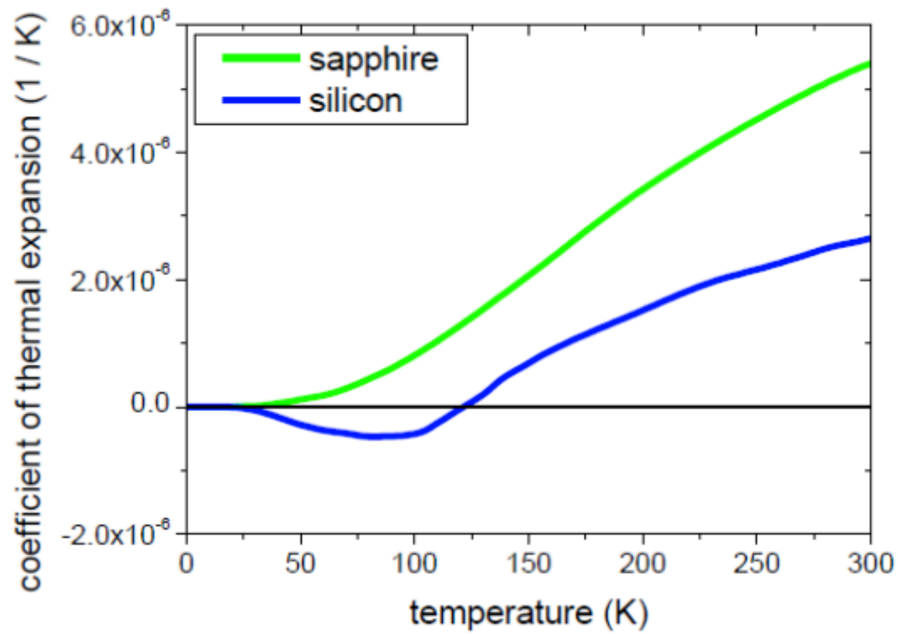
3rd Generation LIGO Detectors

- Longer interferometer arms
- Cryogenically cooled test mass
- New possible materials: Silicon, Sapphire, Fused Silica Hybrid
- New possible geometries: Suspension ribbons (rectangular cross-section) instead of fibers (circular cross-section)

3rd Generation LIGO Detectors





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Thermal Noise

- The main laser is used to make precise distance measurements between the ITM and ETM
- Noise is produced by fluctuations of molecules within the test mass material; heat energy dissipated in the form of kinetic motion from collisions
- Fluctuation-Dissipation Theorem: Thermal noise is a direct result of Brownian motion

$$S_x(\omega) = \frac{4k_BT}{\omega^2} \cdot \Re[Y(\omega)]$$

Callen et al.

Thermal Noise

$$S_x(\omega) = \frac{4k_BT}{\omega^2} \cdot \Re[Y(\omega)]$$

- The real part of the admittance of the material can be expressed in two ways

$$x^2(\omega) = \frac{4k_BT}{m\omega}\left(\frac{\omega_0^2\phi(\omega)}{\omega_0^4\phi^2(\omega) + (\omega_0^2 - \omega^2)^2}\right)$$

Modal Summation

$$S_x(f) = \frac{2k_BT}{\pi^2 f^2}\frac{W_{diss}}{F_0^2}$$

Levin's Direct Approach

Thermal Noise

- Loss and Dissipation
  - » $m\,x''(t) + b\,x'(t) + k\,x(t) = F(t)$
  - » Loss angle ϕ is the fractional energy lost per cycle due to damping

$$\phi_{\text{fibre}}(\omega) = \phi_{\text{surface fibre}} + \phi_{\text{bulk}}(\omega) + \phi_{\text{thermoelastic}}(\omega),$$

$$\phi_i(\omega) = \frac{8h\phi_s}{d_i} + 1.2\times 10^{-11} f^{0.77} + \frac{\omega\tau_i}{1+(\omega\tau_i)^2}\frac{YT}{\rho C}\left(\alpha - \sigma_i\frac{\beta}{Y}\right)^2,$$

- Energy put into the system is either stored in the conservative (non-dissipative) gravitational field or bending of the material
  - » Ratio is called the dilution factor which we seek to maximize

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Levin's Direct Approach

$$S_x(f) = \frac{2k_BT}{\pi^2 f^2}\frac{W_{diss}}{F_0^2}$$

$$W_{diss} = 2\pi f U_{max}\phi(f)$$

- Can handle inhomogeneous dissipation and arbitrary geometries
- All we need is the strain energy at maximum displacement and the loss angle

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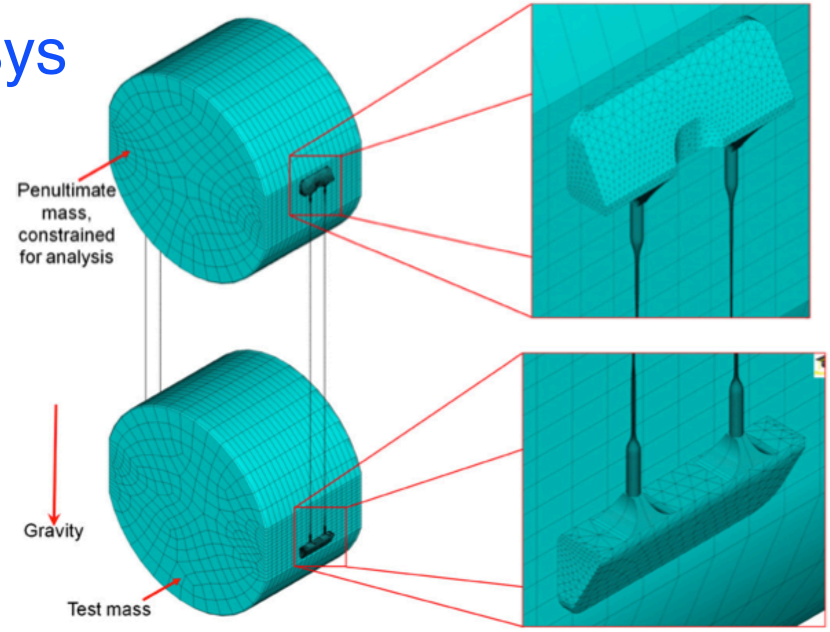
Finite Element Analysis

- Numerical technique for solving large problems with complicated geometries and physical properties.
- Subdivide into smaller elements via meshing
- Moving from Comsol to Ansys



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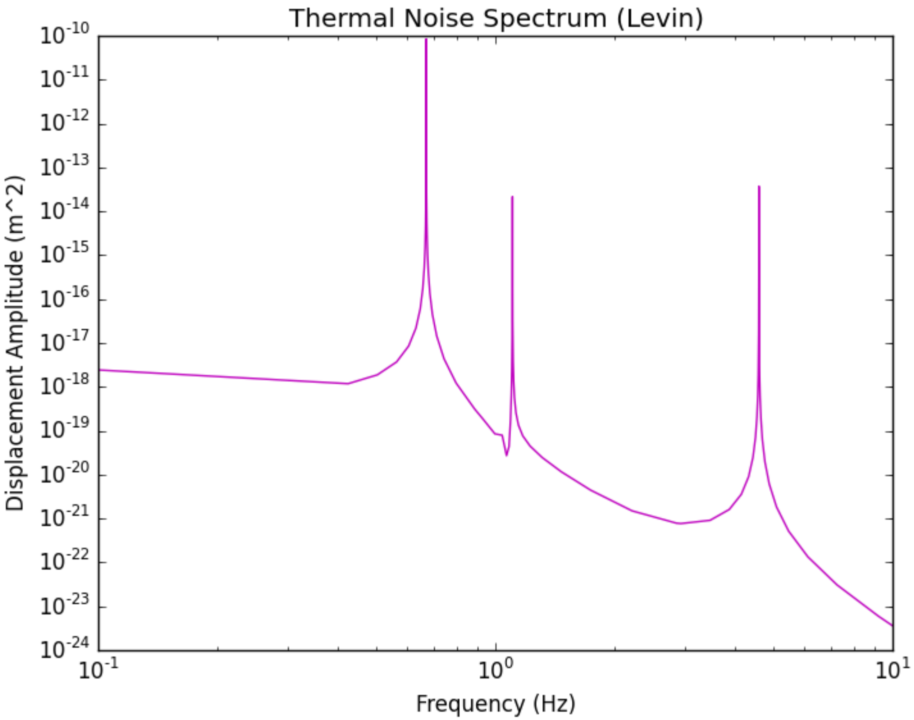
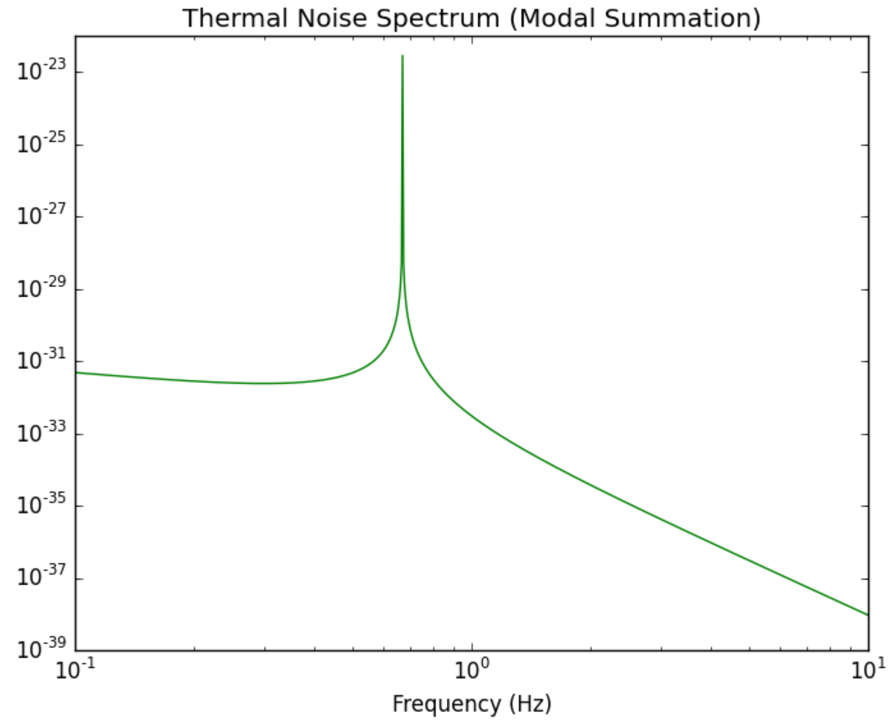
Finite Element Analysis

- Frequency Convergence
- In a harmonic analysis, Ansys can give stress and strain data values at each element, but we need strain energy

$$U = \frac{1}{2} V \sigma \epsilon = \frac{1}{2} V Y \epsilon^2 = \frac{1}{2} \frac{V}{Y} \sigma^2$$

Conclusions





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Thank you!