



Developing a Coherent Search Algorithm to Identify Phase Modulated Continuous Gravitational Wave Sources

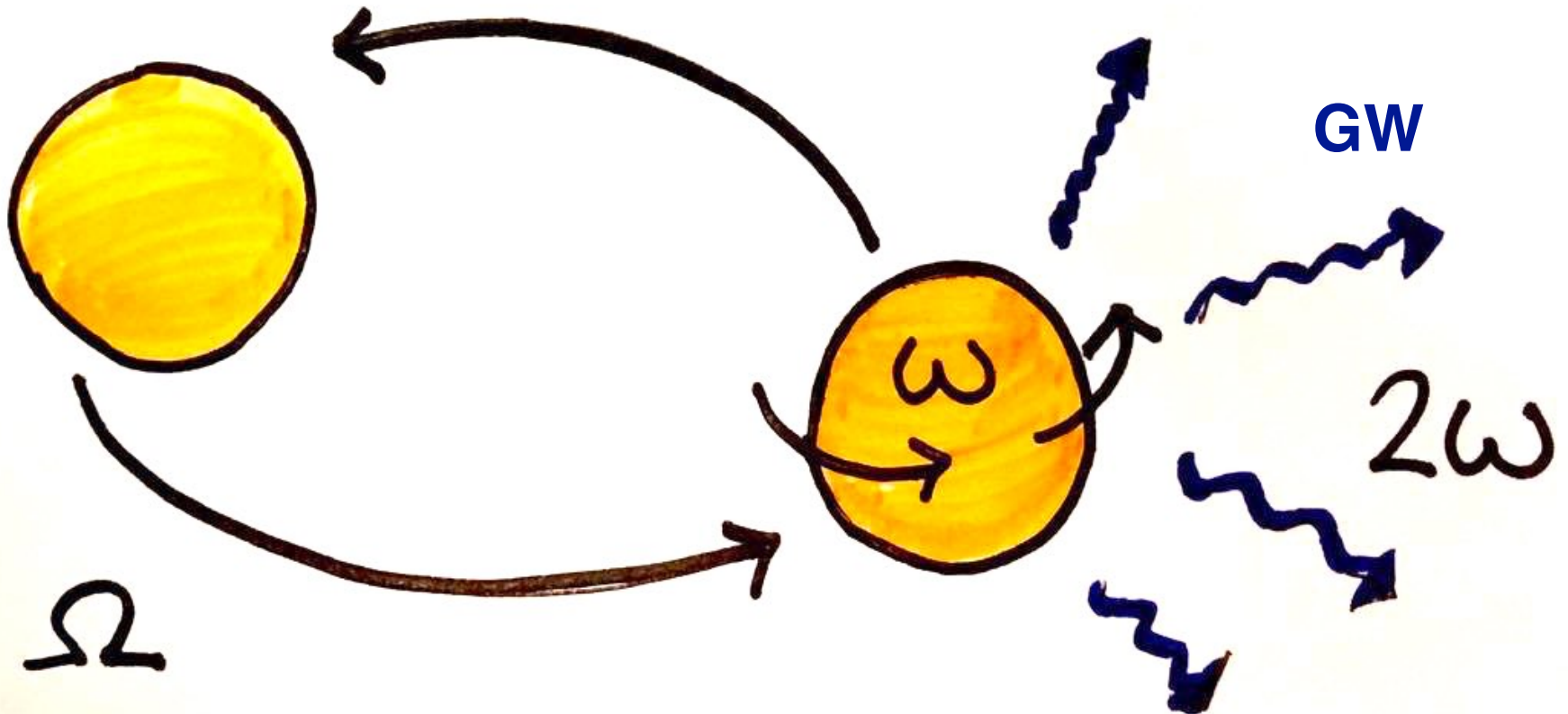
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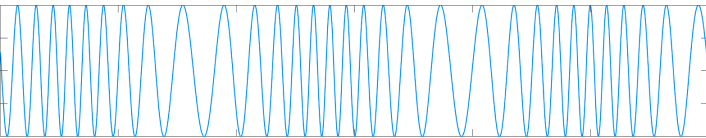
Aim: To develop a coherent search algorithm to identify gravitational waves from binary neutron star systems.



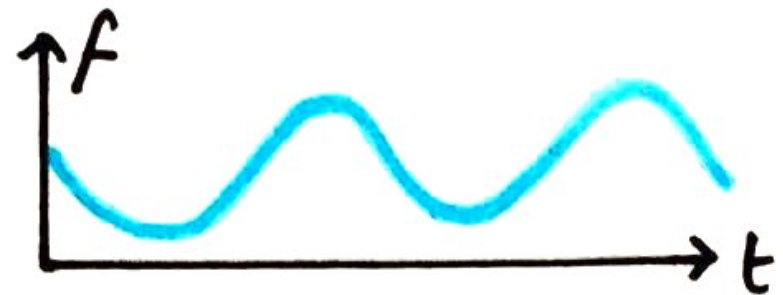
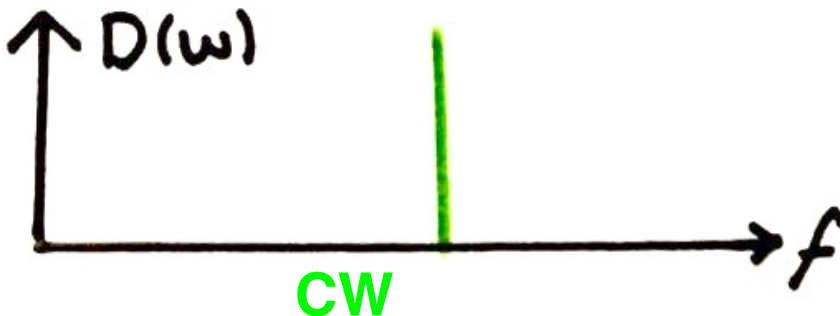
$$h \approx 10^{-27} \cdot \left(\frac{I^3}{10^{38} \text{kg m}^2} \right) \cdot \left(\frac{10 \text{kPc}}{R} \right) \cdot \left(\frac{\omega}{100 \text{Hz}} \right) \cdot \left(\frac{\epsilon}{10^{-6}} \right)$$

The Search Algorithm

A search is performed in three dimensions, over carrier frequency, ω , modulation frequency, Ω , and modulation index, Γ .



$$d(t) = \cos(\omega_0 t + \Gamma \cos(\Omega t))$$



The Search Algorithm

The expected phase modulated gravitational wave is of the form

$$d(t) = e^{i(\omega_0 t + \Gamma \cos(\Omega t))}.$$

It is dependant on carrier frequency, ω , modulation frequency, Ω , and modulation index, Γ .

$$d(t).e^{-i\Gamma \cos(\Omega t)} = e^{i\omega_0 t}$$

$$\text{FT}\{d(t).e^{-i\Gamma \cos(\Omega t)}\} = \text{FT}\{e^{i\omega_0 t}\}$$

Using the Convolution principle:

$$\text{FT}\{A.B\} = \text{FT}\{A\} * \text{FT}\{B\}$$

$$\text{FT}\{d(t)\} * \text{FT}\{e^{-i\Gamma \cos(\Omega t)}\} = \text{FT}\{e^{i\omega_0 t}\}$$

The Search Algorithm

$$\text{FT}\{d(t)\} * \text{FT}\{e^{-i\Gamma \cos(\Omega t)}\} = \text{FT}\{e^{i\omega_0 t}\}$$

Using the Jacobi-Anger expansion:

$$e^{i\Gamma \cos(\Omega t)} = \sum_{n=-\infty}^{\infty} i^n J_n(\Gamma) e^{in\Omega t}$$

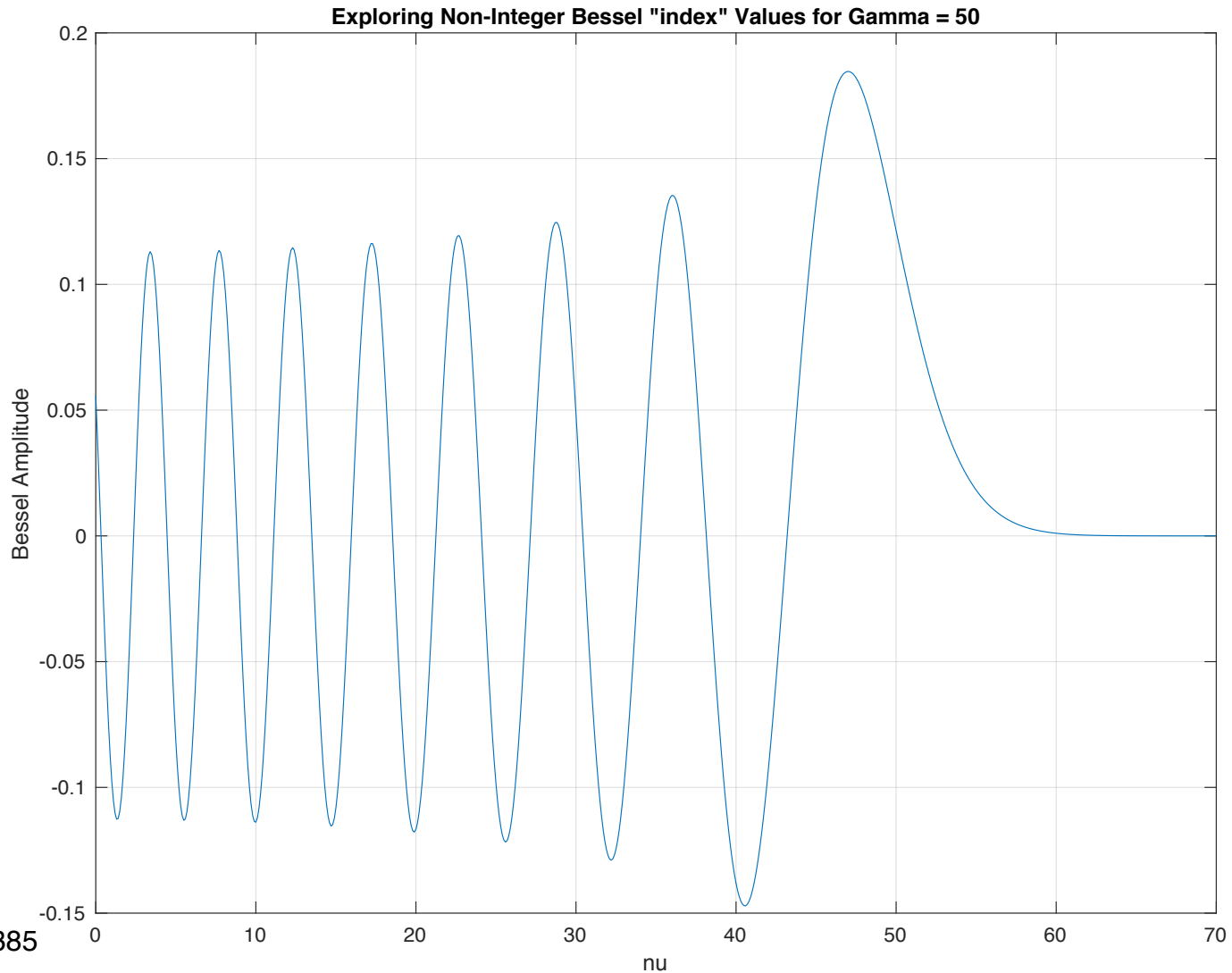
Where $D(\omega)$ is the Fourier transform of the data, $d(t)$, at ω .

$$D(\omega) * \text{FT}\left\{ \sum_{n=-\infty}^{\infty} (-i)^n J_n(\Gamma) e^{in\Omega t} \right\} = \text{FT}\{e^{i\omega_0 t}\}$$

Final Search is over carrier frequency, ω , modulation frequency, Ω , and modulation index, Γ :

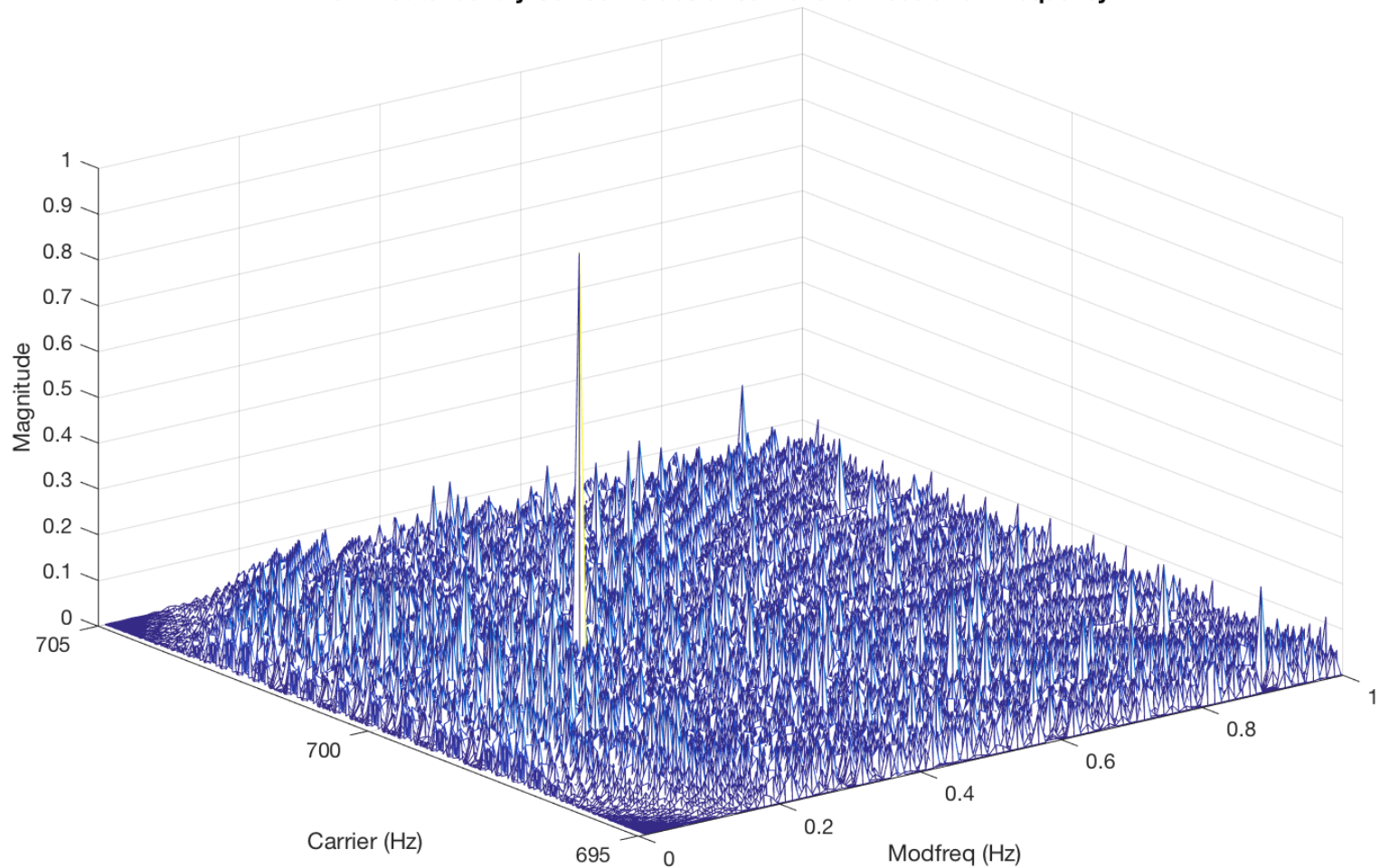
$$\sum_{n=-\infty}^{\infty} (-i)^n J_n(\Gamma) D(\omega - n\Omega) = \delta(\omega - \omega_0)$$

How Bessel Terms Control the Sidebands



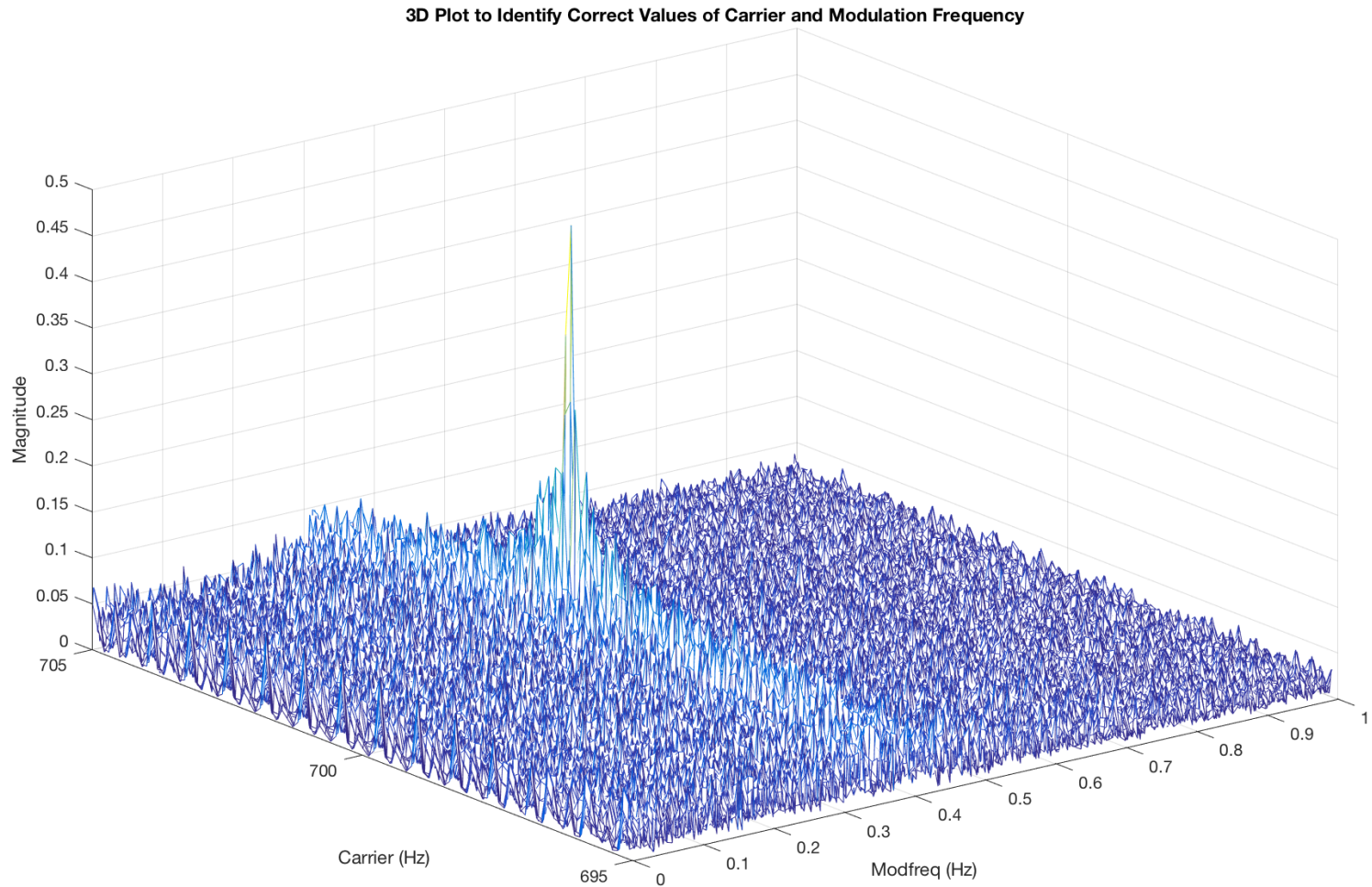
The Perfect Result

3D Plot to Identify Correct Values of Carrier and Modulation Frequency



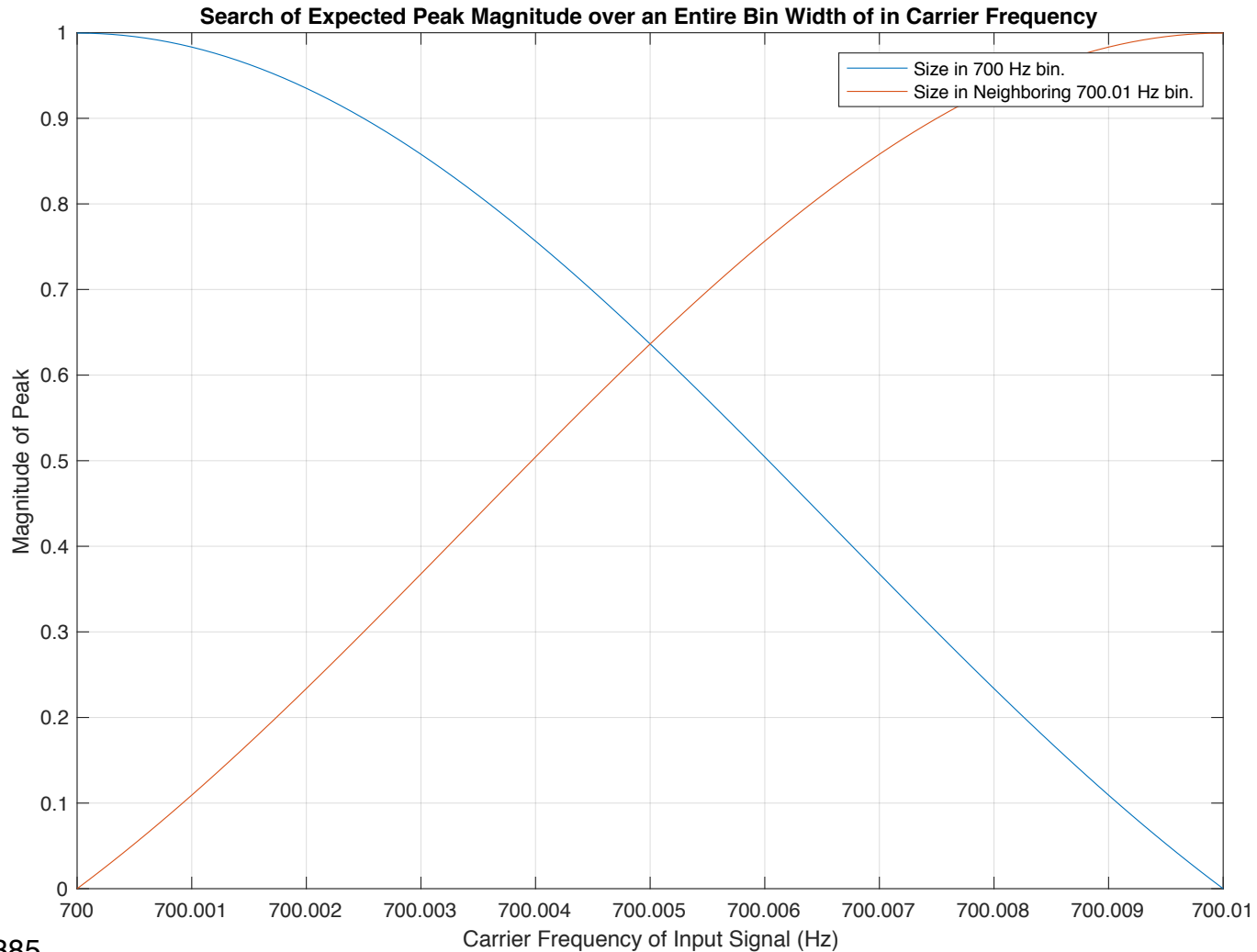


The Result Currently Achieved

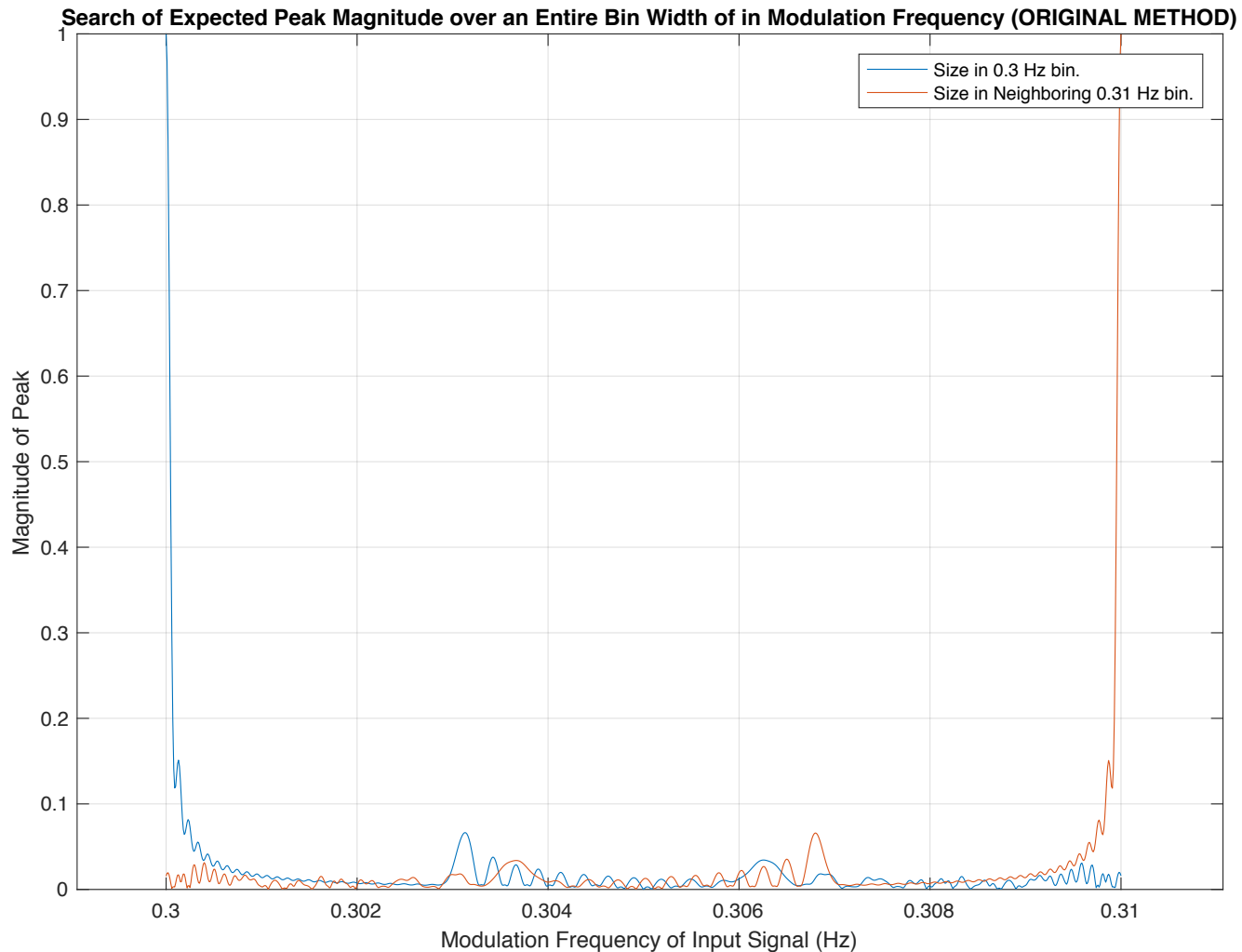




Searching over Carrier Frequency

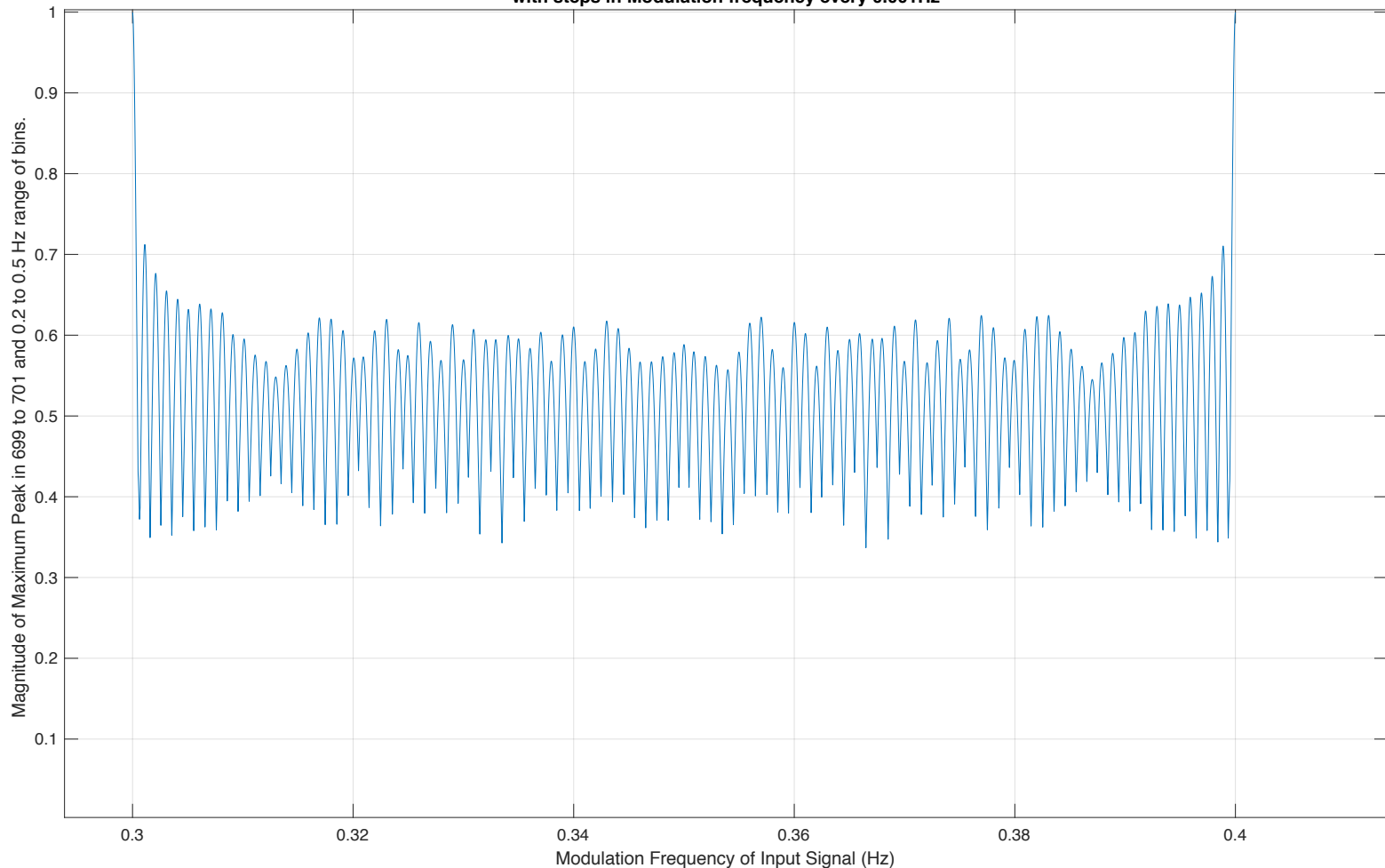


Searching over Modulation Frequency



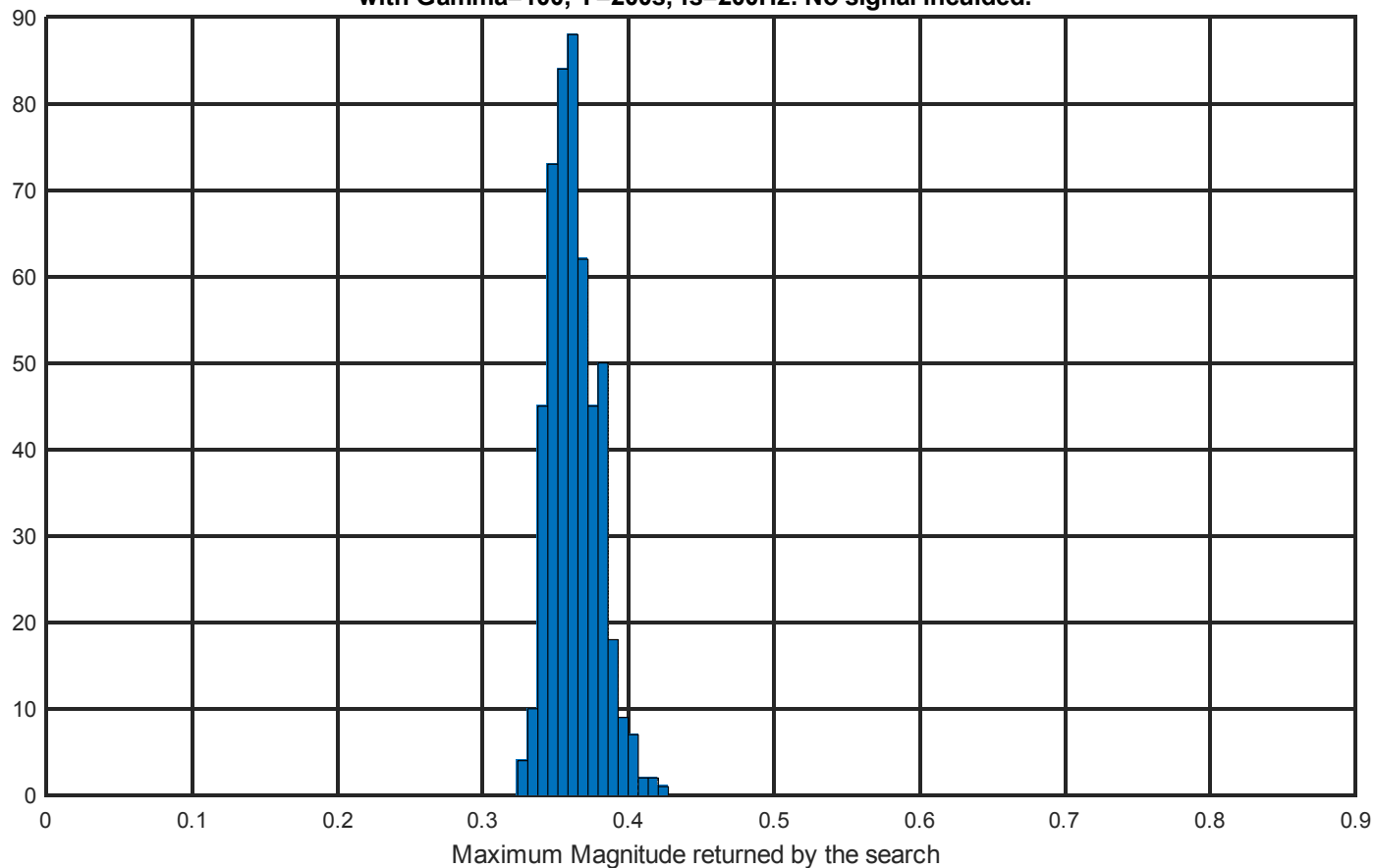
Searching over Modulation Frequency

Search of Maximum Peak Magnitude over an Entire Bin Width of in Modulation Frequency (ORIGINAL METHOD)
with steps in Modulation frequency every 0.001Hz



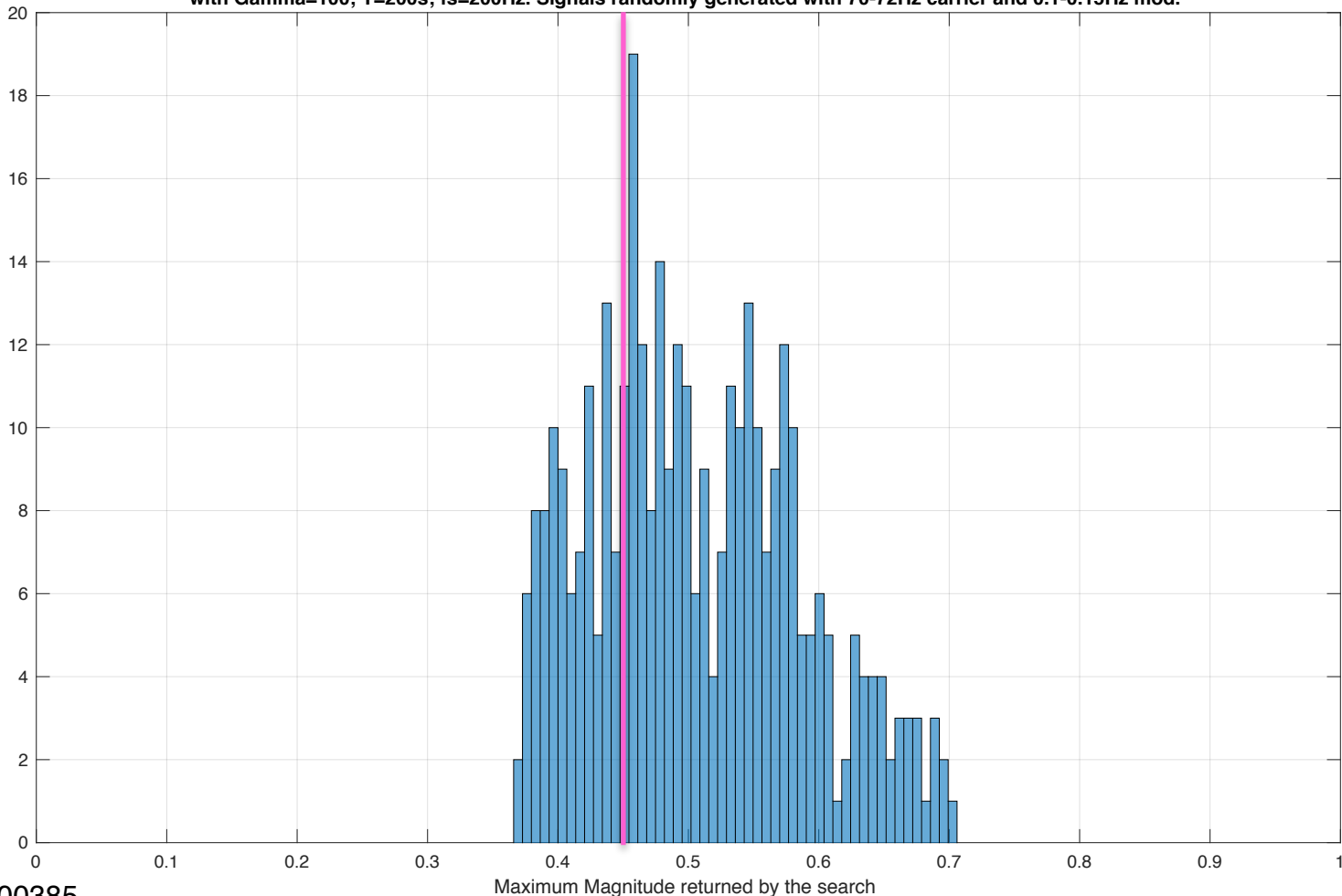
Histogram of Search Results: Only Noise

Histogram of the maximum Magnitude in Search over range 50-80Hz and 0-0.25Hz.
SIGMA = 10 (TIME DOMAIN NOISE). With injected phase modulated signal of size 1 in the time domain
with Gamma=100; T=200s; fs=200Hz. No signal included.



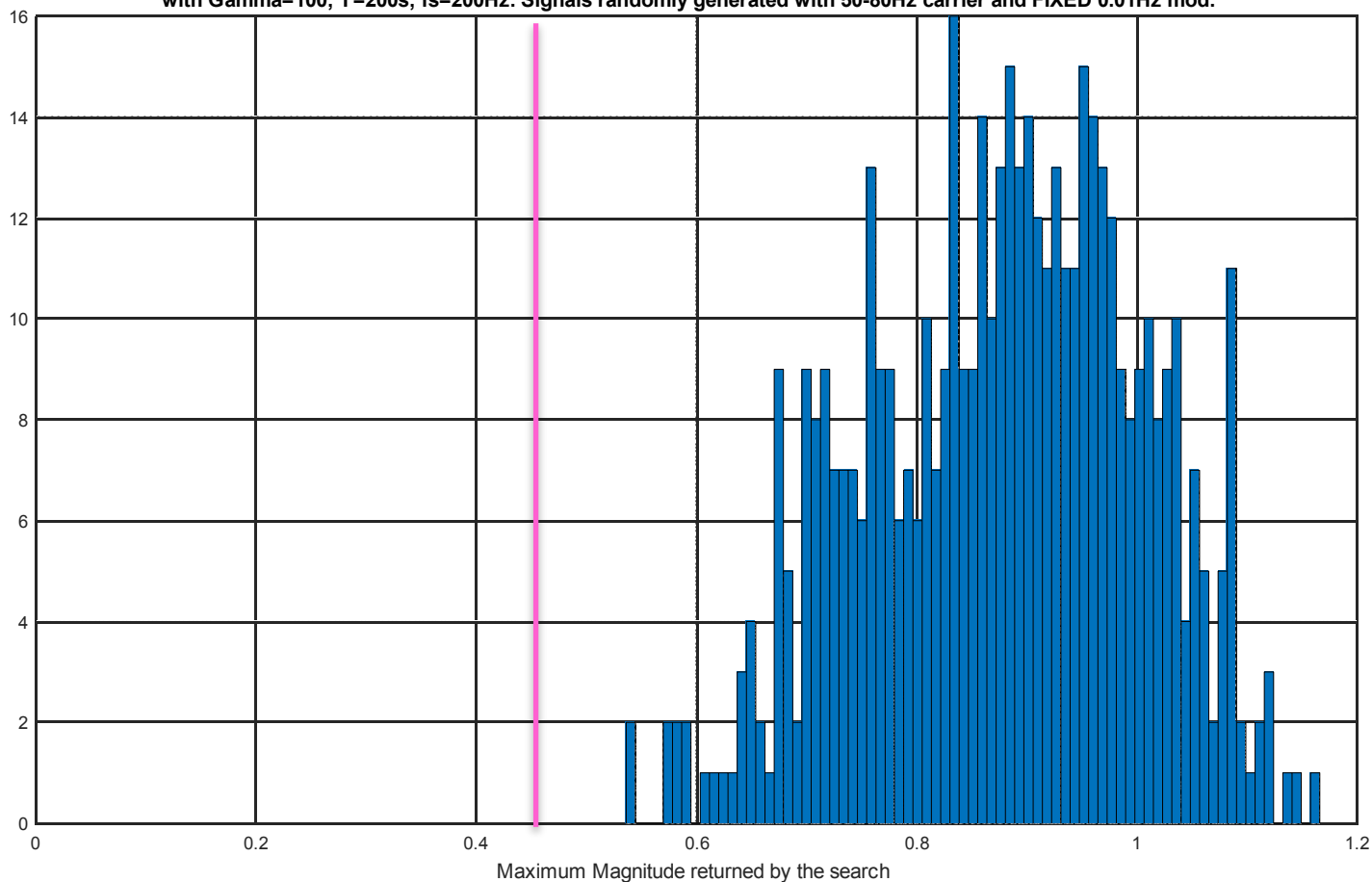
Histogram of Search Results: Noise + Injected Signals

Histogram of the maximum Magnitude in Search over range 70-72Hz and 0-0.2Hz.
SIGMA = 10. With injected phase modulated signal of size 1 in the time domain
with Gamma=100; T=200s; fs=200Hz. Signals randomly generated with 70-72Hz carrier and 0.1-0.15Hz mod.



Searching over Modulation Frequency

Histogram of the maximum Magnitude in Search over range 50-80Hz and 0-0.25Hz.
SIGMA = 10 (TIME DOMAIN NOISE). With injected phase modulated signal of size 1 in the time domain
with Gamma=100; T=200s; fs=200Hz. Signals randomly generated with 50-80Hz carrier and FIXED 0.01Hz mod.



Conclusion

A search algorithm has been developed that is coherent.

The searches are convolution based, where the convolution is performed between the data in frequency space, which has only been Fourier transformed once, and its expected modulated form.

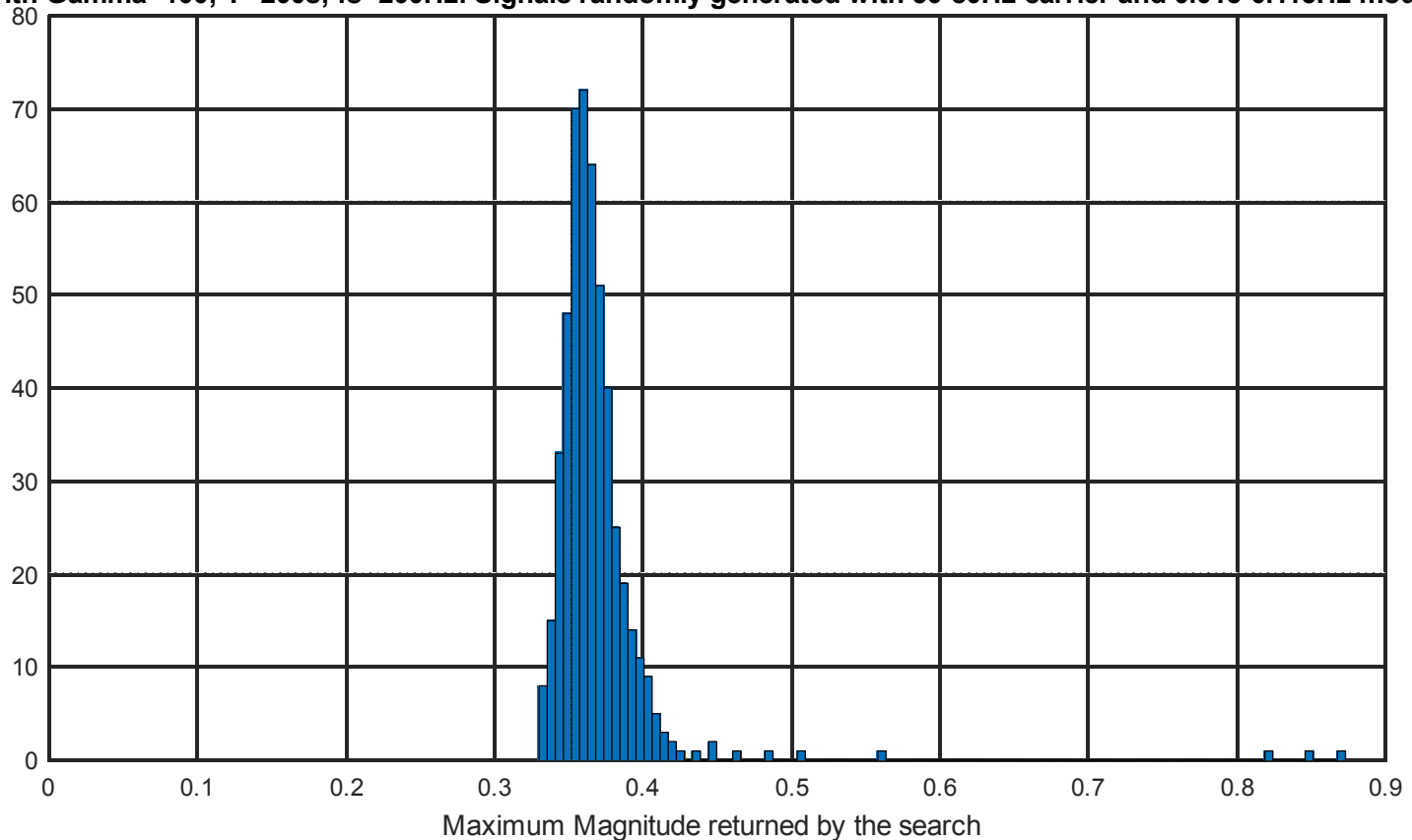
Some undesirable features are still present in the search over Modulation frequency thought to be due to the application of the Discrete Fourier transform to the data.



Thank you for Listening!

Histogram of Search Results: Noise + Injected Signals

Histogram of the maximum Magnitude in Search over range 50-80Hz and 0-0.25Hz.
SIGMA = 10 (TIME DOMAIN NOISE). With injected phase modulated signal of size 1 in the time domain
with Gamma=100; T=200s; fs=200Hz. Signals randomly generated with 50-80Hz carrier and 0.015-0.115Hz mod.



Modulation index Γ

$$\Gamma = \omega R / c$$

Derived from Doppler shift.

How much it swings in phase.

Coherent?

Estimate phase resolution of a few degrees.

4deg/60deg $\sim 1/15$

Square root ~ 4 times better resolution

This leads to increased sensitivity in the range

$\sim 4^3$ more signals received.

ie. 60x more detections