Developing a Coherent Search Algorithm to Identify Phase Modulated Continuous Gravitational Wave Sources

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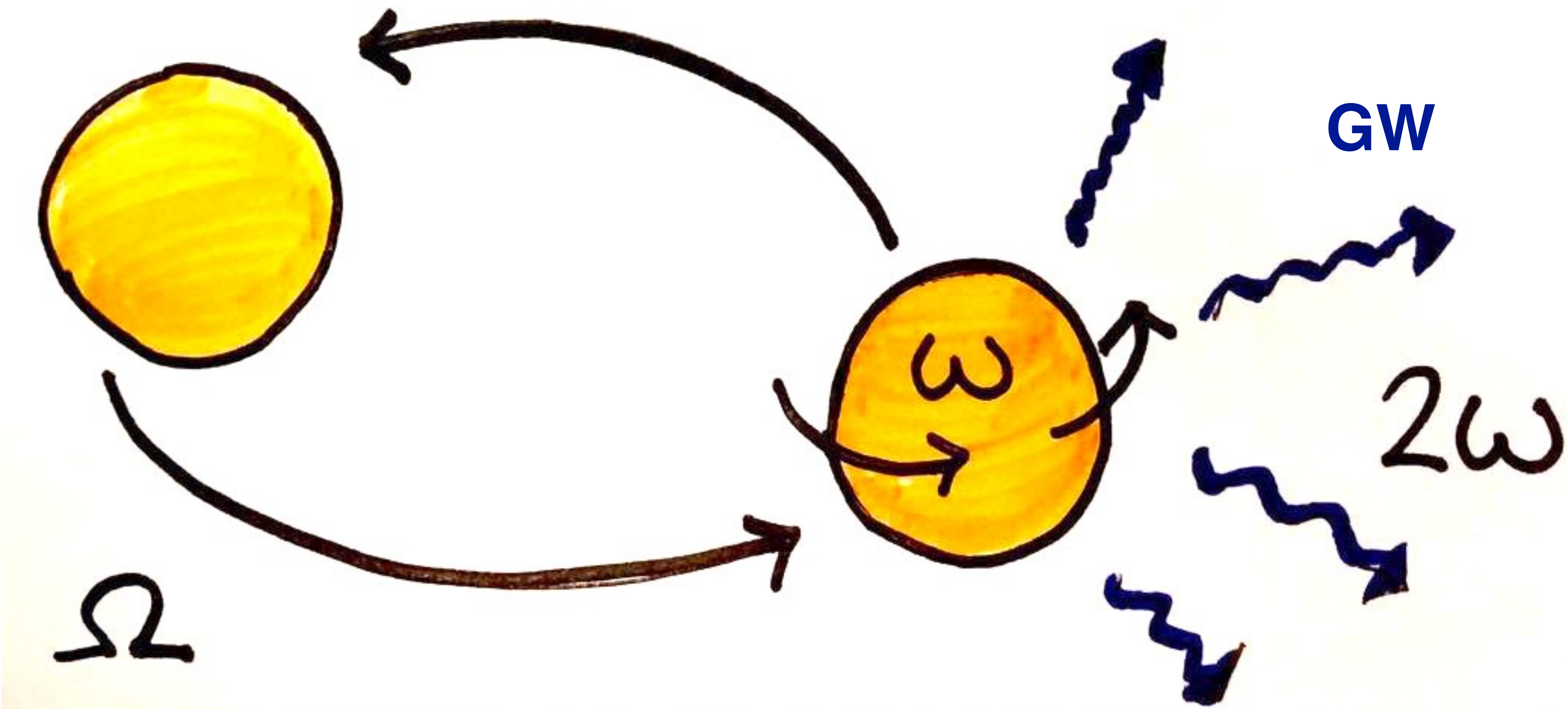
18th August 2016

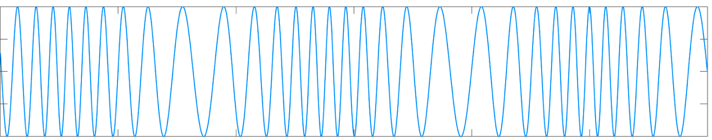
Aim: To develop a coherent search algorithm to identify gravitational waves from binary neutron star systems.



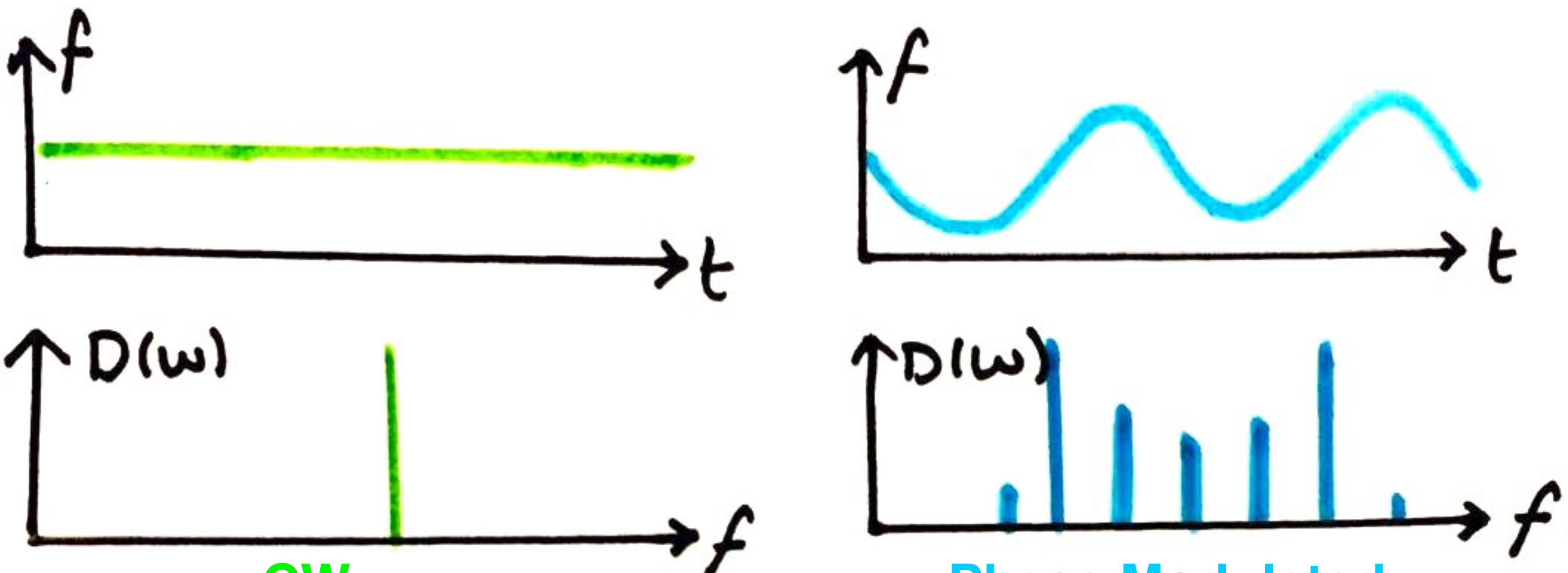
$$h \approx 10^{-27} \cdot \left(\frac{\mathrm{I3}}{10^{38}\mathrm{kg\ m}^2} \right) \cdot \left(\frac{10\mathrm{kPc}}{\mathrm{R}} \right) \cdot \left(\frac{\omega}{100\mathrm{Hz}} \right) \cdot \left(\frac{\epsilon}{10^{-6}} \right)$$

The Search Algorithm

A search is performed in three dimensions, over carrier frequency, ω, modulation frequency, Ω, and modulation index, Γ.



$$d(t) = cos(\omega_0 t + \Gamma cos(\Omega t))$$



CW

Phase Modulated

The Search Algorithm

The expected phase modulated gravitational wave is of the form

$$d(t) = e^{i(\omega_0 t + \Gamma cos(\Omega t))}.$$

It is dependant on carrier frequency, ω, modulation frequency, Ω, and modulation index, Γ.

$$d(t).e^{-i\Gamma cos(\Omega t)} = e^{i\omega_0 t}$$

$$\mathrm{FT}\{d(t).e^{-i\Gamma cos(\Omega t)}\} = \mathrm{FT}\{e^{i\omega_0 t}\}$$

Using the Convolution principle:

$$\mathrm{FT}\{A.B\} = \mathrm{FT}\{A\} * \mathrm{FT}\{B\}$$

$$\mathrm{FT}\{d(t)\} * \mathrm{FT}\{e^{-i\Gamma cos(\Omega t)}\} = \mathrm{FT}\{e^{i\omega_0 t}\}$$

LIGO

The Search Algorithm

$$\mathrm{FT}\{d(t)\} * \mathrm{FT}\{e^{-i\Gamma cos(\Omega t)}\} = \mathrm{FT}\{e^{i\omega_0 t}\}$$

Using the Jacobi-Anger expansion:

$$e^{i\Gamma cos(\Omega t)} = \sum_{n=-\infty}^{\infty} i^n J_n(\Gamma)\, e^{in\Omega t}$$

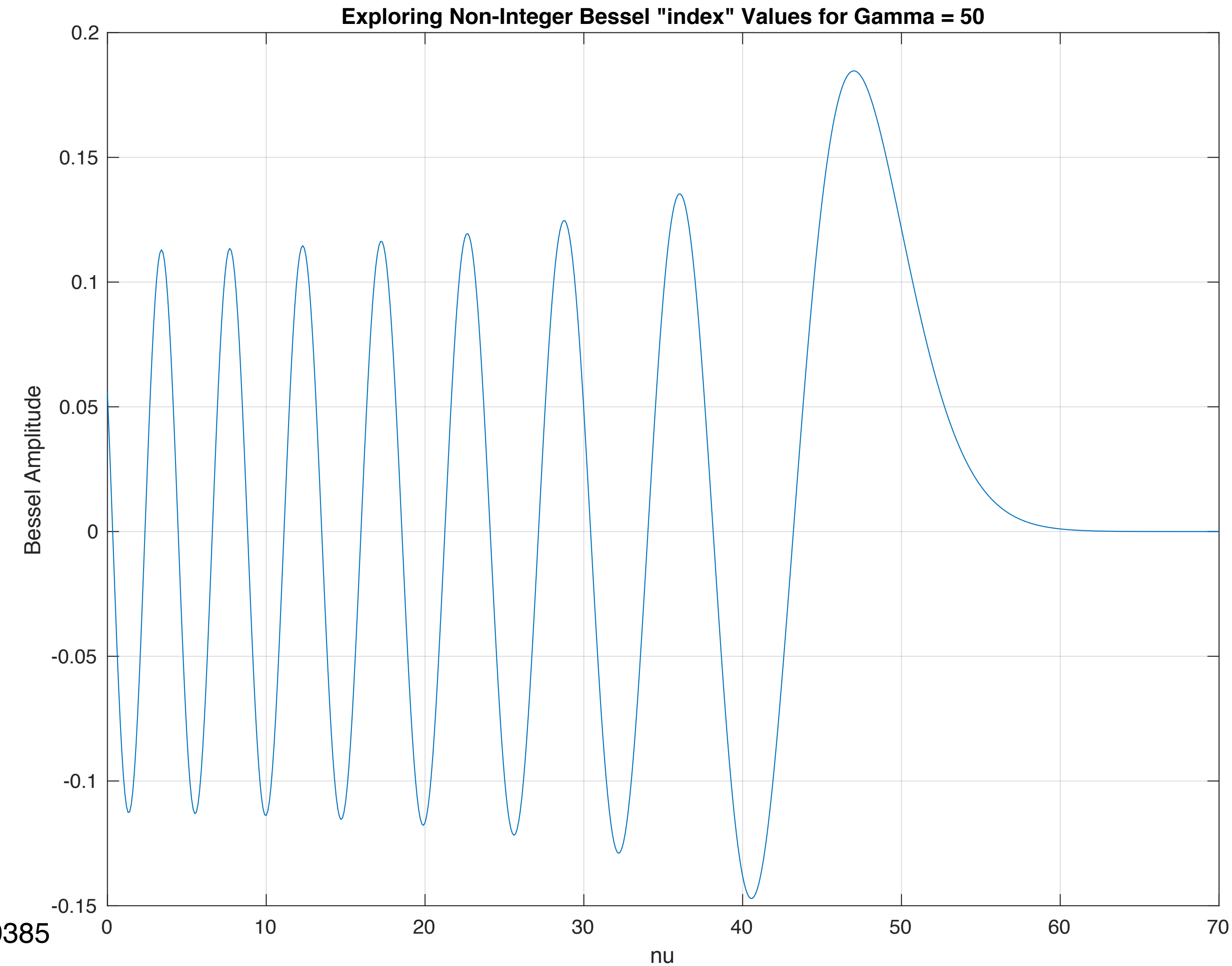
Where $D(\omega)$ is the Fourier transform of the data, $d(t)$, at ω.

$$D(\omega) * \mathrm{FT}\{\sum_{n=-\infty}^{\infty} (-i)^n J_n(\Gamma)\, e^{in\Omega t}\} = \mathrm{FT}\{e^{i\omega_0 t}\}$$

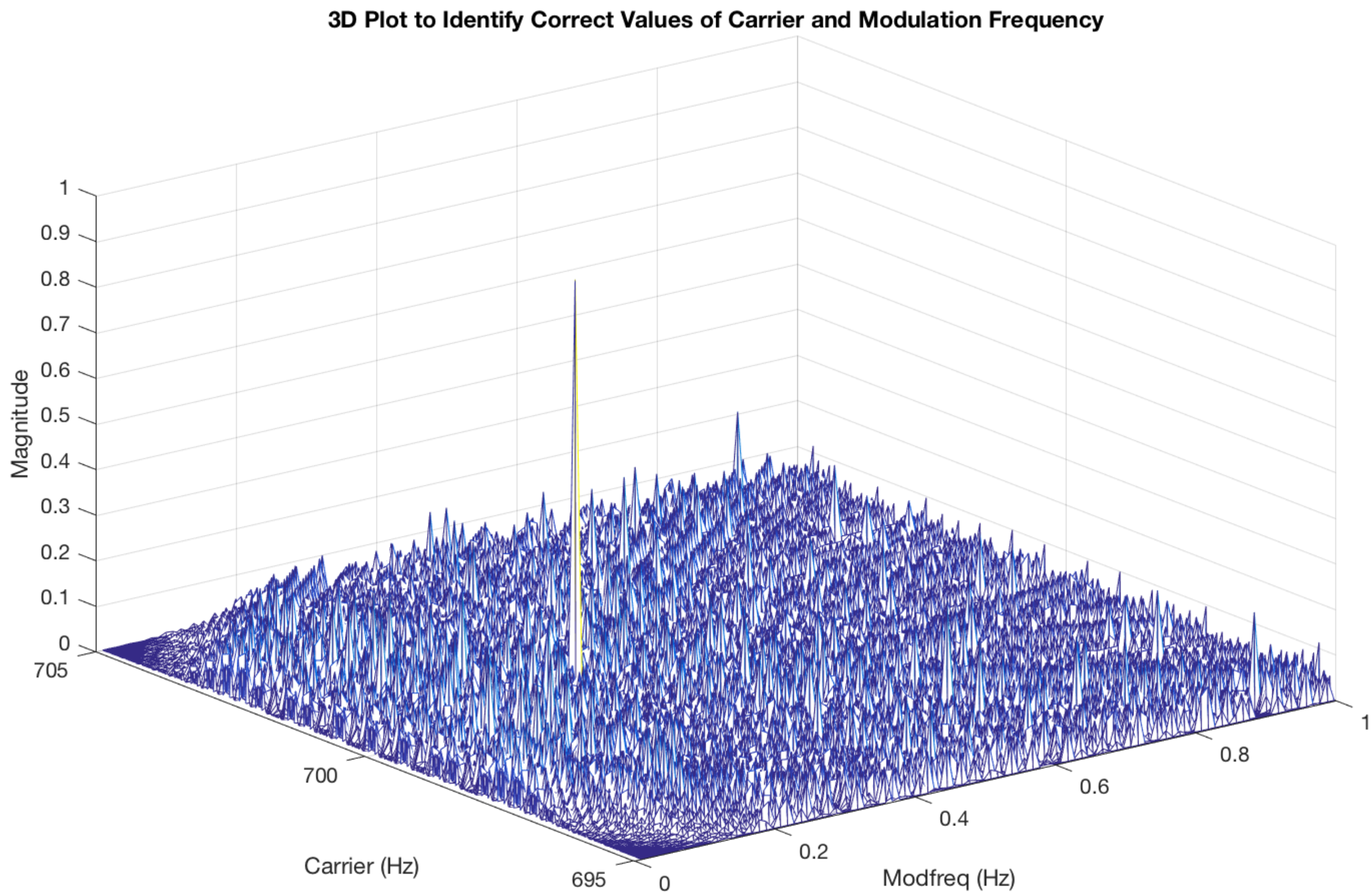
Final Search is over carrier frequency, ω, modulation frequency, Ω, and modulation index, Γ :

$$\sum_{n=-\infty}^{\infty} (-i)^n J_n(\Gamma) D(\omega - n\Omega) = \delta(\omega - \omega_0)$$

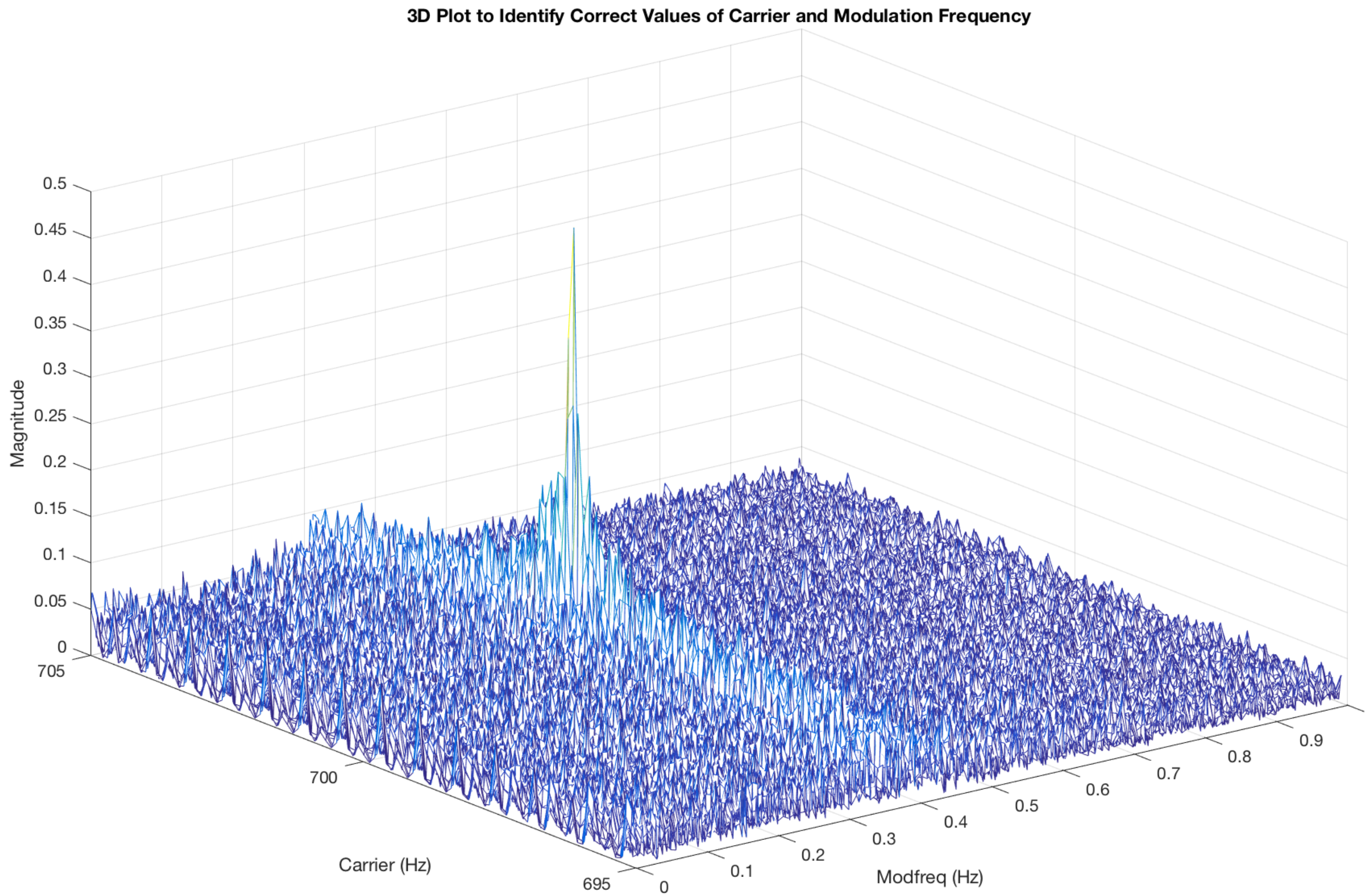
How Bessel Terms Control the Sidebands



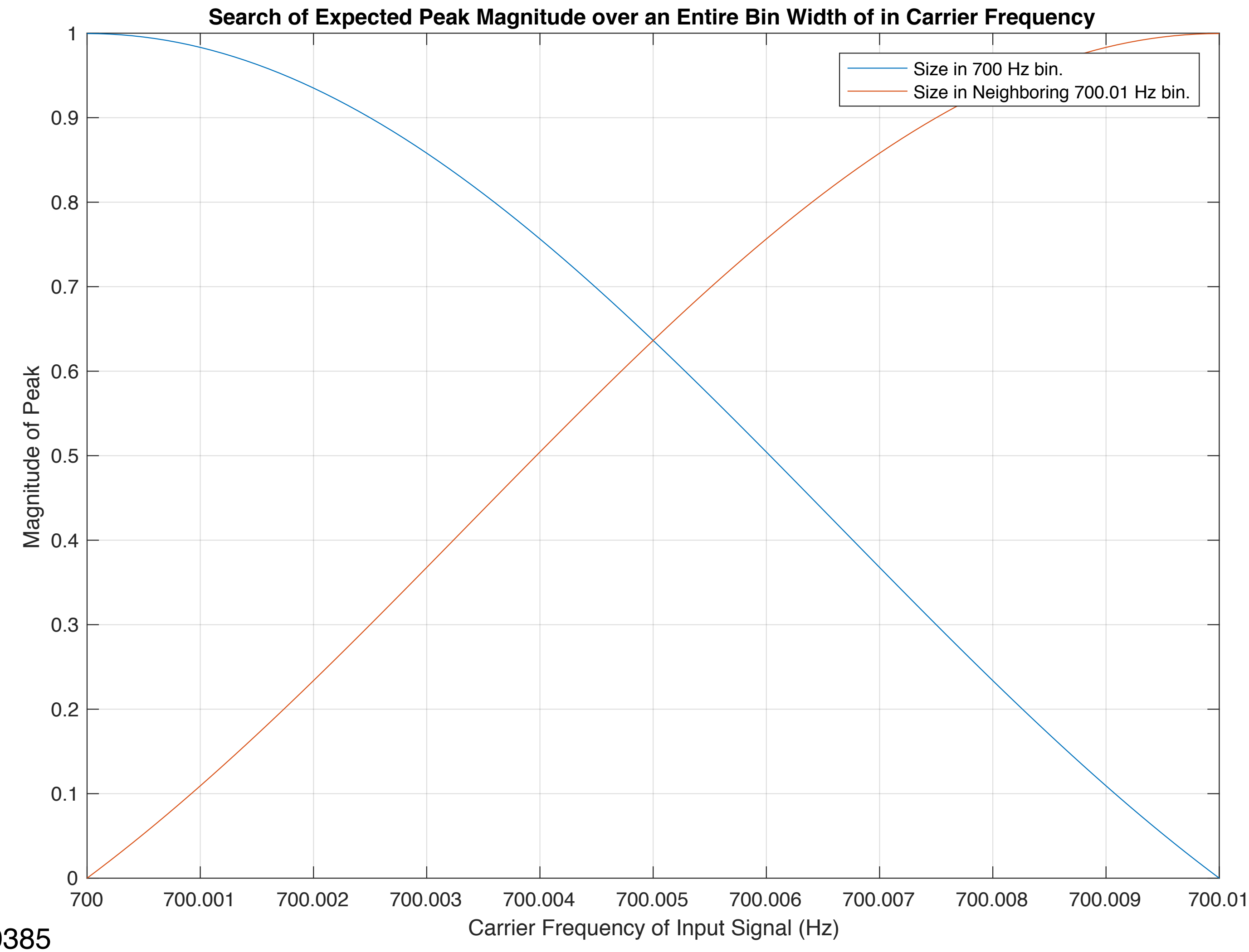
The Perfect Result



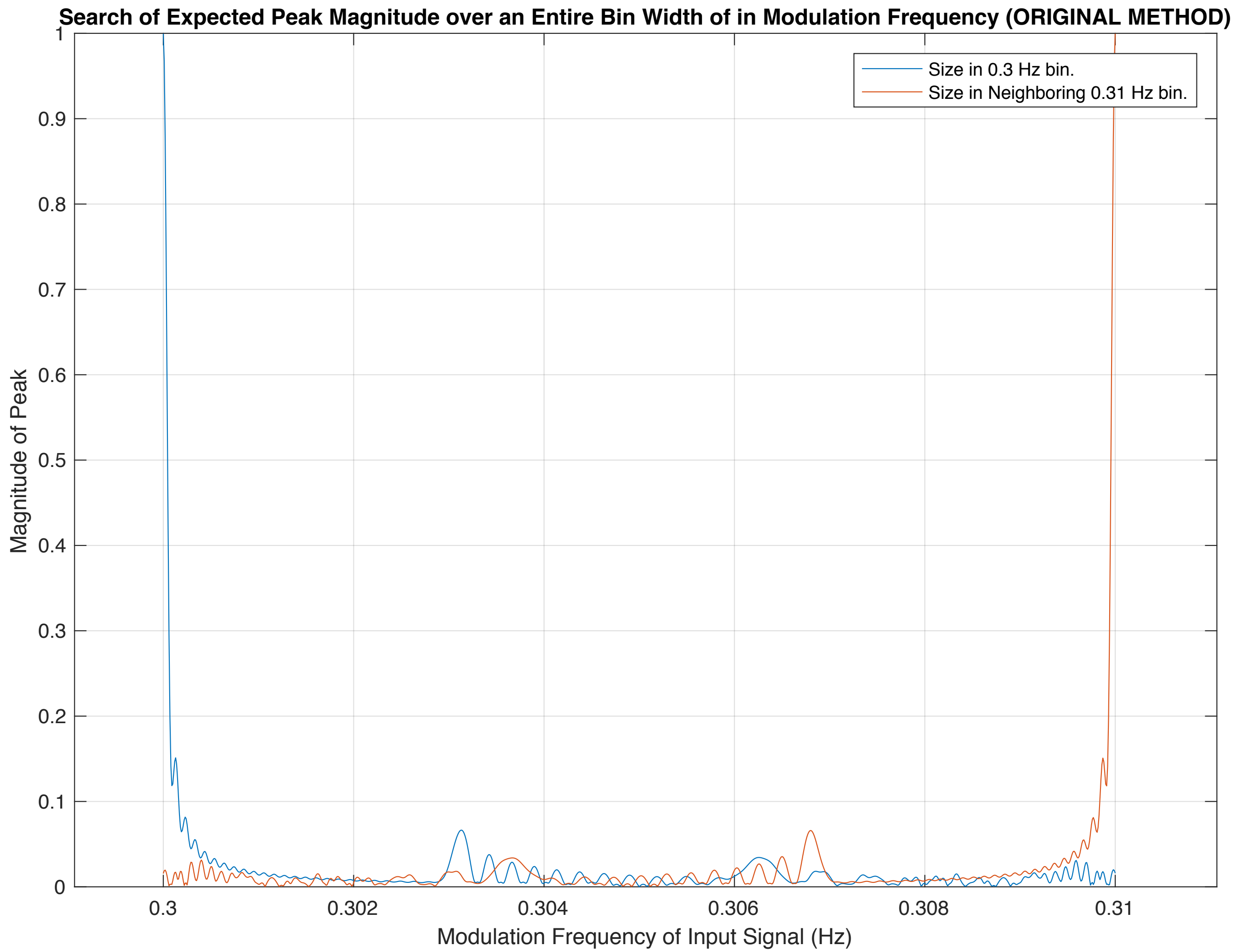
The Result Currently Achieved



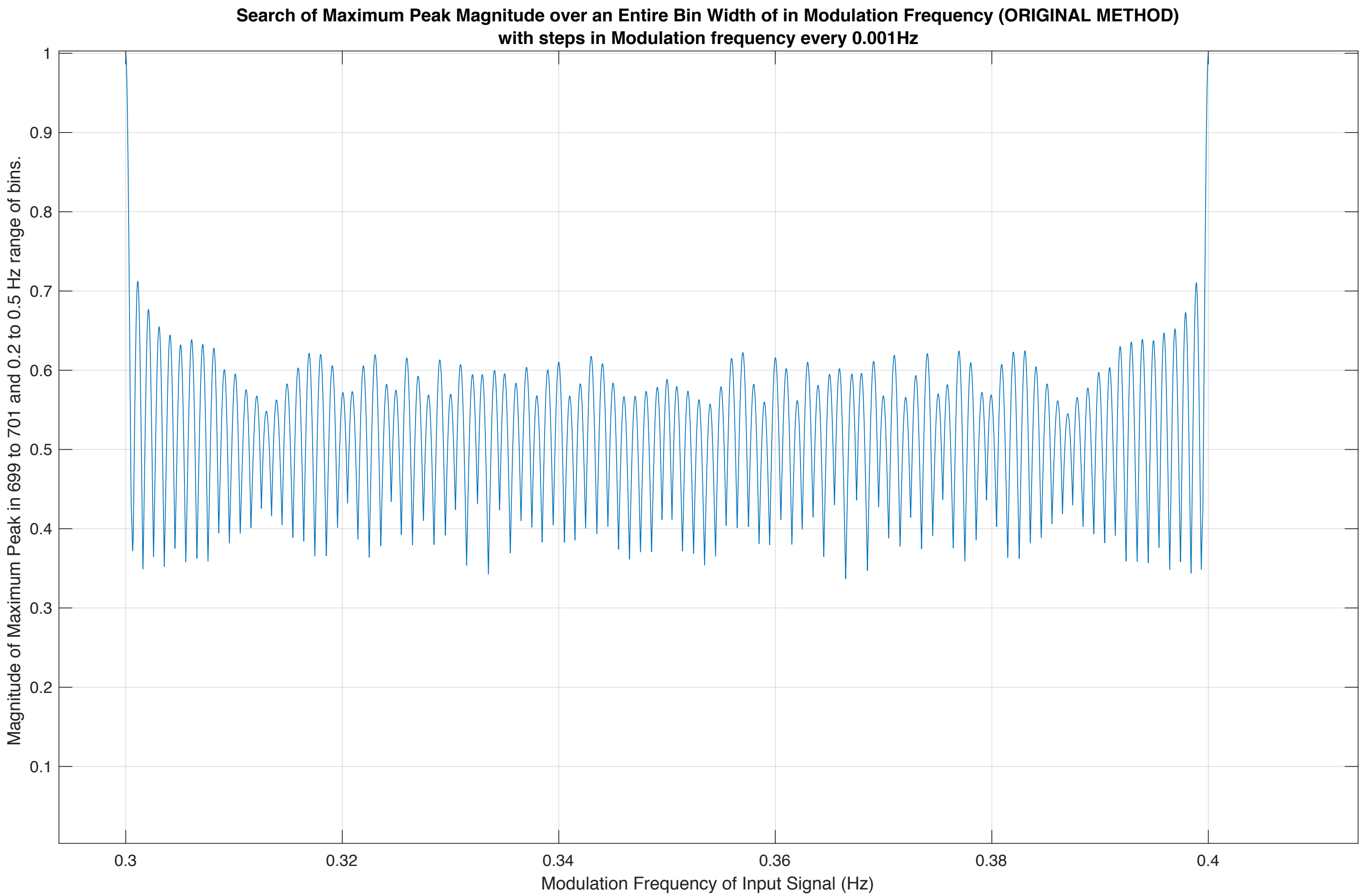
Searching over Carrier Frequency



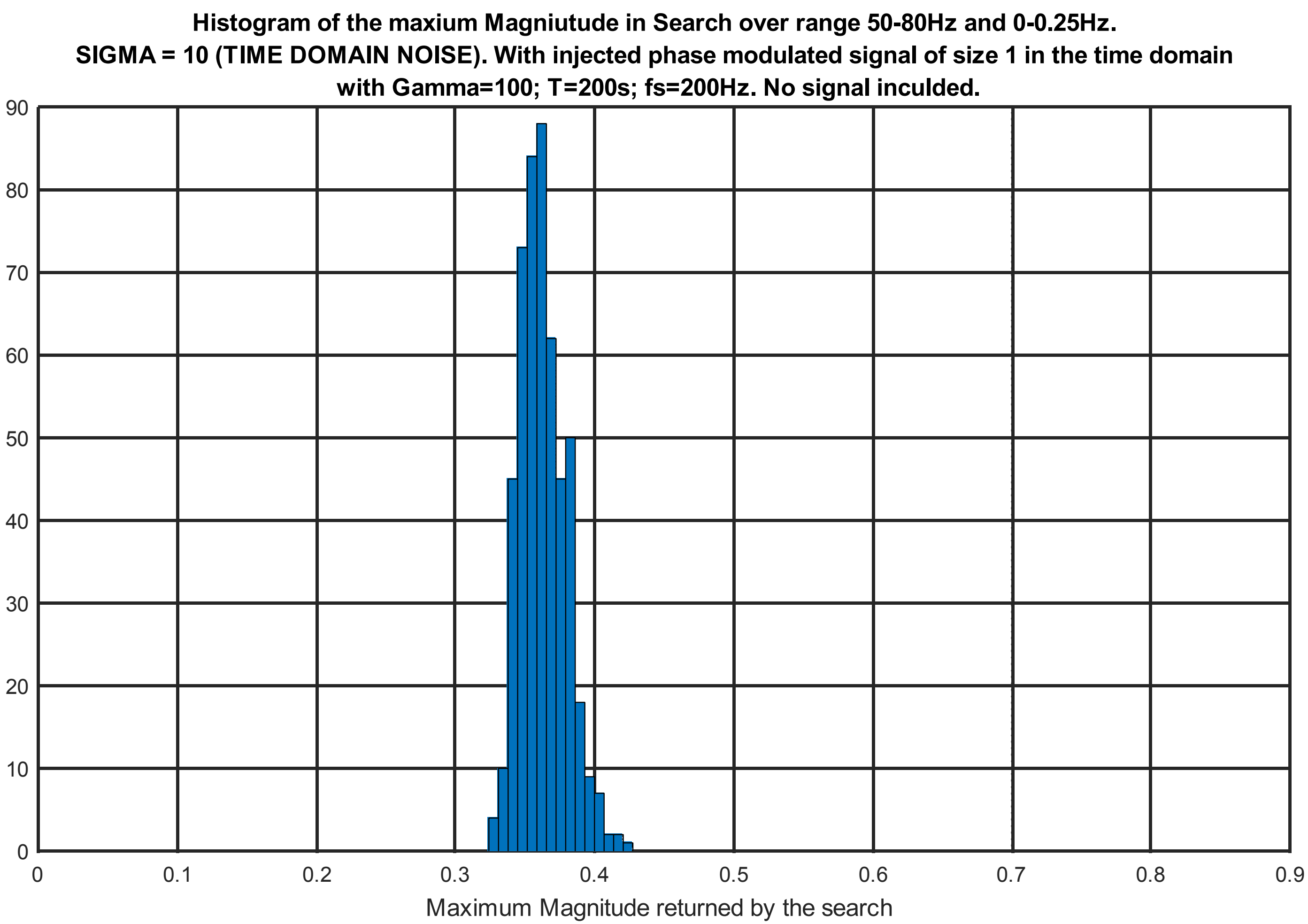
Searching over Modulation Frequency



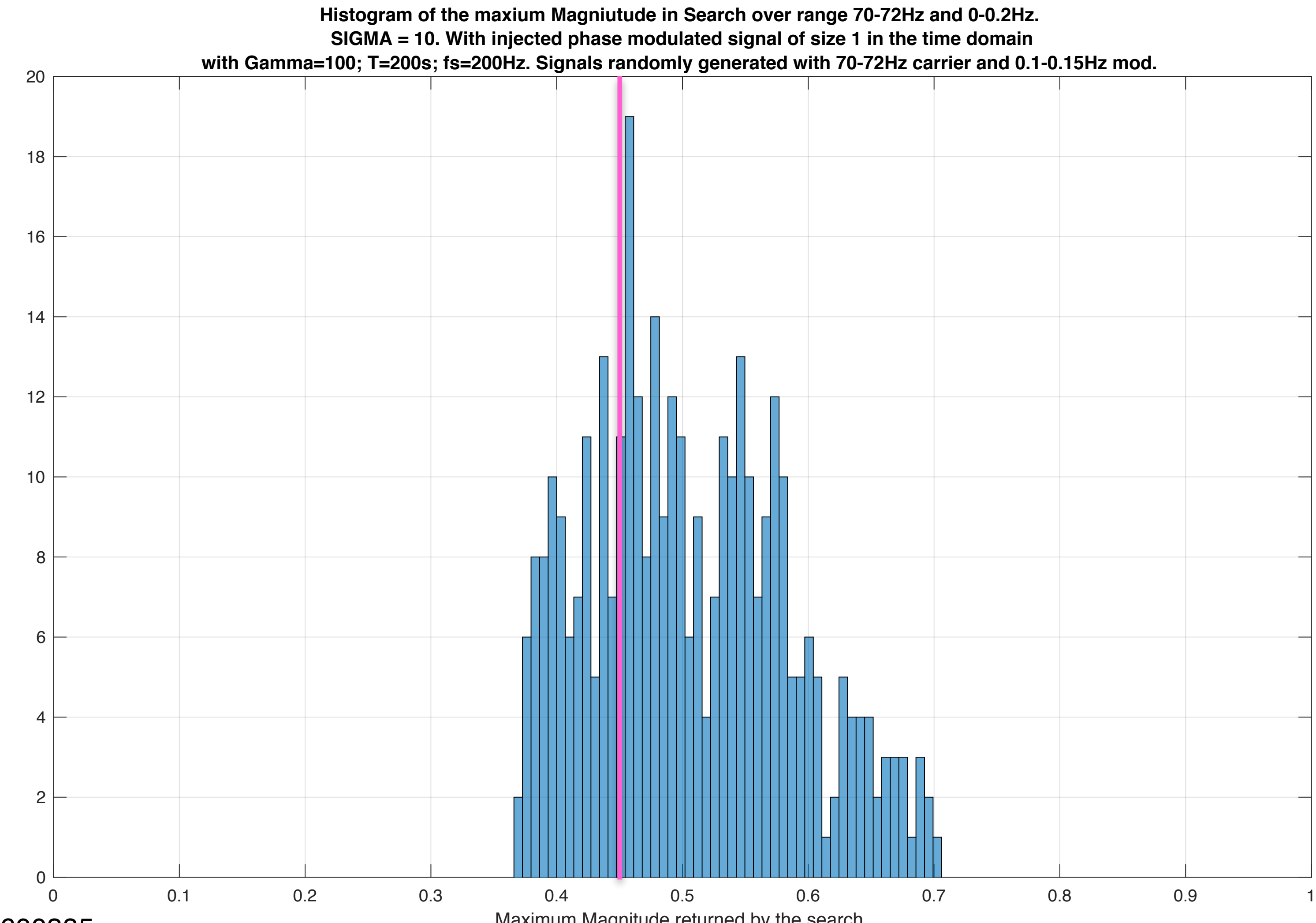
Searching over Modulation Frequency



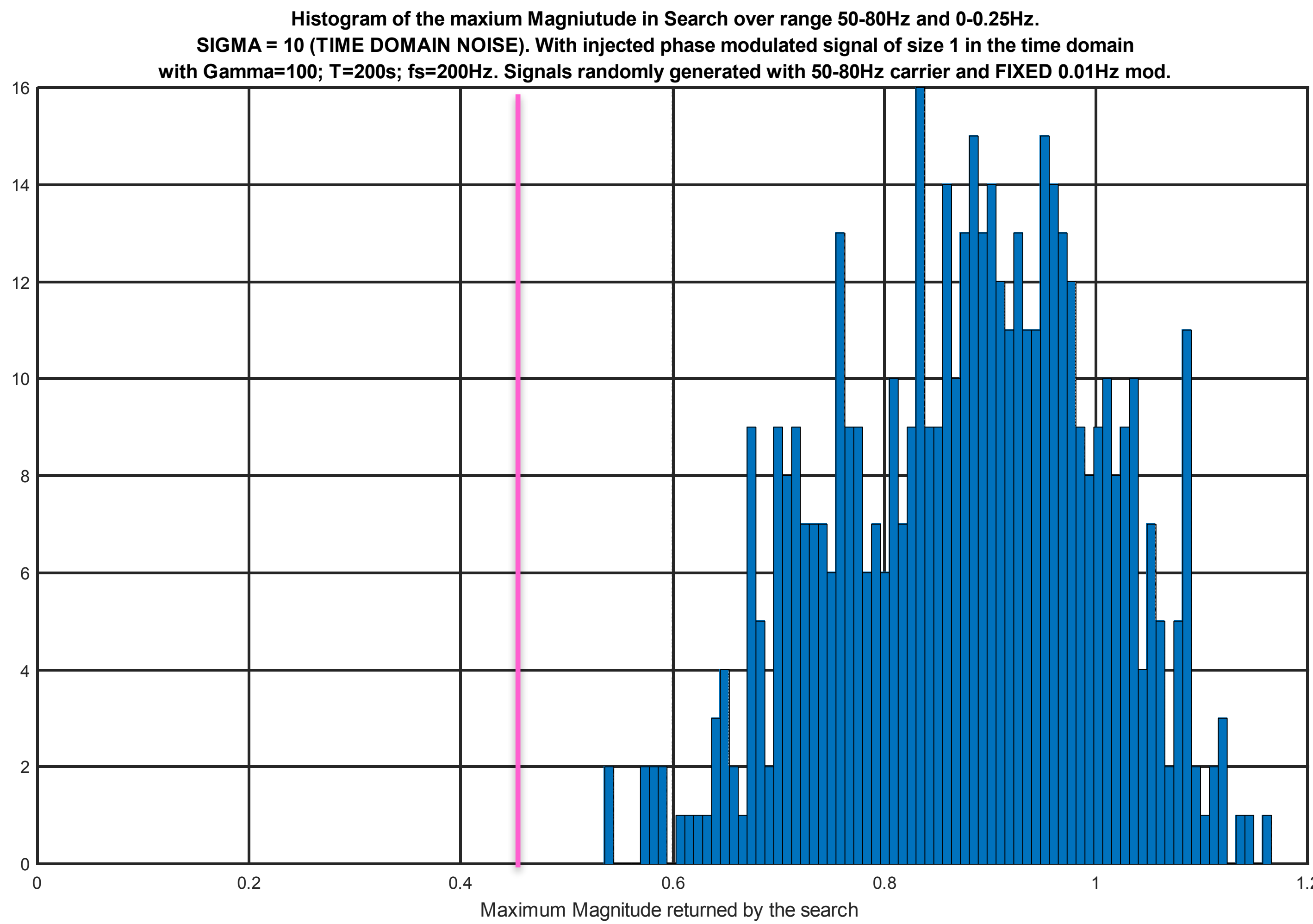
Histogram of Search Results: Only Noise



Histogram of Search Results: Noise + Injected Signals



Searching over Modulation Frequency



Conclusion

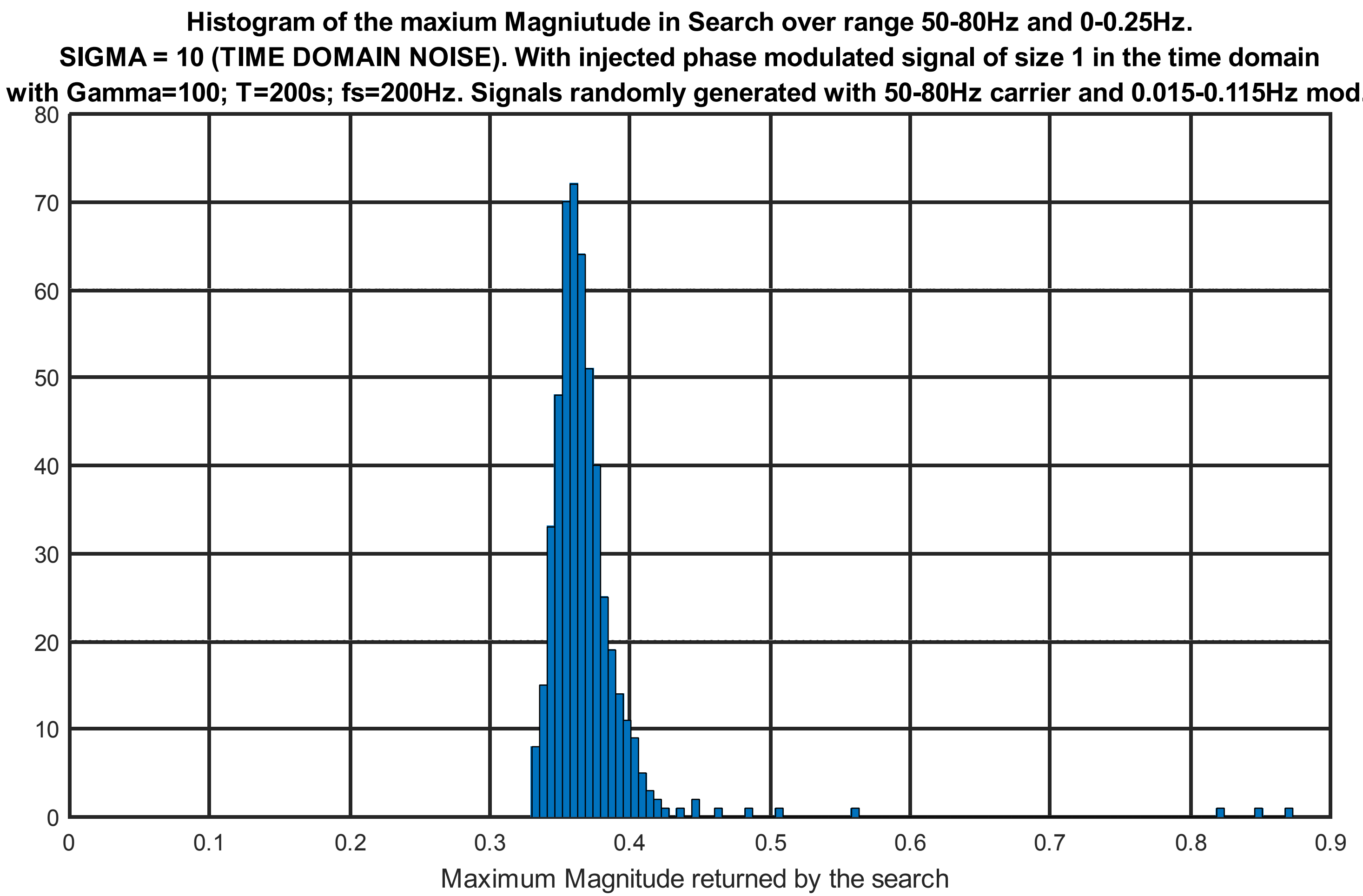
A search algorithm has been developed that is coherent.

The searches are convolution based, where the convolution is performed between the data in frequency space, which has only been Fourier transformed once, and its expected modulated form.

Some undesirable features are still present in the search over Modulation frequency thought to be due to the application of the Discrete Fourier transform to the data.

Thank you for Listening!

Histogram of Search Results: Noise + Injected Signals



Modulation index Γ

$\Gamma = wR / c$

Derived from Doppler shift.

How much it swings in phase.

Coherent?

Estimate phase resolution of a few degrees.

4deg/60deg ~ 1/15

Square root ~ 4 times better resolution

This leads to increased sensitivity in the range

~ 4^3 more signals received.

ie. 60x more detections