



Science & Technology
Facilities Council



Current and future ground-based gravitational-wave detectors

Haixing Miao

University of Birmingham

Key reference: LIGO Instrument Science White Paper (2015-2016)

Outline

❖ **Background**

- Gravitational waves and their detection

❖ **Basic of noise**

- Noise spectral density and transfer function

❖ **Environmental noise**

- Passive isolation and active cancellation

❖ **Thermal noise**

- Fluctuation-dissipation theorem
- How to reduce thermal noise

❖ **Quantum noise**

- Standard Quantum Limit
- Frequency-dependent squeezing

❖ **Current and future detectors**

- Timeline and sensitivity

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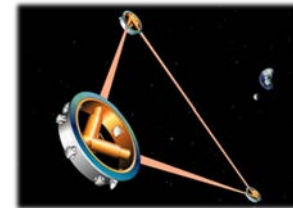
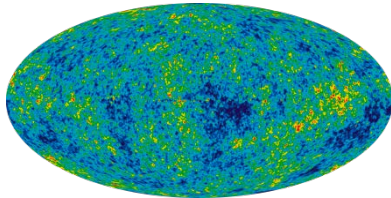
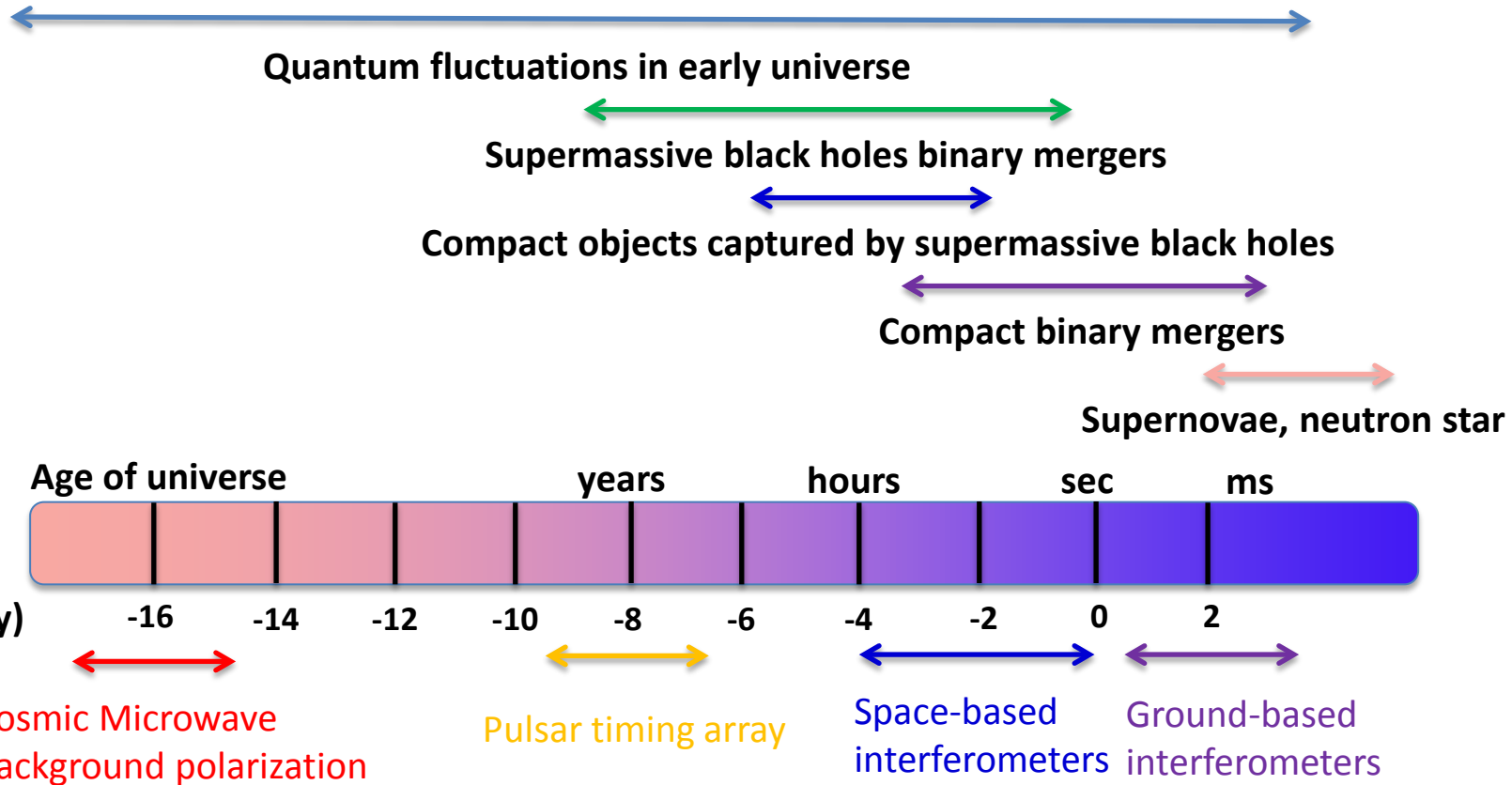
❖ **Quantum noise**

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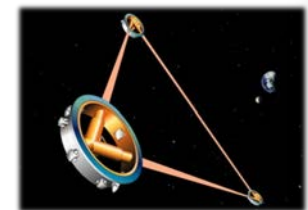
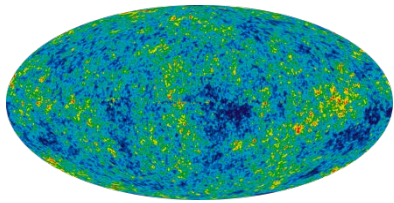
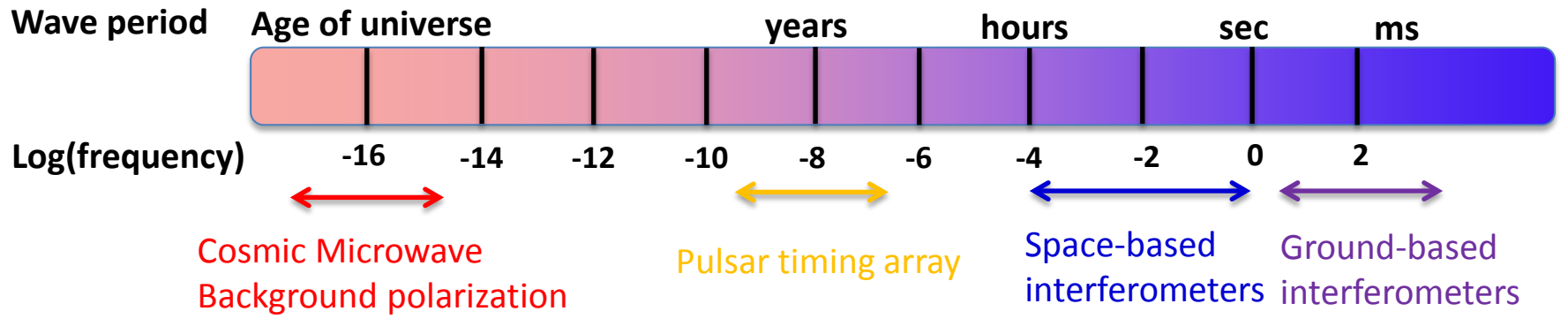
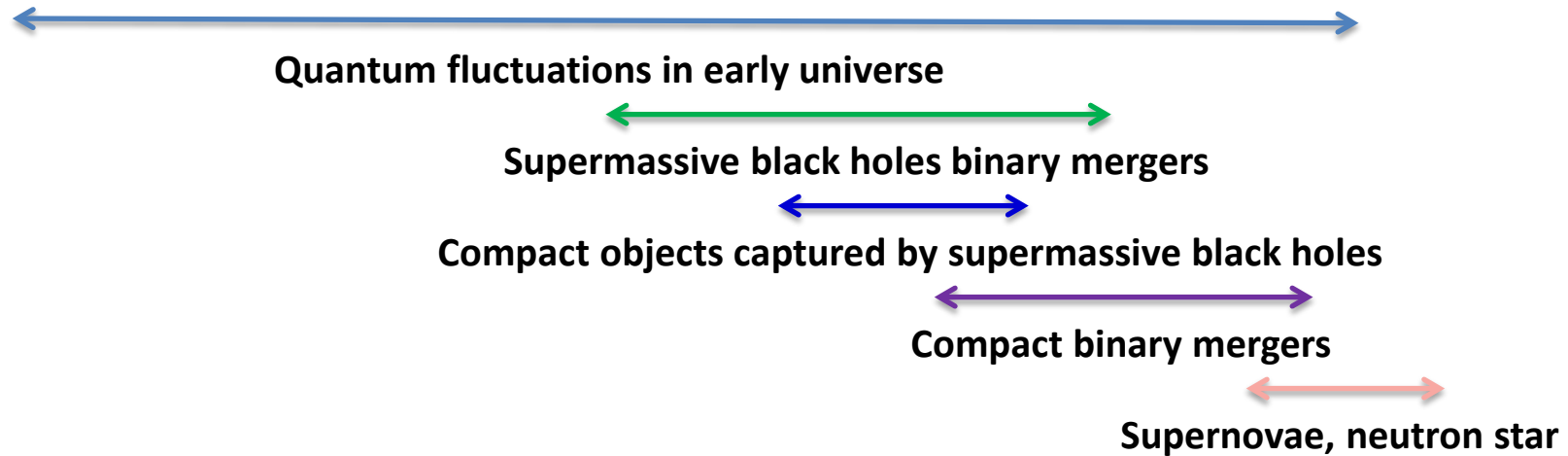
❖ **Current and future detectors**

- Timeline and sensitivity

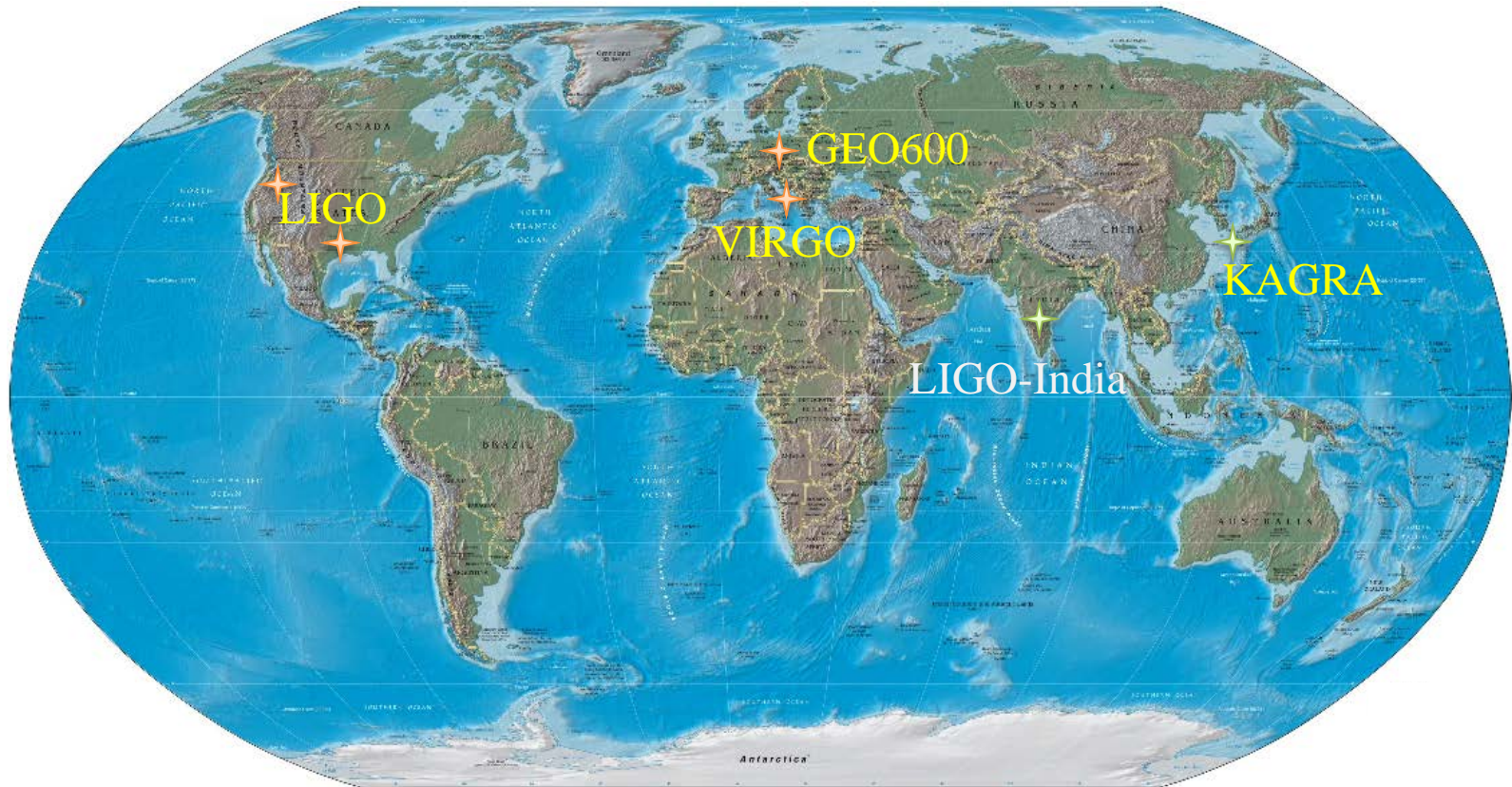
Gravitational waves and their detection



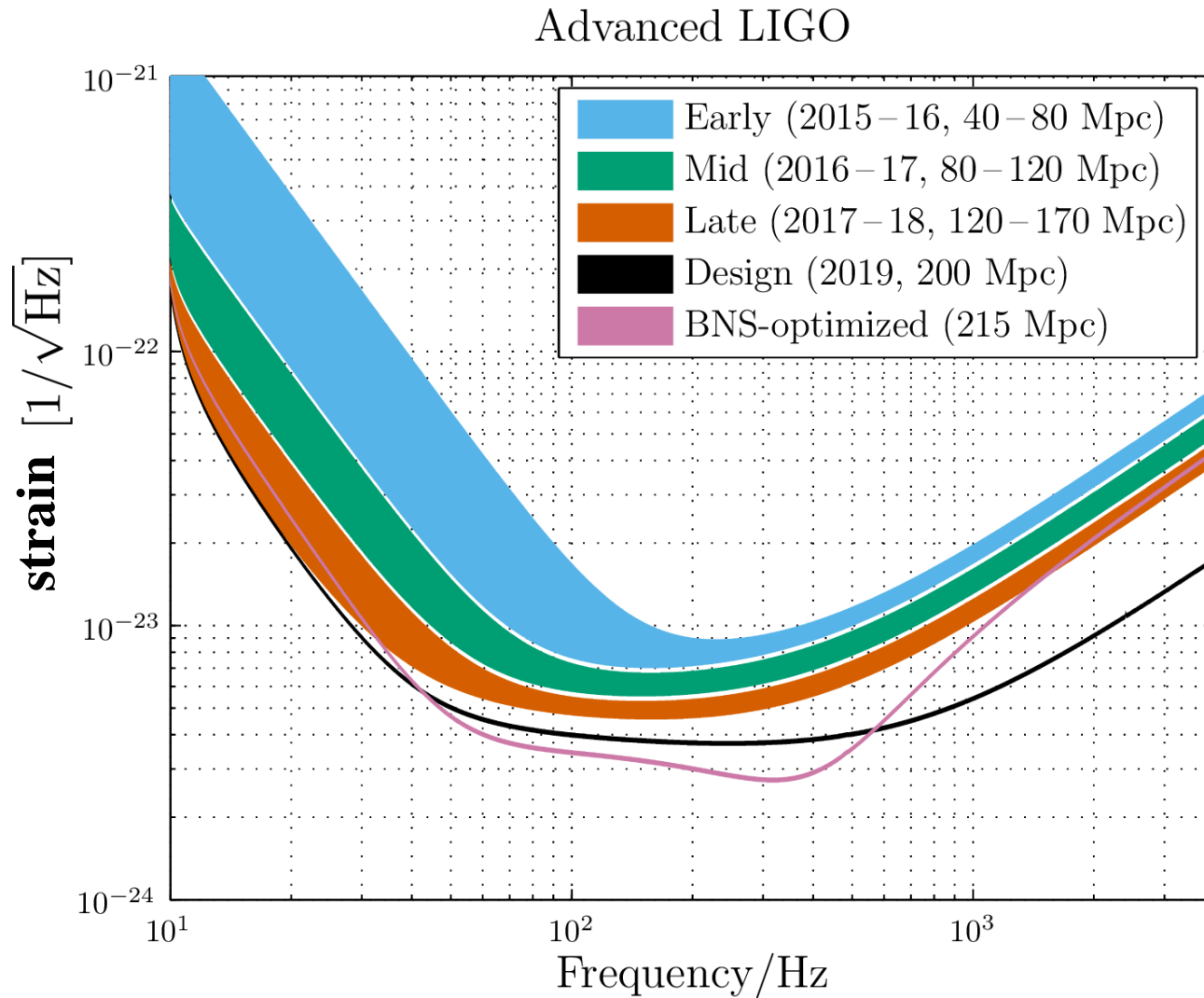
Gravitational waves and their detection



A Global Network

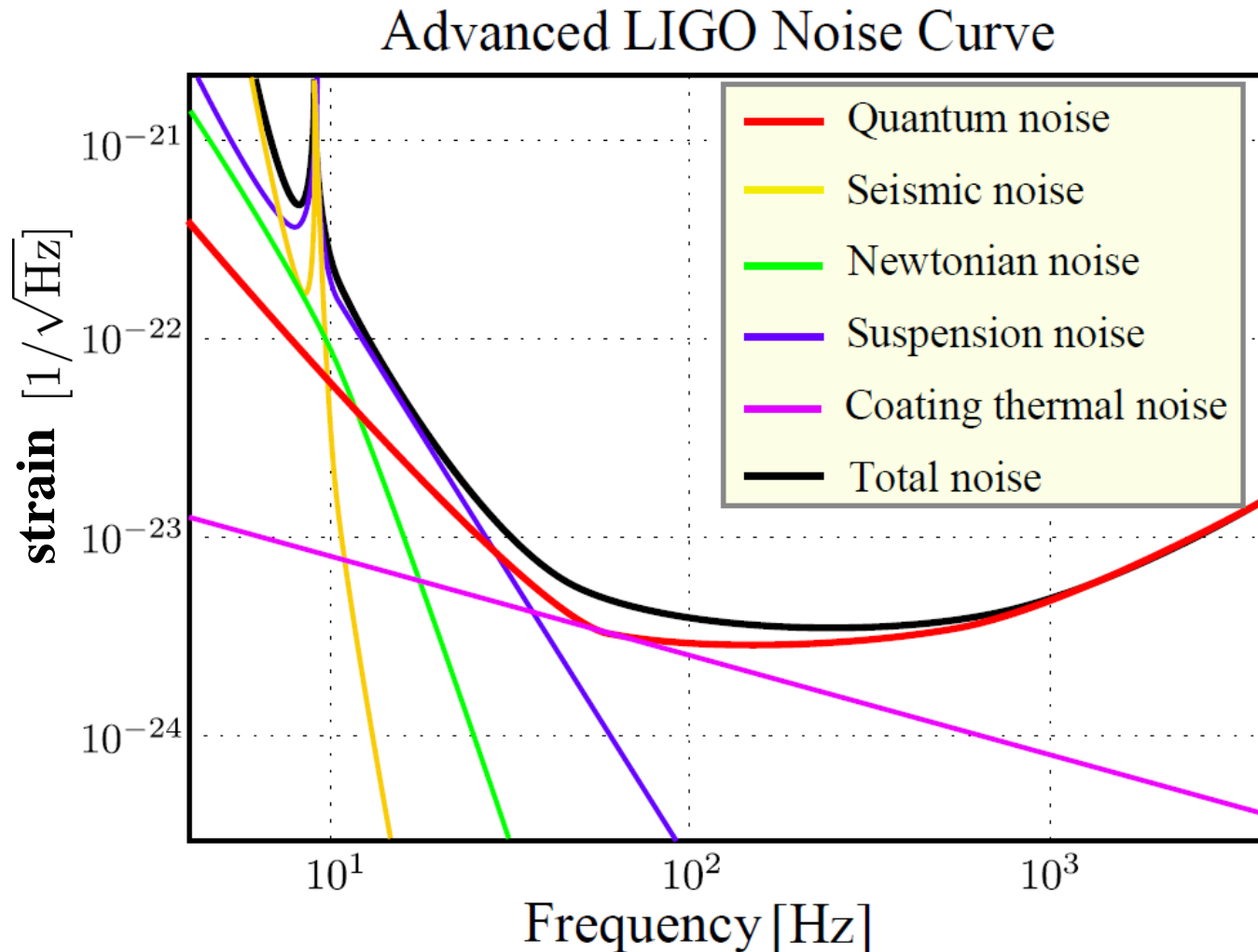


Different stages of Advanced LIGO



LSC, Living Reviews in Relativity 19, 1 (2016)

Design sensitivity of Advanced LIGO



LSC, Class. Quantum Grav. 32, 074001 (2015)

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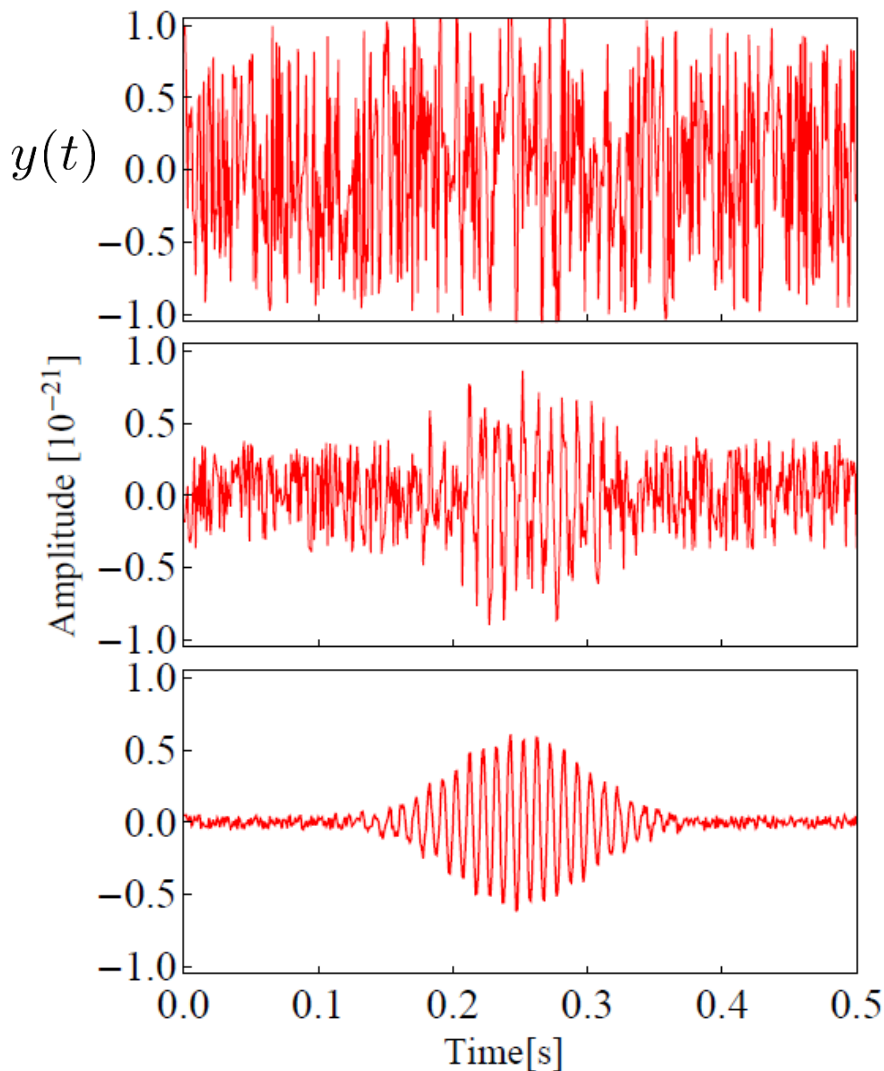
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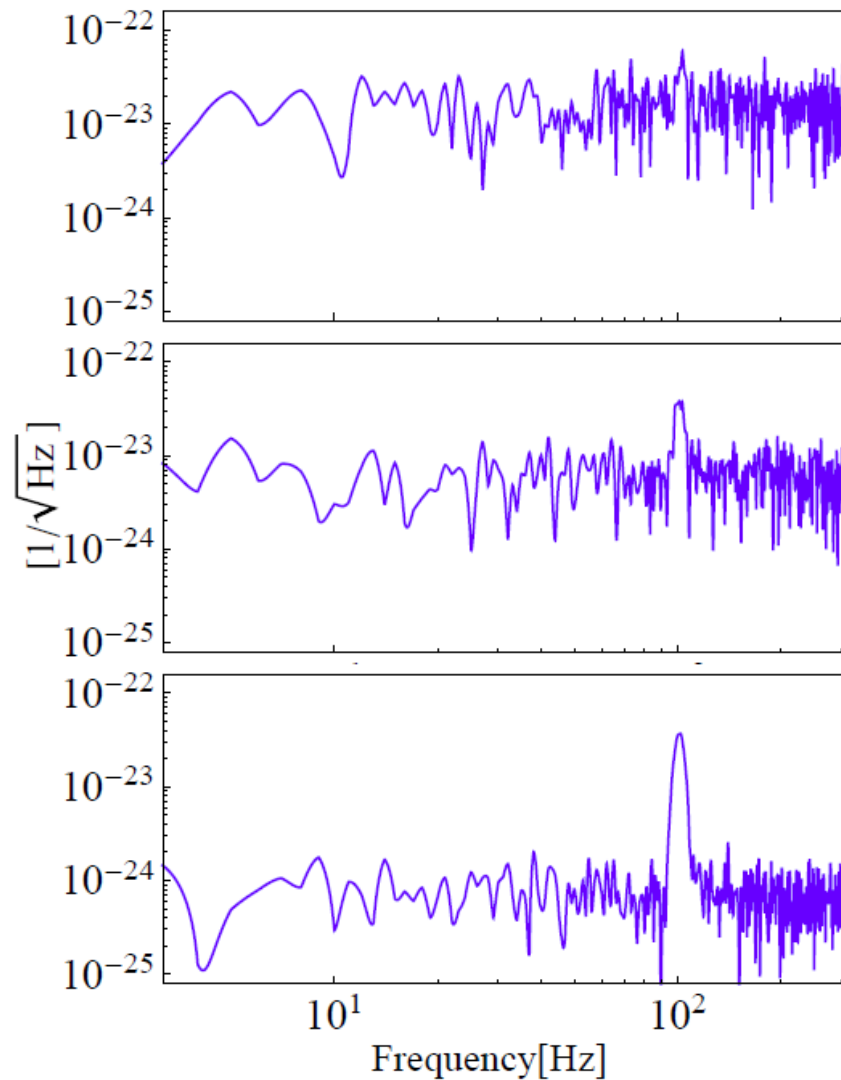
Signal and noise

$$y(t) = n(t) + h(t)$$

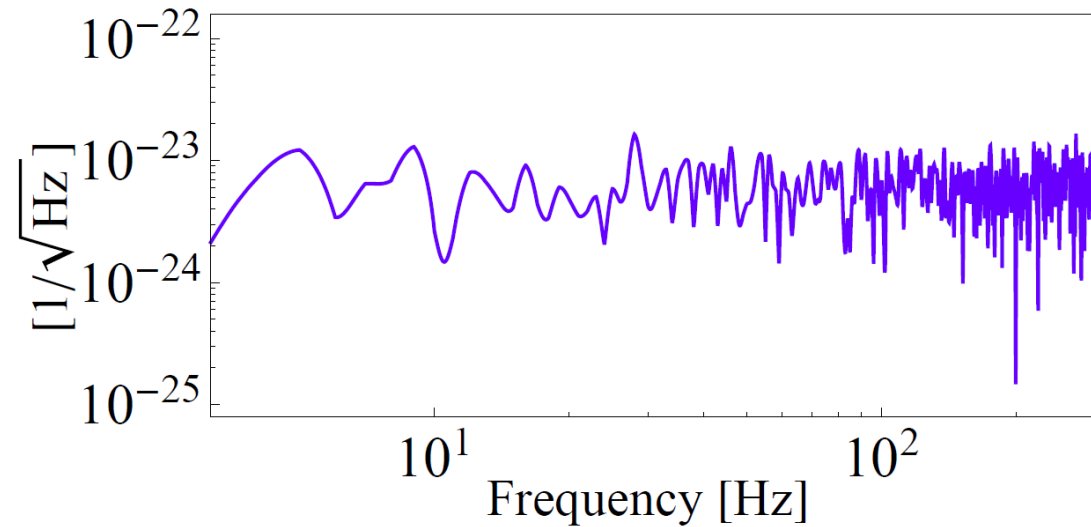
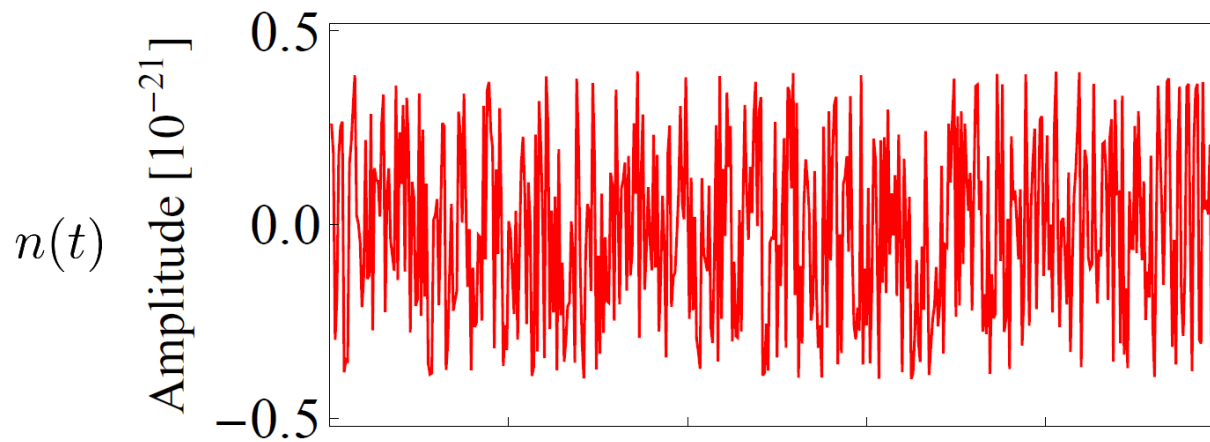
Time Domain



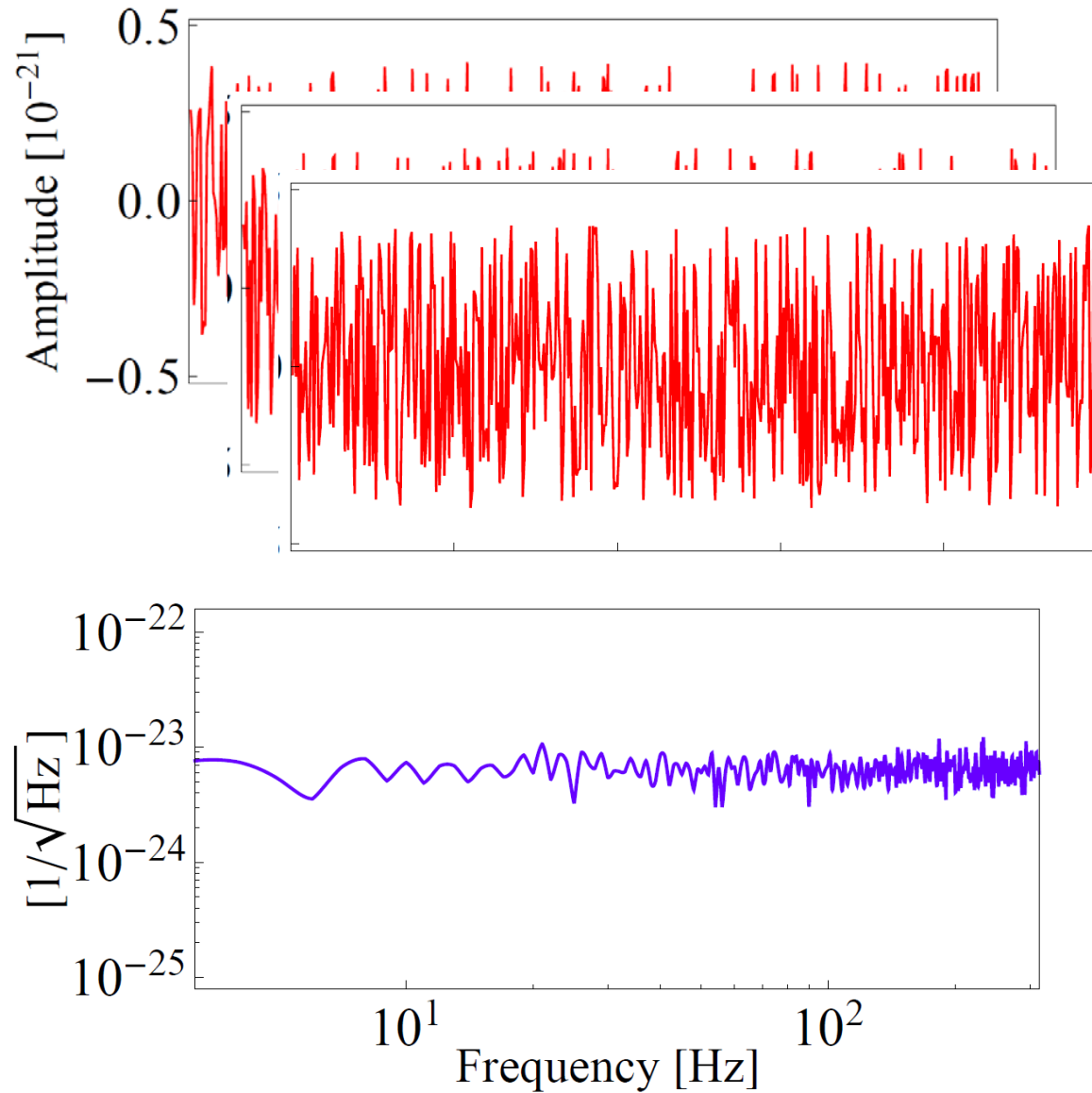
Frequency Domain



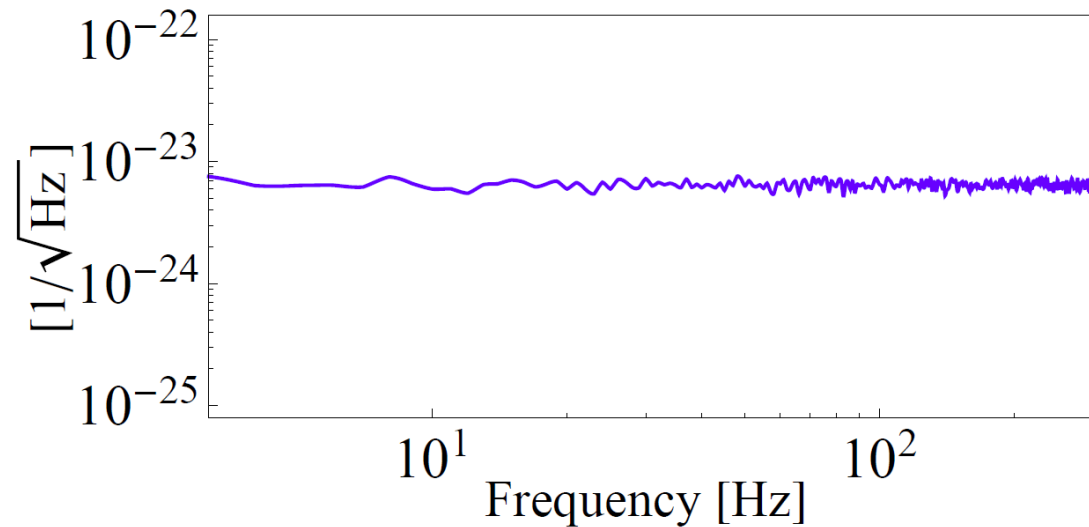
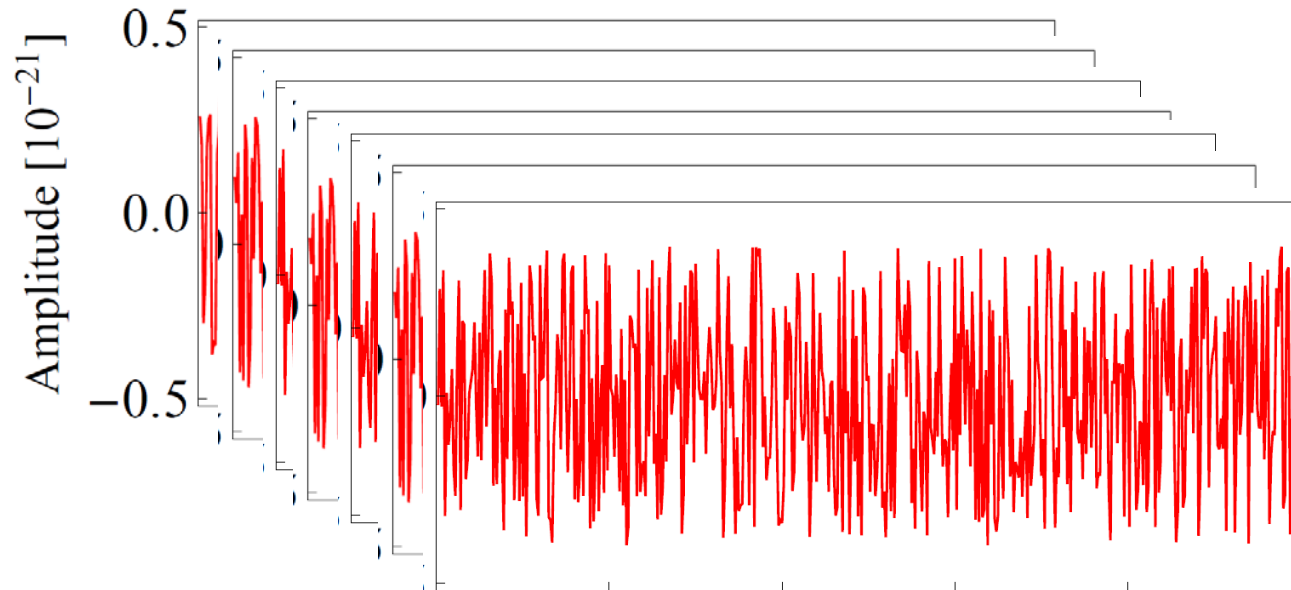
Noise spectral density (spectrum)



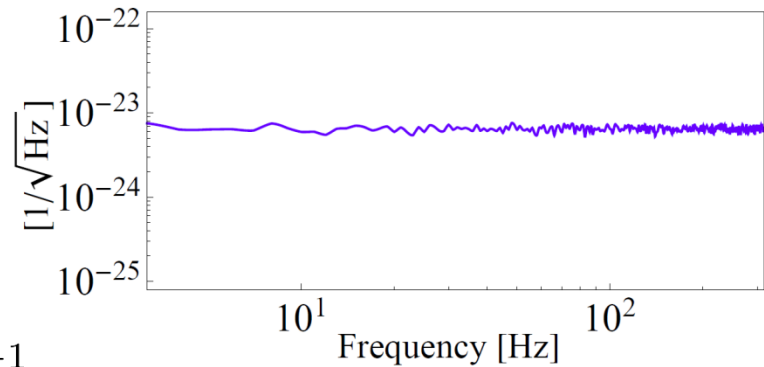
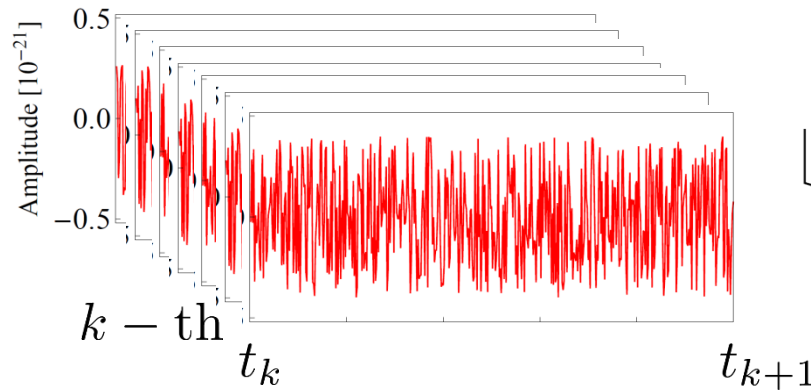
Noise spectral density (spectrum)



Noise spectral density (spectrum)



Noise spectral density (spectrum)



$$\tilde{n}_k(f) \equiv \int_{t_k}^{t_{k+1}} dt [n(t) - \langle n \rangle] e^{2\pi i f t}$$

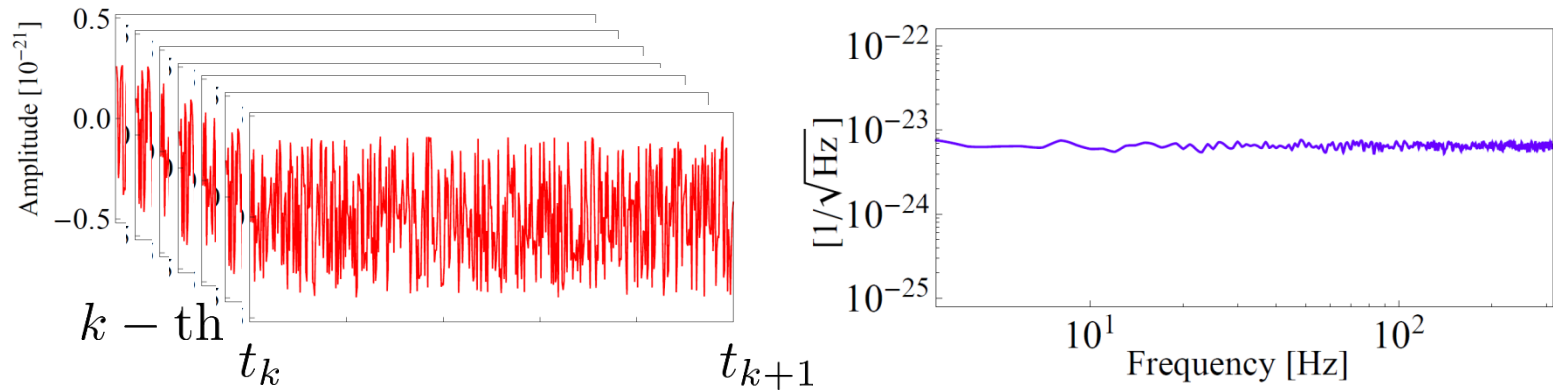
Spectral density: $S_{nn}(f) \equiv \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N \frac{1}{\Delta t} |\tilde{n}_k(f)|^2$ $\Delta t = t_{k+1} - t_k$

$$\approx \lim_{\tau \rightarrow \infty} \frac{1}{\tau} |\tilde{n}(f)|^2$$

$\tau \equiv N \Delta t$

$$\tilde{n}(f) \equiv \int_{-\tau/2}^{+\tau/2} dt [n(t) - \langle n \rangle] e^{2\pi i f t}$$

Noise spectral density (spectrum)



$$\tilde{n}_k(f) \equiv \int_{t_k}^{t_{k+1}} dt [n(t) - \langle n \rangle] e^{2\pi i f t}$$

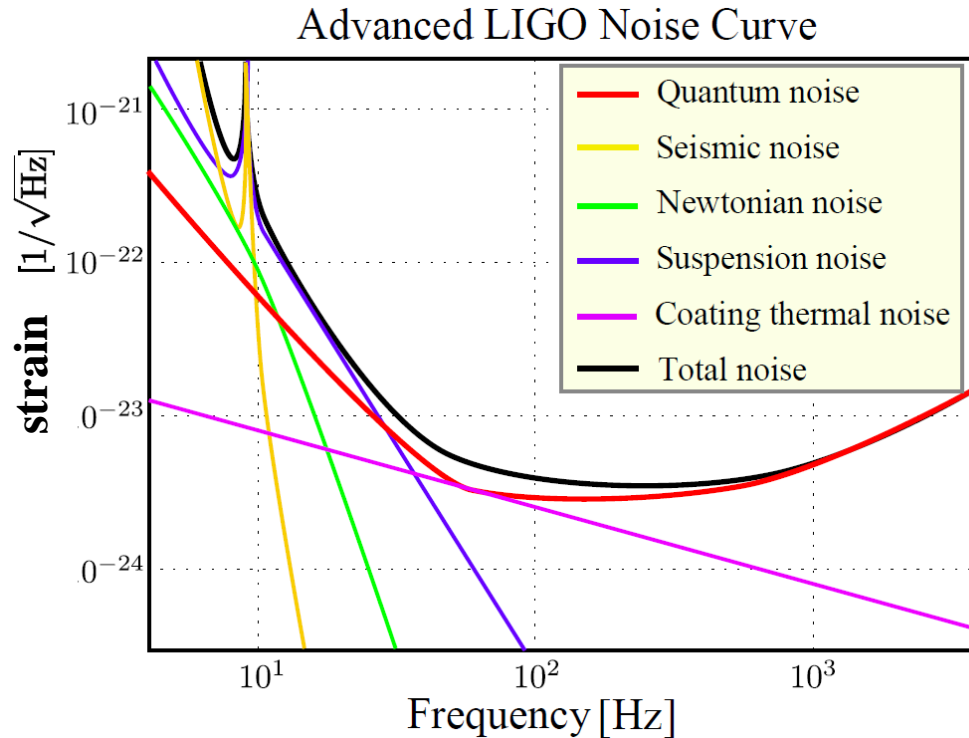
Spectral density: $S_{nn}(f) \equiv \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N \frac{1}{\Delta t} |\tilde{n}_k(f)|^2 \approx \lim_{\tau \rightarrow \infty} \frac{1}{\tau} |\tilde{n}(f)|^2$

Linear version: $\sqrt{S_{nn}(f)}$ [1/√Hz] resolution bandwidth

Order of magnitude: $h_{\min}|_{f_0} \approx \sqrt{S_{nn}(f_0) \Delta f}$ $\Delta f \equiv 1/\Delta t$

Chap 6: Random Process in *Applications of Classical Physics* by Blandford & Thorne

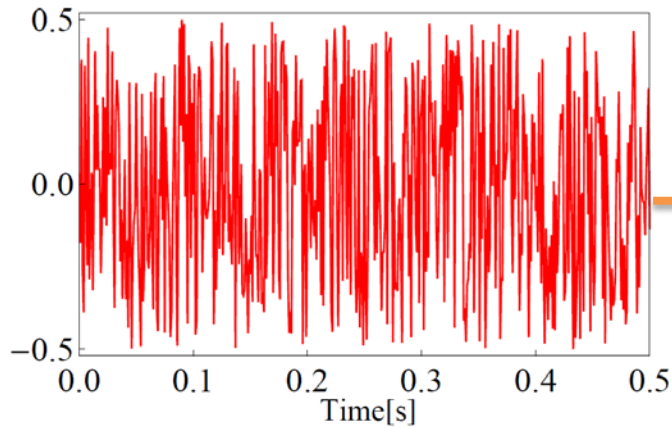
Noise spectral density (spectrum)



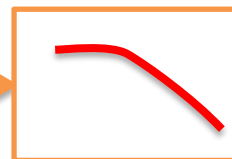
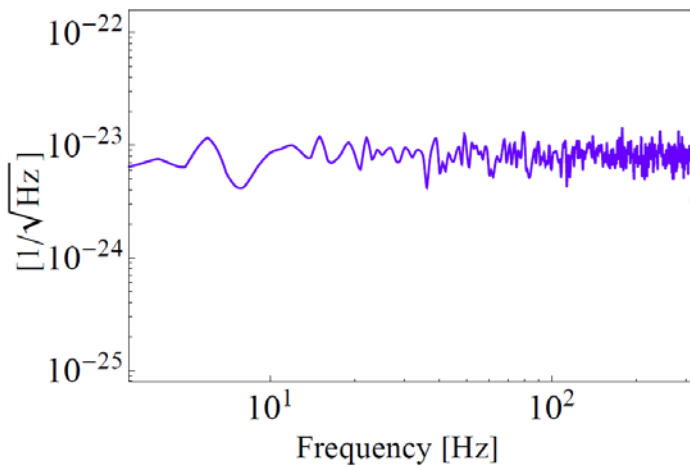
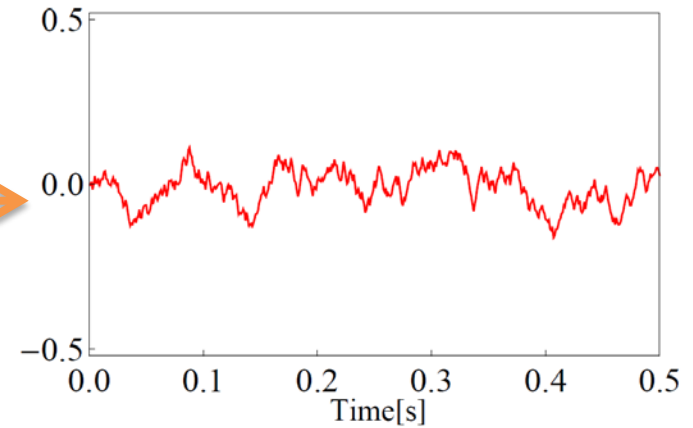
100Hz signal (1sec long): $h_{\min}|_{100\text{Hz}} \approx \sqrt{S_{nn}(100\text{Hz})\Delta f}$
 $\Delta f = 1/\Delta t = 1\text{Hz} \approx 10^{-23}$

In general: $\text{SNR} \equiv \int_{f_{\min}}^{f_{\max}} df \frac{|\tilde{h}(f)|^2}{S_{nn}(f)}$ (with matched filtering)

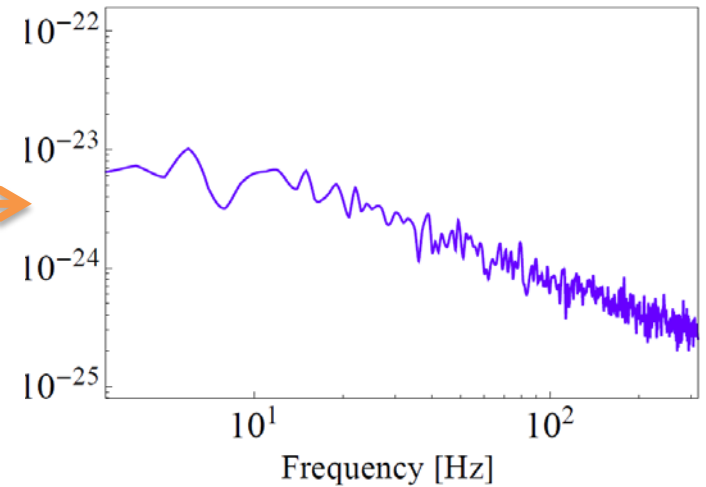
Transfer function



Physical System



Transfer Function



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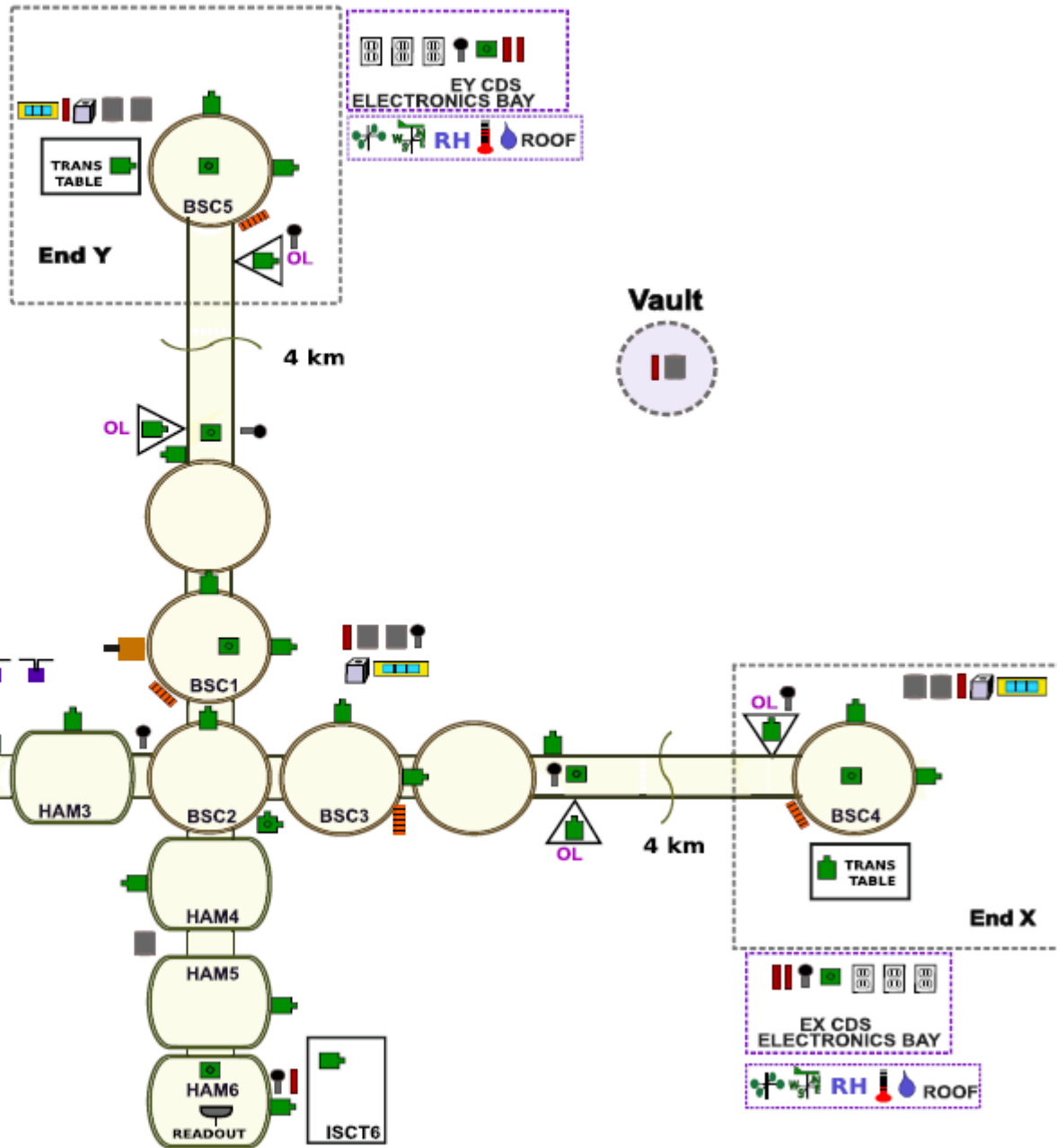
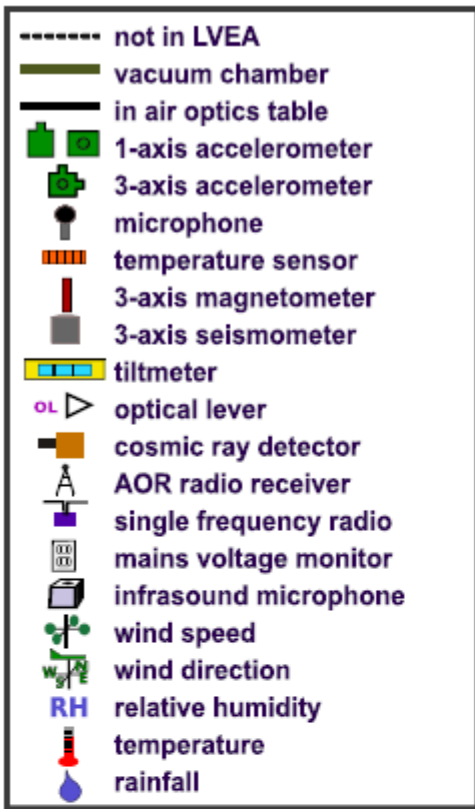
❖ Current and future detectors

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Environmental noise

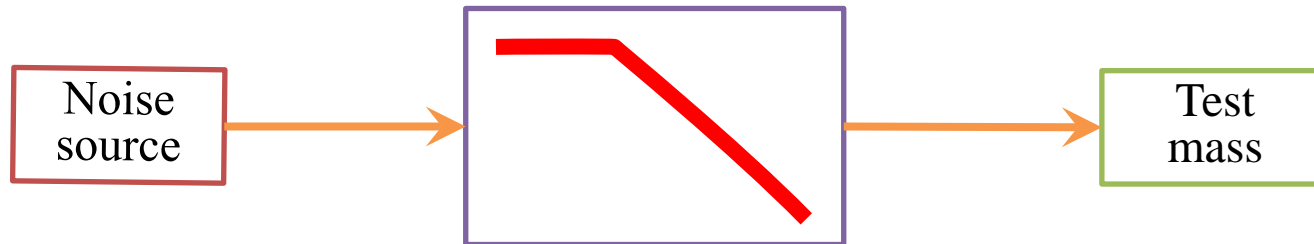


- **Seismic noise**
- **Newtonian noise**
- **Acoustic noise**
- **EM interference**

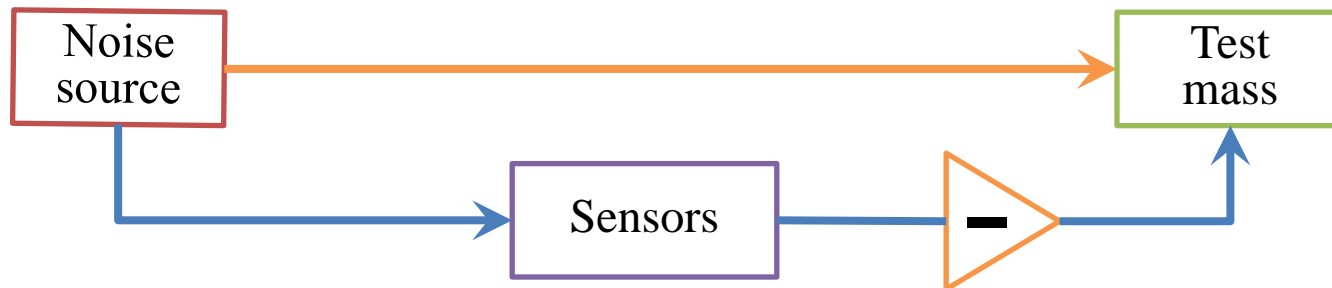


How to reduce environmental noise

1. Passive isolation:

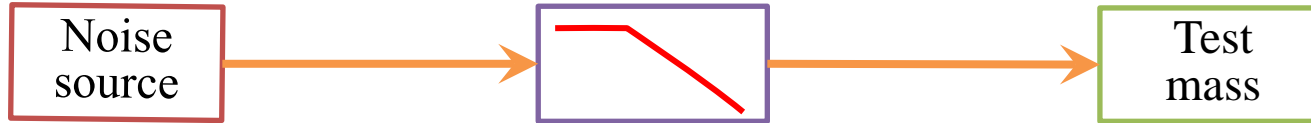


2. Active cancelation: (on-line or off-line)

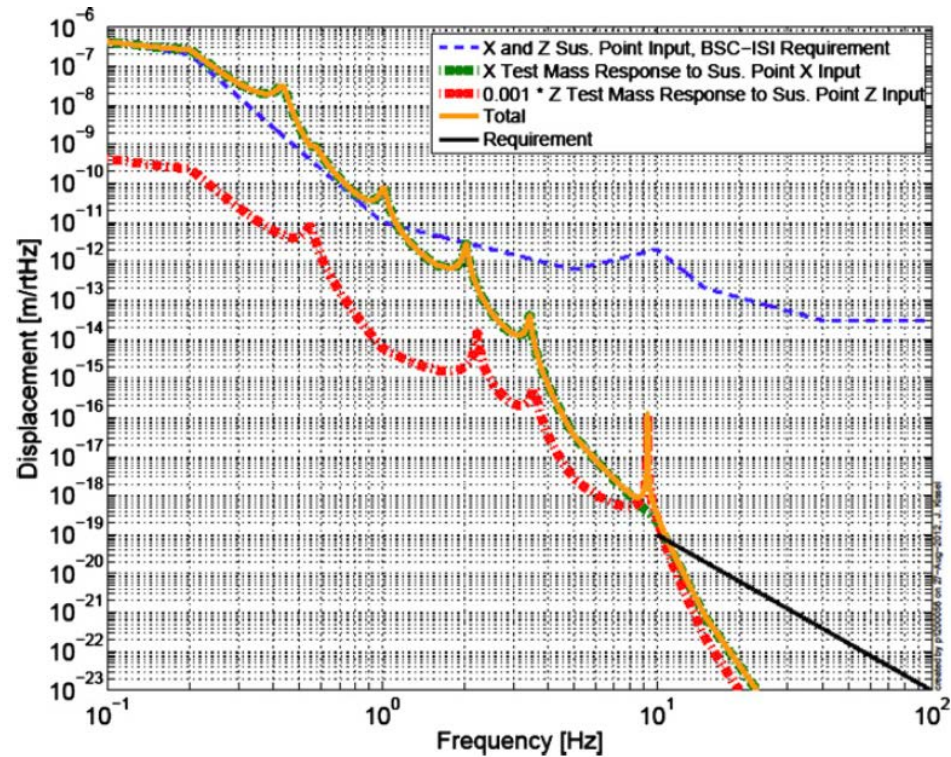
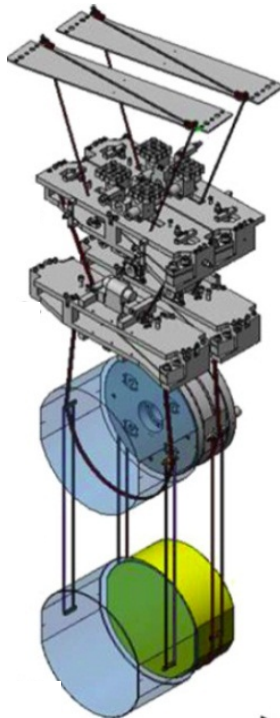


Example: Seismic noise

1. Passive isolation:



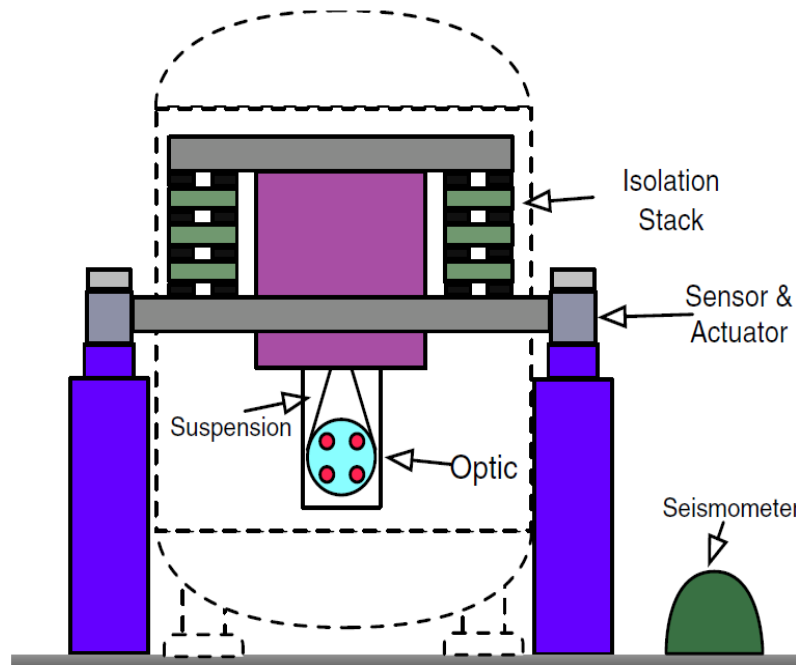
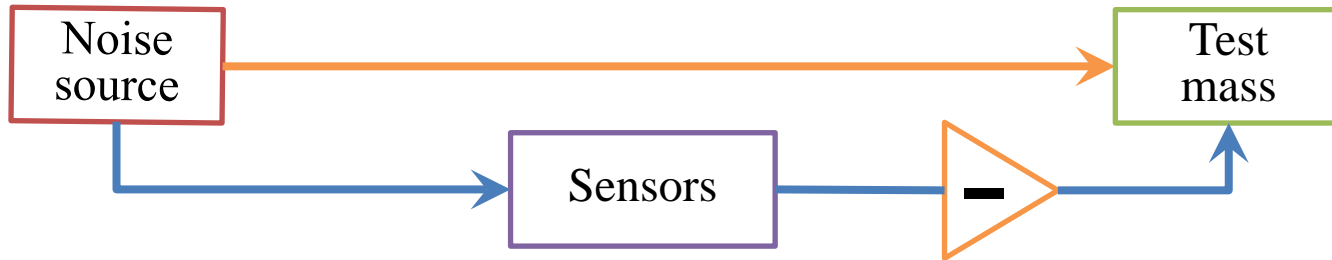
Advanced LIGO quadruple suspension:



Seven orders of magnitude passive isolation

Example: Seismic noise

2. Active cancellation: (on-line or off-line)



Similar technique can be used for **cancelling Newtonian noise**

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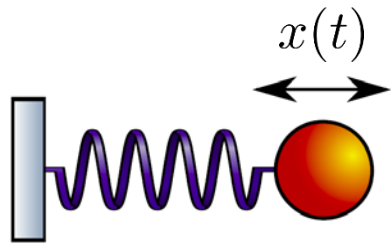
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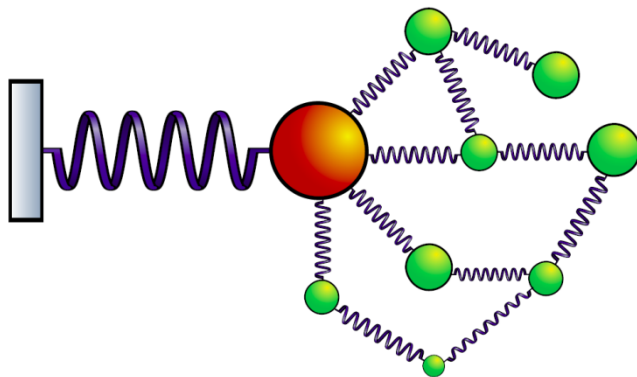
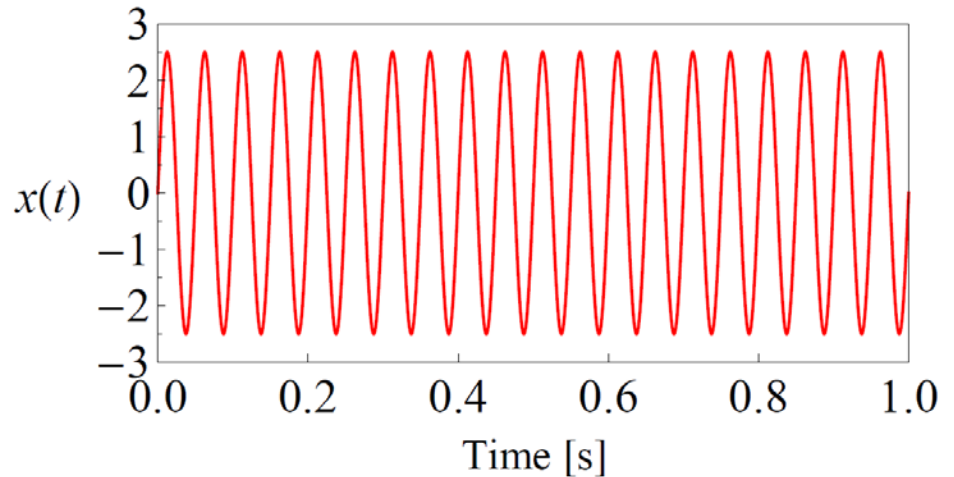
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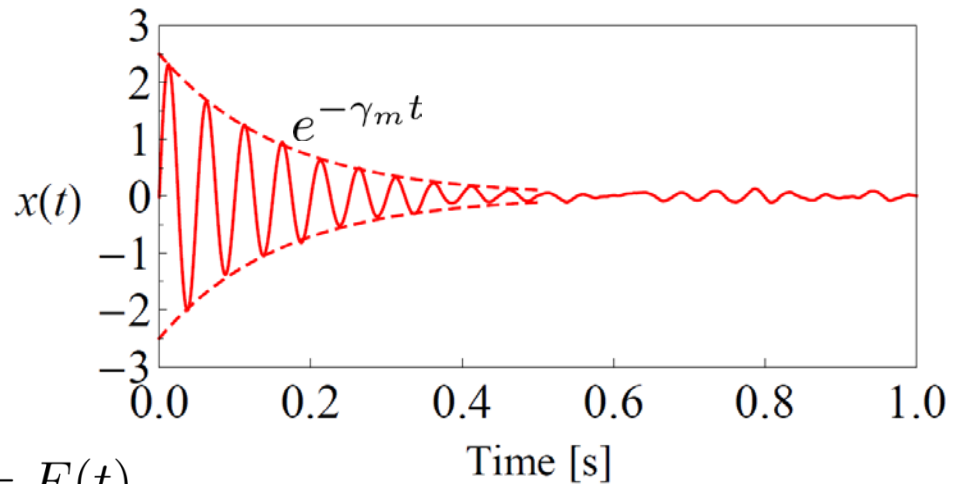
Fluctuation-dissipation theorem (FDT)



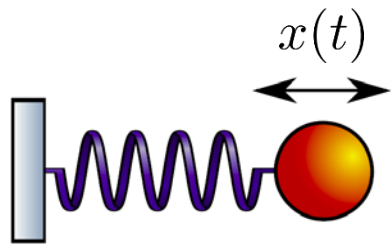
$$m[\ddot{x}(t) + \omega_m^2 x(t)] = F(t)$$



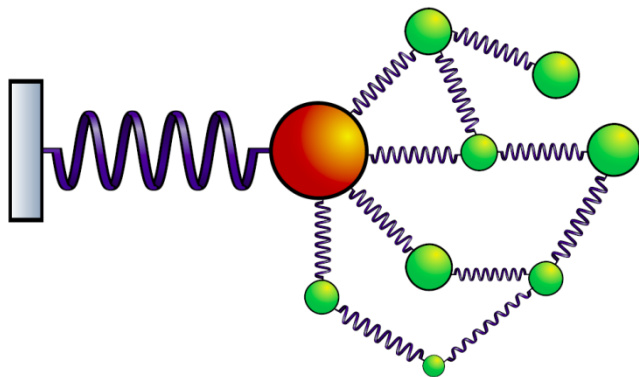
$$m[\ddot{x}(t) + \gamma_m \dot{x}(t) + \omega_m^2 x(t)] = F(t)$$



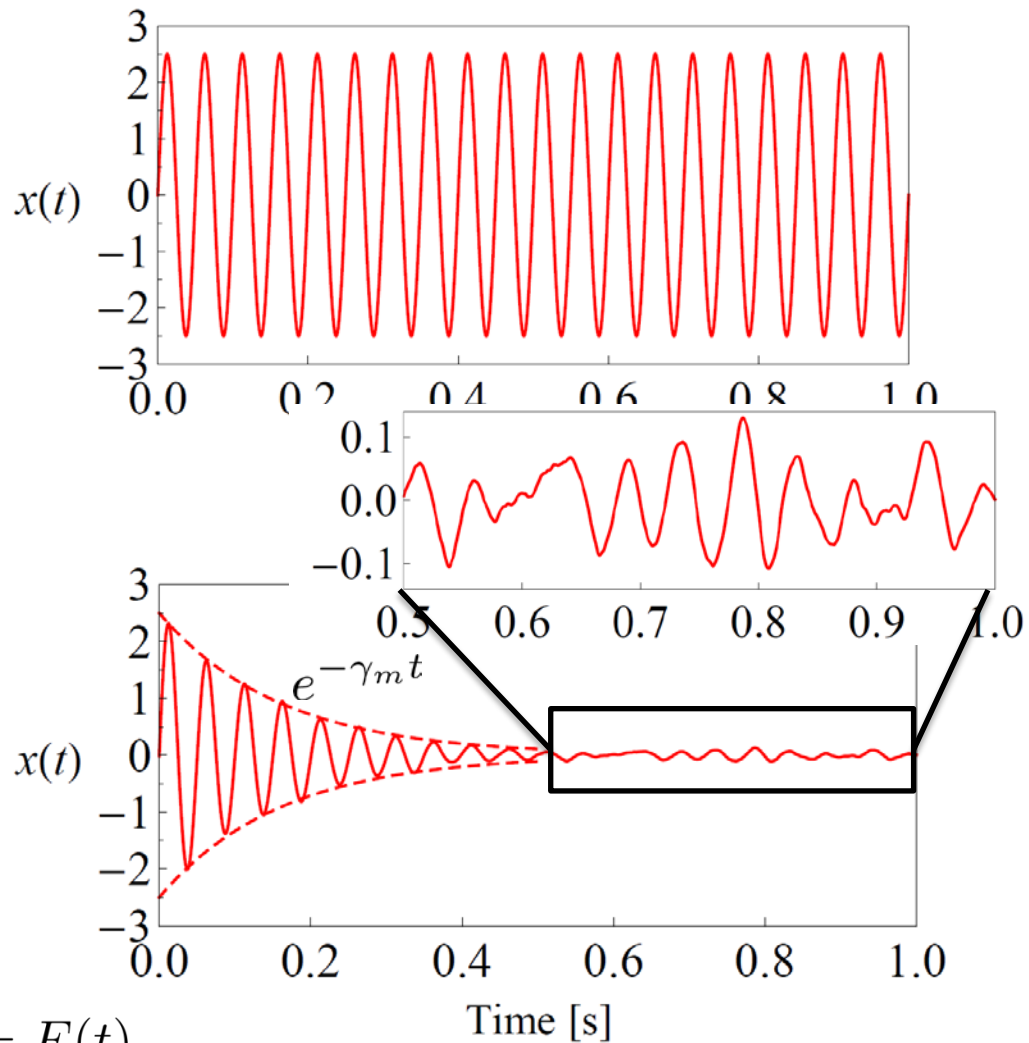
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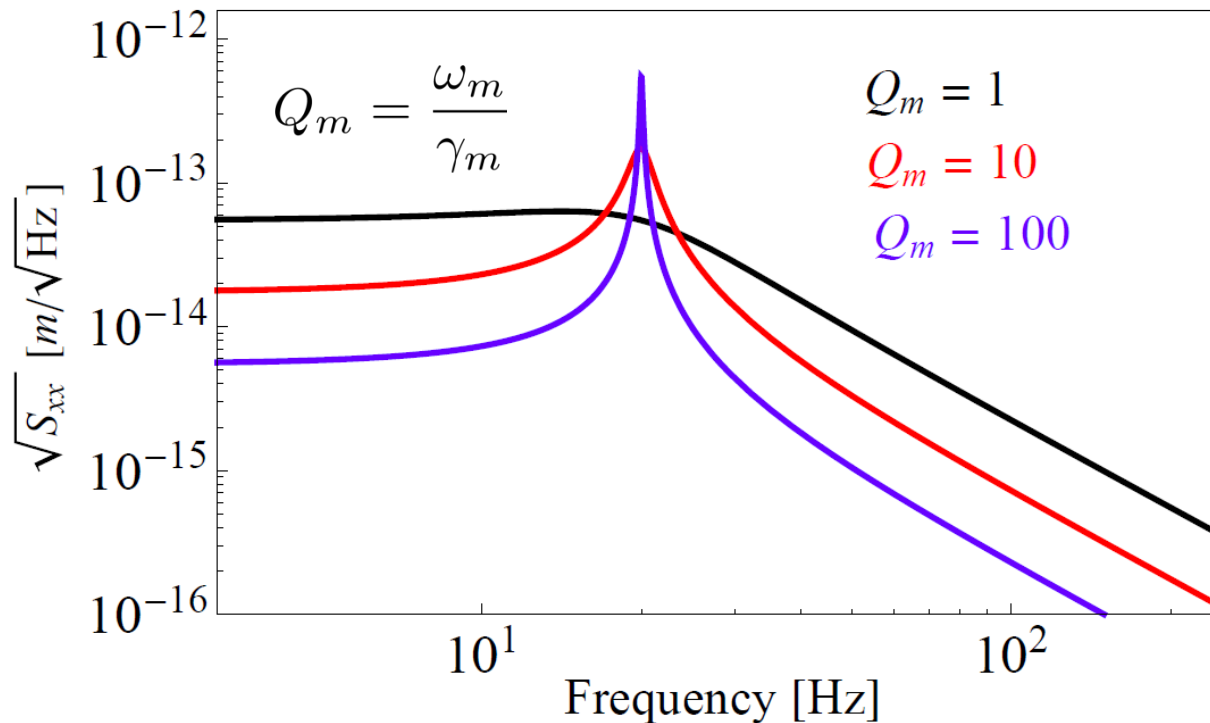
$$m[\ddot{x}(t) + \gamma_m \dot{x}(t) + \omega_m^2 x(t)] = F(t)$$



Fluctuation-dissipation theorem (FDT)

$$\text{FDT: } S_{xx}(f) = \frac{k_B T}{2\pi f} \text{Im}[\chi_{xx}(f)]$$

$$\text{Susceptibility: } \chi_{xx}(f) = \frac{\tilde{x}(f)}{\tilde{F}(f)} = \frac{1}{m[-(2\pi f)^2 - i\gamma_m(2\pi f) + \omega_m^2]}$$

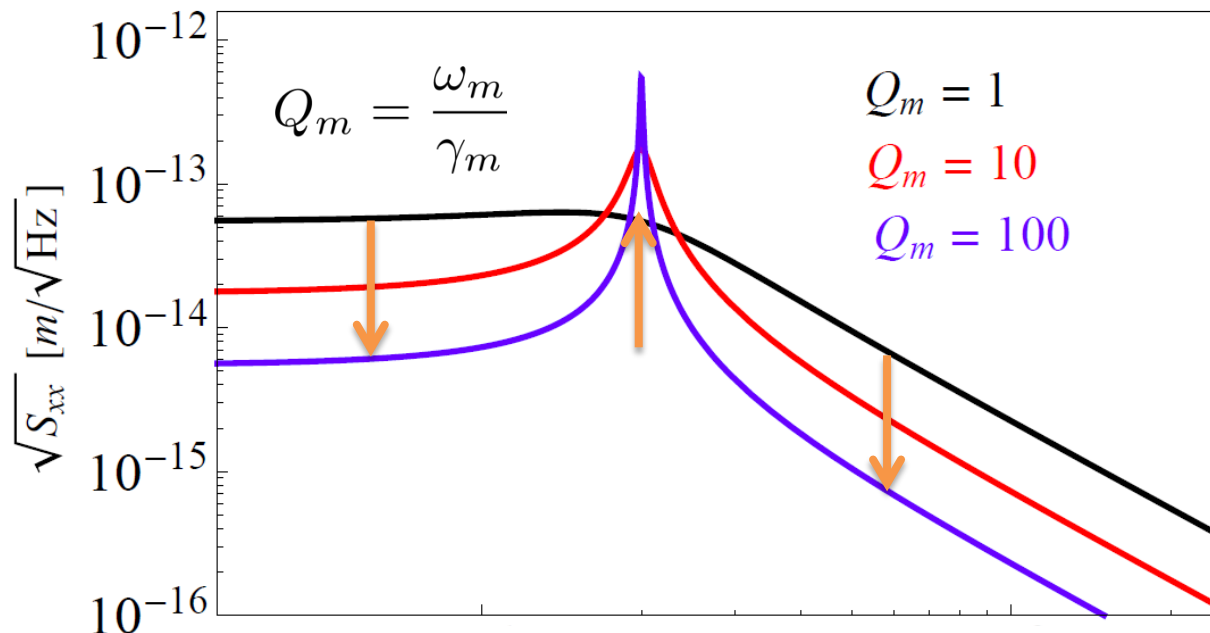


H. Callen, and T. Welton, Phys. Rev. 83, 34 (1951)

Fluctuation-dissipation theorem (FDT)

FDT: $S_{xx}(f) = \frac{k_B T}{2\pi f} \text{Im}[\chi_{xx}(f)]$

Susceptibility: $\chi_{xx}(f) = \frac{\tilde{x}(f)}{\tilde{F}(f)} = \frac{1}{m[-(2\pi f)^2 - i\gamma_m(2\pi f) + \omega_m^2]}$



$$\frac{1}{2} m \omega_m^2 \langle x^2 \rangle = \frac{1}{2} m \omega_m^2 \int df S_{xx}(f) = \frac{1}{2} k_B T \quad (\text{Equipartition})$$

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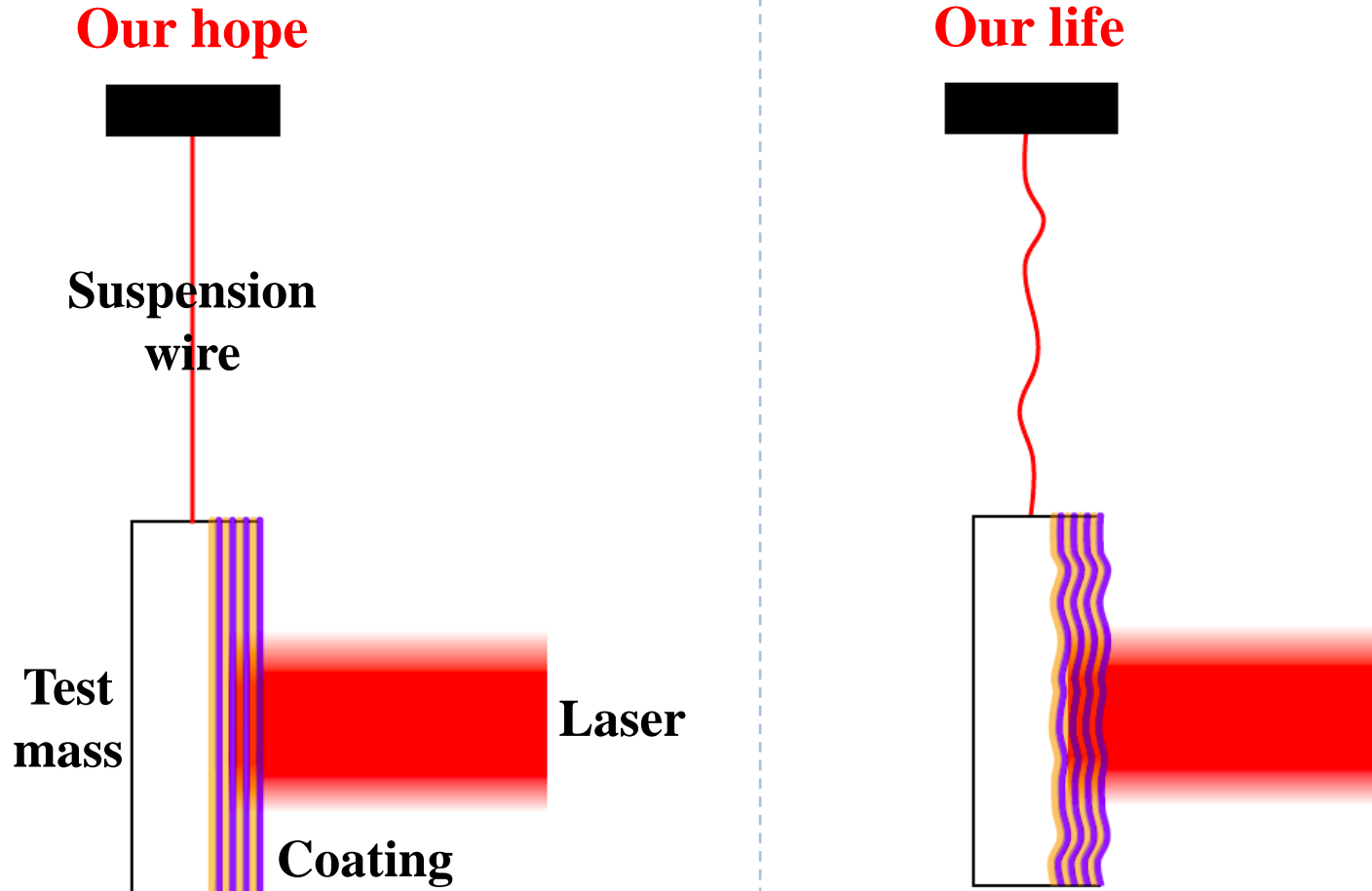
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❖ Current and future detectors

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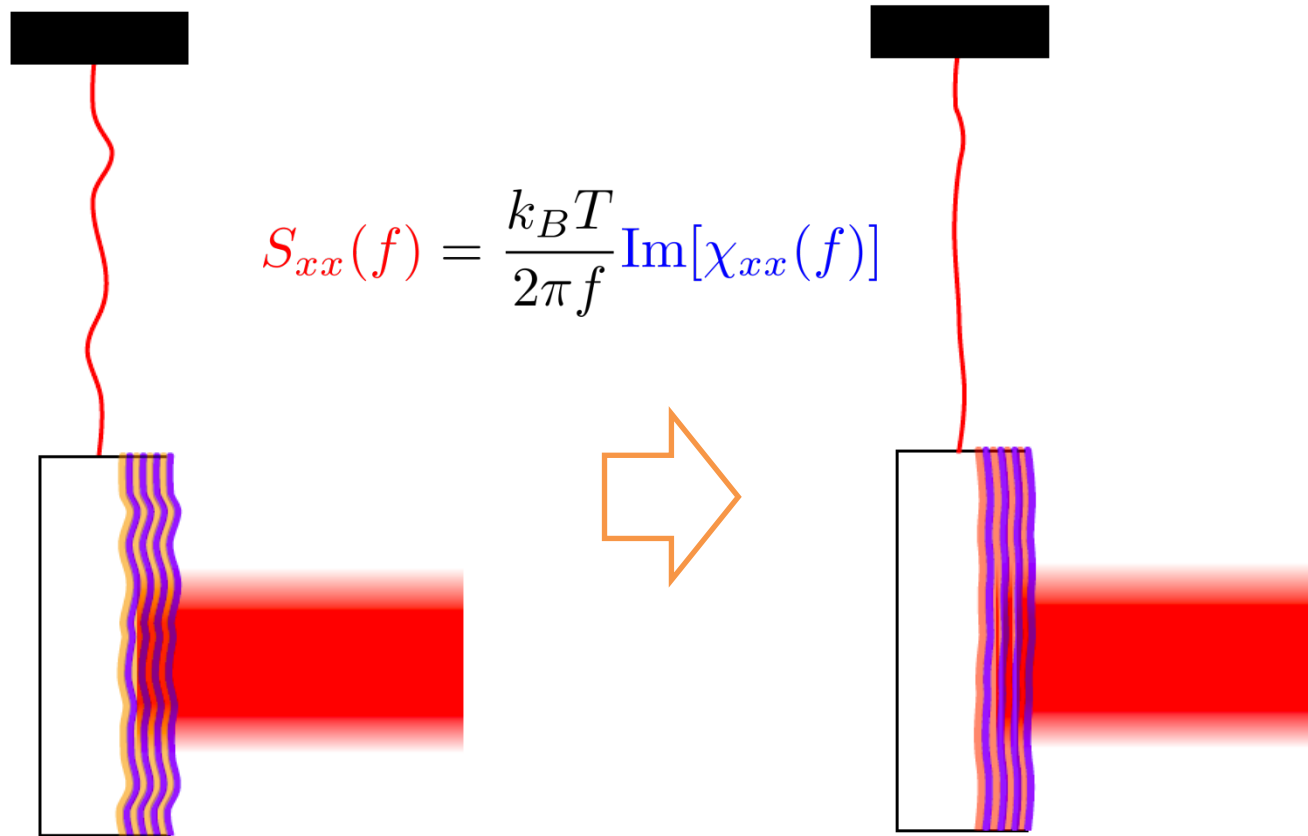
Sources of thermal noise



Thermal noises are ubiquitous. Dominant: suspension and coating.

How to reduce thermal noise

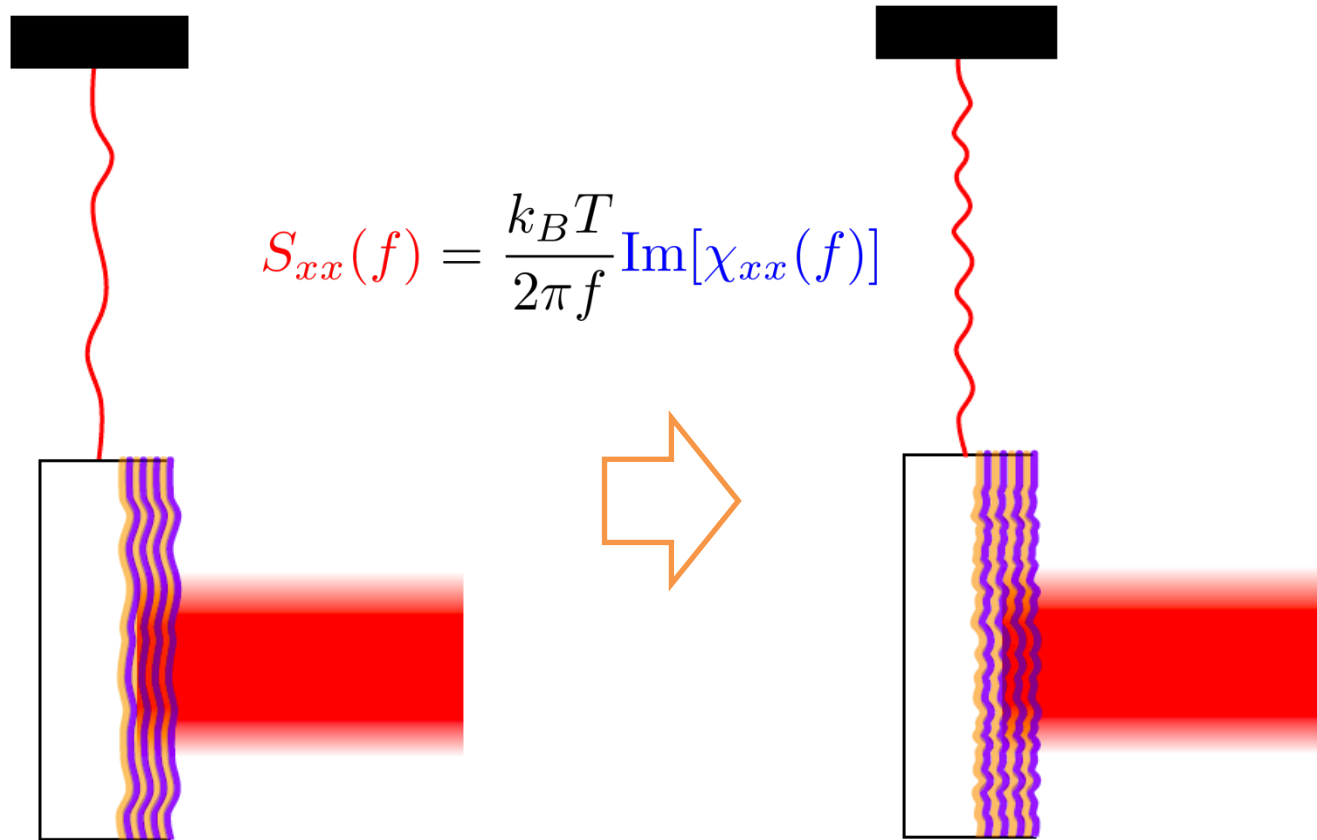
1. Using low temperature



KAGRA in JAPAN will operated at cryogenic temperature.
Some designs of future detectors also incorporates cryogenic.

How to reduce thermal noise

2. Using high-quality material



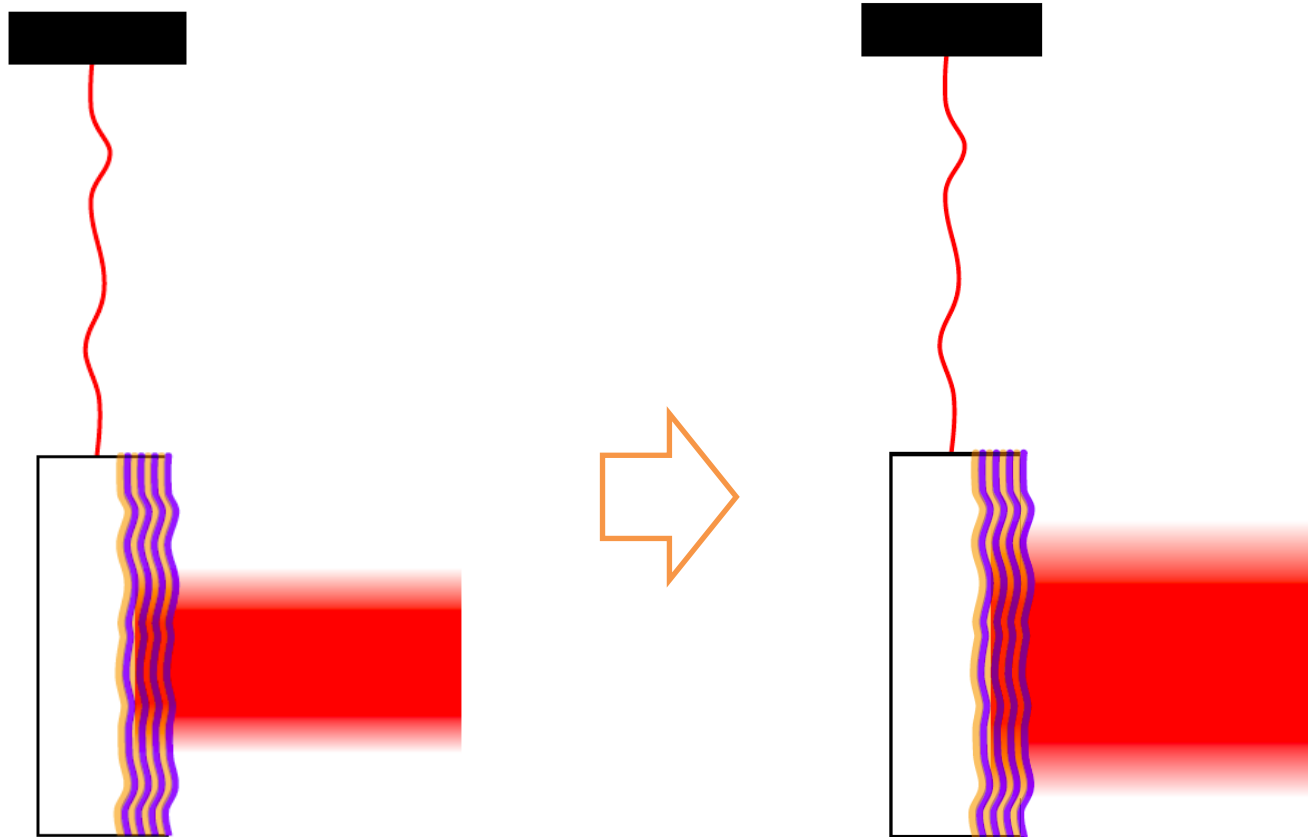
$$S_{xx}(f) = \frac{k_B T}{2\pi f} \text{Im}[\chi_{xx}(f)]$$

FDT

Concentrating thermal energy; pushing noise outside band of interest.

How to reduce thermal noise

3. Using larger beam size



Averaging out the thermal fluctuation.

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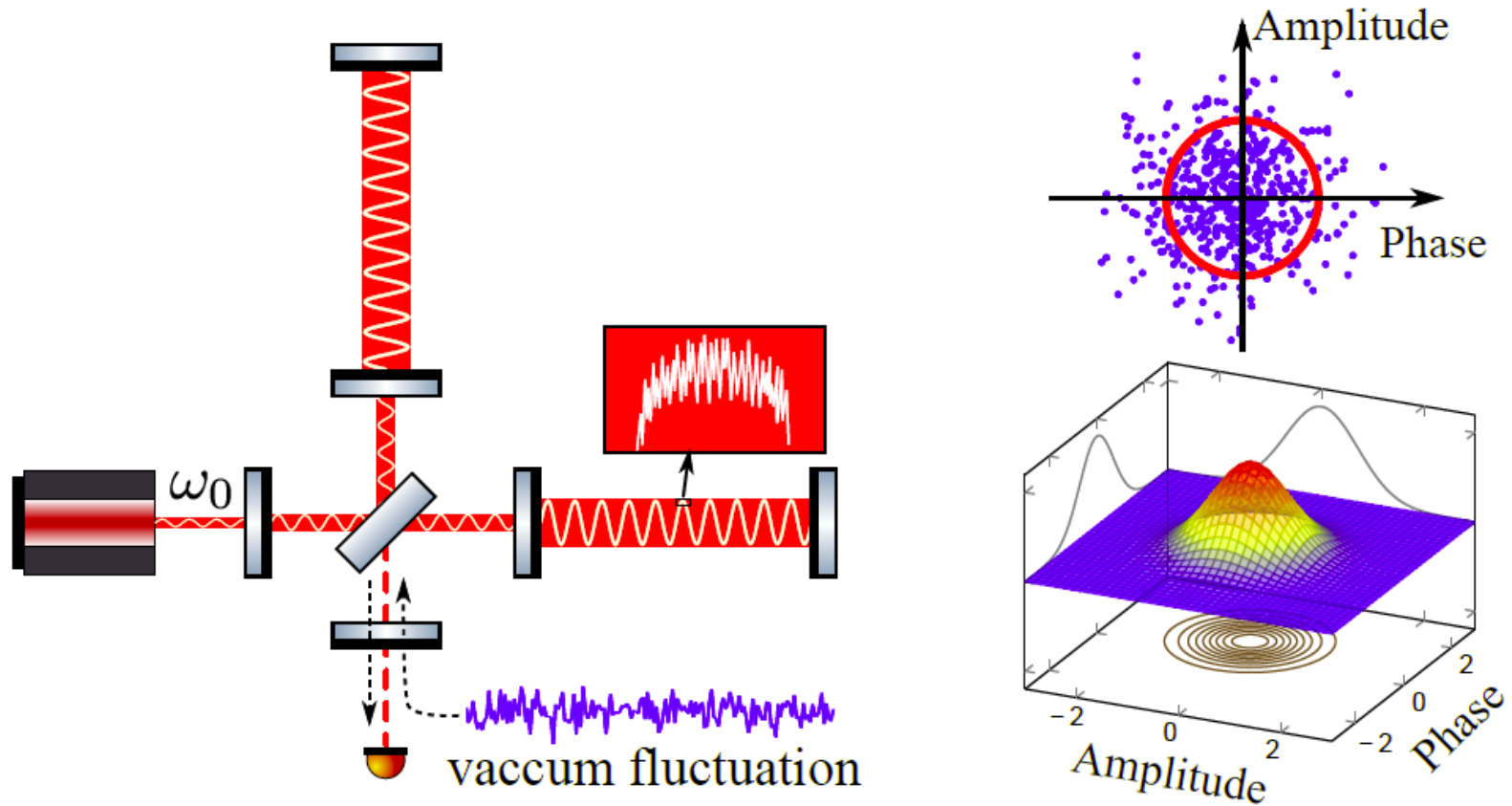
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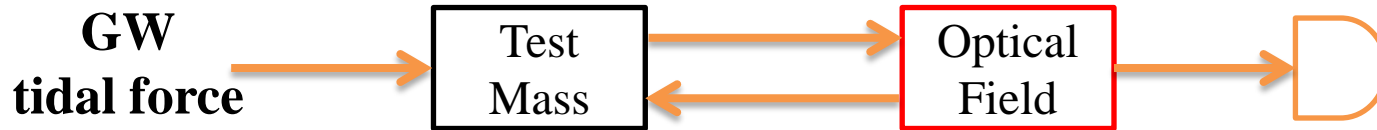
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Origin of quantum noise



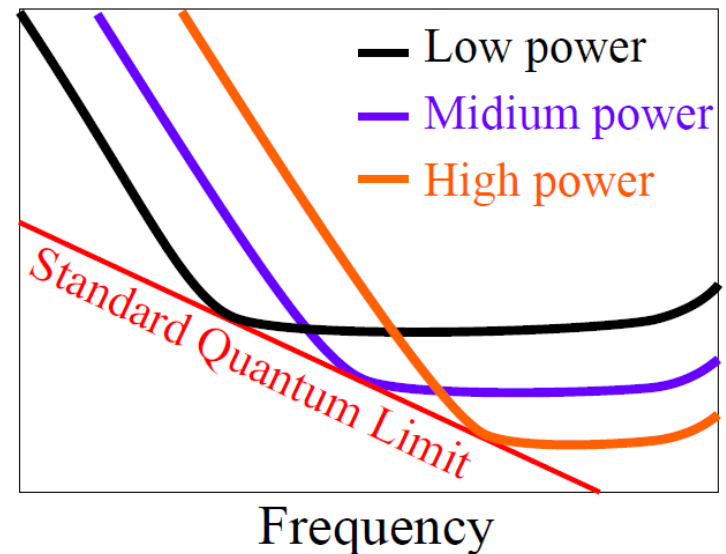
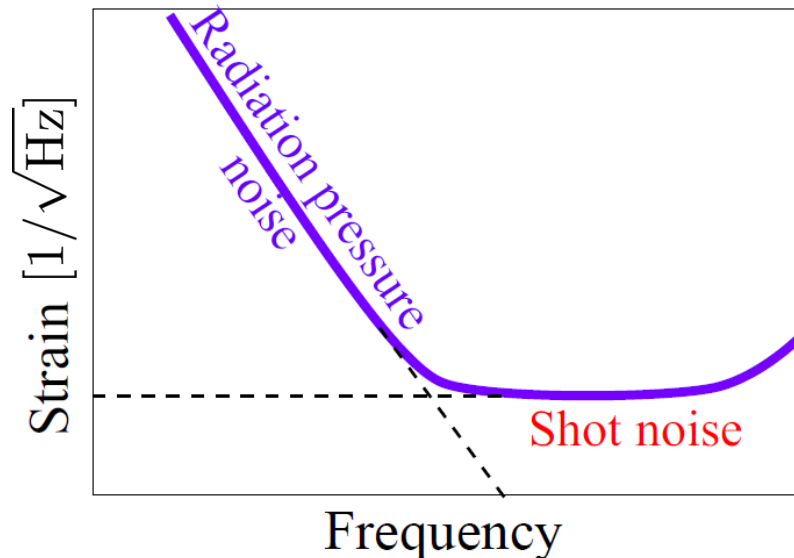
Standard quantum limit



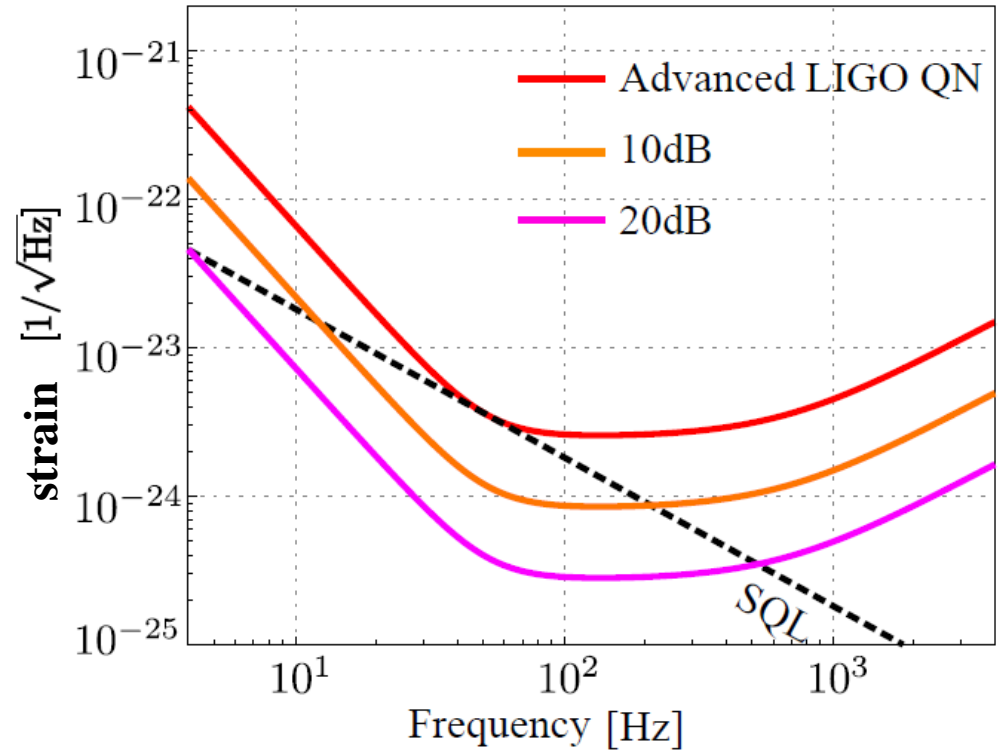
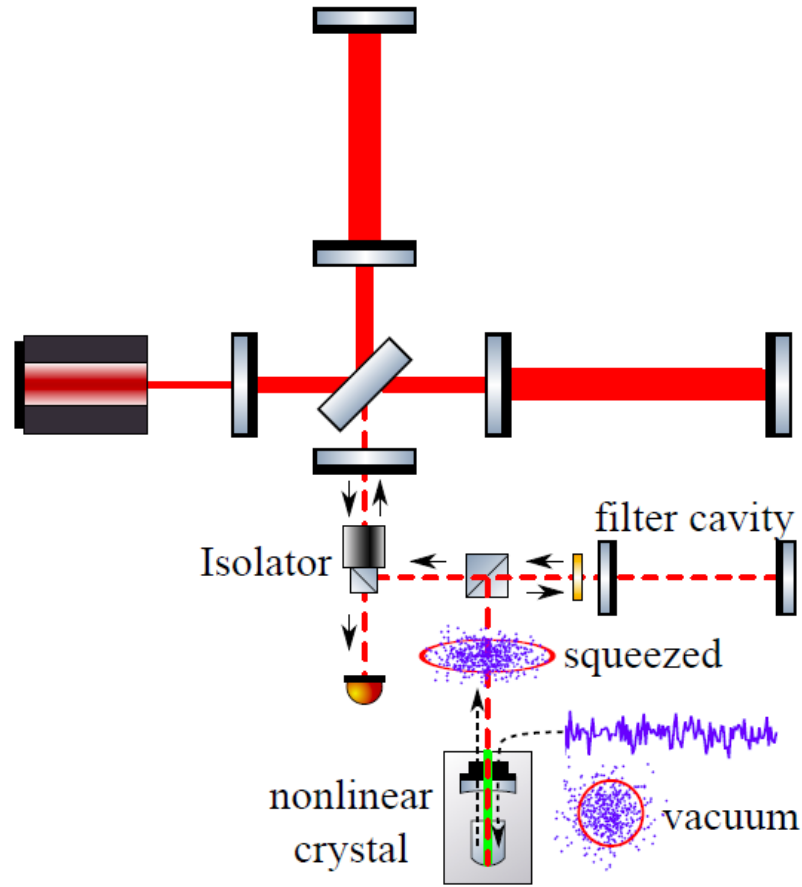
Quantum fluctuation in optical phase \Rightarrow **Shot noise**

In optical amplitude \Rightarrow **Power fluctuation** \Rightarrow **Radiation pressure noise**

Standard Quantum Limit:



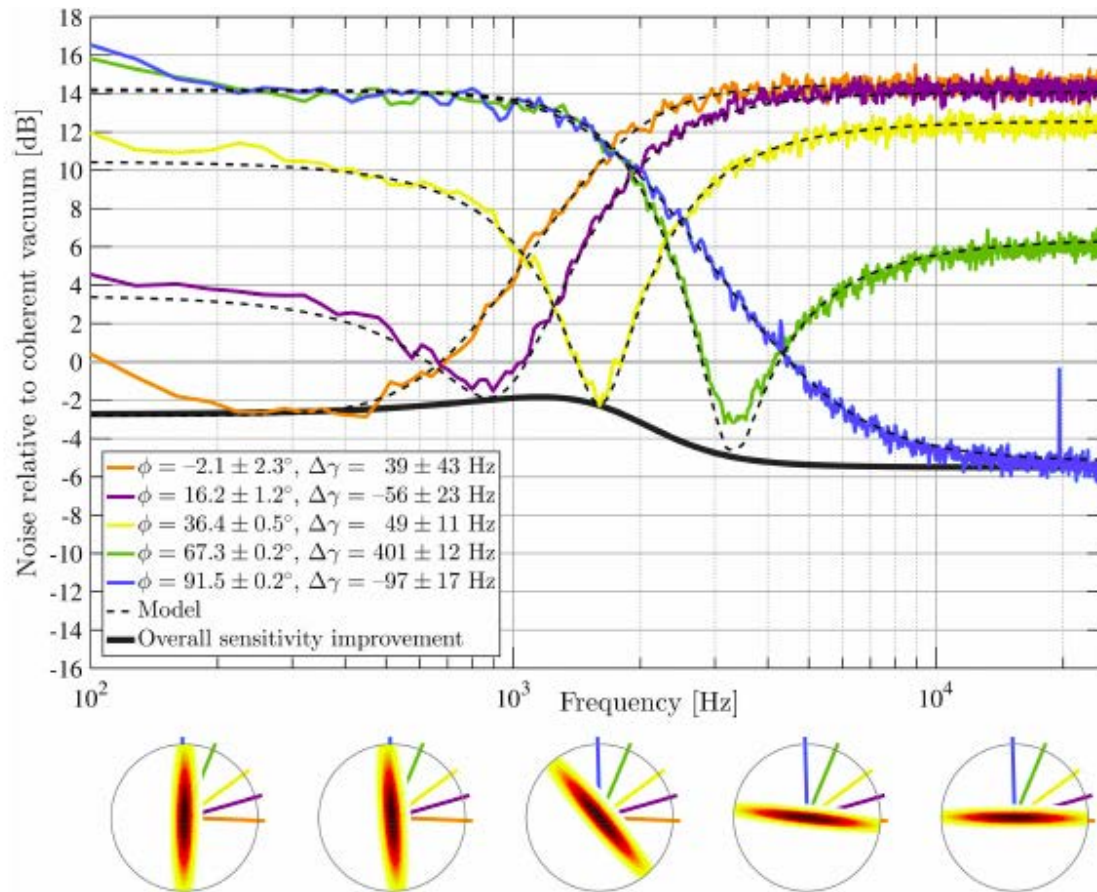
Frequency-dependent squeezing



J. Kimble, et al. Conversion of conventional GW interferometers into QND by modifying their input and/or output optics, PRD 65, 022002 (2001)

State-of-the-art

MIT proof-of-principle demonstration



E. Oelker et al., *Audio-Band Frequency-Dependent Squeezing for Gravitational-Wave Detectors*, PRL 116, 041102 (2016)

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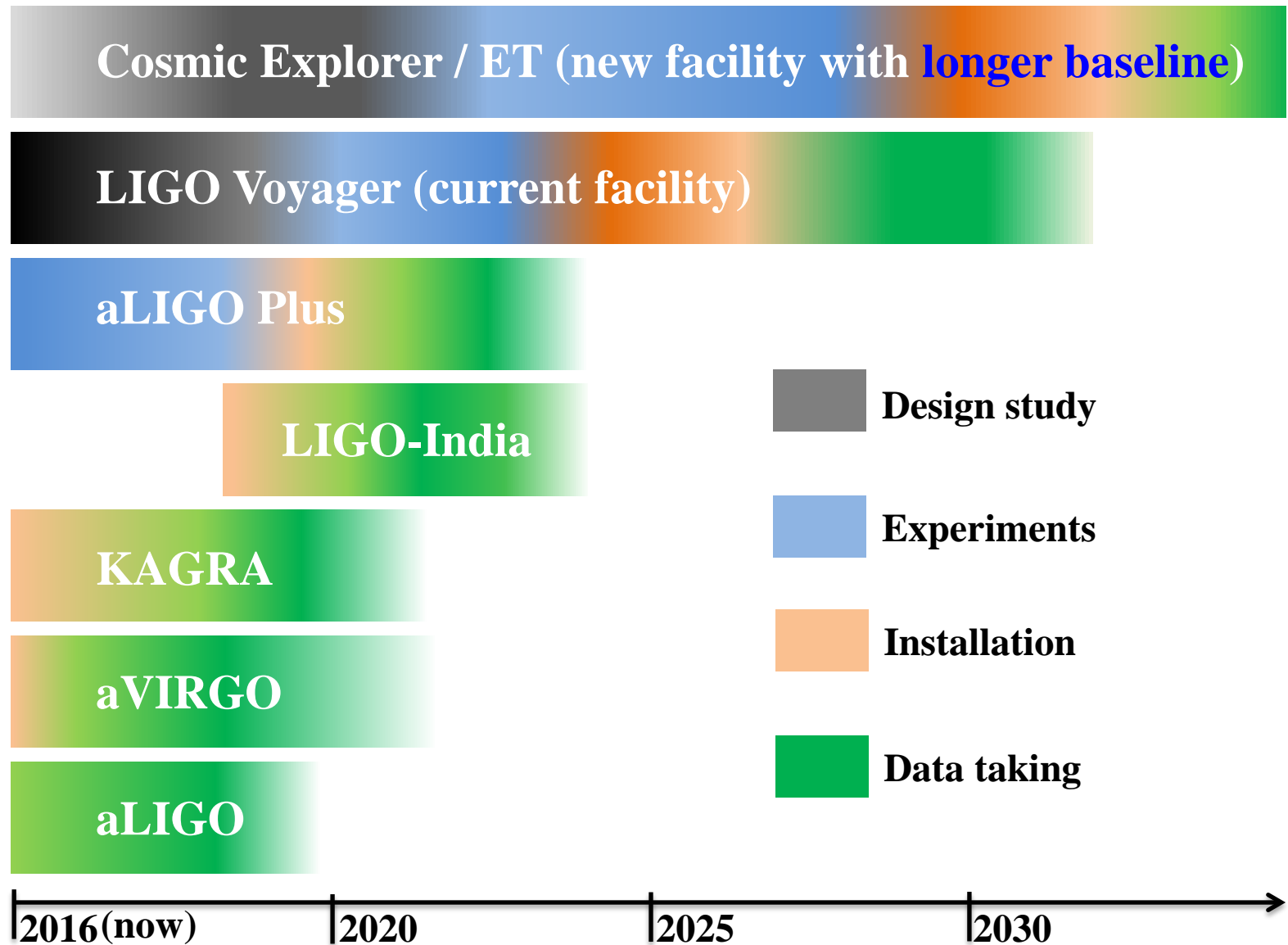
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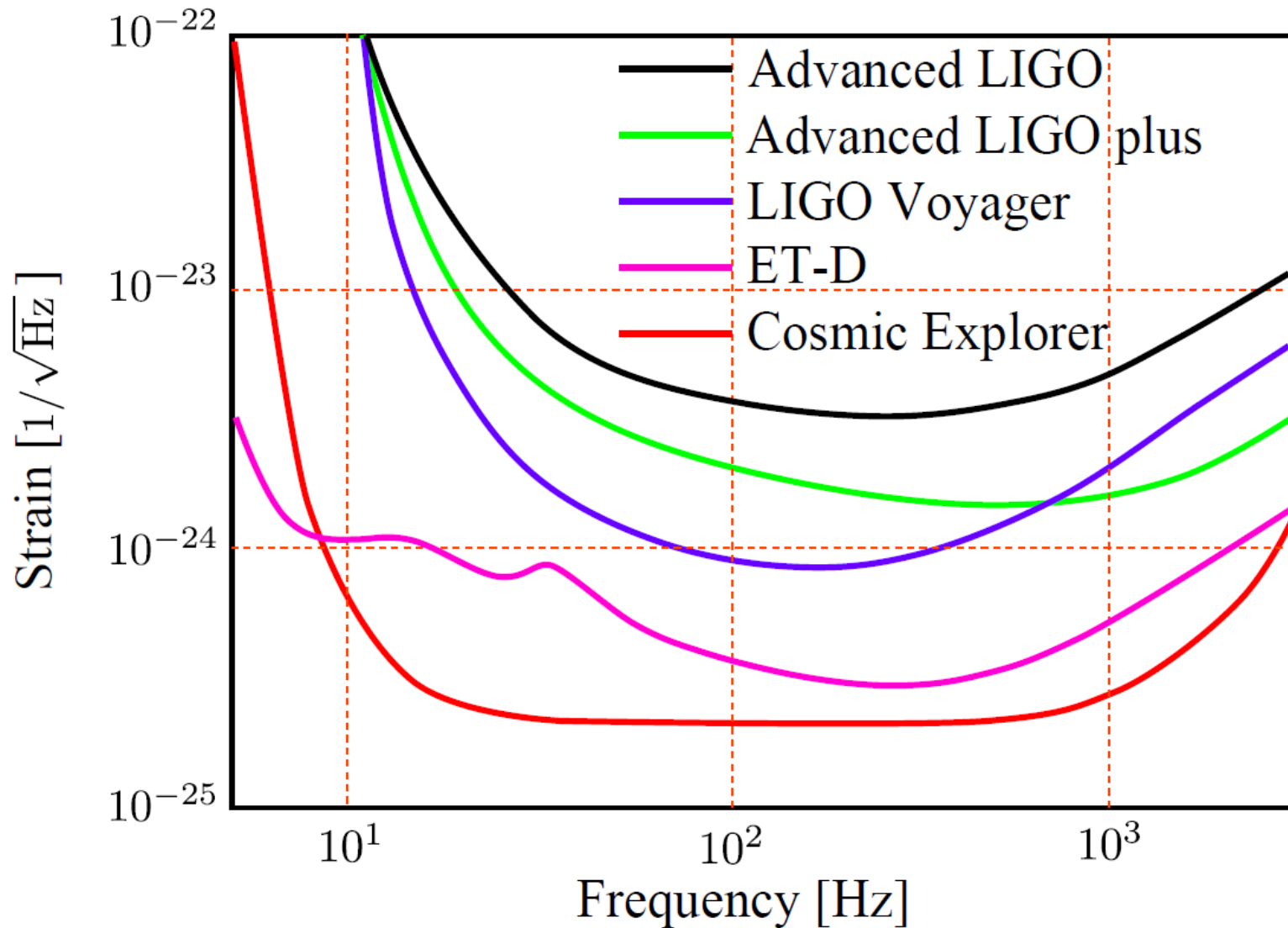
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Timeline of current and future detectors

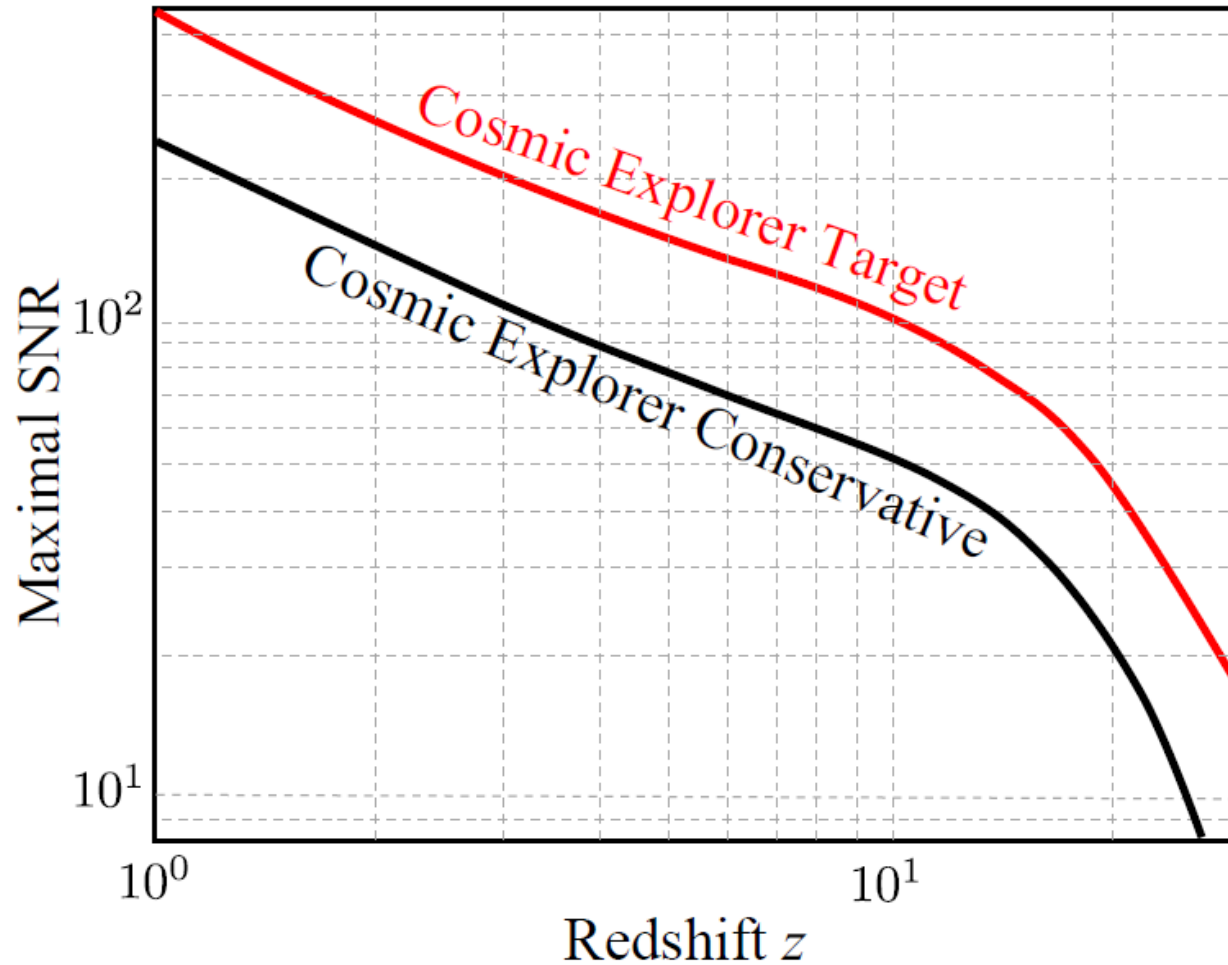


Sensitivity of future detectors



Science enabled by Cosmic Explorer

SNR for $30 M_{\odot}$ binary BHs GW150914



LIGO-DCC: P1600143-v14 (2016)

References for further reading

Advanced LIGO:

- LSC, Class. Quantum Grav. **32**, 074001 (2015)

Advanced LIGO plus:

- M. Evans *et al*, Phys. Rev. D **88**, 022002 (2013)
- J. Miller *et al*, Phys. Rev. D **91**, 062005 (2015)

LIGO Voyager:

- R. X. Adhikari *et al*, LIGO-DCC: T1400226 –v7 (2016)

Einstein Telescope (ET):

- ET science team, Einstein GW Telescope conceptual design study

Cosmic Explorer:

- S. Dwyer *et al*, Phys. Rev. D **91**, 082001 (2015)
- LSC, LIGO-DCC: P1600143-v14 (2016)

Finally, refer to **LIGO Instrument Science White Paper (2015-2016)**

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