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Beam Position from Angle to Length minimization

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The purpose of this note is to summarize the computation of the beam position estimation from the a2l coefficients.
Due to some uncontrolled degrees of freedom (in PRC notably), the beam position on the suspended optics varies from lock to lock at LLO. Because the angular fluctuations easily couple to DARM, it is necessary to apply feedforward compensation to minimize the coupling to length.
A minimization script, called *a2l_min.py*, has been developed to automate this decoupling process. The principle is to apply a dither excitation (pitch or yaw) to a suspension during a lock and to observe the resulting length signal. If the beam is not pointing to the rotation center of the optic, the angular motion creates a difference in length of the cavity: a line appears in the length signal spectrum at the dither frequency. By changing the amount of angular to length (a2l) feedforward, it is possible to reduce the angular coupling to the length signal. When the length coupling is minimal, one can infer the beam position on the optic with respect to the rotation center by knowing the a2l coefficients and the suspension motion transfer functions. The details of the computation are presented hereunder.

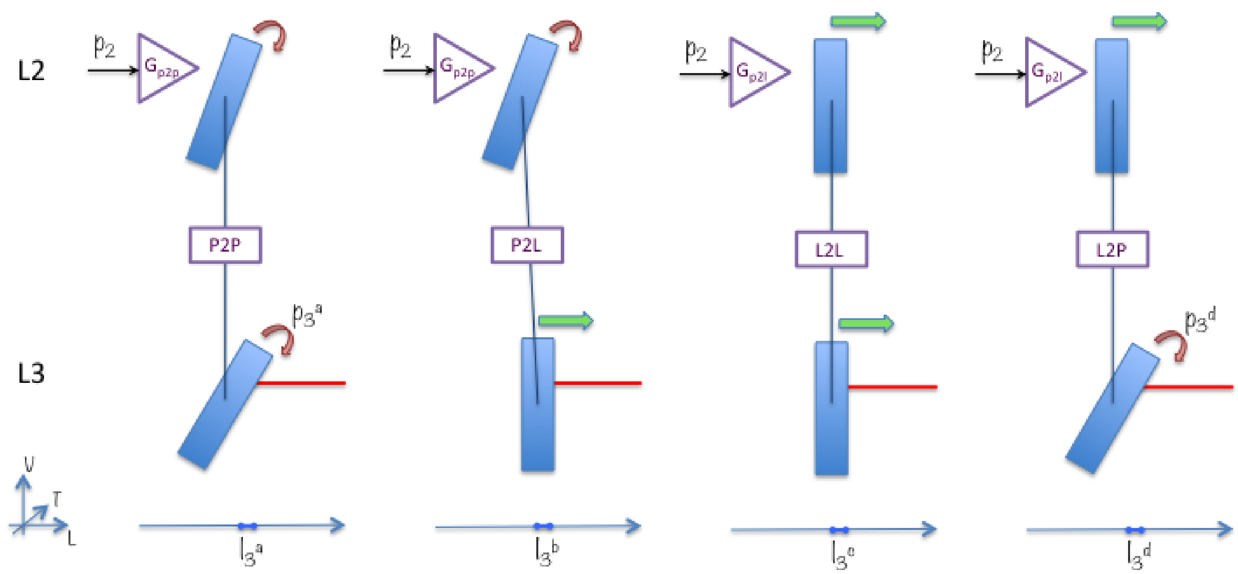

Figure 1: Overview of the four cases leading to the creation of a L3 length motion from a pitch actuation on the L2 stage.

1 Beam Position from a2l coefficients

In Sections 5.2 and 5.3 of [4] there is a short review of the angular noise to length coupling via mis-centered beam position. While [4] approaches the problem through a modal picture, we find it convenient to discuss the problem in a geometrical picture.

We will derive the computation in this note for the quadrupole suspensions, where the feed-forward is applied on the L2 stage. The different cases can be represented in Fig1. Please note that for the simple suspensions we can compute the beam position directly from the torques.

With a positive pitch $\mathbf{p_2}$ actuation on the L2 stage, the L3 mass will experience:

a) a pitch motion $\mathbf{p_3^a}$ given by:

$$\mathbf{p_3^a} = \mathbf{p_2} * G_{p2p} * P2P \tag{1}$$

If the beam is off-centered by a distance $\mathbf{v_0}$, this will also produce a length motion $\mathbf{l_3^a}$:

$$\mathbf{l_3^a} \quad = \mathbf{v_0} * \cos(\tfrac{\pi}{2} - \mathbf{p_3^a}) \approx v_0 * p_2 * G_{p2p} * P2P \tag{2}$$

b) a length motion $\mathbf{l_3^b}$ given by:

$$\mathbf{l_3^b} = \mathbf{p_2} * G_{p2p} * P2L \tag{3}$$

c) a length motion $\mathbf{l_3^c}$ generated by the feedforward path:

$$\mathbf{l_3^c} = \mathbf{p_2} * G_{p2l} * L2L \tag{4}$$

d) a pitch motion $\mathbf{p_3^d}$ generated by the feedforward path:

$$\mathbf{p_3^d} = \mathbf{p_2} * G_{p2l} * L2P \tag{5}$$

If the beam is off-centered by a distance $\mathbf{v_0}$, this will also produce a length motion $\mathbf{l_3^d}$:

$$\mathbf{l_3^d} = v_0 * cos(\frac{\pi}{2} - \mathbf{p_3^d}) \approx v_0 * p_2 * G_{p2l} * L2P \tag{6}$$

with $X2Z$ the transfer function of the suspensions from the L2 stage motion X to the L3 stage motion Z, and (G_{p2p} , G_{p2l}) the feedforward coefficients from pitch to pitch and pitch to length respectively. The transher functions are represented in Fig.3 and 4 for the ETMX QUAD suspensions with O1 damping on M0.

It is worth pointing out the 'bilinear' nature of angular-to-length noise coupling. As the noise proportions to (mis-centering × misalignment) in time-domain, in frequency domain it is the convolution of the two. Yet for AdvLIGO, the mis-centering can be well approximated by a constant, and the angular motion is dominated by the motion at excitation frequency $f_{\rm exc}$, i.e., $p_2(f) = p_2\delta(f - f_{\rm exc})$. Therefore, take I_3^d for example,

$$v_0(f) = v_0 \cdot \delta(f - \mathrm{DC}), \tag{7}$$
$$p_3^d = p_3^d \cdot \delta(f - f_{\rm exc}), \tag{8}$$
$$I_3^d(f) = \int \mathrm{d}f' v_0^* \delta\left[(f - f') - \mathrm{DC}\right] \cdot p_2\delta(f' - f_{\rm exc}) \cdot G_{p2l}(f') L2P(f') \tag{9}$$
$$= \int \mathrm{d}f' \left(v_0 \cdot p_2 \cdot G_{p2l} \cdot L2P\right)(f')\delta(f' - f_{\rm exc}). \tag{10}$$

In other words, in our study, the convolution in frequency domain can be approximated as (DC mis-centering × misalignment at $f_{\rm exc}$. This means that all the beam positions derived in this work are position at 'DC').

The feedforward path is used to compensate for the length motion produced by the $P2L$. The optimization of the feedforward gain $a2l$ is done by minimizing the coupling of the angular motion to the length motion. It means:

$$\sum_{i=a}^{d} \mathbf{l_3^i} = \mathbf{l_3^a} + \mathbf{l_3^a} + \mathbf{l_3^a} + \mathbf{l_3^a} = \mathbf{0} \tag{11}$$

$$\mathbf{p_2} * (\mathbf{v_0} * [G_{a2a} * P2P + G_{a2l} * L2P] + [G_{a2a} * P2L + G_{a2l} * L2L]) = \mathbf{0} \tag{12}$$

$$\mathbf{v_0} = -\frac{G_{p2p} * P2L + G_{p2l} * L2L}{G_{p2p} * P2P + G_{p2l} * L2P} \tag{13}$$

Following the same reasoning, we find that the minimization of the length produced by a positive yaw motion gives:

$$\mathbf{l_3^a} = \quad -\mathbf{t_0} * \cos(\tfrac{\pi}{2} - \mathbf{y_3^a}) \quad \approx -t_0 * y_2 * G_{y2y} * Y2Y \tag{14}$$
$$\mathbf{l_3^b} = \quad \mathbf{p_2} * G_{y2y} * P2L \tag{15}$$
$$\mathbf{l_3^c} = \quad \mathbf{p_2} * G_{y2l} * L2L \tag{16}$$
$$\mathbf{l_3^d} = \quad -\mathbf{t_0} * cos(\tfrac{\pi}{2} - \mathbf{y_3^d}) \quad \approx -\mathbf{t_0} * y_2 * G_{y2l} * L2Y \tag{17}$$

$$\mathbf{t_0} = \frac{G_{y2y} * Y2L + G_{y2l} * L2L}{G_{y2y} * Y2Y + G_{y2l} * L2Y} \tag{18}$$

The beam position (v_0, t_0) is then a function of the feedforward coefficients and the transfer functions at the dither frequency, and can be expressed independently in pitch and yaw. Please note that $G_{p2p} = G_{y2y} = 1$.
We can describe two particular cases:

- If the beam is centered ($v_0 = 0, t_0 = 0$) we have:

$$G_{p2l} = -G_{p2p}\frac{P2L}{L2L} \tag{19}$$

$$G_{y2l} = -G_{y2y}\frac{Y2L}{L2L} \tag{20}$$

  At 10 Hz, for the ETMX suspension with O1 damping, it means $G_{p2l} = +0.7943$ and $G_{y2l} = 0$.

- If the feedforward is zero, the coupling to length is minimal when the beam is at:

$$v_0 = -\frac{P2L}{P2P} \tag{21}$$

$$t_0 = -\frac{Y2L}{Y2Y} \tag{22}$$

  that is (v_0 = 3.5 mm, t_0 = 0 mm) at 10 Hz, for the ETMX suspension with O1 damping.

The position as a function of the a2l coefficients in represented in Fig.2 for the ETMX suspension at 10 Hz.

TO DO: This calibration has to be confirmed by a cross-measurement (by observing the beam position with the cameras for example).

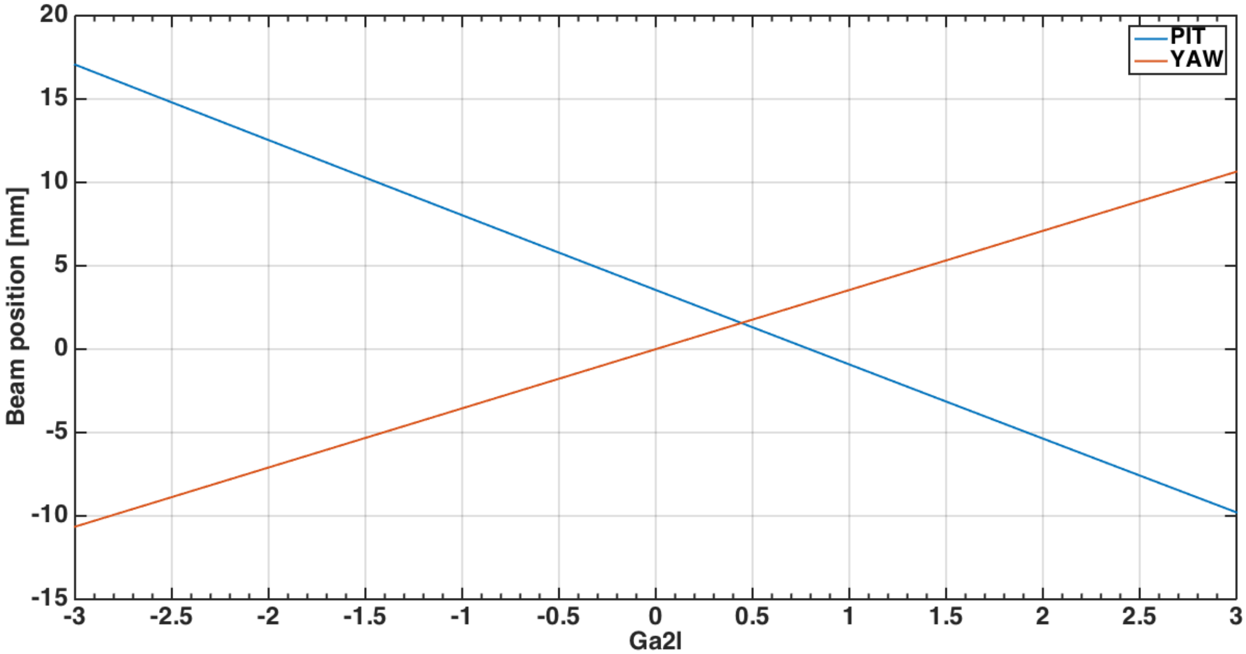

Figure 2: Beam position as a function of the a2l gains as infered from Eq. 13, 18 with the simulated ETMX transfer functions with O1 damping at 10 Hz dithering. The slope of the linear fit of the pit position function is -4.5, and the slope of the yaw position function is 3.5.

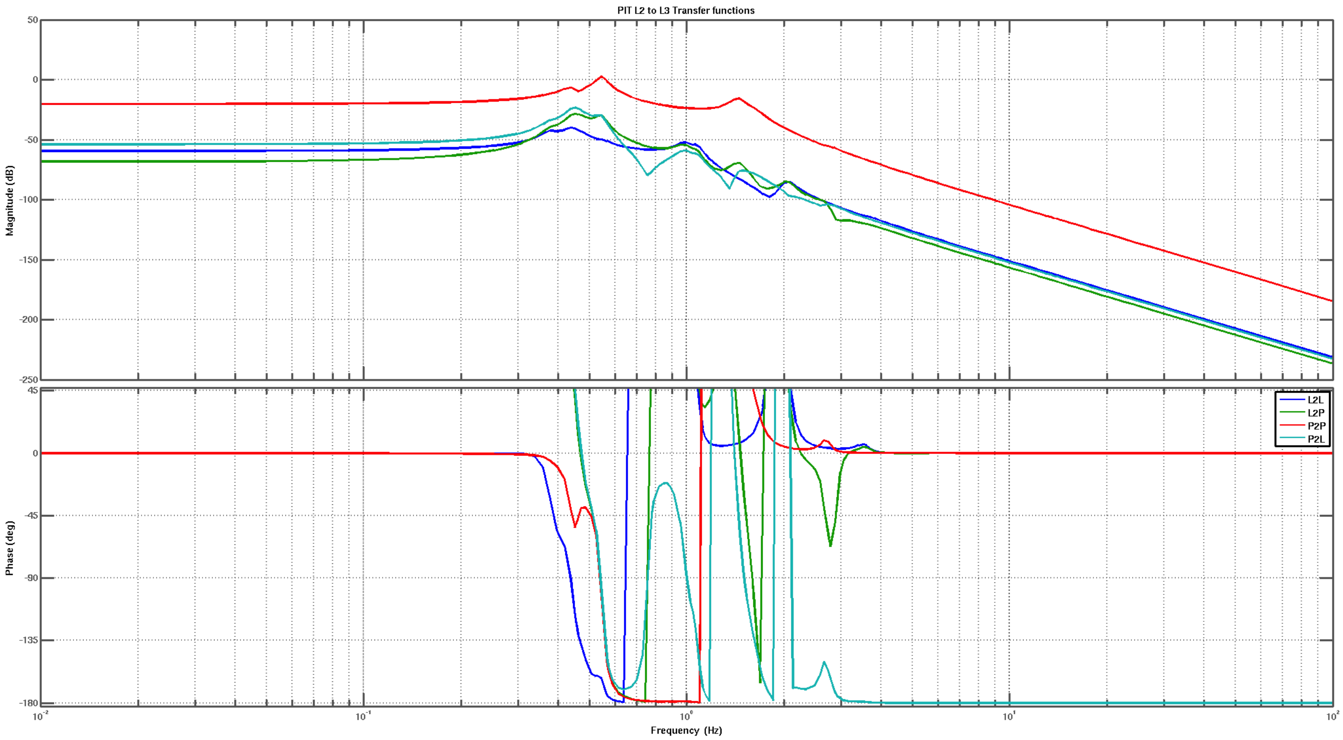

Figure 3: PIT L2 to L3 transfer functions

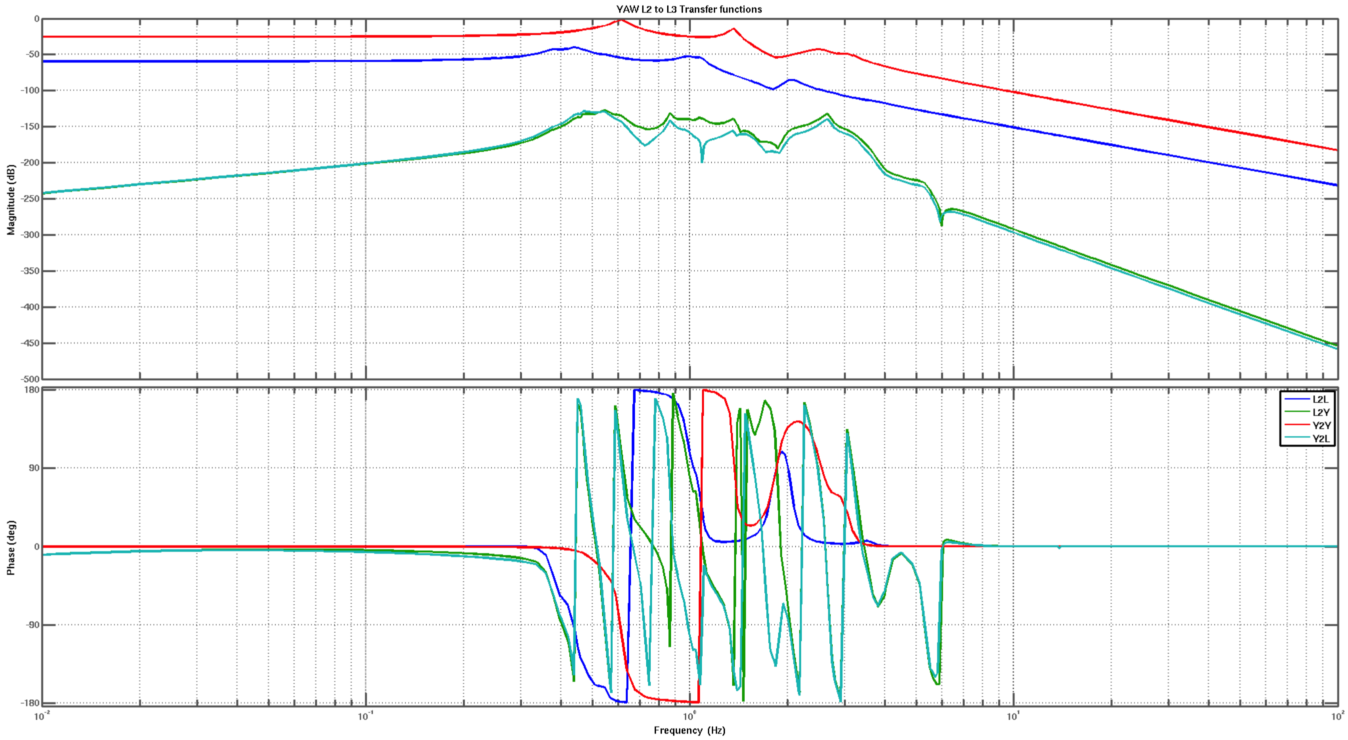

Figure 4: YAW L2 to L3 tranfer functions

2 Torque and Length Actuation

The ASC torque control is applied through the AOSEM on the L2 stage of the quadrupole suspensions (QUAD), and on the last stage M3 of the triple suspensions (BSFM, HLTS, HSTS). Here we check how the torque and length motions are applied to the AOSEM. The conventions are defined in [1] and in Fig. xx.

2.1 Length motion

To create a positive length motion, we apply an equal force on the 4 AOSEMs:

$$\mathbf{F_0^{UL}} = +F_0\ \mathbf{u_L} \quad \mathbf{F_0^{UR}} = +F_0\ \mathbf{u_L} \tag{23}$$

$$\mathbf{F_0^{LL}} = +F_0\ \mathbf{u_L} \quad \mathbf{F_0^{LR}} = +F_0\ \mathbf{u_L} \tag{24}$$

- The equivalent force is:

$$\mathbf{F_L} = \sum_{i=1}^{4} \mathbf{F_0^i} = 4F_0\ \mathbf{u_L} \tag{25}$$

that leads, with the calibration from N to cts, to:

$$F_L\ [cts] = 4 * cal\ [cts/N] * F_0\ [N] \tag{26}$$

- The torque on each AOSEM is, with $\mathbf{r^i}$ the radial coordinate of each actuator:

$$\tau_\mathbf{0}^\mathbf{i} = \mathbf{r^i} \times \mathbf{F_0^i} \tag{27}$$

that is, with the projection on the local coordinates ($\theta = \pi/4$):

$$\tau_\mathbf{0}^\mathbf{UL} = r(\cos\theta\ \mathbf{u_T} + \sin\theta\ \mathbf{u_V}) \times \mathbf{F_0^{UL}} = r\frac{\sqrt{2}}{2}F_0\ (-\mathbf{u_V} + \mathbf{u_T}) \tag{28}$$

$$\tau_\mathbf{0}^\mathbf{UR} = r(-\cos\theta\ \mathbf{u_T} + \sin\theta\ \mathbf{u_V}) \times \mathbf{F_0^{UR}} = r\frac{\sqrt{2}}{2}F_0\ (\mathbf{u_V} + \mathbf{u_T}) \tag{29}$$

$$\tau_\mathbf{0}^\mathbf{LL} = r(\cos\theta\ \mathbf{u_T} - \sin\theta\ \mathbf{u_V}) \times \mathbf{F_0^{LL}} = r\frac{\sqrt{2}}{2}F_0\ (-\mathbf{u_V} - \mathbf{u_T}) \tag{30}$$

$$\tau_\mathbf{0}^\mathbf{LR} = r(-\cos\theta\ \mathbf{u_T} - \sin\theta\ \mathbf{u_V}) \times \mathbf{F_0^{LR}} = r\frac{\sqrt{2}}{2}F_0\ (\mathbf{u_V} - \mathbf{u_T}) \tag{31}$$

$$\tag{32}$$

that leads to the equivalent torque:

$$\mathbf{T_L} = \sum_{i=1}^{4} \tau_\mathbf{0}^\mathbf{i} = r\frac{\sqrt{2}}{2}F_0(-2\mathbf{u_V} + 2\mathbf{u_V} + 2\mathbf{u_T} - 2\mathbf{u_T}) = \mathbf{0} \tag{33}$$

2.2 Torque motion

We derive the equations for PITCH, they are similar for YAW. To create a positive pitch torque motion, we apply an equal force on the 4 AOSEMs with opposite signs for the top and the bottom row:

$$\mathbf{F_0^{UL}} = +F_0\ \mathbf{u_L} \quad \mathbf{F_0^{UR}} = +F_0\ \mathbf{u_L} \tag{34}$$

$$\mathbf{F_0^{LL}} = -F_0\ \mathbf{u_L} \quad \mathbf{F_0^{LR}} = -F_0\ \mathbf{u_L} \tag{35}$$

- The torque on each AOSEM is then:

$$\tau_\mathbf{0}^\mathbf{i} = \mathbf{r} \times \mathbf{F_0^i} \tag{36}$$

that is, with the projection on the local coordinates and $\theta = \pi/4$:

$$\tau_\mathbf{0}^\mathbf{UL} = r(\cos\theta\ \mathbf{u_T} + \sin\theta\ \mathbf{u_V}) \times \mathbf{F_0^{UL}} = r\frac{\sqrt{2}}{2}F_0\ (-\mathbf{u_V} + \mathbf{u_T}) \tag{37}$$

$$\tau_\mathbf{0}^\mathbf{UR} = r(-\cos\theta\ \mathbf{u_T} + \sin\theta\ \mathbf{u_V}) \times \mathbf{F_0^{UR}} = r\frac{\sqrt{2}}{2}F_0\ (\mathbf{u_V} + \mathbf{u_T}) \tag{38}$$

$$\tau_\mathbf{0}^\mathbf{LL} = r(\cos\theta\ \mathbf{u_T} - \sin\theta\ \mathbf{u_V}) \times \mathbf{F_0^{LL}} = r\frac{\sqrt{2}}{2}F_0\ (\mathbf{u_V} + \mathbf{u_T}) \tag{39}$$

$$\tau_\mathbf{0}^\mathbf{LR} = r(-\cos\theta\ \mathbf{u_T} - \sin\theta\ \mathbf{u_V}) \times \mathbf{F_0^{LR}} = r\frac{\sqrt{2}}{2}F_0\ (-\mathbf{u_V} + \mathbf{u_T}) \tag{40}$$

(41)

The equivalent torque in [N.m] is expressed by:

$$\mathbf{T_{pit}} = \textstyle\sum_{i=1}^4 \tau_\mathbf{0}^\mathbf{i} = r\frac{\sqrt{2}}{2}F_0\ (-2\mathbf{u_V} + 2\mathbf{u_V} + 4\mathbf{u_T}) \tag{42}$$

$$= 2\sqrt{2}\ r\ F_0\ \mathbf{u_T} \tag{43}$$

$$T_{pit}\ [cts.m] = 2\sqrt{2} * r\ [m] * F_0\ [N] * cal\ [cts/N] \tag{44}$$

In the case of the quadrupole suspension (r = 0.1 m [2] [3]), the force on each AOSEM is:

$$\mathbf{F_0} = \frac{1}{2\sqrt{2}\ r} = 3.5355\ \mathbf{T_{pit}}\ [N] \tag{45}$$

- The equivalent force along the longitudinal direction is:

$$\mathbf{F_L} = \sum_{i=1}^{4} \mathbf{F_0^i} = \mathbf{0} \tag{46}$$

2.3 Driving the DOFs

The input drives are converted to AOSEM actuation through the EUL2OSEM matrix, that is, for the QUAD:

	L	P	Y
UL	0.25	3.5355	-3.5355
LL	0.25	-3.5355	-3.5355
UR	0.25	3.5355	3.5355
LR	0.25	-3.5355	3.5355

The matrix therefore conserves the relative actuation strengths:

- L drive: 1 unit of input drive [N] gives 1 unit of equivalent length drive [N]
- P drive: 1 unit of input drive [N.m] gives 1 unit of equivalent pit drive [N.m]

The simulated transfer functions of the QUADs are given in [rad]/[N.m] or [m]/[N]. The conversion from [counts] to [N] is the same for each AOSEM and is referenced in []. Therefore we can use the equations 13, 18 without any further conversion to compute the beam position.

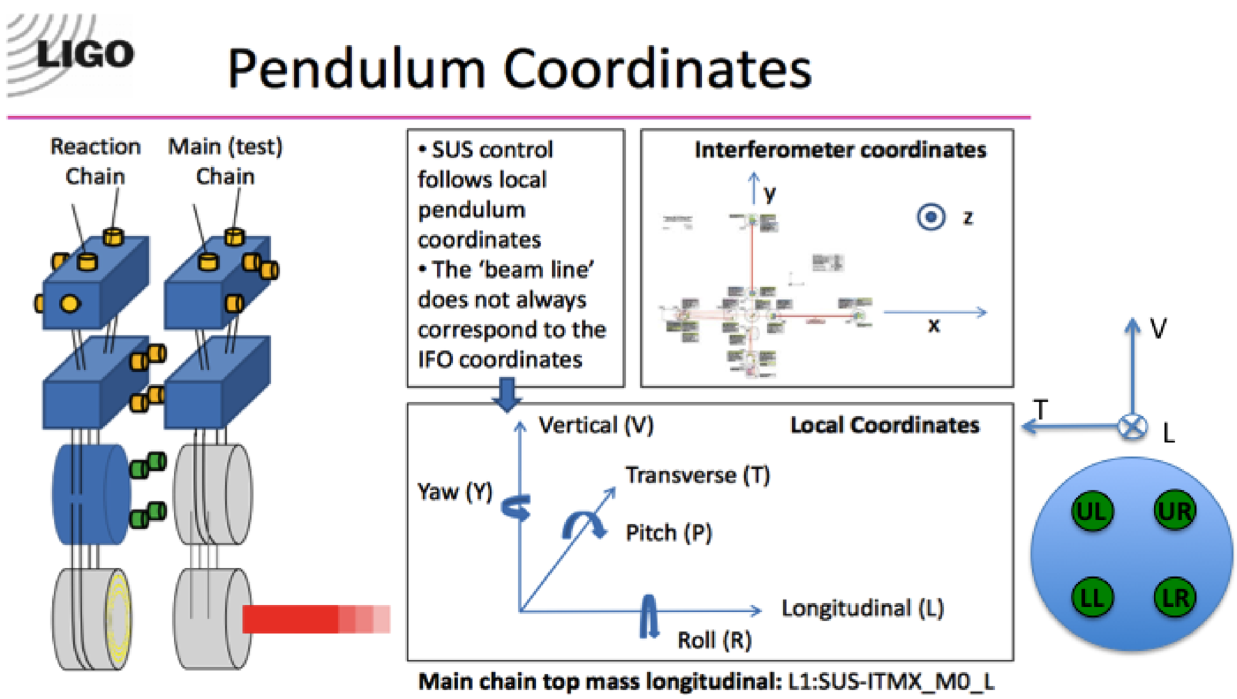

Figure 5: From G1100866-v8: definition of the vectors $\mathbf{u_L}, \mathbf{u_T}, \mathbf{u_V}$ with respect to the AOSEM convention: *UL, UR, LL, LR*

3 Where to find the scripts?

- a2l minimization script:
  /opt/rtcds/userapps/trunk/isc/common/scripts/decoup
- Beam position computation:
  /opt/rtcds/userapps/trunk/isc/common/scripts/decoup/BeamPosition
  This is a Matlab tool that has been developed to facilitate the computation of the beam position. A screenshot of the Simulink model in presented in Fig.6.

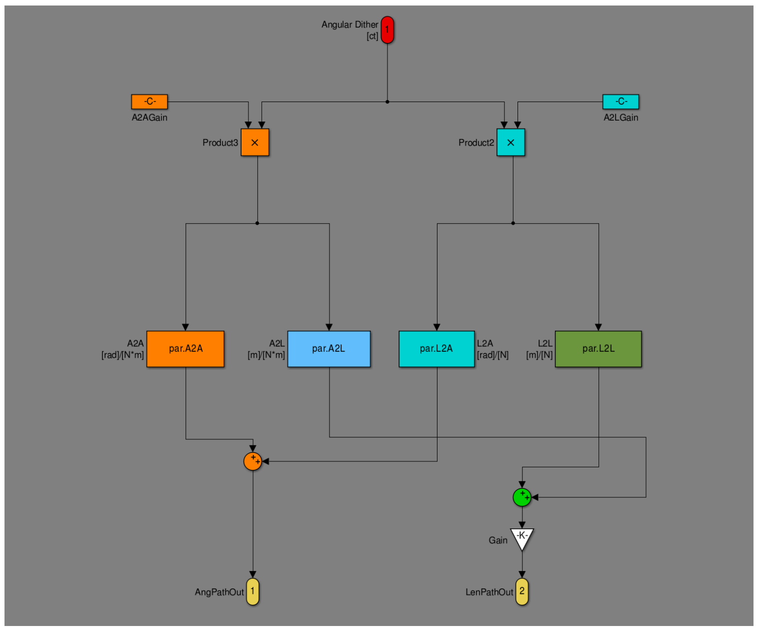

Figure 6: Screenshot of the Simulink model that represents the beam position computation

4 How the a2l script works

The a2l script is an empirical automation to minimize the angular-to-length noise coupling as described in [4] Sec. 5.2-5.3 in the modal picture, and in this document in the geometrical picture. The code will drive an optic in angle (pitch or yaw) at a given frequency, and demodulate the darm response at the excitation frequency. The responses in both the I and the Q phases will be recorded, and thus the user do not need to specify the demodulation phase. The code will vary the a2l feedforward coefficient in small steps and fit the darm I and Q responses, respectively, as linear functions of the a2l coefficient. A rotation of the I and Q responses will be preformed, such that the rotated response will be constant in the new Q' phase, and then the minimum in the new I' phase will correspond to the optimal a2l coefficient, as it correspond to the minimal angular motion coupling to darm. While the code does the optimization using a single line, usually in practice a broad-band (10-20 Hz) noise reduction can be achieved.

References

[1] E1000617, *Quad Suspension Controls Arrangement Poster*

[2] D080128, *Penultimate Mass, ITM Quad*

[3] T0900403, *Advanced LIGO SYS Summary of COC/SUS OPTIC Substrates & associated attachments*

[4] T0900511-v4, *Modeling of Alignment Sensing and Control for Advanced LIGO*