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Update to Tracking Temporal Variations in DARM Loop Model Parameters: Individual Actuation Stage Tracking, Cancelled Lines, and SRC Detuning

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1 Introduction

In this note, the methodology outlined in [1] and [2] is revised to include several updates as necessitated by the identification of the additional time dependence of the Advanced LIGO DARM loop model parameters. Updates include the independent computation of each actuation stage's strength relative to the reference parameter, and the inclusion of signal recycling cavity detuning and the potential time dependence of the new parameters that represent it.

2 Time Dependent Correction Factors

2.1 Definitions

We assume that there is a DARM loop model that was generated for a reference time, t_0, and includes the following parameters valid at $t = t_0$:

- A_0^T, the analog frequency-dependent complex actuation function for the *L3* (TST) ETM actuation stage
- A_0^P, the analog frequency-dependent complex actuation function for the *L2* (PUM) ETM actuation stage
- A_0^U, the analog frequency-dependent complex actuation function for the *L1* (UIM) ETM actuation stage
- C_0, the analog and digital frequency-dependent complex sensing function
- D_0, the digital frequency-dependent complex filter transfer function
- $F^T = F_{ISC} F_{LK}^T F_{DA}^T F_{OUT}^T$, the digital frequency-dependent filter function associated with the *L3* (TST) ETM actuation stage
- $F^P = F_{ISC} F_{LK}^T F_L^P F_{DA}^P F_{OUT}^P$, the digital frequency-dependent filter function associated with the *L2* (PUM) ETM actuation stage
- $F^U = F_{ISC} F_{LK}^T F_L^P F_L^U F_{DA}^U F_{OUT}^U$, the digital frequency-dependent filter function associated with the *L1* (UIM) ETM actuation stage

The DARM open loop transfer function at $t = t_0$ can thus be written as

$$G_0 = C_0 D_0 (F^T A_0^T + F^P A_0^P + F^U A_0^U) \tag{1}$$

For $t > t_0$, we can write the actuation and sensing functions as

$$A(t) = \kappa_T(t) F^T A_0^T + \kappa_P(t) F^P A_0^P + \kappa_U(t) F^U A_0^U \tag{2}$$

$$C(t) = \kappa_C(t) \mathcal{K}_C \mathcal{E}_C \left(\frac{1}{1 + if/f_c(t)} \right) \left(\frac{f^2}{f^2 - iff_s(t)/Q(t) + f_s^2(t)} \right) \tag{3}$$

where $\kappa_T(t)$, $\kappa_P(t)$ and $\kappa_U(t)$ are dimensionless, scalar, complex, time-dependent correction factors for the *L3*, *L2* and *L1* actuation functions (with magnitude typically close to unity and zero phase), $\kappa_C(t)$ is the dimensionless, scalar, real, time-dependent correction factor for the sensing function gain (also close to unity in magnitude), $f_c(t)$ is the time-dependent coupled-cavity pole frequency (in Hz), and f_s and Q represent the complex pole pair and the detuned spring frequency (in Hz) with dimensionless quality factor. $\mathcal{K}_C$ is the interferometer's optical gain (including any gain from the hybrid analog and digital readout chain of the OMC's DCPDs) in d_{err} counts per differential arm length change in meters, ct/m, and $\mathcal{E}_C$ are the electronics readout chain's frequency dependence normalize to unity at DC.

At the reference time, we define $\kappa_C(t_0) = 1$, and the sensing function has no time dependence, and is defined as

$$C_0 \equiv \mathcal{K}_C \mathcal{E}_C \left(\frac{1}{1 + if/f_c} \right) \left(\frac{f^2}{f^2 - iff_s/Q + f_s^2} \right) \tag{4}$$

Comparing Eq. 3 against Eq. 4, we can define a new "residual" time-independent function, C_{res} which normalizes out the components of C_0 that we expect to change as a function of time,

$$C_{res} = C_0 \left(\frac{1}{1 + if/f_c} \right)^{-1} \left(\frac{f^2}{f^2 - iff_s/Q + f_s^2} \right)^{-1} \tag{5}$$

(where the quantities f_c, fs, and Q are that of the reference time) such that 3 can be re-written more conveniently as

$$C(t) = C_{res} \left(\frac{\kappa_C(t)}{1 + if/f_c(t)} \right) \left(\frac{f^2}{f^2 - iff_s(t)/Q(t) + f_s^2(t)} \right) \tag{6}$$

where the latter two terms are corrective to the reference time. Note that C_{res} contains the reference time optical gain because we only extract the time dependent variables.

The time-dependent DARM open loop transfer function, $G(t)$, can thus be written as

$$\begin{aligned} G(t) &= C(t)\ D_0\ A(t) \\ G(t) &= C_{res} \left(\frac{\kappa_C(t)}{1 + if/f_c(t)} \right) \left(\frac{f^2}{f^2 - iff_s(t)/Q(t) + f_s^2(t)} \right) D_0 \left[\kappa_T(t) F^T A_0^T \right. \\ &\quad \left. + \kappa_P(t) F^P A_0^P + \kappa_U(t) F^U A_0^U \right] \end{aligned} \tag{7, 8}$$

We also define the actuation function of the reference actuator, the photon calibrator, as $A_0^{(PC)}$, a frequency dependent, complex transfer function. The front end is typically already calibrated and whitened. The photon calibrator actuation function is 2 1-Hz poles, and compensation for analog and digital anti-aliasing filtering. See G1501518 for further details in "offline photon calibrator corrections".

2.2 Physical Motivation

Below we list the physical processes we believe to be the source of the time dependence in the interferometer's DARM loop model.

- κ_T is the charge accumulation around the test mass or reaction mass, and in the separation between the main chain and reaction chain. These must be continuously tracked; the separation of the chains creates a tens-of-minutes-scale change in actuation strength periodically at the beginnings or ends of lock stretched when large tidal force corrections are applied, and charge accumulates very slowly on the days-to-weeks time scale.

- κ_P and κ_U are not expected to change continuously as function of time. We track the change in actuation strength to ensure we're covered against electronics chain changes and/or failures, which we expect to happen suddenly and very infrequently.

- κ_C represents changes on the few-minutes time scale that are from drifts of the optical alignment in the arm cavities.

- f_{cc}, f_s, and Q are all parameters believed to change on the same few-minutes time-scale again due to optical alignment drifts. As of yet it is unclear which, but it is likely to be relative alignment drifts between the arm cavities and the signal recycling cavity.

The photon calibrator is used as a stable reference, but there may be situations that the actuation strength may change due to unintended laser beam clipping between the transmitter module and the ETM or between the ETM and the receiver module. This would change the actuation strength (m/W as measured by receiver or transmitter PD), thus rendering a calibrated photon calibrator channel (e.g., `${ifo}:CAL-PCALY_RX_PD_OUT_DQ`) in accurate. *It is important to note that if using a calibrated channel for the photon calibrator*[1]*, then* $A_0^{(PC)}(f)$ *is simply the Pcal correction factor at frequency* f*. Here, we will continue to explicitly write* $A_0^{(PC)}(f)\widetilde{x}^{(PC)}(f)$ *for the calibrated displacement readout of the photon calibrator.* If not using a calibrated photon calibrator readout channel, then the full calibration of the digital signal will need to be applied (m/ct).

2.3 Calculating the relative actuation strengths

For each stage, we calculate the time-dependent actuation strength κ_i by exciting the ith actuation stage ($i \in \{U, P, T\}$) at a single, sinusoidal frequency f_i, and using the reference actuator, the photon calibrator, to excite at a nearby frequency f. The amplitude of the lines are tracked in the interferometer's DARM readout signal `${ifo}:CAL-DARM_ERR_WHITEN_OUT_DQ`.

The excitations, $x_i^{(SUS)}(f_i)$ and $x^{(PC)}(f)$, are generated in both the SUS actuator system and PCAL actuator system, respectively, with synchronized oscillators. These actuations are driven by calibration lines into the `DRIVE_ALIGN` bank, so the filtering is only what is seen downstream. However, the actuation paths between these excitations and the test mass displacement differ by their time-independent transfer functions, $A_0^i(f_i)$ and $A_0^{(PC)}(f)$, so we define them independently.

We also express the additive noise to the signal in the DARM error line at each sinusoidal frequency as $N(f_i)$. We assume this noise to be stationary (i.e. it does not vary much with

[1] e.g., `${ifo}:CAL-PCALY_RX_PD_OUT_DQ`

respect time) so we show it only as frequency-dependent.

$$\widetilde{d}_{err}(f_T) = \frac{C(f_T,t)}{1+G(f_T,t)} \; \kappa_T(t) \; F^T_{DA}(f_T)F^T_{OUT}(f_T)A^T_0(f_T) \; \widetilde{x}^{(SUS)}_T(f_T)+N(f_T) \quad (9a)$$

$$\widetilde{d}_{err}(f_P) = \frac{C(f_P,t)}{1+G(f_P,t)} \; \kappa_P(t) \; F^P_{DA}(f_P)F^P_{OUT}(f_P)A^P_0(f_P) \; \widetilde{x}^{(SUS)}_P(f_P)+N(f_P) \quad (9b)$$

$$\widetilde{d}_{err}(f_U) = \frac{C(f_U,t)}{1+G(f_U,t)} \; \kappa_U(t) \; F^U_{DA}(f_U)F^U_{OUT}(f_U)A^U_0(f_U) \; \widetilde{x}^{(SUS)}_U(f_U)+N(f_U) \quad (9c)$$

$$\widetilde{d}_{err}(f) = \frac{C(f,t)}{1+G(f,t)} \; A^{(PC)}_0(f) \; \widetilde{x}^{(PC)}(f)+N(f)\,. \quad (10)$$

2.4 Actuation strengths: separate lines for each stage

For any of the UIM, PUM, or TST stages ($i \in \{U,P,T\}$), the calculation proceeds similarly. The transfer function between the photon calibrator and the DARM readout signal is given by the ratio of Eqns. 9 and 10 and rearranging to solve for κ_i:

$$\kappa_i(t) = \frac{A^{(PC)}_0(f)}{A^i_0(f_i)F^i_{DA}(f_i)F^i_{OUT}(f_i)} \left(\frac{\widetilde{x}^{(PC)}(f)}{\widetilde{d}_{err}(f)-N(f)}\right)\left(\frac{\widetilde{d}_{err}(f_T)-N(f_T)}{\widetilde{x}^{(SUS)}_T(f_T)}\right) \frac{C_0(f)}{1+G_0(f)}\frac{1+G_0(f_T)}{C_0(f_T)} \quad (11a)$$

An assumption was made here that the frequencies are near enough to each other (∼1 Hz) and that the response function slope between the two lines doesn't change substantially with time.

2.5 Actuation strengths: special case DARM line and no UIM line

For the UIM stage ($i = U$), the DARM excitation could be used in conjunction with the PUM, TST, and Pcal lines in order to work out the value for κ_U. First, the DARM excitation can be calculated and written in terms of a transfer function between $\widetilde{x}_d$ and $\widetilde{d}_{err}$

$$\widetilde{d}_{err} = \frac{-A(f_d,t)C(f_d,t)}{1+G(f_d,t)}\widetilde{x}_d + N(f_d) \quad (12)$$

Then, expanding $A(f_d,t)$ and utilizing the Pcal transfer function, we can write this as

$$\frac{\widetilde{d}_{err}-N(f_d)}{\widetilde{x}_d} = \frac{-C(f_d,t)}{1+G(f_d,t)}\left[\kappa_T(t)F^T(f_d)A^T_0(f_d)+\kappa_P(t)F^P(f_d)A^P_0(f_d)+ \kappa_U(t)F^U(f_d)A^U_0(f_d)\right] \quad (13)$$

$$\kappa_U(t) = \frac{-A^{(PC)}_0(f)}{F^U(f_d)A^U_0(f_d)}\left[\frac{1+G_0(f_d)}{C_0(f_d)}\frac{C_0(f)}{1+G_0(f)}\frac{\widetilde{d}_{err}(f_d)-N(f_d)}{\widetilde{x}_d}\frac{\widetilde{x}^{(PC)}}{\widetilde{d}_{err}(f)-N(f)} +\kappa_T(t)F^T(f_d)A^T_0(f_d)+\kappa_P(t)F^P(f_d)A^P_0(f_d)\right] \quad (14)$$

2.6 Calculating the Sensing Function Parameters

After calculating all time-dependent, relative actuation strengths, $\kappa_i(t)$, we have a complete time-dependent description of $A(t) = \kappa_T(t) F^T A_0^T + \kappa_P(t) F^P A_0^P + \kappa_U(t) F^U A_0^U$. This means we can use photon calibrator excitations at other frequencies, $f_k \in \{f_1,\ f_2\}$ (sufficiently far away in frequency from f to minimize covariance) to characterize the sensing function parameters.

Starting from a re-arrangement of Eq. 10, we can solve for the time dependent sensing function $C(t)$ in terms of the transfer function $\widetilde{d}_{err}(f_k)/\widetilde{x}_k^{(PC)}(f_k)$ and known quantities,

$$\begin{aligned}
\widetilde{d}_{err}(f_k) &= \frac{C(f_k,t)}{1+G(f_k,t)}\ A_0^{(PC)}(f_k)\ \ \widetilde{x}_k^{(PC)}(f_k) + N(f_k) \\
C(f_k,t) &= \frac{1}{A_0^{(PC)}(f_k)}\left(1+G(f_k,t)\right)\left(\frac{\widetilde{d}_{err}(f_k) - N(f_k)}{\widetilde{x}_k^{(PC)}(f_k)}\right) \\
C(f_k,t) &= \frac{1}{A_0^{(PC)}(f_k)}\left(\frac{\widetilde{d}_{err}(f_k) - N(f_k)}{\widetilde{x}_k^{(PC)}(f_k)}\right) + \frac{C(f_k,t)D_0(f_k)A(f_k,t)}{A_0^{(PC)}(f_k)}\left(\frac{\widetilde{d}_{err}(f_k) - N(f_k)}{\widetilde{x}_k^{(PC)}(f_k)}\right) \\
&= \frac{1}{A_0^{(PC)}(f_k)}\left(\frac{\widetilde{d}_{err}(f_k) - N(f_k)}{\widetilde{x}_k^{(PC)}(f_k)}\right)\left[1 - \frac{D_0 A(f_k,t)}{A_0^{(PC)}(f_k)}\left(\frac{\widetilde{d}_{err}(f_k) - N(f_k)}{\widetilde{x}_k^{(PC)}(f_k)}\right)\right]^{-1} \\
C(f_k,t) &= \left[A_0^{(PC)}(f_k)\left(\frac{\widetilde{d}_{err}(f_k) - N(f_k)}{\widetilde{x}_k^{(PC)}(f_k)}\right)^{-1} - D_0 A(f_k,t)\right]^{-1} \qquad (15)
\end{aligned}$$

and using Eq. 6, we can single out only the time dependent correction factors,

$$\left(\frac{\kappa_C(t)}{1+if_k/f_c(t)}\right)\left(\frac{f_k^2}{f_k^2 - if_k f_s(t)/Q(t) + f_s^2(t)}\right) =$$

$$\left[C_{res}(f_k)A_0^{(PC)}(f_k)\left(\frac{\widetilde{d}_{err}(f_k) - N(f_k)}{\widetilde{x}_k^{(PC)}(f_k)}\right)^{-1} - C_{res}(f_k)D_0 A(f_k,t)\right]^{-1} \equiv S(f_k,t) \qquad (16)$$

Given our two sensing function excitation frequencies, we now have two complex measurements, each with a scalar real and imaginary part, $\Re(S(f_k,t))$ and $\Im(S(f_k,t))$, and four scalar unknowns, $\kappa_C(t)$,$f_c(t)$, $f_s(t)$, and $Q(t)$,

$$\Re(S(f_1,t)) + i\ \Im(S(f_1,t)) = \left(\frac{\kappa_C(t)}{1+if_1/f_c(t)}\right)\left(\frac{f_1^2}{f_1^2 - if_1 f_s(t)/Q(t) + f_s^2(t)}\right) \qquad (17)$$

$$\Re(S(f_2,t)) + i\ \Im(S(f_2,t)) = \left(\frac{\kappa_C(t)}{1+if_2/f_c(t)}\right)\left(\frac{f_2^2}{f_2^2 - if_2 f_s(t)/Q(t) + f_s^2(t)}\right) \qquad (18)$$

Rather than solve this full system of equations for a complete expression for $\kappa_C(t)$, $f_c(t)$, $f_s(t)$, and $Q(t)$, in terms of $\Re(S(f_k,t))$ and $\Im(S(f_k,t))$, we use our knowledge that for small negative detuning, as in the lowish-power aLIGO IFOs [10, 11, 12, 13], $f_s \approx 10$ Hz and $Q \approx 20$ and insist that $f_1 \gg f_2$, with $f_2 \approx 10$ Hz such that Eq. 17 becomes,

$$\Re(S(f_1,t)) + i\,\Im(S(f_1,t)) \approx \left(\frac{\kappa_C(t)}{1 + if_1/f_c(t)}\right) \equiv S_c(f_1,t)$$

which yields much more simple expressions for the relative optical gain and cavity pole frequency,

$$\kappa_C(t) = \frac{|S_c(f_1,t)|^2}{\Re(S_c(f_1,t))} \quad (19)$$

$$(20)$$

$$f_c(t) = -f_1\,\frac{\Re(S_c(f_1,t))}{\Im(S_c(f_1,t))} \quad (21)$$

$$(22)$$

which are identical to that of [1].

Once we obtain the optical gain and cavity pole frequency, we can plug these back into 18,

$$S_s(f_2,t) \equiv \frac{S(f_2,t)}{S_c(f_2,t)} = S(f_2,t)\left(\frac{1 + if_2/f_c(t)}{\kappa_C(t)}\right) = \left(\frac{f_2^2}{f_2^2 - if_2 f_s(t)/Q(t) + f_s^2(t)}\right) \quad (23)$$

and solve for the remaining parameters,

$$f_s(t) = f_2\,\frac{\sqrt{\Re[S_s(f_2,t)] - |S_s(f_2,t)|^2}}{\sqrt{|S_s(f_2,t)|^2}} \quad (24)$$

$$(25)$$

$$Q(t) = \frac{\sqrt{|S_s(f_2,t)|^2}}{\Im[S_s(f_2,t)]}\,\sqrt{\Re[S_s(f_2,t)] - |S_s(f_2,t)|^2}\,. \quad (26)$$

These formulae can be further simplified as

$$f_s(t) = f_2\left[\frac{\Re[S_s(f_2,t)]}{|S_s(f_2,t)|^2} - 1\right]^{1/2} \quad (27)$$

$$(28)$$

$$Q(t) = \frac{|S_s(f_2,t)|^2}{\Im(S_s(f_2,t))}\left[\frac{\Re(S_s(f_2,t))}{|S_s(f_2,t)|^2} - 1\right]^{1/2}. \quad (29)$$

Here, the math works out simply that $\Re[S_s(f_2,t)]/|S_s(f_2,t)|^2 - 1 = f_s^2/f_2^2$. Note the definition of the complex, frequency dependent correction factors, $S_c(f_1,t)$ and $S_s(f_2,t)$ which will be used later in Section 4 to represent the implementation of frequency-dependent, time-dependent correction factors calibrated output stream, $h(t)$.

3 Proposed Implementation in CAL-CS and GDS

Here we will describe the utilization of EPICS records that, together with interferometer data, are used to calculate the time-dependent correction factors parameters derived in

Sec. 2. The relevant time-independent functions from Sec. 2 are translated to include the EPICS records that are pre-calculated from the reference model. EPICS records cannot be complex; though they are abbreviated below as `EPn` to simplify the notation in equation form, they are actually installed as `EPn` = `EPn_R` + i `EPn_I`. The convention for defining our `EPn` coefficients are for the purpose of backwards compatibility with EPICS records.

For each of the time-dependent actuation coefficients, $\kappa_i(t)$, we'll need a record for $A_0^i(f_i)$, such that,

$$\kappa_T = \widetilde{\texttt{EP1}} \; \frac{\widetilde{d}_{err}(f_T) - N(f_T)}{\widetilde{x}_T^{(SUS)}(f_T)} \left(\frac{1}{\widetilde{\texttt{EP0}}} \frac{\widetilde{d}_{err}(f) - N(f)}{\widetilde{x}^{(PC)}(f)} \right)^{-1} \tag{30}$$

$$\kappa_P = \widetilde{\texttt{EP2}} \; \frac{\widetilde{d}_{err}(f_P) - N(f_P)}{\widetilde{x}_P^{(SUS)}(f_P)} \left(\frac{1}{\widetilde{\texttt{EP0}}} \frac{\widetilde{d}_{err}(f) - N(f)}{\widetilde{x}^{(PC)}(f)} \right)^{-1} \tag{31}$$

$$\kappa_U = \widetilde{\texttt{EP3}} \; \frac{\widetilde{d}_{err}(f_U) - N(f_U)}{\widetilde{x}_U^{(SUS)}(f_U)} \left(\frac{1}{\widetilde{\texttt{EP0}}} \frac{\widetilde{d}_{err}(f) - N(f)}{\widetilde{x}^{(PC)}(f)} \right)^{-1} \tag{32}$$

where

$$\widetilde{\texttt{EP0}} = A_0^{(PC)}(f) \tag{33}$$

$$\widetilde{\texttt{EP1}} = \frac{1}{F_{DA}^T(f_T) F_{OUT}^T(f_T) A_0^T(f_T)} \frac{C_0(f)}{1+G_0(f)} \frac{1+G_0(f_T)}{C_0(f_T)} \tag{34}$$

$$\widetilde{\texttt{EP2}} = \frac{1}{F_{DA}^P(f_P) F_{OUT}^P(f_P) A_0^P(f_P)} \frac{C_0(f)}{1+G_0(f)} \frac{1+G_0(f_P)}{C_0(f_P)} \tag{35}$$

$$\widetilde{\texttt{EP3}} = \frac{1}{F_{DA}^U(f_U) F_{OUT}^U(f_U) A_0^U(f_U)} \frac{C_0(f)}{1+G_0(f)} \frac{1+G_0(f_U)}{C_0(f_U)} \tag{36}$$

Moving on to the sensing function,

$$S(f_1, t) = \left[\widetilde{\texttt{EP4}} \left(\frac{1}{\widetilde{\texttt{EP9}}} \frac{\widetilde{d}_{err}(f_1) - N(f_1)}{\widetilde{x}_2^{(PC)}(f_1)} \right)^{-1} - \widetilde{\texttt{EP4}}\; \widetilde{\texttt{EP5}} \left(\kappa_T \widetilde{\texttt{EP6}} + \kappa_P \widetilde{\texttt{EP7}} + \kappa_U \widetilde{\texttt{EP8}} \right) \right]^{-1} \tag{37}$$

with

$$\widetilde{\texttt{EP4}} = C_{res}(f_1) \tag{38}$$

$$\widetilde{\texttt{EP5}} = D_0(f_1) \tag{39}$$

$$\widetilde{\texttt{EP6}} = F^T(f_1) A_0^T(f_1) \tag{40}$$

$$\widetilde{\texttt{EP7}} = F^P(f_1) A_0^P(f_1) \tag{41}$$

$$\widetilde{\texttt{EP8}} = F^U(f_1) A_0^U(f_1) \tag{42}$$

$$\widetilde{\texttt{EP9}} = A_0^{(PC)}(f_1) \tag{43}$$

and

$$S(f_2, t) = \left[\widetilde{\texttt{EP10}} \left(\frac{1}{\widetilde{\texttt{EP15}}} \frac{\widetilde{d}_{err}(f_2) - N(f_2)}{\widetilde{x}_2^{(PC)}(f_2)} \right)^{-1} - \widetilde{\texttt{EP10}}\; \widetilde{\texttt{EP11}} \left(\kappa_T \widetilde{\texttt{EP12}} + \kappa_P \widetilde{\texttt{EP13}} + \kappa_U \widetilde{\texttt{EP14}} \right) \right]^{-1} \tag{44}$$

EP#	Value	Suggested Channel Names	Purpose
EP0	$A_0^{(PC)}(f)$	CAL-CS_TDEP_PCAL_LINE1_CORRECTION	compute κ_T, κ_P, κ_U
EP1	$\frac{1}{F_{DA}^T(f_T)F_{OUT}^T(f_T)A_0^T(f_T)}\frac{C_0(f)}{1+G_0(f)}\frac{1+G_0(f_T)}{C_0(f_T)}$	CAL-CS_TDEP_SUS_LINE3_REF_INVA_TST_RESPRATIO	compute κ_T
EP2	$\frac{1}{F_{DA}^P(f_P)F_{OUT}^P(f_P)A_0^P(f_P)}\frac{C_0(f)}{1+G_0(f)}\frac{1+G_0(f_P)}{C_0(f_P)}$	CAL-CS_TDEP_SUS_LINE2_REF_INVA_PUM_RESPRATIO	compute κ_P
EP3	$\frac{1}{F_{DA}^U(f_U)F_{OUT}^U(f_U)A_0^U(f_U)}\frac{C_0(f)}{1+G_0(f)}\frac{1+G_0(f_U)}{C_0(f_U)}$	CAL-CS_TDEP_SUS_LINE1_REF_INVA_UIM_RESPRATIO	compute κ_U
EP4	$C_{res}(f_1)$	CAL-CS_TDEP_PCAL_LINE2_REF_C_NOCAVPOLE	compute κ_C, f_{cc}
EP5	$D_0(f_1)$	CAL-CS_TDEP_PCAL_LINE2_REF_D	compute κ_C, f_{cc}
EP6	$F^T(f_1)A_0^T(f_1)$	CAL-CS_TDEP_PCAL_LINE2_REF_A_TST	compute κ_C, f_{cc}
EP7	$F^P(f_1)A_0^P(f_1)$	CAL-CS_TDEP_PCAL_LINE2_REF_A_PUM	compute κ_C, f_{cc}
EP8	$F^U(f_1)A_0^U(f_1)$	CAL-CS_TDEP_PCAL_LINE2_REF_A_UIM	compute κ_C, f_{cc}
EP9	$A_0^{(PC)}(f_1)$	CAL-CS_TDEP_PCAL_LINE2_CORRECTION	compute κ_C, f_{cc}
EP10	$C_{res}(f_2)$	CAL-CS_TDEP_PCAL_LINE4_REF_C_NOCAVPOLE	compute f_s, Q
EP11	$D_0(f_2)$	CAL-CS_TDEP_PCAL_LINE4_REF_D	compute f_s, Q
EP12	$F^T(f_2)A_0^T(f_2)$	CAL-CS_TDEP_PCAL_LINE4_REF_A_TST	compute f_s, Q
EP13	$F^P(f_2)A_0^P(f_2)$	CAL-CS_TDEP_PCAL_LINE4_REF_A_PUM	compute f_s, Q
EP14	$F^U(f_2)A_0^U(f_2)$	CAL-CS_TDEP_PCAL_LINE4_REF_A_UIM	compute f_s, Q
EP15	$A_0^{(PC)}(f_2)$	CAL-CS_TDEP_PCAL_LINE4_CORRECTION	compute f_s, Q
EP16	$F_{DA}^U(f_U)F_{OUT}^U(f_U)A_0^U(f_U)$	CAL-CS_TDEP_SUS_LINE1_REF_A_UIM_NOLOCK	remove $x_P^{(SUS)}(f_P)$
EP17	$F_{DA}^P(f_P)F_{OUT}^P(f_P)A_0^P(f_P)$	CAL-CS_TDEP_SUS_LINE2_REF_A_PUM_NOLOCK	remove $x_P^{(SUS)}(f_P)$
EP18	$F_{DA}^T(f_T)F_{OUT}^T(f_T)A_0^T(f_T)$	CAL-CS_TDEP_SUS_LINE3_REF_A_TST_NOLOCK	remove $x_U^{(SUS)}(f_U)$

Table 1: EPICS records needed for O3 calibration. Note: All channel names must end in _REAL or _IMAG.

with

$$\widetilde{\texttt{EP10}} = C_{res}(f_2) \tag{45}$$
$$\widetilde{\texttt{EP11}} = D_0(f_2) \tag{46}$$
$$\widetilde{\texttt{EP12}} = F^T(f_2)A_0^T(f_2) \tag{47}$$
$$\widetilde{\texttt{EP13}} = F^P(f_2)A_0^P(f_2) \tag{48}$$
$$\widetilde{\texttt{EP14}} = F^U(f_2)A_0^U(f_2) \tag{49}$$
$$\widetilde{\texttt{EP15}} = A_0^{(PC)}(f_2) \tag{50}$$

Three other EPICS records are saved to allow for line subtraction:

$$\widetilde{\texttt{EP16}} = F_{DA}^U(f_U)F_{OUT}^U(f_U)A_0^U(f_U) \tag{51}$$
$$\widetilde{\texttt{EP17}} = F_{DA}^P(f_P)F_{OUT}^P(f_P)A_0^P(f_P) \tag{52}$$
$$\widetilde{\texttt{EP18}} = F_{DA}^T(f_T)F_{OUT}^T(f_T)A_0^T(f_T) \tag{53}$$

A complete list of EPICS records needed by the calibration pipelines during O3 is seen in table 1.

4 Application to $h(t)$

The strain output, $h(t)$, of the calibration pipeline – if time-dependent correction factors to the model are included – can be expressed as

$$h(t) = \frac{1}{S_c(f,t)\ S_s(f,t) C_{res}(f)}\ d_{err}(t) + \left[\kappa_T\ F^T(f) A_0^T(f) + \kappa_P\ F^P(f) A_0^P(f) + \kappa_u\ F^U(f) A_0^U(f)\right]\ d_{ctrl}(t) \tag{54}$$

where, as defined in section 2.6,

$$S_c(f,t) = \left(\frac{\kappa_C(t)}{1 + if/f_c(t)}\right)$$

$$S_s(f,t) = \left(\frac{f^2}{f^2 - iff_s(t)/Q(t) + f_s^2(t)}\right)$$

Ifo.	#	Freq. (Hz)	Sym.	Type	Target SNR[2]	Purpose
H1	1	7.93	f_2	PC Only	10	$f_s,\ Q$
H1	2	(tbd ≈ 30 Hz)	f_T	PC and TST SUS	10	κ_T
H1	3	(tbd ≈ 20 Hz)	f_P	PC and PUM SUS	10	κ_P
H1	4	(tbd ≈ 10 Hz)	f_U	PC and UIM SUS	10	κ_U
H1	5	331.9	f_1	Pcal	10	f_{cc}, κ_C
H1	6	1083.7	f_3	Pcal	10	check

Table 2: Calibration lines for both the L1 and the H1 interferometers.

5 Calibration lines

Six calibration lines are injected for calculation and tracking of time-dependent parameters. Their frequencies, target signal-to-noise ratios (SNRs), injection points, and purpose are detailed in Table 2. Lines for both H1 and L1 are shown, and those lines which characterize the actuation strength have frequencies yet to be determined.

6 Signal-to-Noise Ratio and Ratios between Actuation Coefficients - work in progress

In this section, we take one of the calibration lines, assume its signal-to-noise ratio to be at least 100, and demonstrate how to calculate the contribution of each of the actuation stages to the overall signal-to-noise ratio at that frequency. Here, we choose to take measurements of the 16.5 Hz DARM error line, where this assumption holds true. We report the values of the actuation and filter coefficients in the table below.

Quantity	TST	PUM	UIM
A_0	$1.355x10^{-15}$	$1.9x10^{-15}$	$2.7x10^{-18}$
F	1	0.9283	1.989

Table 3: Calculated actuation and filter coefficients from measurements of the DARM line excitation at 16.5 Hz.

We can express the equation for $\widetilde{d}_{err}$ as a combination of signal and noise, where $\widetilde{S}(f)$ is the signal component, and *N(f)* is the noise component. In order to calculate the contribution to the overall signal-to-noise ratio from each of the actuation stages, we assume that the

[2] using 10-sec.-long FFTs

ratio $\widetilde{S}(f)/N(f) = 100$.

$$\widetilde{d}_{err}(f) = \widetilde{S}(f) + N(f) \tag{55a}$$

$$\widetilde{S}_i^{(SUS)}(f_i) \equiv \frac{C(f_i,t)}{1+G(f_i,t)} \; \kappa_i(t) F^i(f_i) A_0^i \; \widetilde{x}_i^{(SUS)}(f_i) \tag{55b}$$

We give a more general expression for $\widetilde{S}(f)$ based on Eqn. 14. However, our assumptions about the signal-to-noise ratio hold true for each of the actuation stages individually, and for the combined DARM error equation. As we will be performing the excitation at the same frequency of 16.5 Hz for all of the actuation stages and for the Pcal DARM error line, we drop the subscript for the frequencies.

Likewise, for the Pcal excitation, we have

$$\widetilde{S}^{(PC)}(f) = \frac{C(f,t)}{1+G(f,t)} A_0^{(PC)}(f) \;\; \widetilde{x}^{(PC)}(f) \tag{56}$$

Now we write Eqn. 11a in terms of their corresponding signal terms. Note here that all of the noise terms drop out of the κ_i equations because of the way we defined the quantity $\frac{\widetilde{d}_{err} - N(f_d)}{\widetilde{x}_d}$ in Eqn. 14.

$$\kappa_T(t) = \frac{A_0^{(PC)}(f)}{A_0^T(f) F_T(f)} \; \frac{\widetilde{x}^{(PC)}(f)}{\widetilde{x}_d(f)} \frac{\widetilde{S}(f)}{\widetilde{S}^{(PC)}(f)} \; \frac{C_0(f)}{1+G_0(f)} \frac{1+G_0(f)}{C_0(f)} \tag{57a}$$

$$\kappa_P(t) = \frac{A_0^{(PC)}(f)}{A_0^P(f) F_T(f) F_P(f)} \; \frac{\widetilde{x}^{(PC)}(f)}{\widetilde{x}_d(f)} \frac{\widetilde{S}(f)}{\widetilde{S}^{(PC)}(f)} \; \frac{C_0(f)}{1+G_0(f)} \frac{1+G_0(f)}{C_0(f)} \tag{57b}$$

(57c)

We make the simplifying assumptions that $\frac{\widetilde{S}(f)}{\widetilde{S}^{(PC)}} \approx 1$, and $\frac{\widetilde{x}^{(PC)}(f)}{\widetilde{x}_d(f)} \approx 1$ because we can treat the signal-to-noise ratios and excitation strengths of the DARM and Pcal lines as being the same at the same frequency, under our earlier assumptions. Thereby we can reduce the expressions for κ_T and κ_P to a combination of the actuation and filter function terms,

$$\kappa_T(t) \approx \frac{A_0^{(PC)}(f)}{A_0^T(f) F_T(f)} \tag{58a}$$

$$\kappa_P(t) \approx \frac{A_0^{(PC)}(f)}{A_0^P(f) F_T(f) F_P(f)} \tag{58b}$$

Using the measurements given in Table 3, we calculate $\kappa_T \approx 7.38x10^{14} A_0^{PC}(f)$ and $\kappa_P \approx 5.67x10^{14} A_0^{(PC)}(f)$. We will refer back to these when calculating the relative strengths of the κ_i terms.

Now, using Eqn. 14 we can also express κ_U in terms of $A_0^{PC}(f)$, in a similar fashion, by substituting the reduced expressions for κ_T and κ_P from Eqn. 58.

$$\kappa_U(t) = -\frac{A_0^{(PC)}(f)}{A_0^U(f)F_T(f)F_P(f)F_U(f)} \frac{C_0(f)}{1+G_0(f)} \frac{1+G_0(f)}{C_0(f)} \frac{\widetilde{S}(f)}{\widetilde{S}^{(PC)}(f)} \frac{\widetilde{x}^{(PC)}}{\widetilde{x}_d} \quad (59)$$

$$-\kappa_T \frac{A_0^T(f)}{A_0^U(f)F_P(f)F_U(f)} - \kappa_P \frac{A_0^P(f)}{A_0^U(f)F_U(f)} \quad (60)$$

$$\kappa_U(t) = -\frac{A_0^{(PC)}(f)}{A_0^U(f)F_T(f)F_P(f)F_U(f)} - \left(\frac{A_0^{(PC)}(f)}{A_0^T(f)F_T(f)}\right) \frac{A_0^T(f)}{A_0^U(f)F_P(f)F_U(f)} \quad (61)$$

$$-\left(\frac{A_0^{(PC)}(f)}{A_0^P(f)F_T(f)F_P(f)}\right) \frac{A_0^P(f)}{A_0^U(f)F_U(f)} \quad (62)$$

$$\kappa_U(t) = -\frac{3A_0^{(PC)}(f)}{A_0^U(f)F_T(f)F_P(f)F_U(f)} \quad (63)$$

Thus, using Table 3, $\kappa_U = -6.02x10^{17} A_0^{(PC)}(f)$. Next we calculate the actuation function of the Pcal in terms of the actuation gains in different stages, and equate the different expressions. We drop the minus sign on κ_U as we are only concerned about the magnitude of the κ_i terms.

$$A_0^{(PC)}(f) \approx 1.355x10^{-15}\kappa_T = 1.763x10^{-15}\kappa_P = 1.661x10^{-18}\kappa_U \quad (64)$$

Then we can determine ratios between the actuation gains:

numerator	κ_T	κ_P	κ_U
κ_T	1	1.3	$1.225x10^{-3}$
κ_P	0.769	1	$9.421x10^{-4}$
κ_U	816	1061	1

Table 4: Ratios between actuation gains at different stages at a frequency of 16.5 Hz. The quantities displayed are in the format row/column.

We can use the information in Table 4 to calculate the contributions of each of the actuation stages to the overall DARM error signal-to-noise ratio. Instead using κ_i, we notate the actuation stage contributions with ρ_T ρ_P, and ρ_U. We break down our overall signal-to-noise ratio into components:

$$\frac{\widetilde{S}(f)}{N(f)} = [\rho_T + \rho_P + \rho_U] = 100 \quad (65)$$

We assume here that $\frac{C(f,t)}{1+G(f,t)}\widetilde{x}_d$ from Eqn. **??** does not contribute towards the overall signal-to-noise ratio of the DARM line. Combining Eqn. 65 and Table 4, we form a system of equations that we can use to solve for the individual ρ_i values:

$$\rho_T + \rho_P + \rho_U = 100 \quad (66a)$$
$$\rho_T = 1.3\rho_P \quad (66b)$$
$$\rho_U = 1061\rho_T \quad (66c)$$
$$\rho_U = 816\rho_T \quad (66d)$$

Solving this system, we find $\rho_T = 0.122$, $\rho_P = 0.094$, and $\rho_U = 99.784$. Therefore, we see that the UIM stage provides overwhelmingly the largest contribution towards the signal-to-noise ratio of the DARM line. The TST stage contributes a fractionally higher amount to the overall signal-to-noise ratio than the PUM stage, but both stages together contribute less than one percent of the total. We conclude that one can obtain a nearly accurate measurement of κ_U at the 16.5 Hz calibration line frequency. This is just one example using the 16.5 Hz calibration line, but similar calculations can be performed to determine the ρ_i of different actuation stages at another frequency, given that all actuation stages have excitations at the same frequency. Furthermore, we demonstrate that while adding in a noise term allows us to perform this calculation, the noise term itself does not enter into the calculation of the contribution of each actuation stage to the signal-to-noise ratio of the DARM line.

7 Calculating the Relative Actuation Strengths with cancellation

For each stage, we calculate the time-dependent actuation strength κ_i, by exciting the ith actuation stage at a single frequency, f_i, and using the reference actuator, the photon calibrator, to excite at the same frequency but in exactly opposite in phase cancelling the ith stage actuator line. The residual amplitude of the canceled line is tracked in the interferometer's DARM readout signal `${ifo}:CAL-DARM_ERR_WHITEN_OUT_DQ` .

The excitations, $x_i^{(SUS)}(f_i)$ and $x_i^{(PC)}(f_i)$, are generated in both the SUS actuator system and PCAL actuator system, respectively, with synchronized oscillators. However, the actuation paths between these excitations and the test mass displacement differ by their time-independent transfer functions, $A_0^i(f_i)$ and $A_0^{(PC)}$, so we define them independently,

$$\widetilde{d}_{err}(f_T) = \frac{C(f_T,t)}{1+G(f_T,t)} \kappa_T(t) \; A_0^T(f_T) F_T(f_T) \widetilde{x}_T^{(SUS)}(f_T) + N(f_T) \quad (67a)$$
$$\widetilde{d}_{err}(f_P) = \frac{C(f_P,t)}{1+G(f_P,t)} \kappa_P(t) \; A_0^P(f_P) F_T(f_P) F_P(f_P) \widetilde{x}_P^{(SUS)}(f_P) + N(f_P) \quad (67b)$$
$$\widetilde{d}_{err}(f_U) = \frac{C(f_U,t)}{1+G(f_U,t)} \kappa_U(t) \; A_0^U(f_U) F_T(f_U) F_P(f_U) F_U(f_U) \widetilde{x}_U^{(SUS)}(f_U) + N(f_U) \quad (67c)$$

$$\widetilde{d}_{err}{}^{(PC)}(f_i) = \frac{C(f_i,t)}{1+G(f_i,t)} \rho_i \; A_0^{(PC)}(f_i) \; \widetilde{x}_i^{(PC)}(f_i) + N(f_i) \quad (68)$$

We don't wish *perfect* cancellation of these two excitations, or we will not be able to resolve either excitation in the interferometer's error signal. This, we explicitly apply a small coefficient ρ_i to the PCAL excitation which defines the signal-to-noise ratio of the residual line in $\widetilde{d}_{err}$.

Using the common term, $C(f_i,t)/(1+G(f_i,t))$, we can equate Eq. 9 by Eq. 10 and rearrange terms to arrive at an expression for $\kappa_i(t)$ in terms of the complex, time-dependent, observable transfer functions $\widetilde{d}_{err}(f_i)/\widetilde{x}_i^{(SUS)}(f_i)$ and $\widetilde{d}_{err}(f_i)/\widetilde{x}^{(PC)}(f_i)$,

$$\kappa_T(t) = \rho_T \frac{A_0^{(PC)}(f_T)}{A_0^T(f_T)F_T(f_T)} \frac{\widetilde{d}_{err}(f_T)-N(f_T)}{\widetilde{x}_{(SUS)}(f_T)} \left(\frac{\widetilde{d}_{err}^{(PC)}(f_T)-N^{(PC)}(f_T)}{\widetilde{x}_T^{(PC)}(f_T)}\right)^{-1} \tag{69a}$$

$$\kappa_P(t) = \rho_P \frac{A_0^{(PC)}(f_P)}{A_0^P(f_P)F_T(f_P)F_P(f_P)} \frac{\widetilde{d}_{err}(f_P)-N(f_P)}{\widetilde{x}_{(SUS)}(f_P)} \left(\frac{\widetilde{d}_{err}^{(PC)}(f_P)-N^{(PC)}(f_P)}{\widetilde{x}_P^{(PC)}(f_P)}\right)^{-1} \tag{69b}$$

$$\kappa_U(t) = \rho_U \frac{A_0^{(PC)}(f_U)}{A_0^U(f_U)F_T(f_U)F_P(f_U)F_U(f_U)} \frac{\widetilde{d}_{err}(f_U)-N(f_U)}{\widetilde{x}^{(SUS)}(f_U)} \tag{69c}$$

$$\left(\frac{\widetilde{d}_{err}^{(PC)}(f_U)-N^{(PC)}(f_U)}{\widetilde{x}_U^{(PC)}(f_U)}\right)^{-1} \tag{69d}$$

In general, because the transfer functions $\widetilde{d}_{err}(f_i)/\widetilde{x}_{(SUS)}(f_i)$ and $\widetilde{d}_{err}(f_i)/\widetilde{x}_i^{(PC)}(f_i)$ are complex, so are the correction factors, κ_i. While we only expect the real part and/or magnitude to change as a function of time, we also track the imaginary part, looking for effects that may only affect the phase of the actuator, like timing inconsistencies in the SUS actuator's front-end computer.

A Alternative methods for calculating time dependent variables

A.1 Alternative κ_U computation

Alternatively, if no excitation $x_U^{(SUS)}(f_U)$ is injected into the UIM stage of actuation, κ_U can be computed using the DARM injection, κ_T, and κ_P, and the photon calibrator injection:

$$\kappa_U = -\widetilde{\mathrm{EP22}}\left[\widetilde{\mathrm{EP19}}\,\frac{\widetilde{d}_{err}(f_U)-N(f_U)}{\widetilde{x}_d(f_U)}\left(\frac{1}{\widetilde{\mathrm{EP0}}}\frac{\widetilde{d}_{err}(f)-N(f)}{\widetilde{x}^{(PC)}(f)}\right)^{-1}+\kappa_T\widetilde{\mathrm{EP20}}+\kappa_P\widetilde{\mathrm{EP21}}\right] \tag{70}$$

where

$$\widetilde{\text{EP19}} = \frac{1+G_0(f_d)}{C_0(f_d)}\frac{C_0(f)}{1+G_0(f)} \tag{71}$$
$$\widetilde{\text{EP20}} = F^T(f_d)A_0^T(f_d) \tag{72}$$
$$\widetilde{\text{EP21}} = F^P(f_d)A_0^P(f_d) \tag{73}$$
$$\widetilde{\text{EP22}} = \frac{1}{F^U(f_d)A_0^U(f_d)} \tag{74}$$

A.2 Alternative κ_{PU} computation and impact on sensing function computation

If no excitation to $x_U^{(SUS)}(f_U)$ or $x_P^{(SUS)}(f_P)$ is injected into the UIM and PUM stages of actuation, κ_{PU} can be computed using the DARM injection, κ_T, and the photon calibrator injection:

$$\kappa_{PU} = -\widetilde{\text{EP23}}\left[\widetilde{\text{EP19}}\,\frac{\tilde{d}_{err}(f_U)-N(f_U)}{\tilde{x}_d(f_U)}\left(\frac{1}{\widetilde{\text{EP0}}}\frac{\tilde{d}_{err}(f)-N(f)}{\tilde{x}^{(PC)}(f)}\right)^{-1}+\kappa_T\widetilde{\text{EP20}}\right] \tag{75}$$

where

$$\widetilde{\text{EP23}} = \frac{1}{F^P(f_d)A_0^P(f_d)+F^U(f_d)A_0^U(f_d)} \tag{76}$$

Since κ_{PU} is fundamentally different than separately calculating κ_P and κ_U, this changes the calculation using EPICS records for the sensing function parameters:

$$S(f_1,t) = \left[\widetilde{\text{EP4}}\left(\frac{1}{\widetilde{\text{EP9}}}\frac{\tilde{d}_{err}(f_1)-N(f_1)}{\tilde{x}_2^{(PC)}(f_1)}\right)^{-1}-\widetilde{\text{EP4}}\;\widetilde{\text{EP5}}\left(\kappa_T\widetilde{\text{EP6}}+\kappa_{PU}\left(\widetilde{\text{EP7}}+\widetilde{\text{EP8}}\right)\right)\right]^{-1} \tag{77}$$
$$S(f_2,t) = \left[\widetilde{\text{EP10}}\left(\frac{1}{\widetilde{\text{EP15}}}\frac{\tilde{d}_{err}(f_2)-N(f_2)}{\tilde{x}_2^{(PC)}(f_2)}\right)^{-1}-\widetilde{\text{EP10}}\;\widetilde{\text{EP11}}\left(\kappa_T\widetilde{\text{EP12}}+\kappa_{PU}\left(\widetilde{\text{EP13}}+\widetilde{\text{EP14}}\right)\right)\right]^{-1} \tag{78}$$

A.3 Alternative κ_A computation and impact on sensing function computation

If no excitation to $x_U^{(SUS)}(f_U)$, $x_P^{(SUS)}(f_P)$, or $x_T^{(SUS)}(f_T)$ is injected into the UIM, PUM, and TST stages of actuation, κ_A can be computed using the DARM injection and the photon calibrator injection:

$$\kappa_A = -\widetilde{\text{EP24}}\left[\widetilde{\text{EP19}}\,\frac{\tilde{d}_{err}(f_U)-N(f_U)}{\tilde{x}_d(f_U)}\left(\frac{1}{\widetilde{\text{EP0}}}\frac{\tilde{d}_{err}(f)-N(f)}{\tilde{x}^{(PC)}(f)}\right)^{-1}\right] \tag{79}$$

where

$$\widetilde{\text{EP24}} = \frac{1}{F^T(f_d)A_0^T(f_d)+F^P(f_d)A_0^P(f_d)+F^U(f_d)A_0^U(f_d)} \tag{80}$$

EP#	Value	Suggested Channel Names	Purpose
EP19	$\frac{1+G_0(f_d)}{C_0(f_d)}\frac{C_0(f)}{1+G_0(f)}$	CAL-CS_TDEP_REF_CLGRATIO_CTRL	compute κ_U
EP20	$F^T(f_d)A_0^T(f_d)$	CAL-CS_TDEP_DARM_LINE1_REF_A_TST	compute κ_U
EP21	$F^P(f_d)A_0^P(f_d)$	CAL-CS_TDEP_DARM_LINE1_REF_A_PUM	compute κ_U
EP22	$\frac{1}{F^U(f_d)A_0^U(f_d)}$	CAL-CS_TDEP_DARM_LINE1_REF_A_UIM_INV	compute κ_U
EP23	$\frac{1}{F^P(f_d)A_0^P(f_d)+F^U(f_d)A_0^U(f_d)}$	CAL-CS_TDEP_DARM_LINE1_REF_A_USUM_INV	compute κ_{PU}
EP24	$\frac{1}{F^T(f_d)A_0^T(f_d)+F^P(f_d)A_0^P(f_d)+F^U(f_d)A_0^U(f_d)}$	CAL-CS_TDEP_DARM_LINE1_REF_A_USUM_INV	compute κ_A

Table 5: EPICS records needed for O1/O2 calibration. Note: All channel names must end in _REAL or _IMAG.

Since κ_A is fundamentally different than separately calculating κ_T, κ_P, and κ_U (or computing κ_{PU}), this changes the calculation using EPICS records for the sensing function parameters:

$$S(f_1,t) = \left[\widetilde{\mathsf{EP4}}\left(\frac{1}{\widetilde{\mathsf{EP9}}}\frac{\widetilde{d}_{err}(f_1)-N(f_1)}{\widetilde{x}_2^{(PC)}(f_1)}\right)^{-1} - \widetilde{\mathsf{EP4}}\;\widetilde{\mathsf{EP5}}\left(\kappa_A\left(\widetilde{\mathsf{EP6}}+\widetilde{\mathsf{EP7}}+\widetilde{\mathsf{EP8}}\right)\right)\right]^{-1} \tag{81}$$

$$S(f_2,t) = \left[\widetilde{\mathsf{EP10}}\left(\frac{1}{\widetilde{\mathsf{EP15}}}\frac{\widetilde{d}_{err}(f_2)-N(f_2)}{\widetilde{x}_2^{(PC)}(f_2)}\right)^{-1} - \widetilde{\mathsf{EP10}}\;\widetilde{\mathsf{EP11}}\left(\kappa_A\left(\widetilde{\mathsf{EP12}}+\widetilde{\mathsf{EP13}}+\widetilde{\mathsf{EP14}}\right)\right)\right]^{-1} \tag{82}$$

References

[1] LIGO-T1500377: Tracking temporal variations in the DARM calibration parameters.

[2] LIGO-P1600063: Improving LIGO calibration accuracy by tracking and compensating for slow temporal variations.

[3] LIGO-T1600278: DARM (differential arm) response with a small detuning.

[4] LIGO-G1601599: Calibrating optical springs.

[5] LIGO-T1400256: Time Domain Calibration in Advanced LIGO.

[6] LIGO-T1500121: aLIGO Front-end Optical Gain Compensation, or $\gamma(t)$.

[7] LIGO-T1500422: Compensating for time variations in DARM actuation.

[8] LHO aLog entry #20361 by J. Kissel, Aug. 9, 2015.

[9] LIGO-T1501014: Pcal signal chain topology.

[10] LIGO-G1601599: Calibrating optical springs.

[11] LIGO-T1600278: DARM (differential arm) response with a small detuning

[12] LLO aLOG entry #32495

[13] LHO aLOG entry #33004