Frequency Domain Binary Black Hole Gravitational Waveforms with Higher Multipoles

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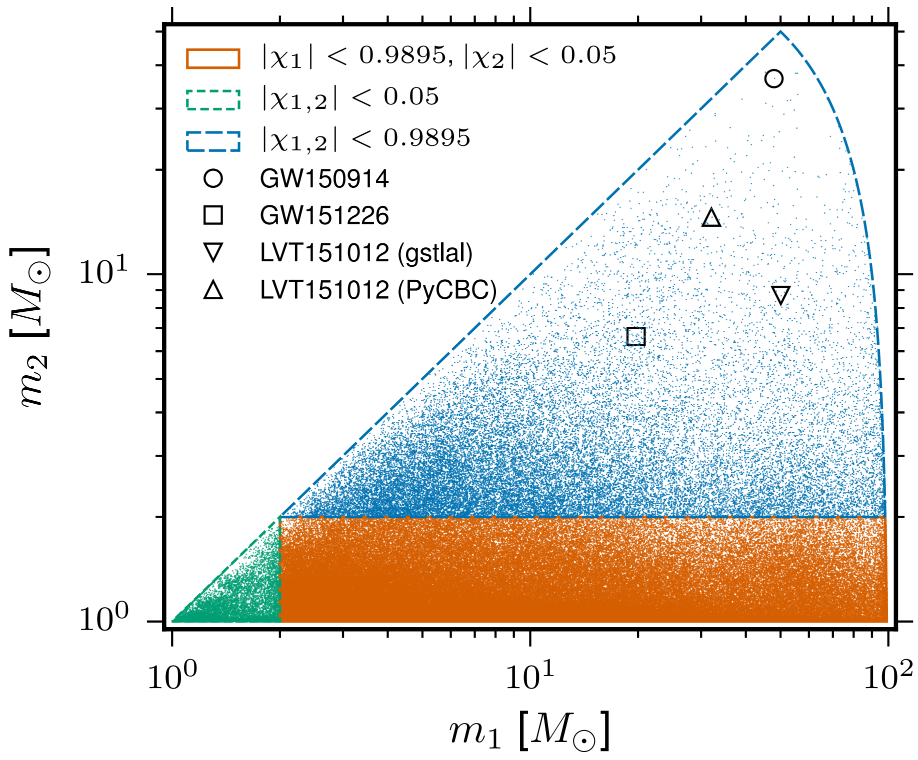
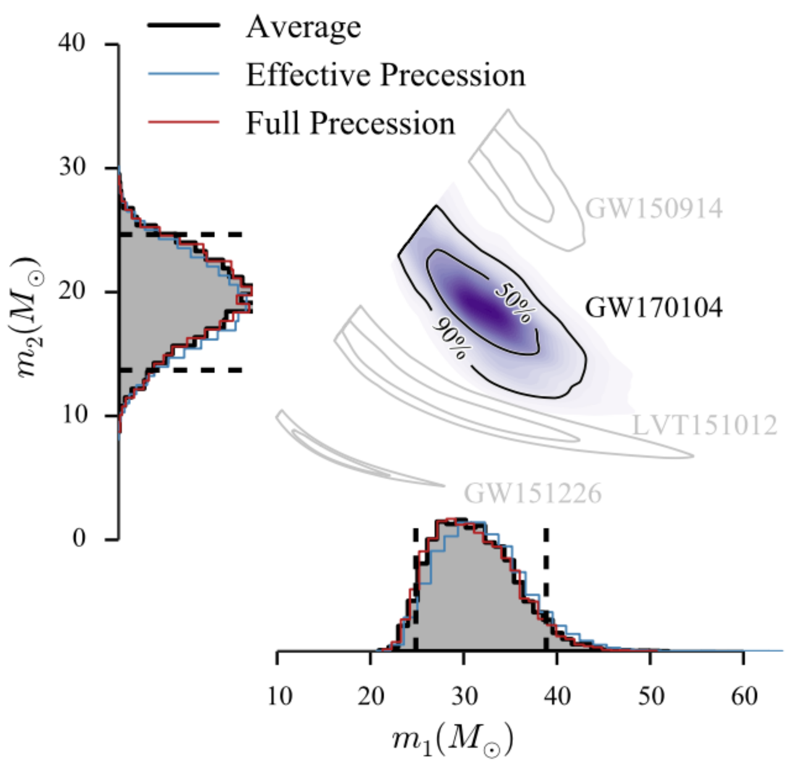
The Need for Gravitational Waveform Models



[Abbott *et al.*, PRX 6, 041015 (2016)]



[Abbott *et al.*, PRL 118, 221101 (2016)]

The Need for Higher-Multipole Gravitational Waveform Models

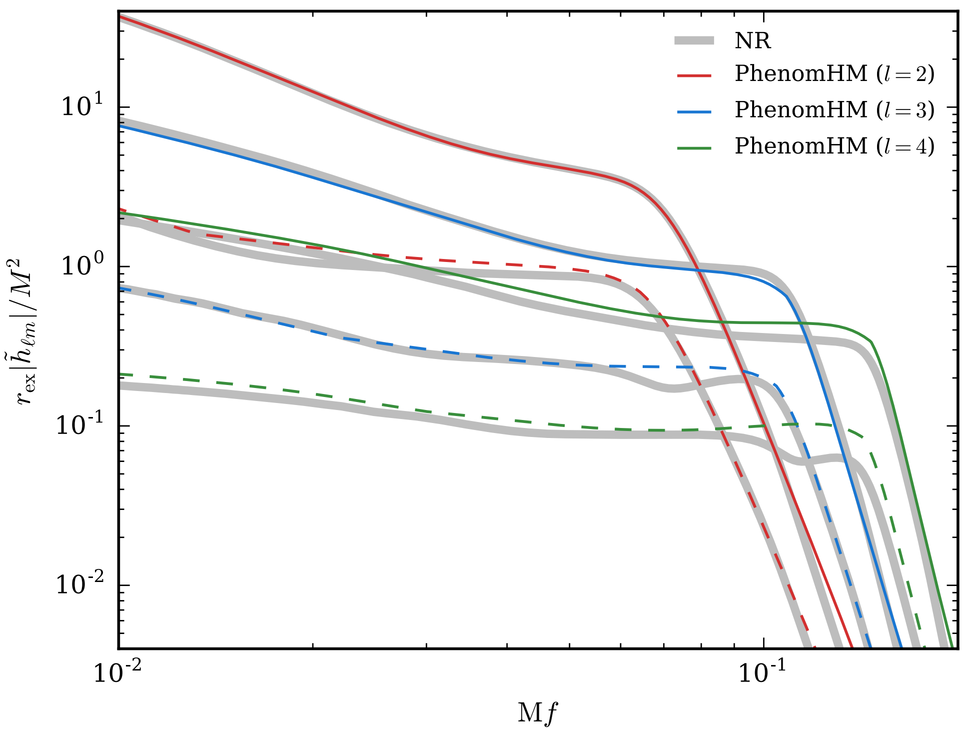
- Currently models include only the dominant multipoles (l=2, |m|=2) of the signal
- Modelling subdominant multipoles can:
  - increase detectable volume (for $m_1/m_2 \gtrsim 4$)
  - improve measurement accuracy
  - avoid large biases in measurements

[Bustillo *et al.*, PRD D93, 084019 (2016)]
[Bustillo *et al.*, PRD 95, 104038 (2017)]
[Capano *et al.*, PRD 89, 102003 (2014)]
[Varma *et al.*, arXiv:1612.05608]

- Current models that include higher multipoles:
  - do not apply to spinning binary black hole (BBH) systems [Pan *et al.*, PRD 84, 124052 (2011)]
  - are limited to corners of the parameter space [Blackman *et al.*, arXiv:1705.07089]

First Spinning Higher-Multipole Gravitational Waveform Model

- Takes an accurate model for the dominant multipole and **maps** it into each of the other multipoles
- Mapping based on results from Post-Newtonian (PN) and perturbation theory
- This approach can be applied to any non-precessing frequency-domain model
- PhenomD ⟶ PhenomHM



Construction: Basic Picture

- GW strain decomposed into spherical harmonics with spin weight –2

$$h(t, \vec{\theta}, \iota, \phi) = \sum_{\ell \geq 2} \sum_{-\ell \geq m \geq \ell} h_{\ell m}(t, \vec{\theta})\, {}^{-2}Y_{\ell m}(\iota, \phi)$$

- Appropriately scale and stretch the dominant multipole to reproduce each subdominant multipole – operate separately on frequency domain values, and phase and amplitude functions:

$$\begin{aligned} \tilde{h}_{\ell m}(f) &= A_{\ell m}(f) \times \exp\{i\, \varphi_{\ell m}(f)\} \\ &\approx \beta_{\ell m}(f)\, A^{22}(f^{\mathrm{A}}_{\ell m}) \times \exp\{i\, [\kappa_{\ell m}\, \varphi_{22}(f^{\varphi}_{\ell m}) + \Delta_{\ell m}]\} \end{aligned}$$

Construction: Foundations

$$f_{lm}(f) = \begin{cases} \frac{2}{m} f, & f \leq f_0 \\ \frac{f_{22}^{\rm RD} - 2 f_0/m}{f_{lm}^{\rm RD} - f_0} (f - f_0) + \frac{2 f_0}{m}, & f_0 < f \leq f_{lm}^{\rm RD} \\ f - f_{lm}^{\rm RD} + f_{22}^{\rm RD}, & f > f_{lm}^{\rm RD}. \end{cases}$$

Post-Newtonian time-domain scaling + stationary phase approximation

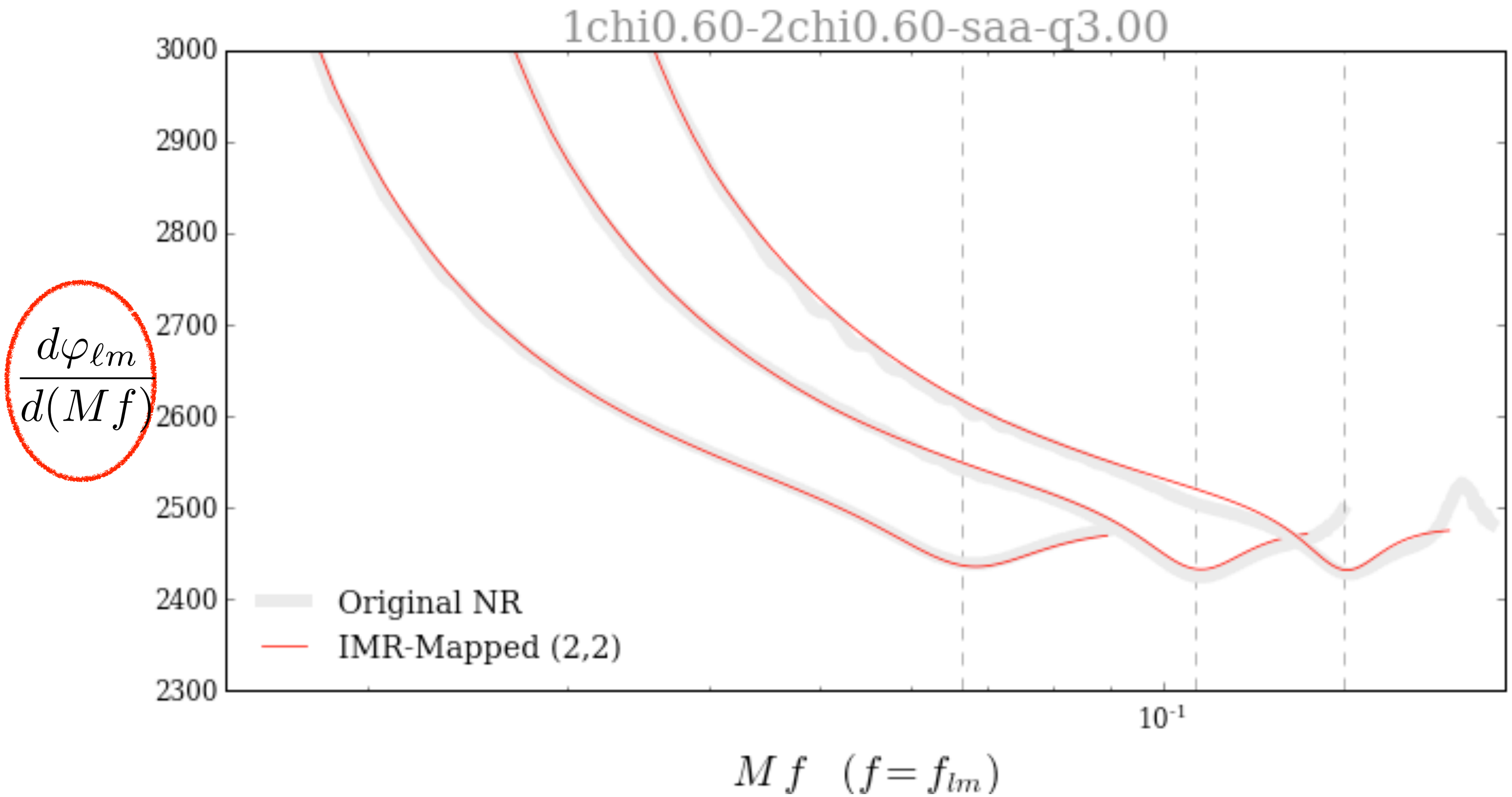
Bridge: linear interpolation

Quasi-Normal Mode theory

- Freedom to optimize the agreement with NR simulations by allowing different values of f_0 for amplitude and phase – PhenomD:

$$f_0^A = 0.018 f_{\ell m}^{\rm RD} / f_{22}^{\rm RD}, \quad f_0^\varphi = 0.014 f_{\ell m}^{\rm RD} / f_{22}^{\rm RD}, \quad f_{\ell m}^{\rm RD} = \omega_{lm0}/2\pi$$

Construction: Foundations



Construction: Foundations

- Integrate once, using PN theory and continuity to obtain the additional, multipole-dependent phase offsets

$$\kappa_{\ell m} = \frac{1}{f'_{lm}(f)}, \qquad \text{(piecewise constant)}$$

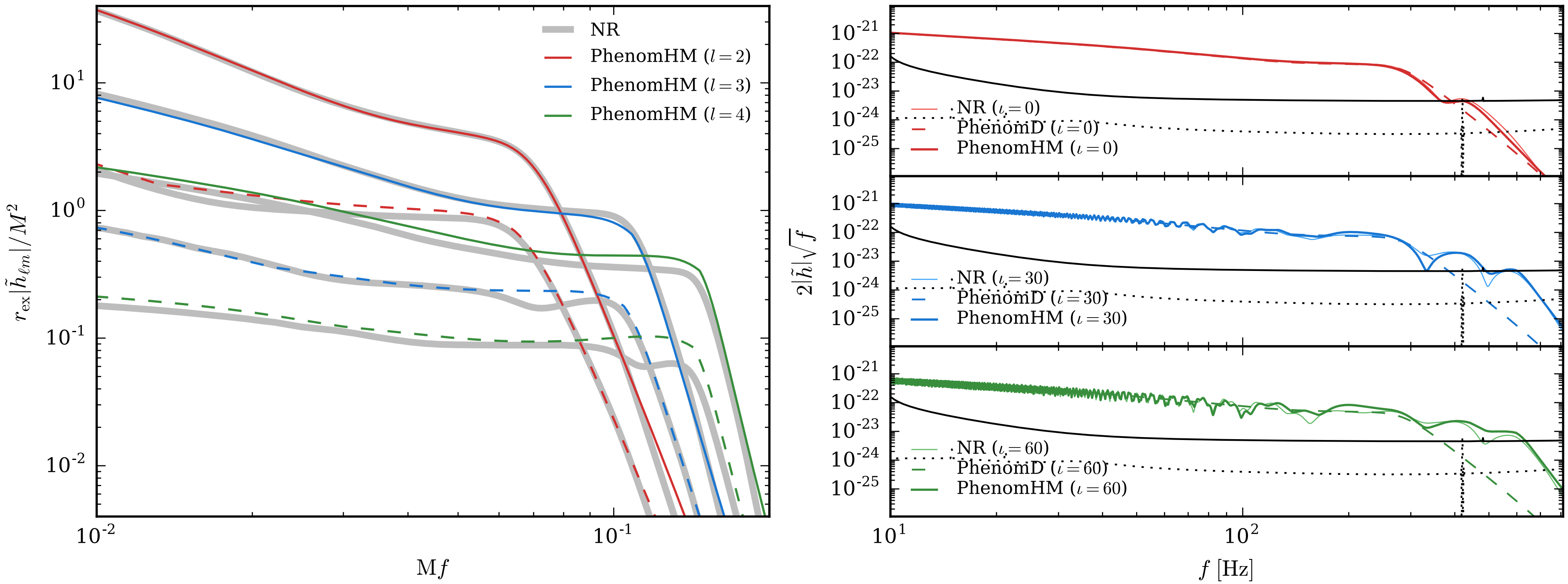
$$\Delta_{\ell m} = \begin{cases} \frac{\pi}{2}\,[\,3l + \mathrm{mod}(l+m,2)\,] - \pi, & f \leq f_0^{\varphi} \\ \varphi_{lm}(f_0^{\varphi}) - \kappa_{\ell m}\,\varphi_{22}[f_{lm}^{\varphi}(f_0^{\varphi})], & f_0^{\varphi} < f \leq f_{lm}^{\mathrm{RD}} \\ \varphi_{lm}(f_{lm}^{\mathrm{RD}}) - \varphi_{22}[f_{lm}^{\varphi}(f_{lm}^{\mathrm{RD}})], & f > f_{lm}^{\mathrm{RD}}. \end{cases}$$

- Completes $\tilde{h}_{\ell m}(f) \approx \beta_{\ell m}(f)\, A^{22}(f^{\mathrm{A}}_{\ell m}) \times \exp\left\{i\left[\kappa_{\ell m}\,\varphi_{22}(f^{\varphi}_{\ell m}) + \Delta_{\ell m}\right]\right\}$

  (the stationary phase approximation predicts the amplitude scalings to leading-order in f)

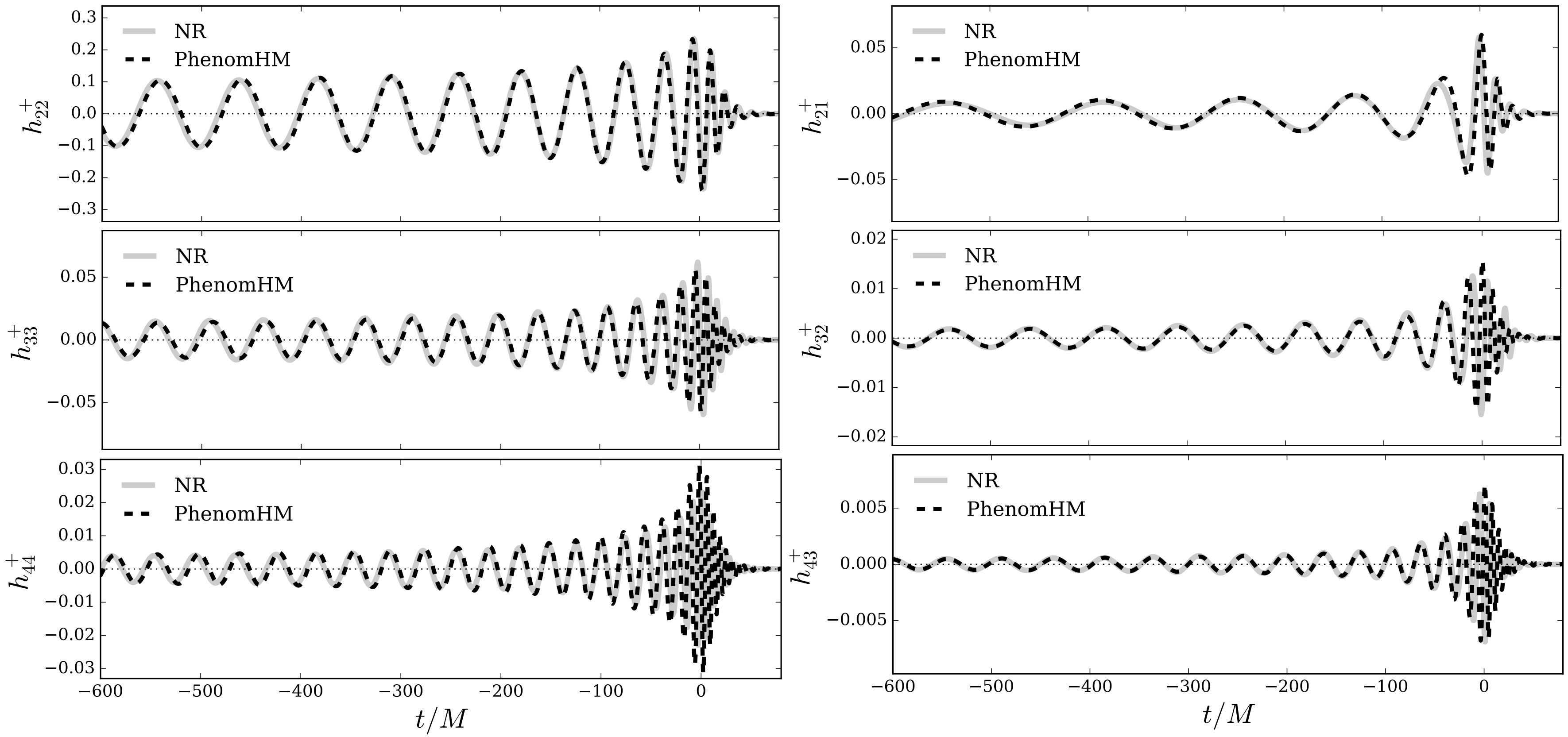
Comparison to NR Data: Frequency Domain

Mass ratio=8, non-spinning



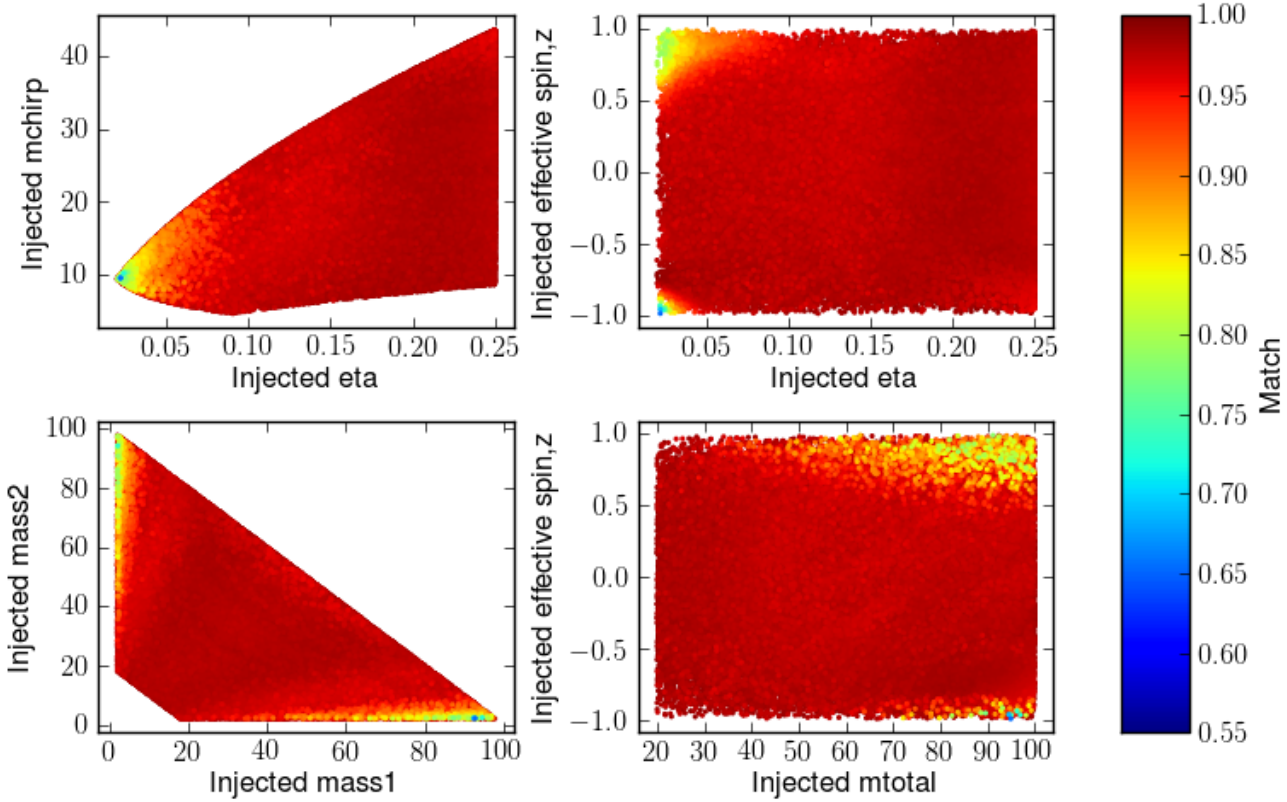
Comparison to NR Data: Time Domain

Mass ratio=4, $\chi_{1,z}=\chi_{2,z}=0.85$

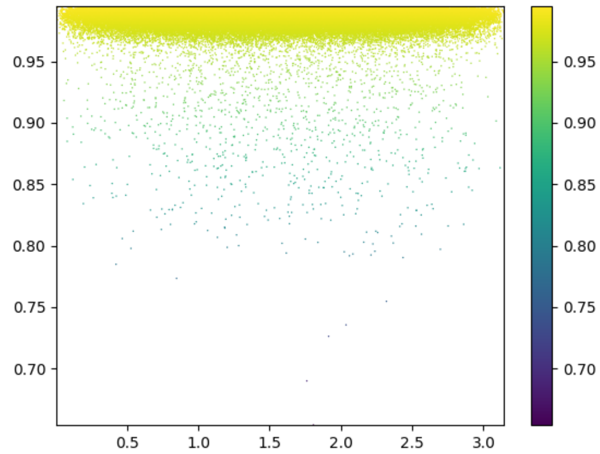


Impact on Searches

PhenomD injections

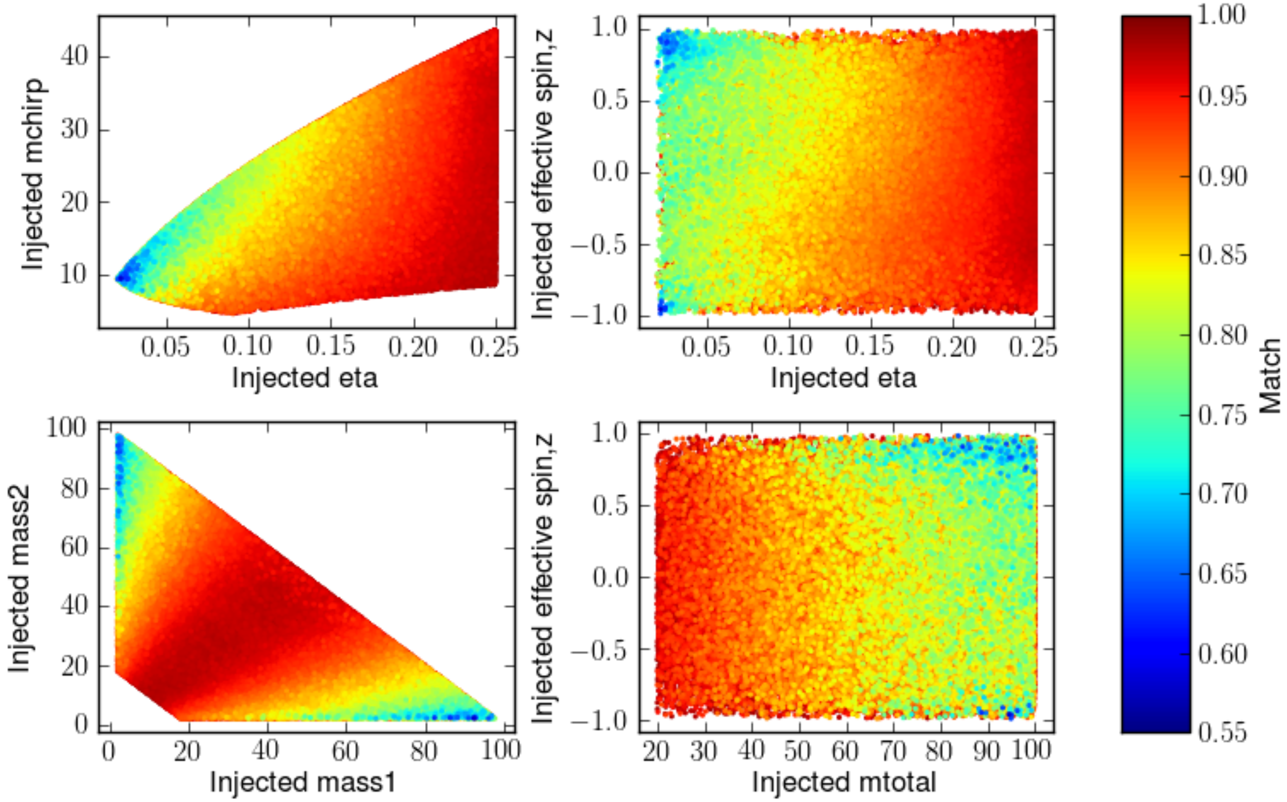


Match vs inclination

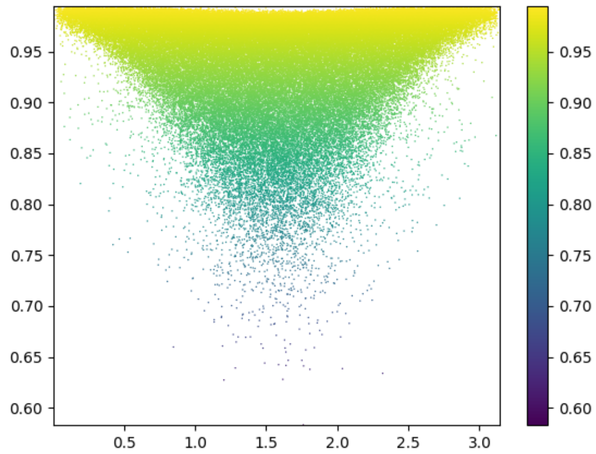


Impact on Searches

PhenomHM injections

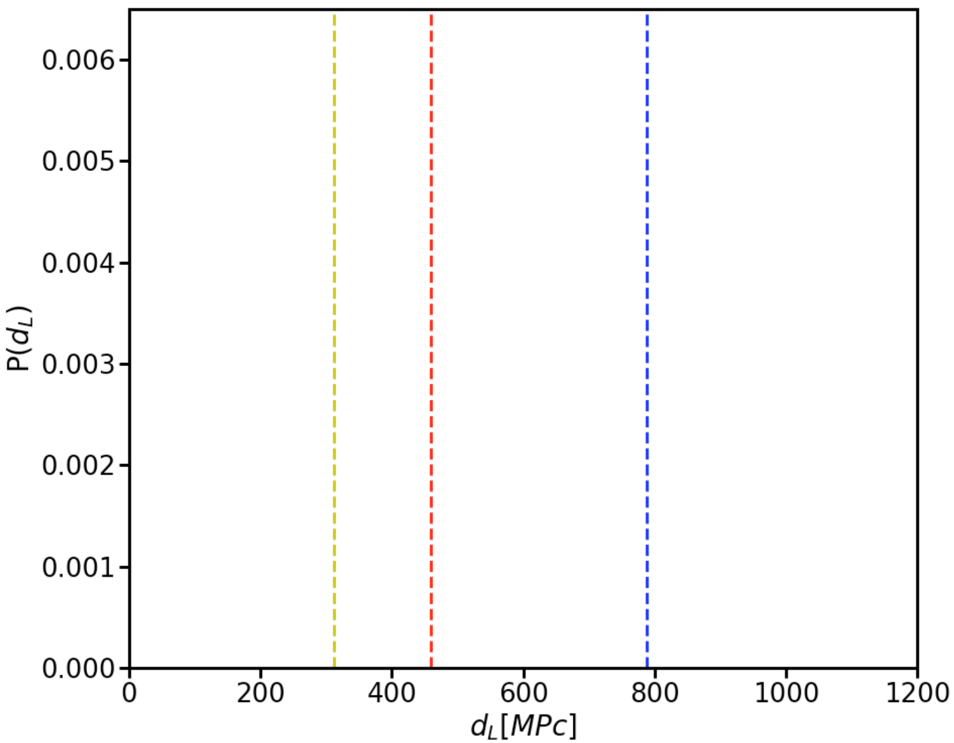


Match vs inclination



Impact on Parameter Estimation: Extrinsic Parameters

Mass ratio=4, $\chi_{1,z}$=0.5, $\chi_{2,z}$=0



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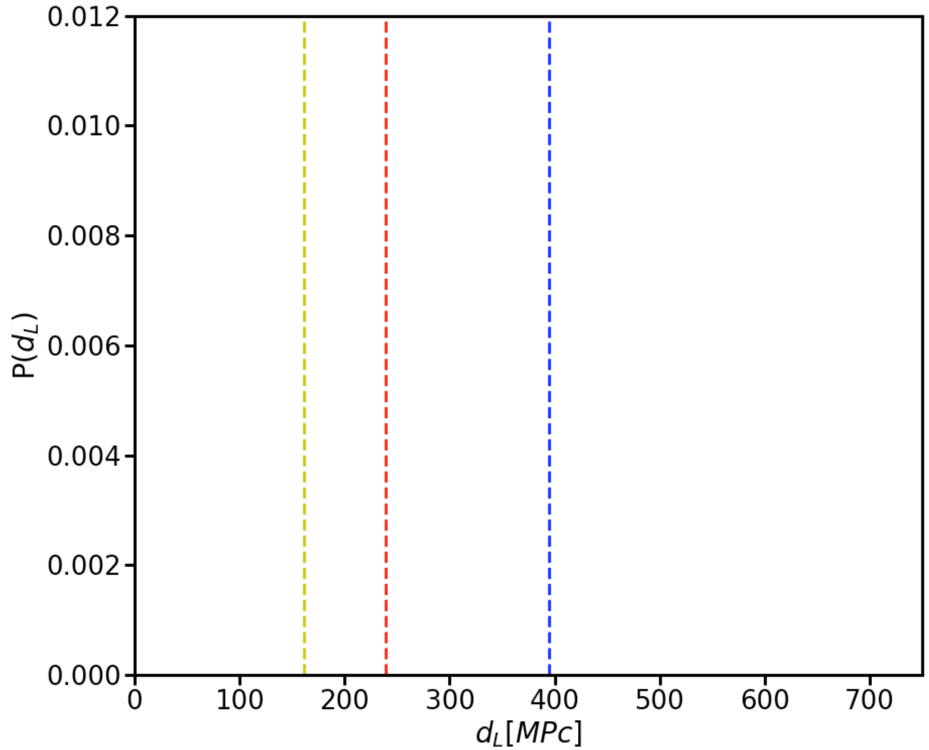

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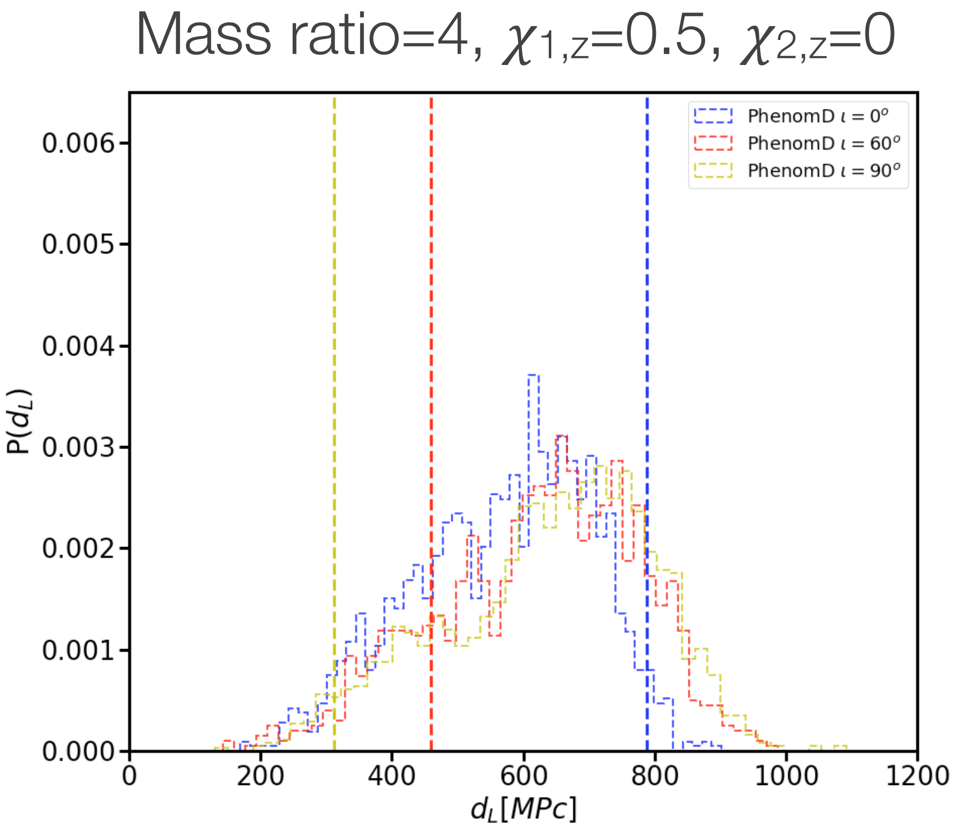

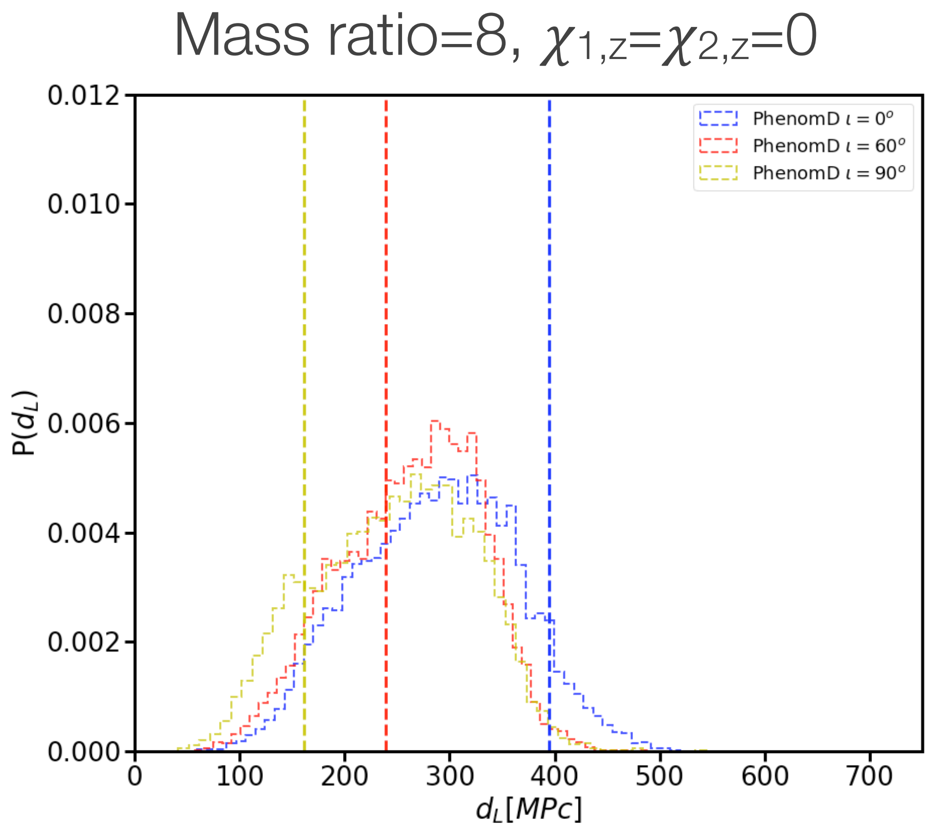

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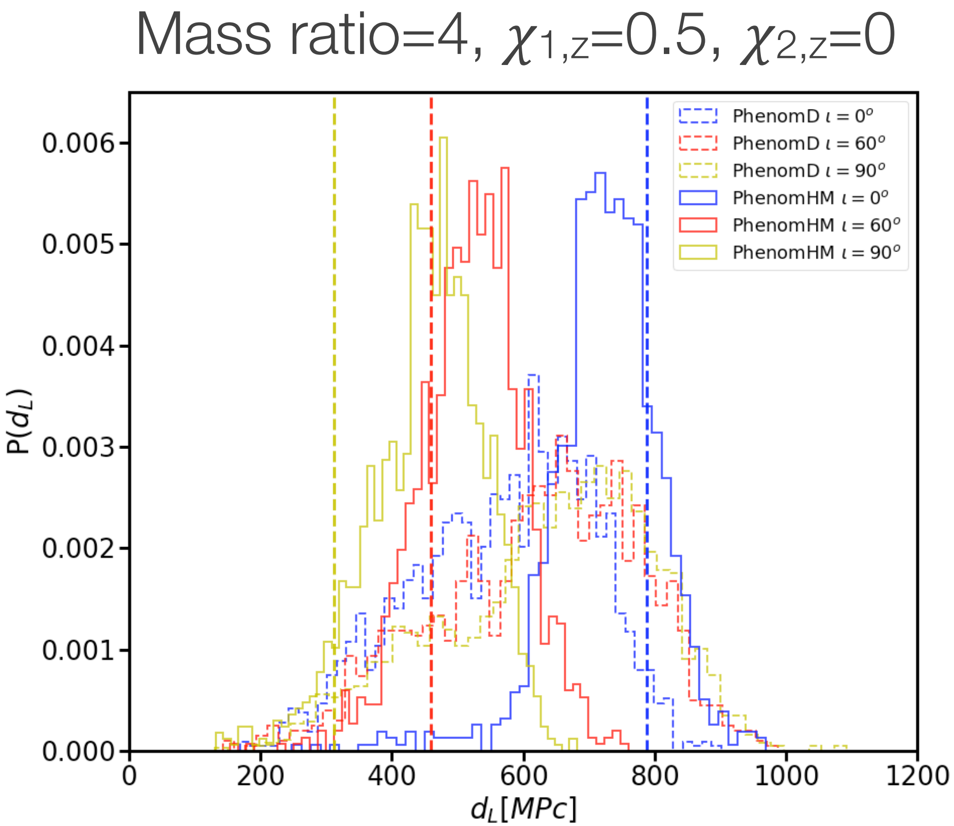
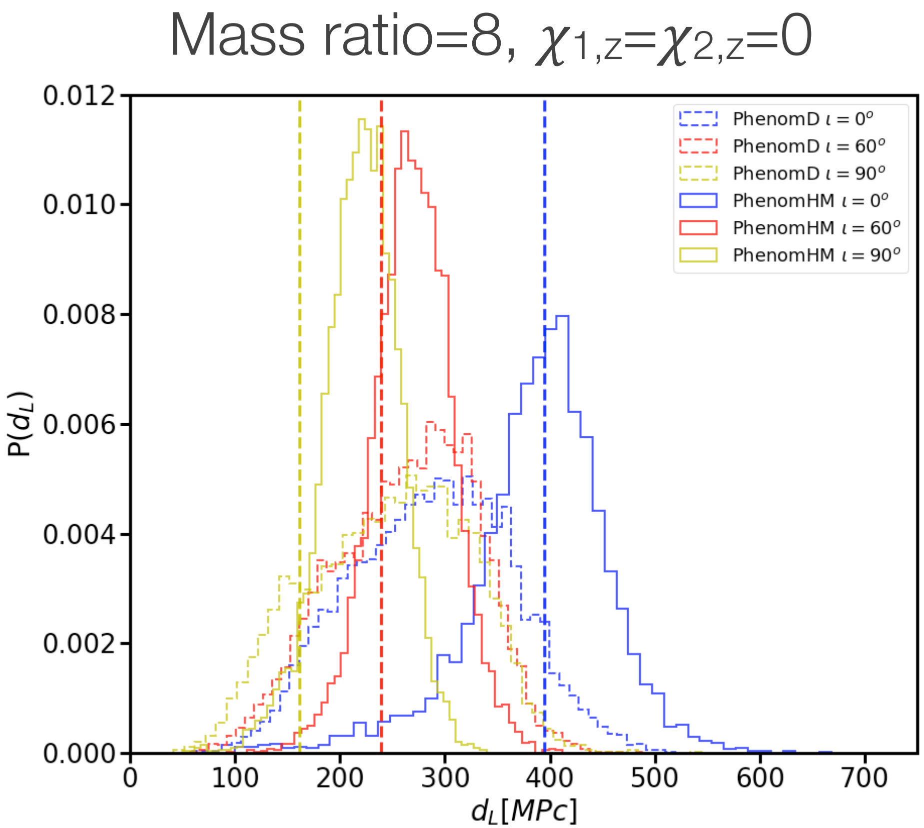
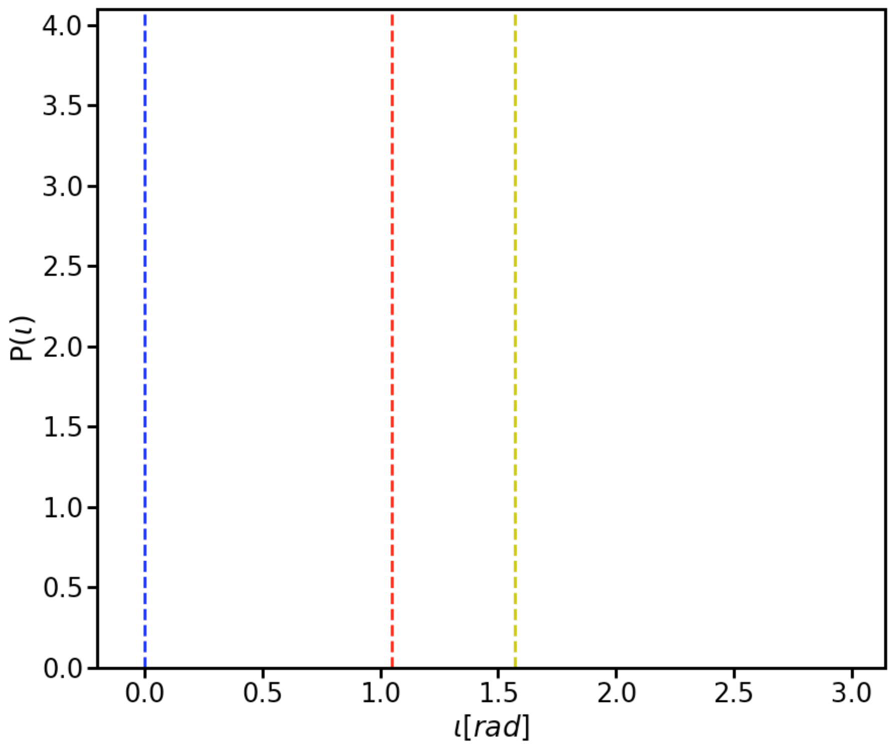


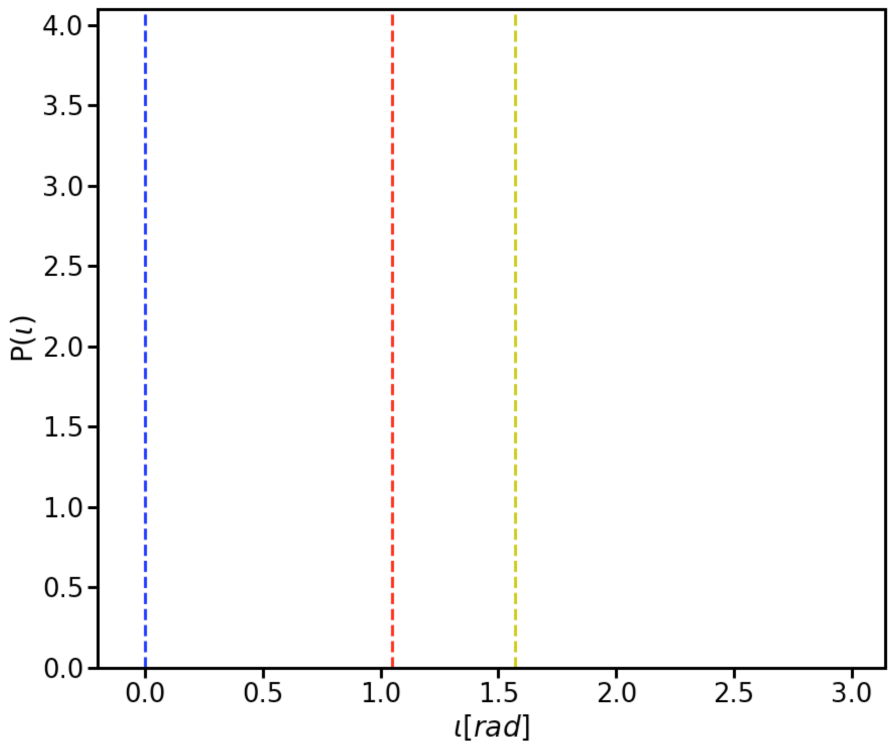


Impact on Parameter Estimation: Extrinsic Parameters

Mass ratio=4, $\chi_{1,z}$=0.5, $\chi_{2,z}$=0



Mass ratio=8, $\chi_{1,z}$=$\chi_{2,z}$=0



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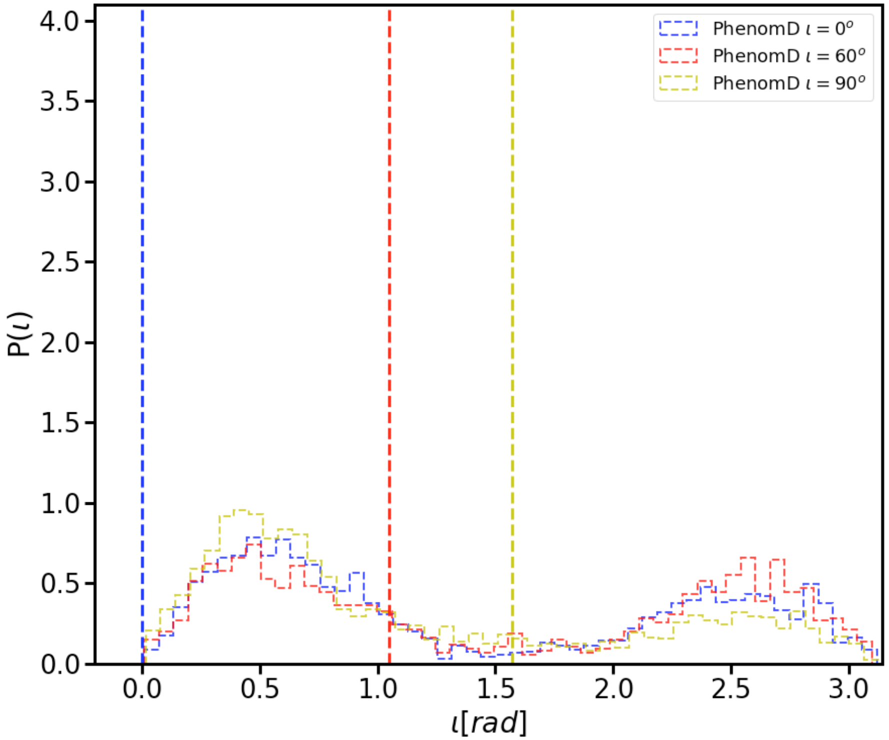

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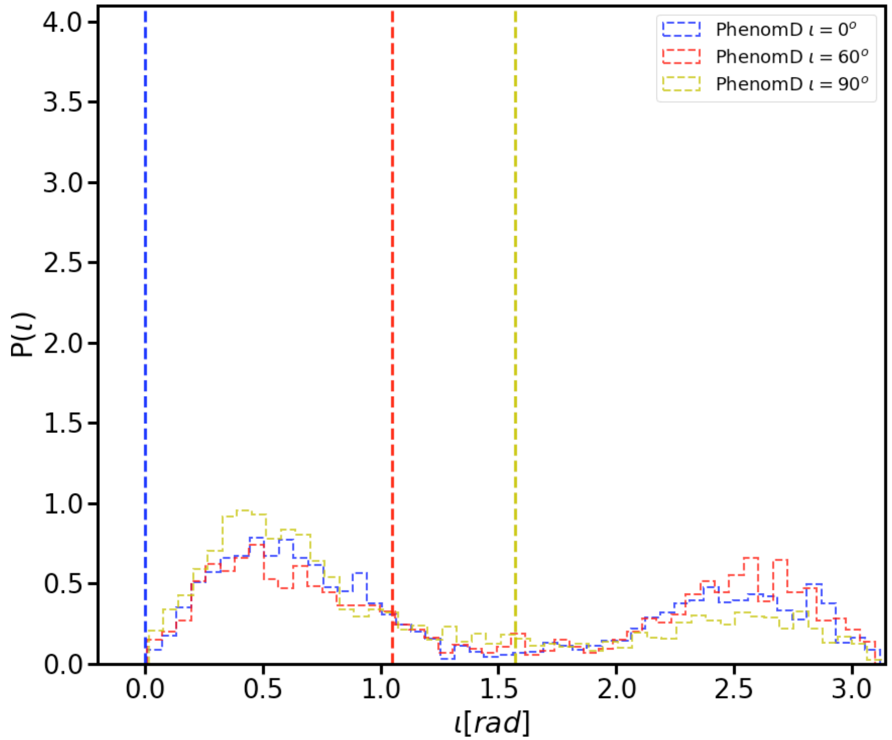

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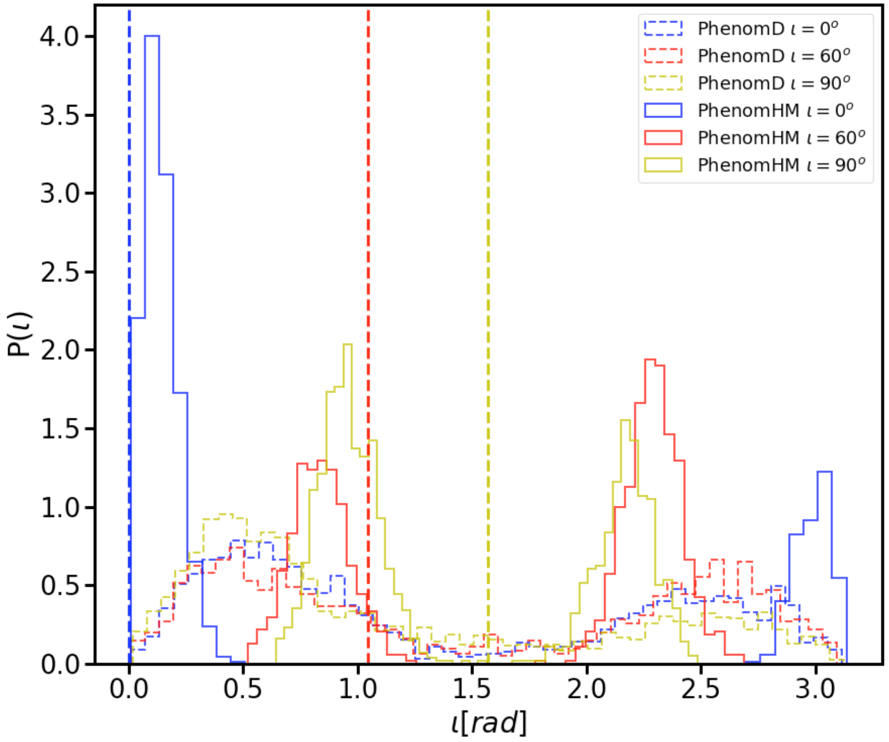

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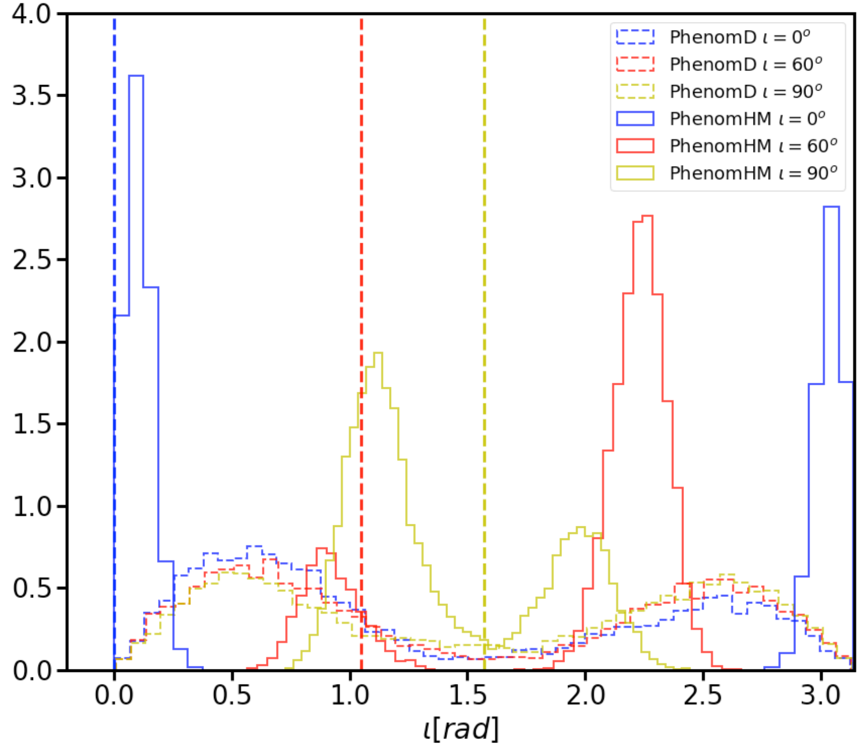
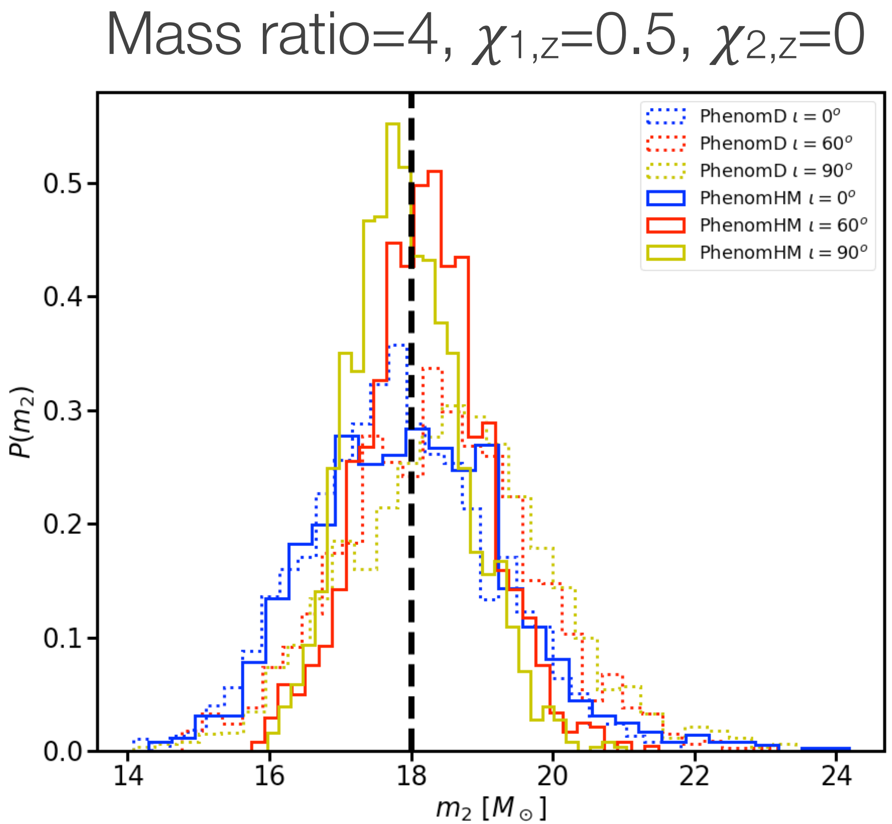
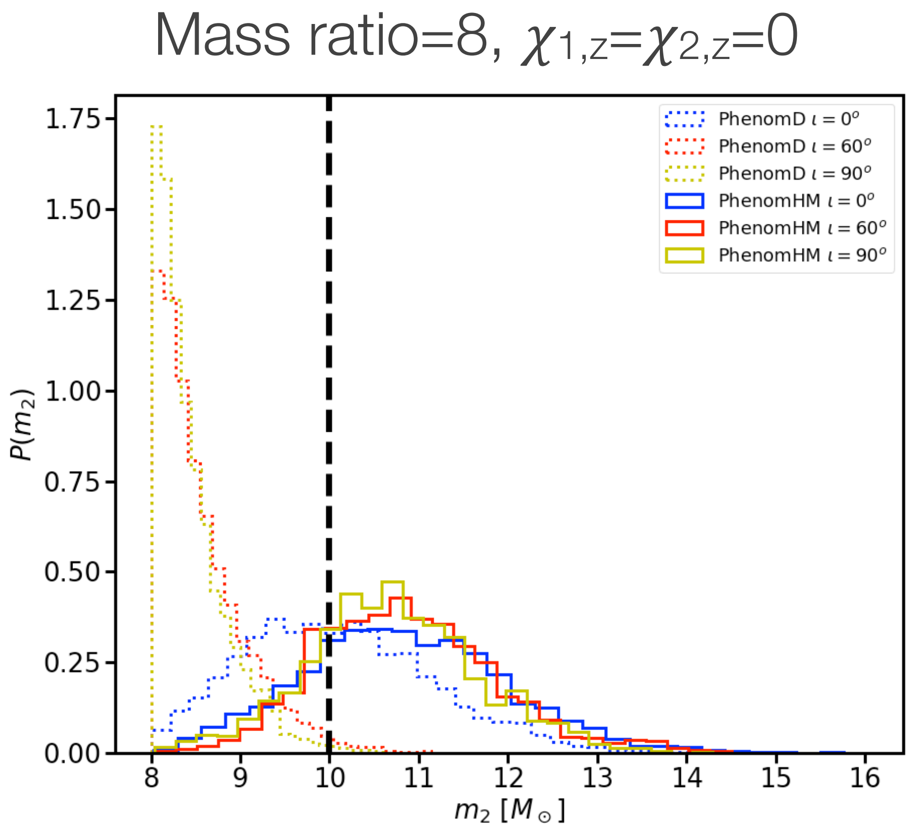


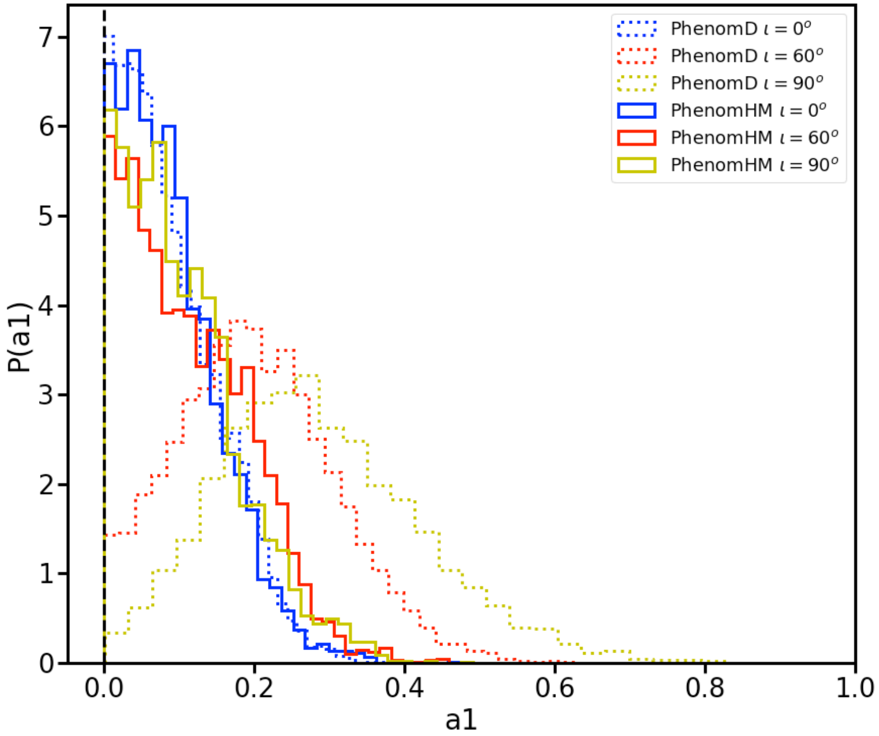
Impact on Parameter Estimation: Masses

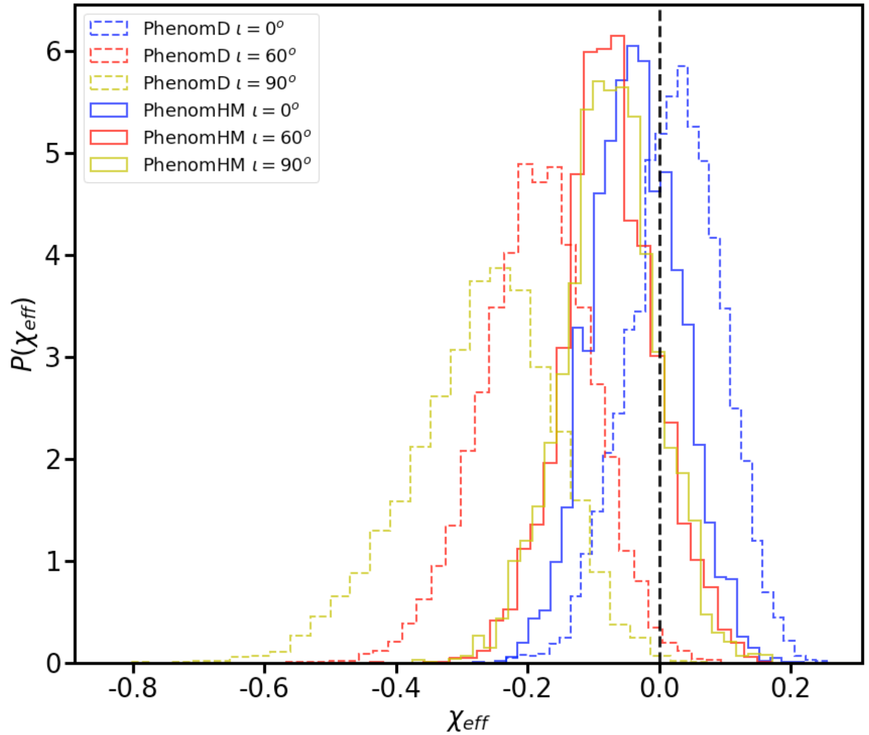




Impact on Parameter Estimation: Spins

Mass ratio=8, $\chi_{1,z}=\chi_{2,z}=0$



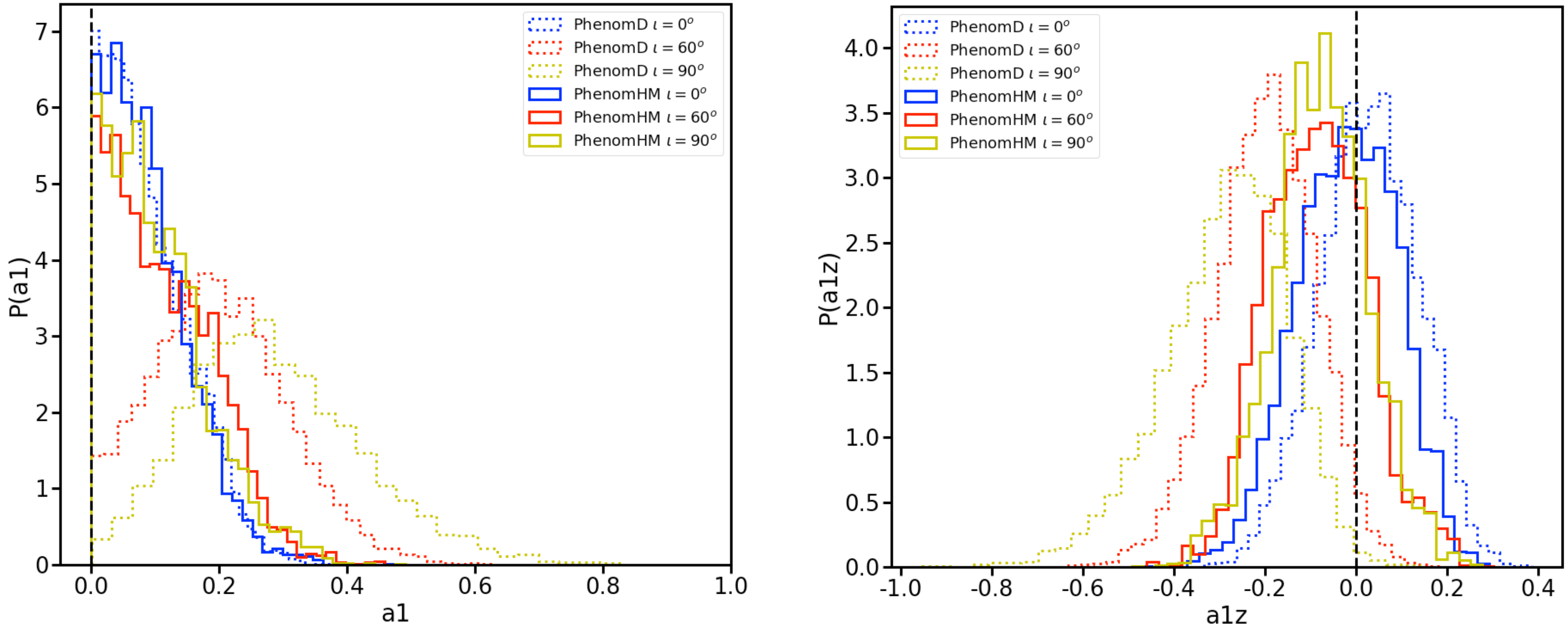


Summary

- A simple, flexible method to include the subdominant multipole contributions to binary black hole gravitational waveforms: benefits for searches and parameter estimation
- Amplitude and phase of the starting frequency-domain model are appropriately stretched and rescaled (guidance from post-Newtonian and perturbation theory)
- No additional tuning to numerical-relativity simulations!
- PhenomD ⟶ PhenomHM: **first higher-multipole spinning binary black-hole model**
  1. More accurate in all comparisons to numerical-relativity data
  2. Typically leads to improved measurements of the binary properties
- Approach can be extended to precessing systems: will enable studies of the impact of higher multipoles on gravitational-wave astronomy, and tests of fundamental physics

Impact on Parameter Estimation: Spins

Mass ratio=8, $\chi_{1,z}$=$\chi_{2,z}$=0



Amplitude Scaling

$$\beta_{22} = 1, \qquad \beta_{21} = \frac{\delta}{3} F^{1/3},$$

$$\beta_{33} = \sqrt{\frac{45}{56}} \delta F^{1/3}, \qquad \beta_{32} = \sqrt{\frac{5}{63}} (\delta^2 + \eta) F^{2/3},$$

$$\beta_{44} = \sqrt{\frac{320}{567}} (\delta^2 + \eta) F^{2/3}, \quad \beta_{43} = \sqrt{\frac{81}{1120}} (\delta^2 + 2\eta) \delta F$$

Current Models

Model	HMs	Spin	Precession	Det.	PE
SEOBNRv2	No	Yes	No	Yes	Yes
SEOBNRv4	No	Yes	No	Yes	Yes
SEOBNRv3	$\ell \leq 2$	Yes	Yes	No	Yes
IMRPhenomD	No	Yes	No	Yes	Yes
IMRPhenomP	$\ell \leq 2$	Yes	Yes	No	Yes
EOBNRv2HM	Yes	No	No	No	Yes

[Bustillo *et al.*, PRD 95, 104038 (2017)]