Laser Mode Spectroscopy for Mirror Metrology

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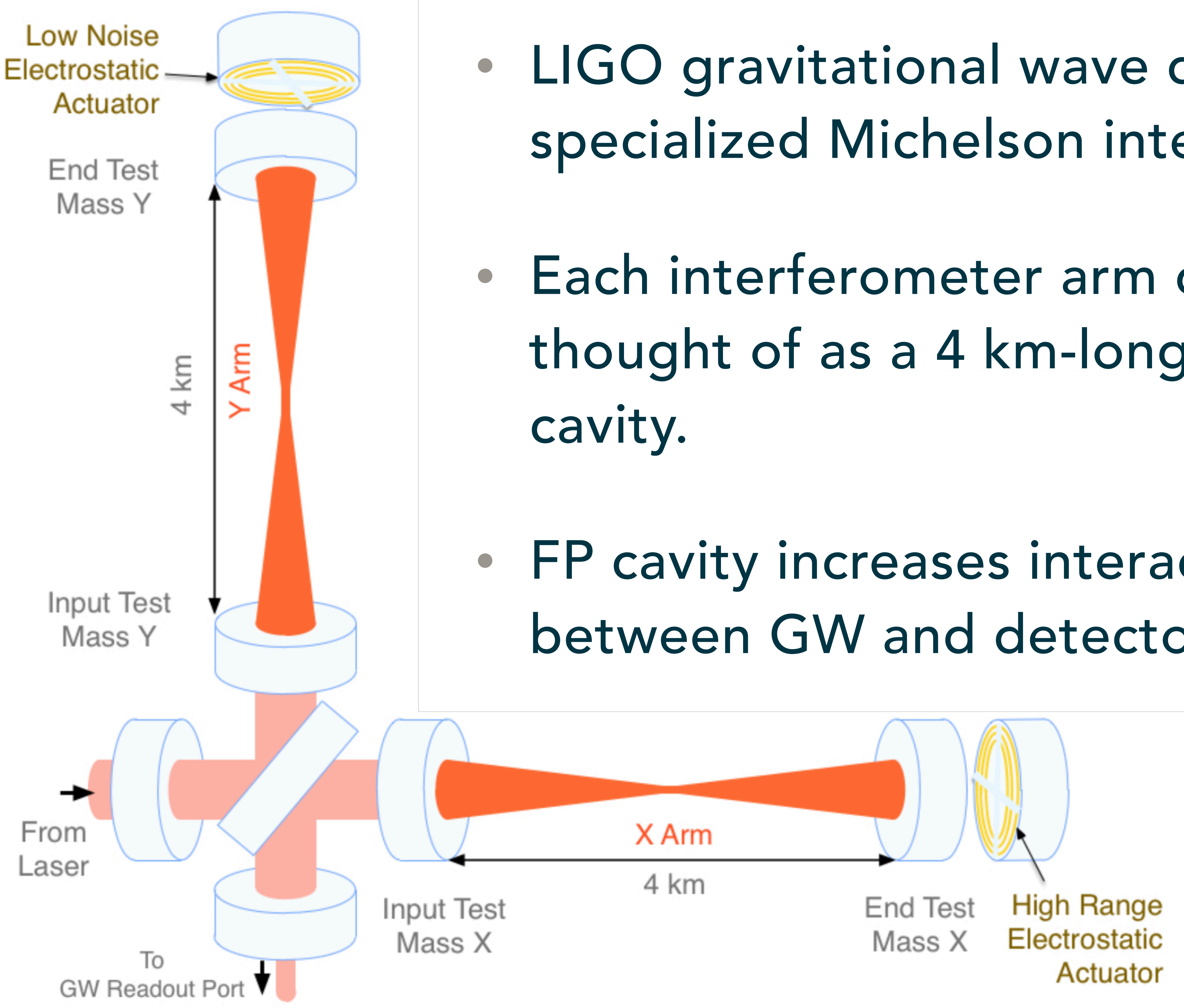
Gravitational Wave Detectors



- LIGO gravitational wave detectors are specialized Michelson interferometers.
- Each interferometer arm can be thought of as a 4 km-long Fabry-Perót cavity.
- FP cavity increases interaction time between GW and detector.

Optical Loss

- Low optical power loss needed to maintain sensitivity of interferometer.

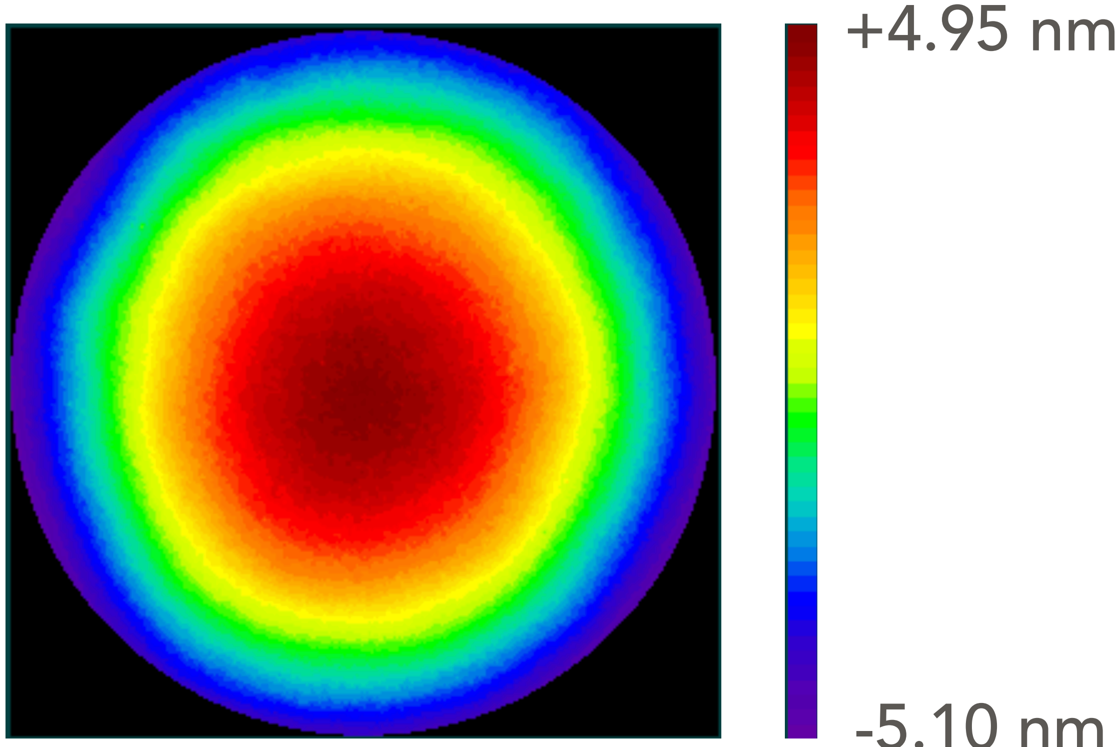
- Optical loss → reduced effective power of input beam → loss of squeezed light → increased shot noise → lower sensitivity to GW

- Some causes of optical loss:
    - Mirror figure error
    - Surface aberrations, scratches, point defects
    - Absorption
    - Microroughness
    - ETM transmission

Mirror Figure Error

- More focused problem: How can we evaluate optical loss due to mirror figure error?

- *Fizeau interferometer* → mirror surface compared to ideal reference piece.
  → Produce *phase map*.

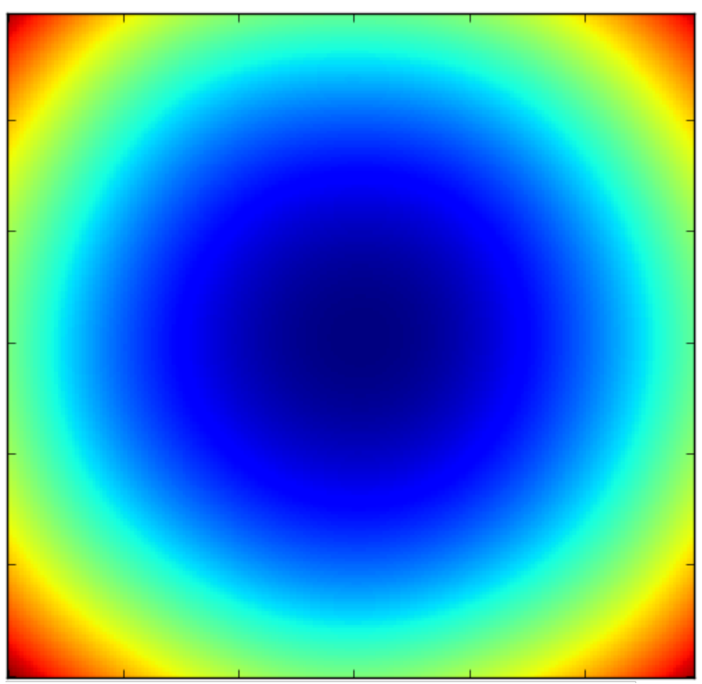


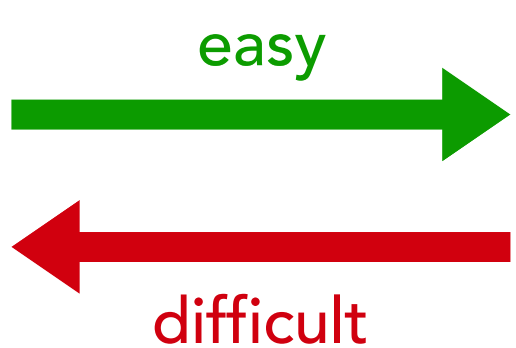
https://dcc.ligo.org/LIGO-E1300196

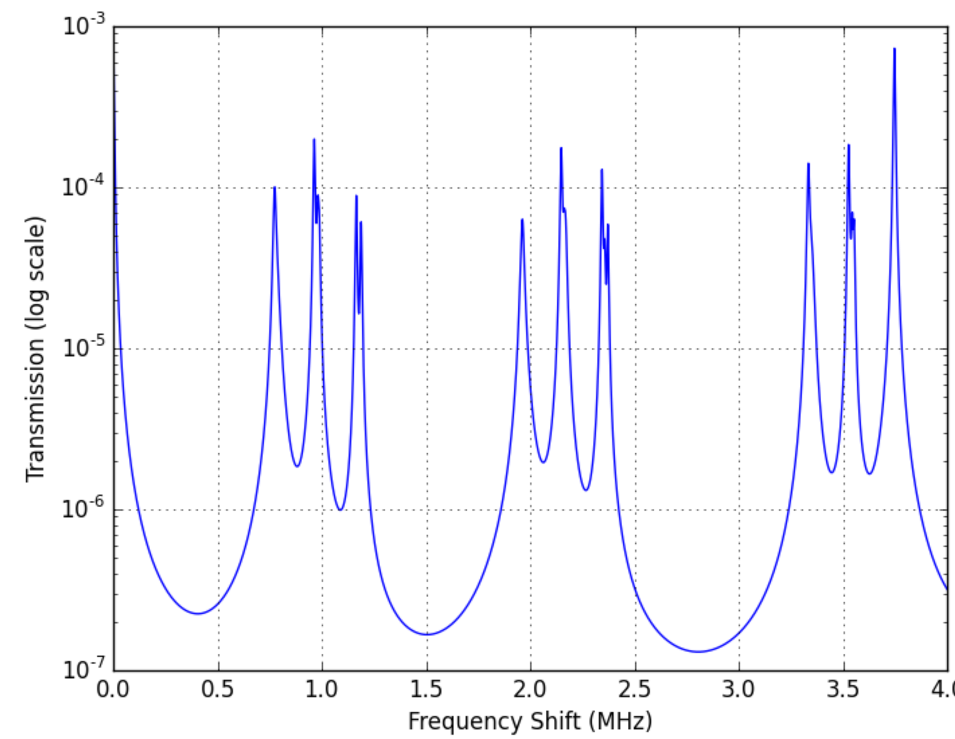
- Instead, want *in-situ* interferometric measurement with actual cavity beam used for GW detection.

Method

- Difficult: In-situ measurement of mirror figure error.
- Easier: Given cavity with some figure error → Measure transmission curve.
- This project: *Can we use cavity transmission of transverse modes (TEM) as a sensor for mirror figure error?*



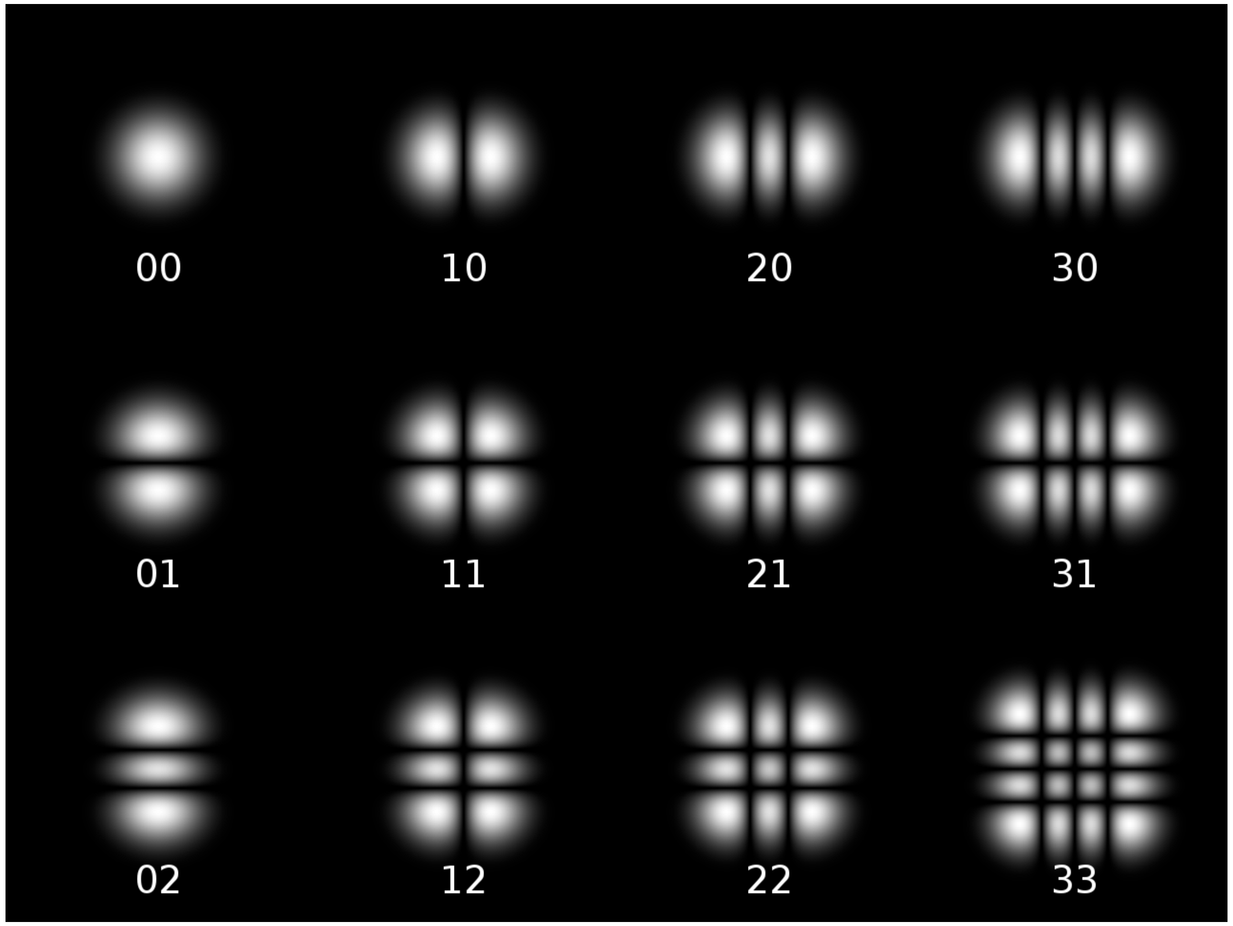




Higher-Order Cavity Modes

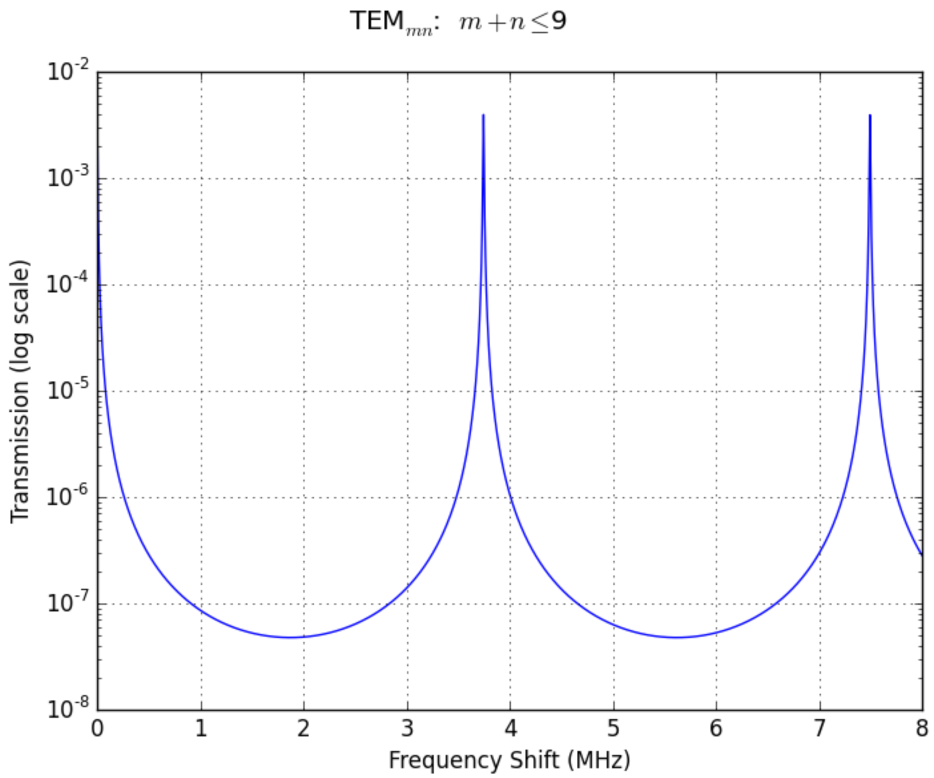
- Hermite-Gaussian modes: Family of solutions to paraxial Helmholtz equation.

- Resonant modes of FP cavity.

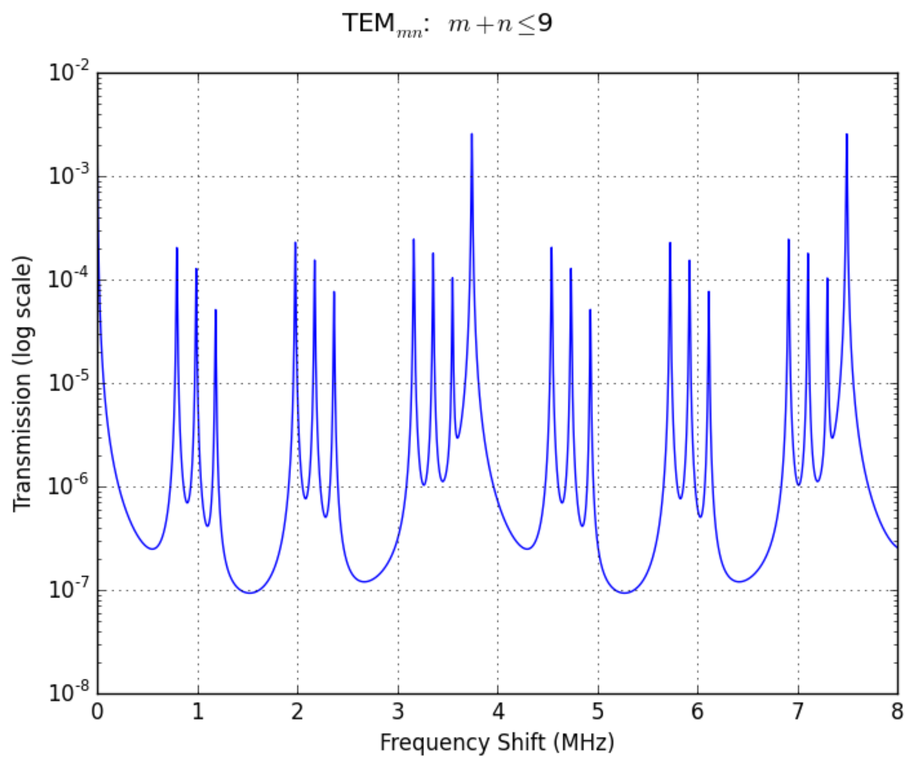


Higher-Order Cavity Modes

- Beam aligned to cavity → only see Gaussian beam, the lowest-order solution (TEM_{00}).



- Misaligned beam → higher-order modes appear.

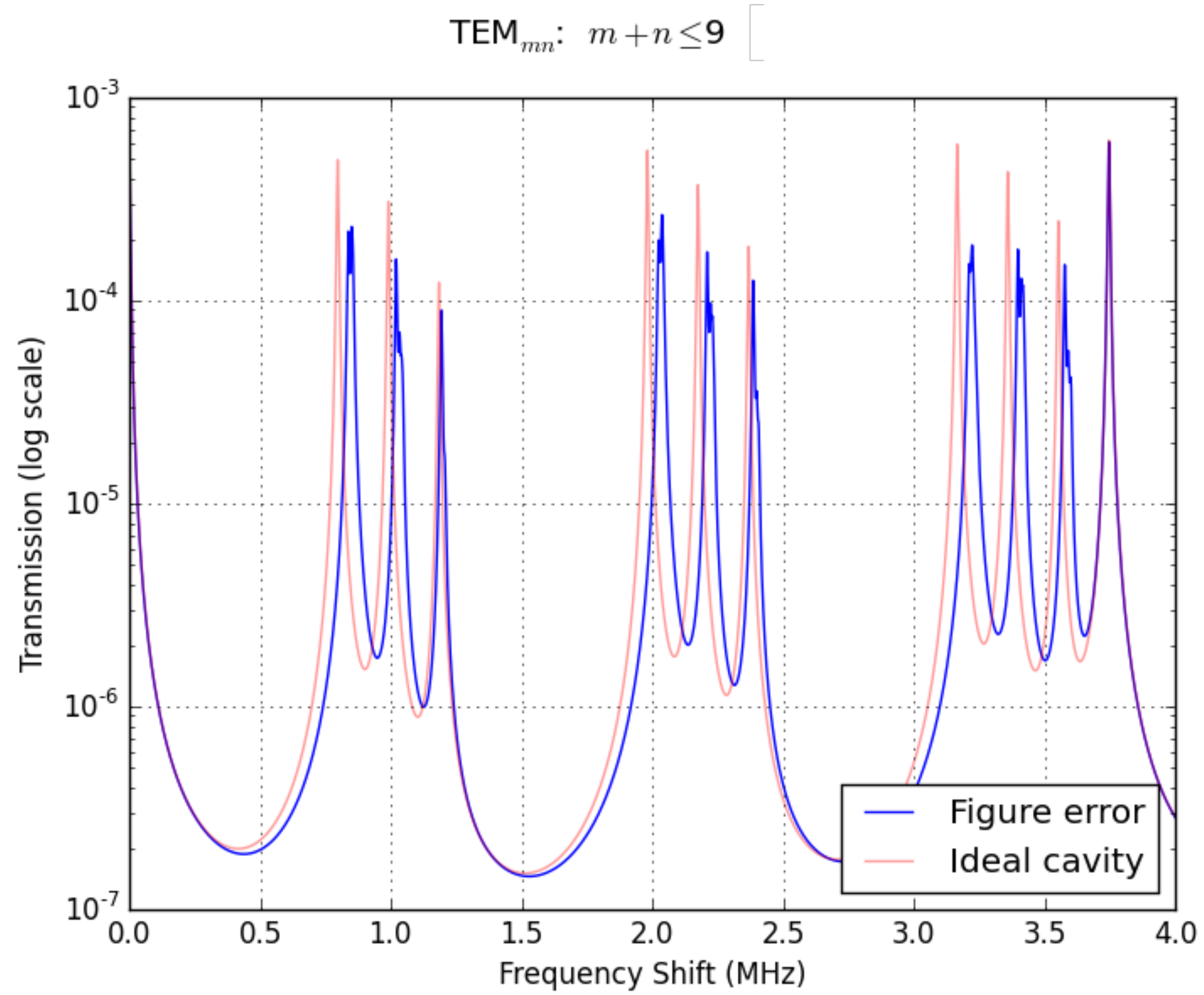


Higher-Order Cavity Modes

- Ideal cavity → resonant frequencies determined by cavity *length* and *radius of curvature.*

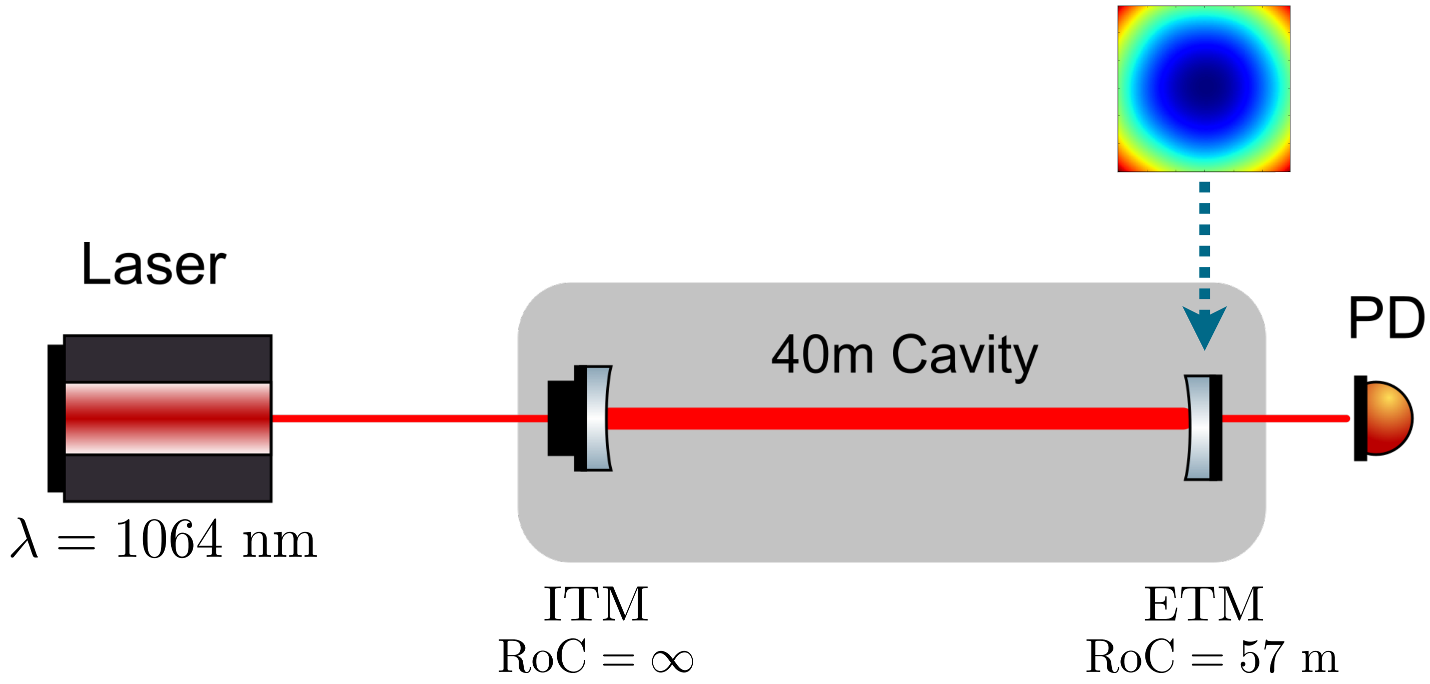
$$\nu_{\mathrm{FSR}} = \frac{c}{2L} \qquad \nu_{\mathrm{TMS}} = \nu_{\mathrm{FSR}} \left(\frac{m+n}{\pi} \right) \cos^{-1} \sqrt{\left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right)}$$

- Real cavity → mirror figure error creates shifts in mode frequencies and amplitudes.



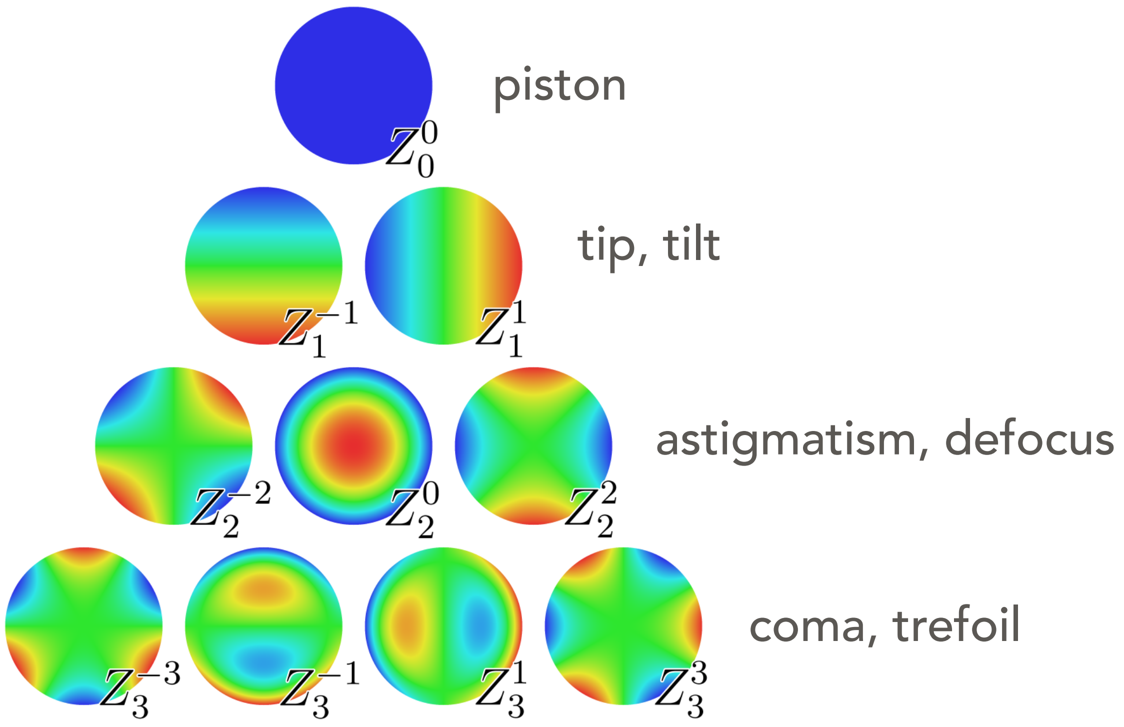
Finesse

- Software package for running simulations of user-defined optical cavities.
- Run *Finesse* simulation of FP cavity with parameters of one arm of LIGO 40m prototype interferometer.
- By default, all mirrors are perfectly smooth → Make simulation more realistic by introducing a phase map to the ETM.

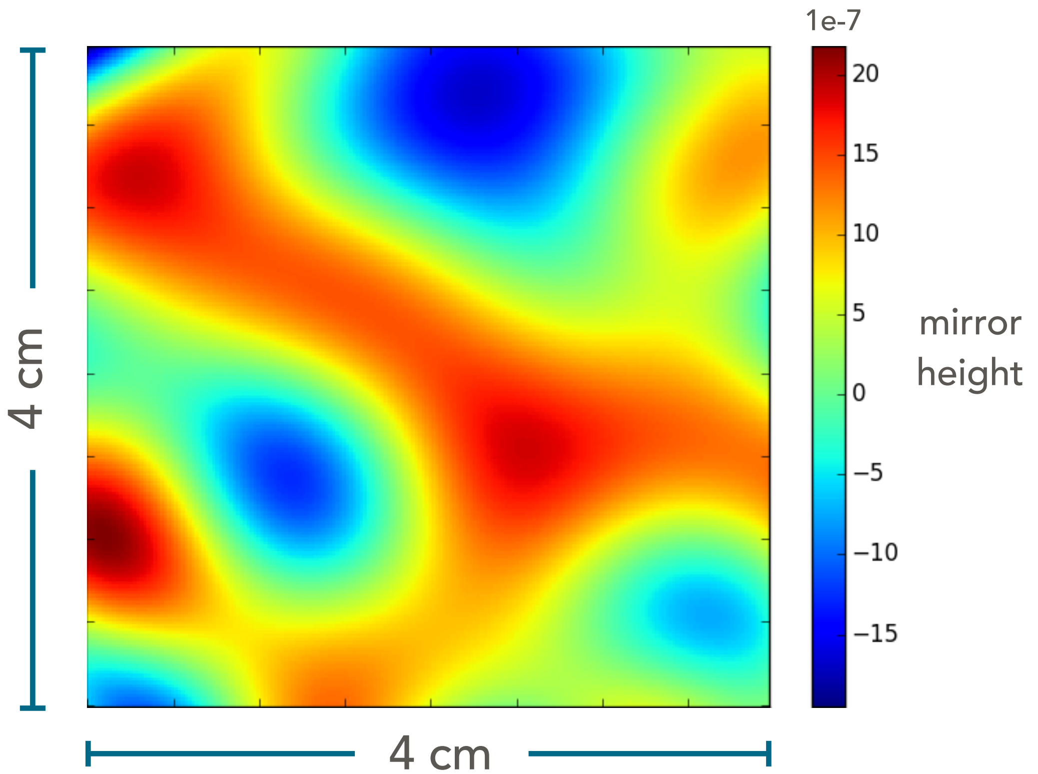


Zernike Polynomials

- Sequence of polynomials orthogonal on unit disk. Each polynomial corresponds to a type of optical aberration.

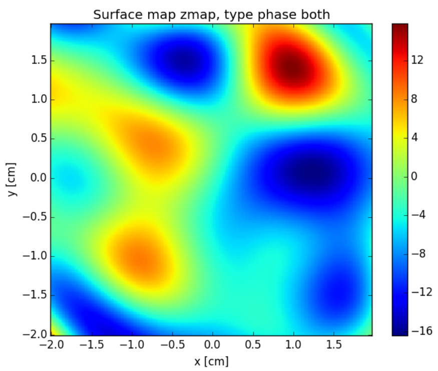


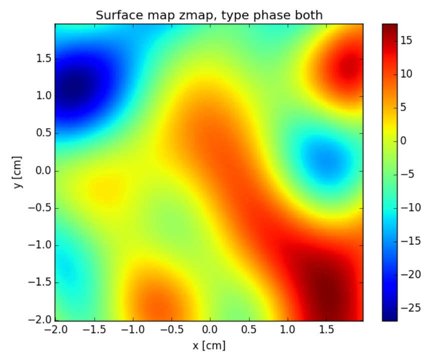
- Simulate mirror figure error:
  - Apply random coefficients to Zernike polynomials
  - Coefficients normally distributed, σ = 4 nm

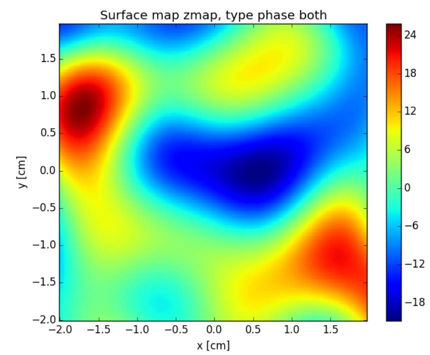


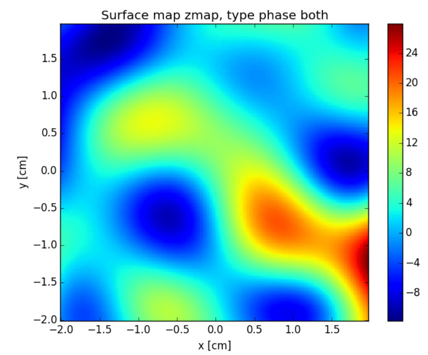
Zernike Polynomials

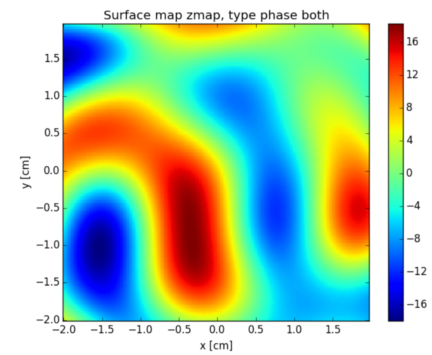
- Run many simulations with different Zernike coefficients → learn how much figure error affects cavity transmission.



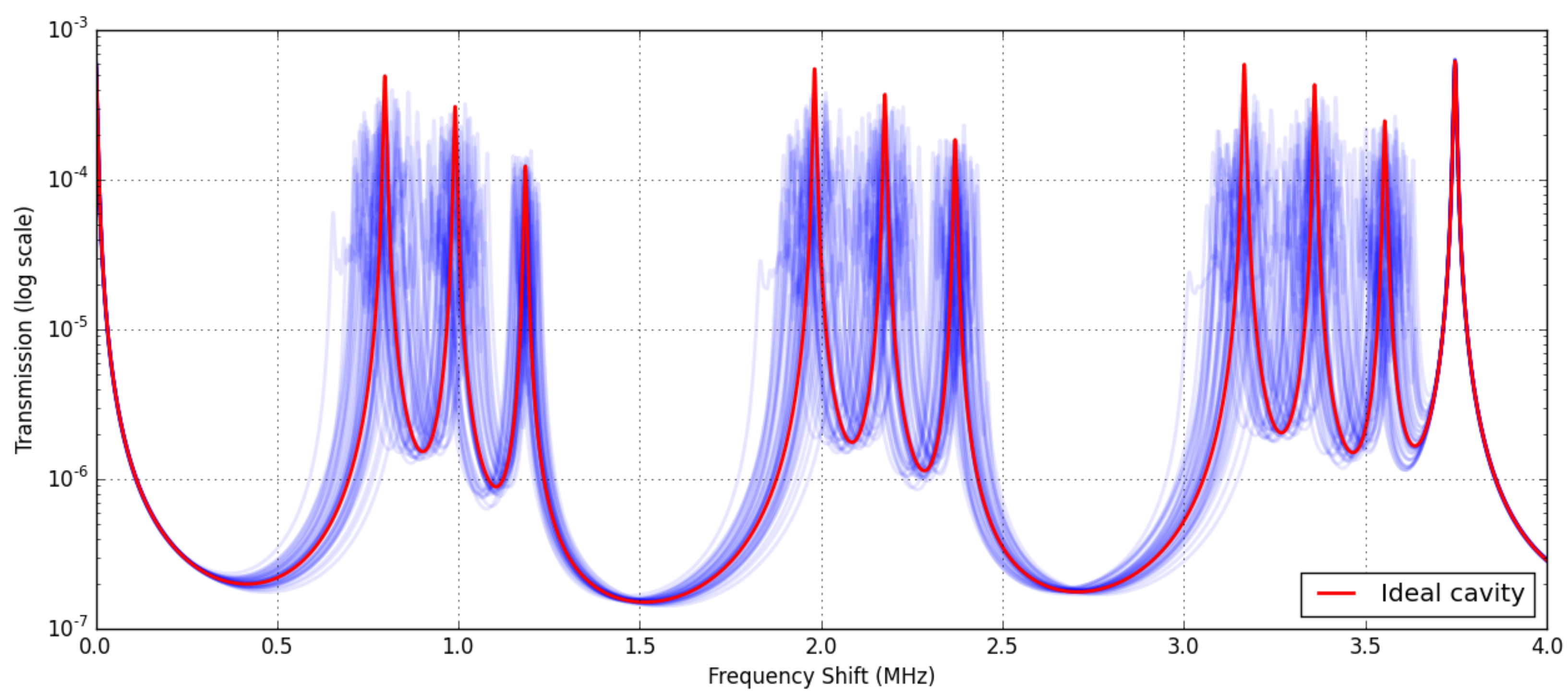






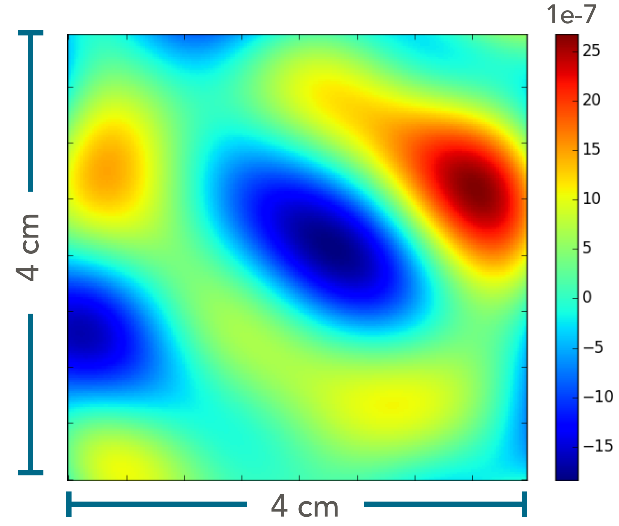


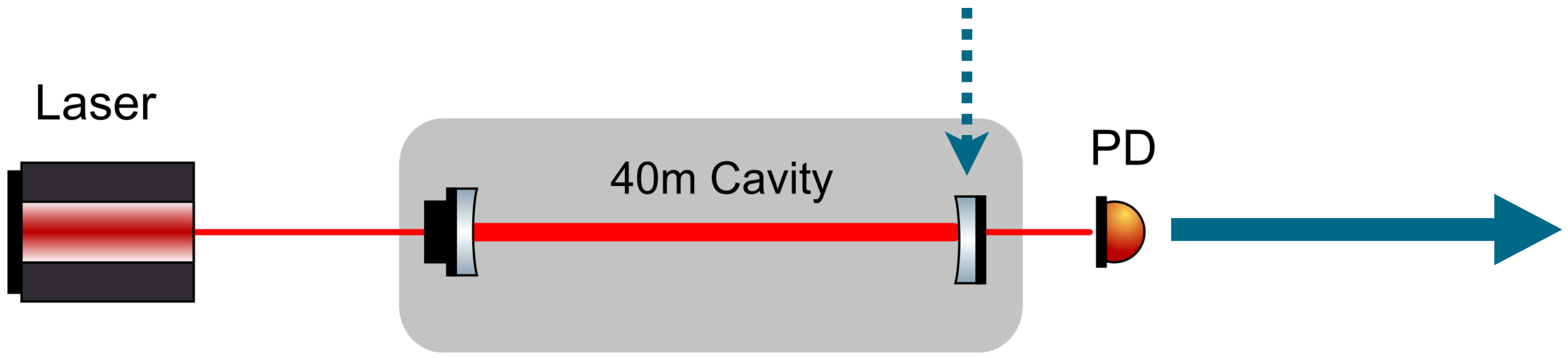
- Compare HOM transmission peaks from many different phase maps:

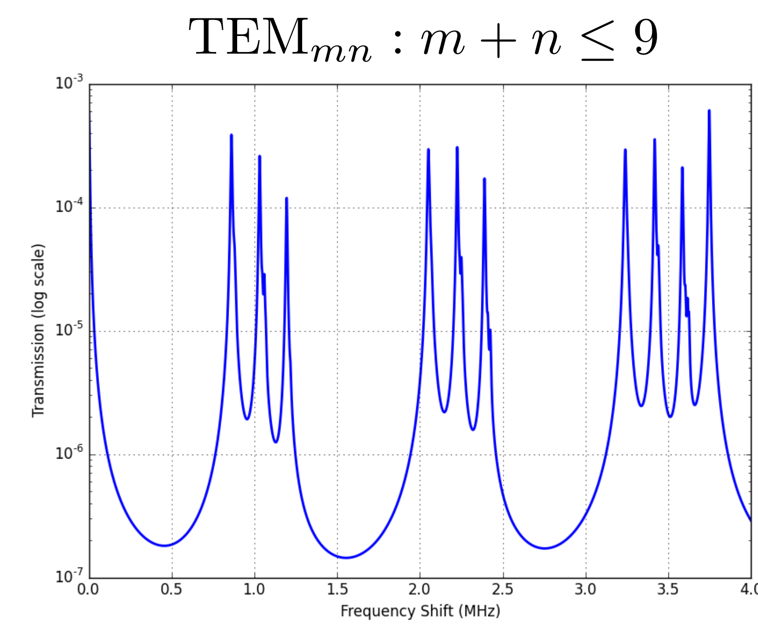


Example

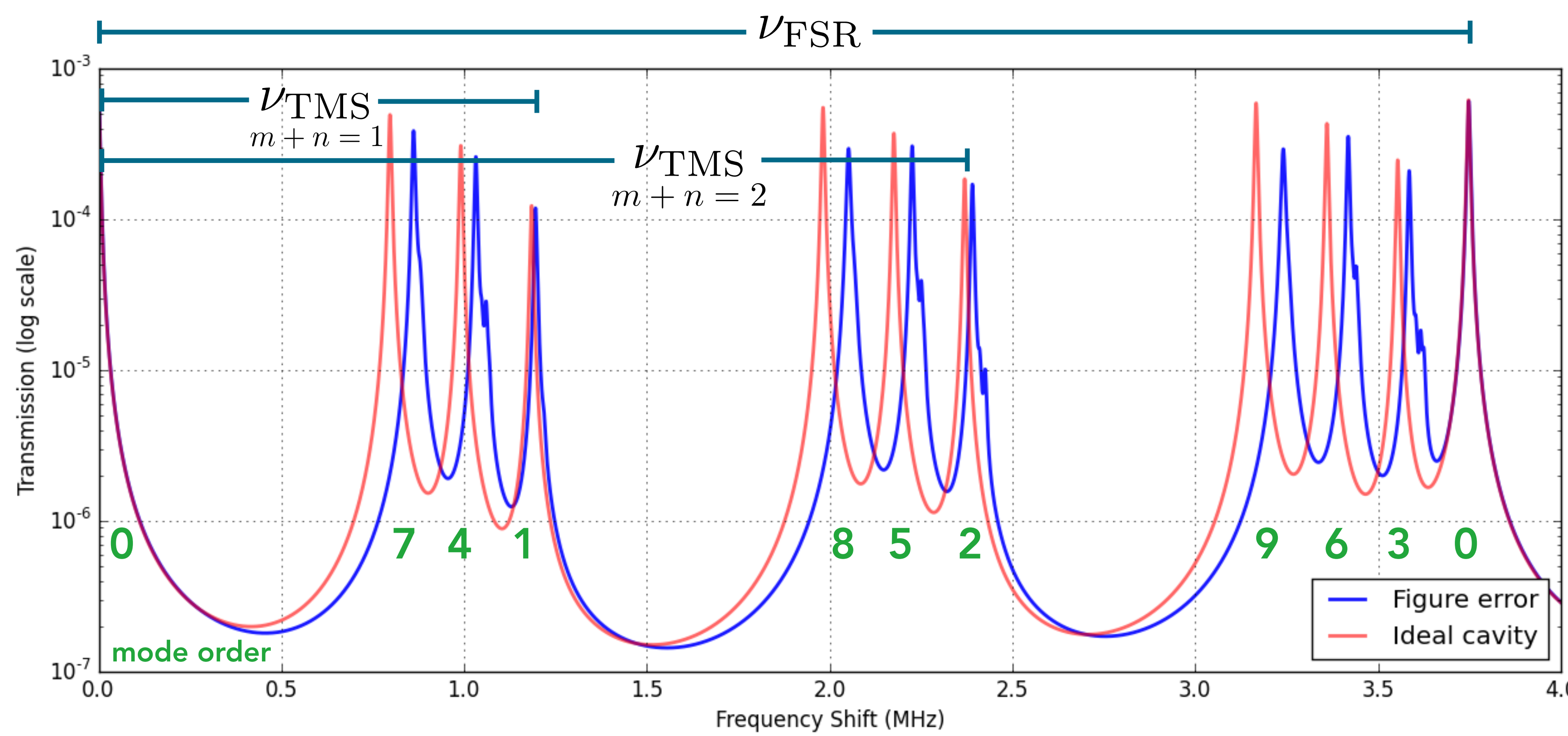
Run *Finesse* simulation with a given ETM mirror map:





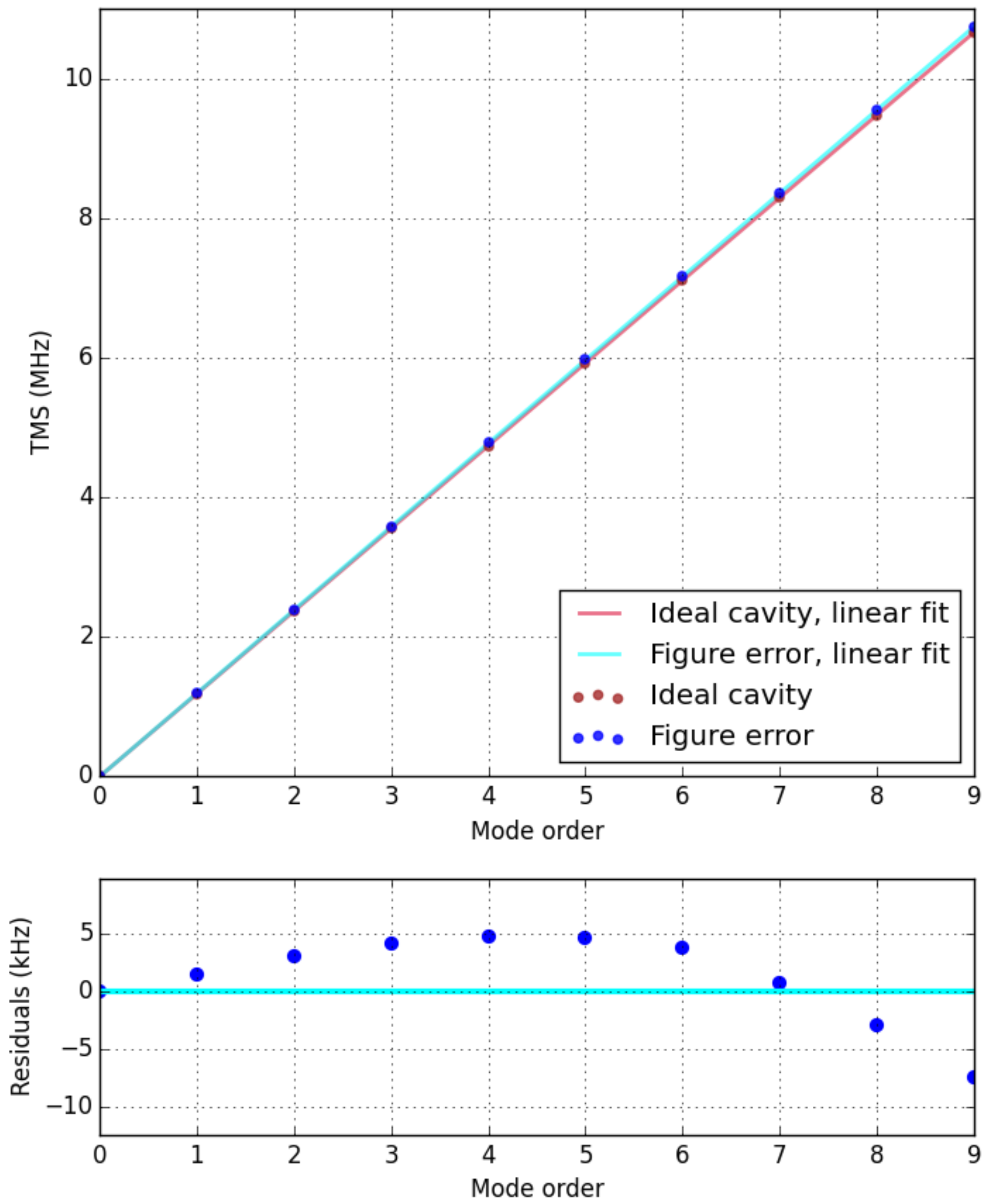


- Compare transmission peaks to ideal cavity. → Changes in ν_{FSR} and ν_{TMS} give information about cavity parameters.



Example

- TMS should vary linearly with mode order:
  $$\nu_{\text{TMS}} = \nu_{\text{FSR}} \left(\frac{m+n}{\pi} \right) \cos^{-1} \sqrt{\left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right)}$$
  → Perform linear fit to find new TMS
  → Calculate R_2, ETM radius of curvature
- FSR varies with cavity length:
  $$\nu_{\text{FSR}} = \frac{c}{2L}$$
  → Find FSR from distance between consecutive TEM_{00} peaks
  → Calculate effective cavity length L
- $\sigma \approx 4\,\text{nm}$ deviation induces $\approx \pm 5\,\text{kHz}$ shift of the TMS

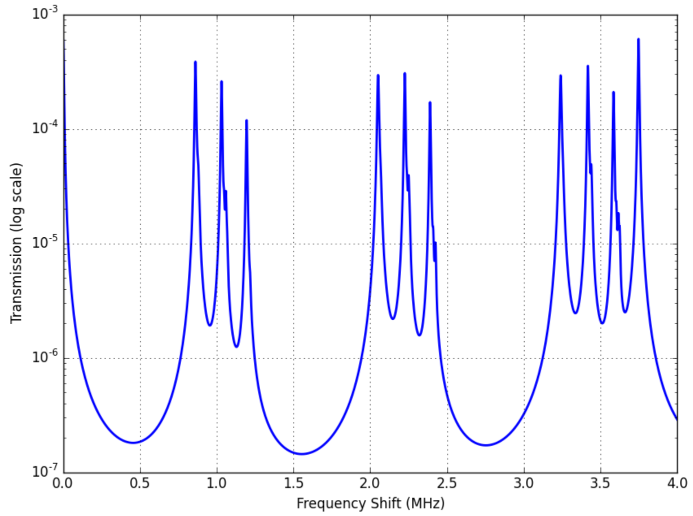


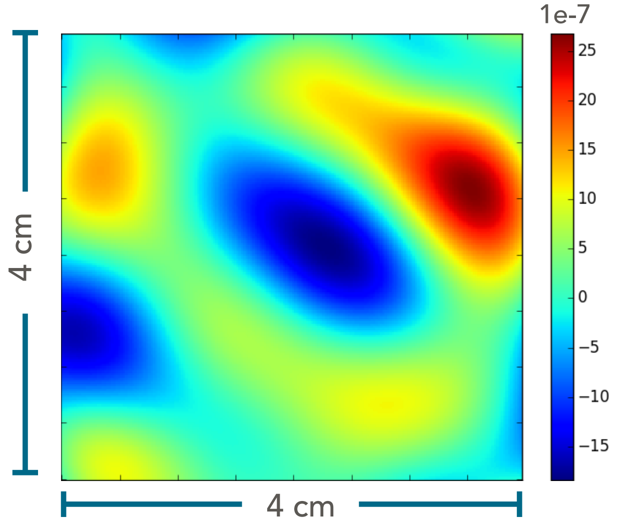
$$R_2 \approx 56.443\,\text{m}$$
$$L \approx 40.002\,\text{m}$$

Summary

- Goal: Determine optical losses in GW detector interferometers due to mirror figure error.

- Method: Use cavity transmission peaks as sensor for figure error.





→ Simulate realistic mirror perturbations with phase maps.

→ Inject higher-order laser modes into simulated cavity.

→ Use shifts in resonant frequencies of HOMs to learn about cavity parameters.

Next Step: Bayesian Inference

- Problem: Identify most probable phase map of a cavity mirror given a certain measurement of its transmission.

- One method: Markov chain Monte Carlo (MCMC)

→ Relies on Markov chain: process with property that, conditional on its *n*th step, its future values do not depend on its previous values.

→ Insert many phase maps and their corresponding transmission curves.

→ Accuracy of approximation for most probable phase map increases as input sample size increases.

Thank you!