Searching for Echoes of Gravitational Waves from the Coalescence of Exotic Compact Objects: A Bayesian Approach

Rico Ka Lok Lo

Department of Physics, The Chinese University of Hong Kong

LIGO SURF 2017
Final Presentation

August 25, 2017

Overview

Introduction

Gravitational Waves from Compact Binary Coalescences

Three stages of gravitational waves from CBCs

- **I**nspiral
- **M**erger
- **R**ingdown

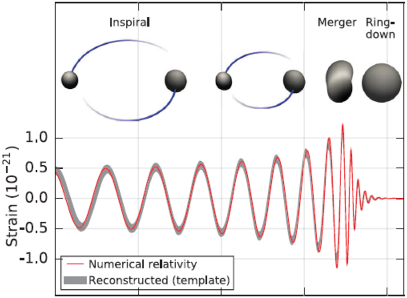


Figure: The numerical relativity simulation (in red) and reconstructed template (in gray) of GW150914. Figure taken from [1].

Echoes from Exotic Compact Objects (ECOs)

- ECOs: Compact objects that aren't White Dwarfs, Neutron Stars nor Black Holes
- Examples of ECOs as alternatives to black holes:
  - Gravastar
  - Fuzzball
- A common feature of these alternatives:
  Some **structure near the would-be event horizon**, at $r = 2M + l$, where $l \ll M$
  $\Rightarrow$ Echoes in late-time ringdown phase

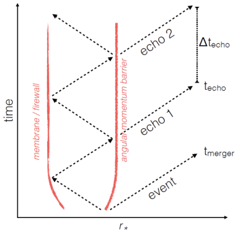


Figure: Echoes due to the reflections between the angular momentum barrier and the reflective surface

Abedi et al.'s Claim of Evidence

- Abedi, Dykaar and Afshordi [2] published a paper on December 2016, claiming that they have found *evidence of Planck-scale structure near the black hole event horizons* at a combined 2.9σ significance level on GW150914, LVT151012 and GW151226 using matched filtering technique.

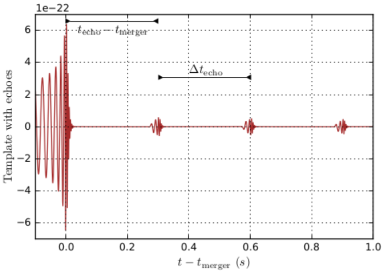


Figure: Best-fit echo template of GW159014 found by Abedi et al. Figure taken from [2].

Objectives of this project

- The data analysis method adopted by Abedi et al. [2] received a lot of critics, especially the background estimation
- In light of the results claimed in [2], we would like to
  1. Verify their results using their phenomenological model with **Bayesian analysis** instead
  2. Ultimately, we aim to verify their results using different models (robust evidence), and extract the physics of ECOs

Throughout the project, we will use geometrized unit $c = G = 1$

Why Bayesian?

- A model that fits the data best **does not imply the model gives the highest evidence**
- A more complicated model (i.e. with more free parameters) often fits noisy data better than a simpler model (i.e. with less free parameters)
- This is similar to *over-fitting* in regression
- Bayesian analysis embodies the **Occam's razor** and penalizes more complicated models automatically

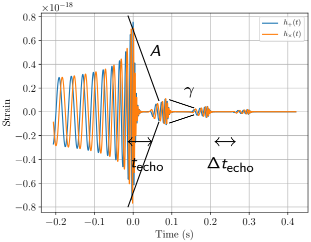
And More ......

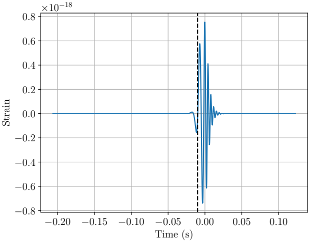
- How to sample the posterior probability distribution?
  Markov Chain Monte Carlo (MCMC)
- How to sample multi-modal distribution well?
  Affine-invariant ensemble sampler + Parallel tempering
- How to calculate the evidence?
  Thermodynamic integration
- How to reduce the correlation between samples?
  Burn-in + Thinning by Autocorrelation Length (ACL)
- How to compute the point estimates of the parameters?
  Maximum Likelihood Estimator (MLE), Maximum A Posteriori Estimator (MAP), Bayes Estimator
- How to estimate the precision of the point estimates?
  Minimum (Bayesian) Credible Interval
- *Refer to my final report for details*/Ask me later

Phenomenological Templates for Echoes

Template by Abedi et al.

Parameter	Description
Δt_{echo}	The time interval between each echo
t_{echo}	The time of arrival of the first echo
t_0	The time of truncation of the GW CBC template $\mathcal{M}_{\text{I}}(t)$ to produce the echo template $\mathcal{M}_{\text{TE,I}}(t)$
γ	The damping factor
A	The (overall) amplitude of the echoes



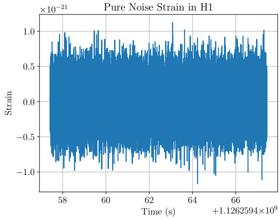


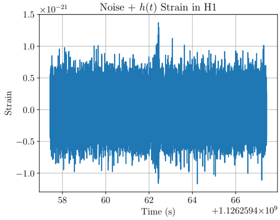
Bayesian Analysis on Fake Strain Data

1. Generating Fake Strain Data
2. Parameter Estimation on Fake Strain Data
3. Hypothesis Testing on Fake Strain Data
4. Estimation of Statistical Significance of Detection in Colored Gaussian Noise

Bayesian Analysis on Fake Strain Data

- Before performing the Bayesian analysis on **actual** strain data, it is educational to perform the same analysis on **fake strain data** first, namely strain data with **colored** Gaussian noise and IMR signal with echoes (which we will refer as *IMRE*) of known parameters
- Taken care of sky location and IFO antenna pattern





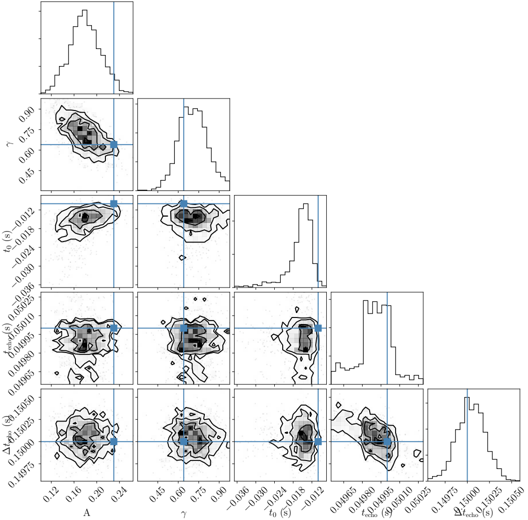
Technical details of the injection run

Generation of fake strain	
IMR approximant	SEOBNRv4_ROM
Analytical PSD	aLIGOZeroDetHighPower
Characteristic SNR in H1	50.0
Characteristic SNR in L1	38.3
Computational resources used	
Run time	17:18:19
Number of CPU cores used	5
Total CPU time	3 days 08:44:35
Memory usage	4.58 GB
Hyperparameters in MCMC	
Number of iterations	20000
Number of walkers	10
Number of temperature chains	8
Fraction of burn-in	0.1

Table: Technical details of the parameter estimation run

Corner Plot

- **Accurate** (close to injected value) and **precise** (narrow posterior distribution)



Prior Range and Summary Statistics

Parameter	Prior range
A	[0.01, 0.99]
γ	[0.01, 0.99]
t_0 (s)	[-0.05, 0.0]
t_{echo} (s)	[0, 0.1]
Δt_{echo} (s)	[0.1, 0.2]

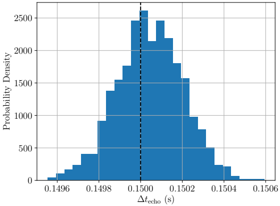
Table: The prior range of the echo parameters

Parameter	MLE/MAP	Mean	Median	SD	90% CI	Injected Value
A	0.184	0.180	0.178	0.0272	[0.133, 0.224]	0.23
γ	0.725	0.708	0.706	0.105	[0.548, 0.887]	0.64
t_0 (s)	-0.0135	-0.0158	-0.0149	0.00427	[-0.0220, -0.0104]	-0.01
t_{echo} (s)	0.0500	0.0499	0.0499	0.000137	[0.0496, 0.0500]	0.05
Δt_{echo} (s)	0.150	0.150	0.150	0.000159	[0.150, 0.150]	0.15

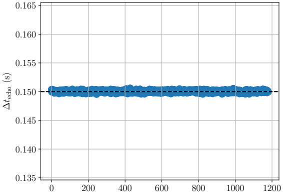
Table: The summary statistics of a parameter estimation run. 90% CI refers to the 90% minimum Bayesian credible interval

A Closer Examination: Δt_{echo}

- The MAP is very close the injected value (represented by the vertical dashed line)
- The 90% Bayesian credible interval ([0.1498, 0.1503]) is much narrower than the prior range ([0.1, 0.2]), that means the range is shrank by about 99.5%.
- The chain converges near the true value and hovers around it.



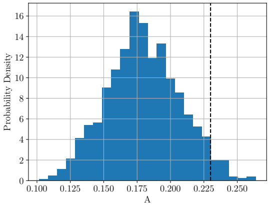
(e) The estimate of 1D marginal posterior probability density for Δt_{echo} by histogram



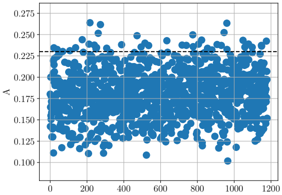
(f) The posterior samples of Δt_{echo}

A Closer Examination: A

- As for amplitude-related parameters A and γ, the parameter estimation is not as accurate and precise as those time-related parameters
- The range is not shrank as much as compared with Δt_{echo}, only by about 90.8%
- The posterior samples for A also did not converge well as compared with Δt_{echo}



(a) The estimate of 1D marginal posterior probability density for A by histogram



(b) The posterior samples of A

Hypothesis Testing on Fake Strain Data

- Ultimately, we want to perform hypothesis testing on a given strain data, and parameter estimation is just an intermediate step
- In the language of Bayesian inference, we have two competing models H_0 and H_1, which are just null hypothesis and alternative hypothesis in frequentist language, and they are

$$H_0 : \neg \text{Echo} \Rightarrow d = n + h_{\text{IMR}}$$
$$H_1 : \text{Echo} \Rightarrow d = n + h_{\text{IMRE}}$$

- Note that we *assumed that there is a gravitational wave signal in the data*, and we are only interested in knowing whether there are echoes in the data or not

Hypothesis Testing of the injection run

- The log Bayes factor of IMRE versus IMR is the **detection statistic** we are going to use to decide whether we claim there is an IMRE signal or IMR signal in the data
- Since the log Bayes factor $\ln B^{\text{Echo}}_{\neg\text{Echo}}$, and equivalently log odds ratio $\ln O^{\text{Echo}}_{\neg\text{Echo}}$ [1], is greater than 1, we can conclude, from the Bayesian point of view, that **the data favors the existence of echoes**, which is indeed the case

	Value
Log Evidence for IMRE in the data, $\ln Z_{\text{IMRE}}$	-40490.980
Log Evidence for IMR in the data, $\ln Z_{\text{IMR}}$	-40529.718
Log Bayes factor for IMRE vs. IMR, $\ln B^{\text{Echo}}_{\neg\text{Echo}}$	38.738

Table: A summary of log evidence and log Bayes factor calculated for hypothesis testing

[1] Since we assume that the prior odds is 1

Estimation of Statistical Significance of Detection in Colored Gaussian Noise

- A natural question to ask is that how significant statistically is the detection statistic
- Simply put, how strong is the evidence[2] of the detection?
- In the Bayesian school, there are different *empirical scales*, such as Jeffreys' scale, to interpret the strength of the evidence. However, they are **subjective and not universally applicable**
- Therefore, we are not going to use them

[2]Here by evidence we refer to the detection statistic, not the marginal likelihood

Posterior Probability of Data Containing IMRE Signal

- The posterior probability that the data (**injected** *with IMRE signal*) contains echoes, given that there is a gravitational wave signal is

$$\Pr(\text{Echo} \,|\, d, \text{GW}) = 1 - 1.50 \times 10^{-17}$$

- This tells you that, in the *Bayesian framework*, the probability of the data containing no echoes is as small as 1.50×10^{-17}!

The Frequentist Approach: p-value

- The Bayesian posterior probability fails to tell us that how likely the so-called evidence is **simply due to random background noise**, since we only consider merely one set of data!
- The frequentist approach can answer the following question: Given the null hypothesis H_0 is true, how likely (i.e. the probability) are the data going to be as *extreme or more extreme than* the observed data?
- The probability that we are looking for is exactly the frequentist **p-value**
- We need the null distribution of a test statistic T to calculate the p-value

Sampling the Null Distribution

- To sample the null distribution of T in the case of colored Gaussian noise, we can perform a lot of runs with **IMR** injection (i.e. assuming that H_0 is true) and different realization of the noise
- In this way, we can see how likely the noise will cause the detection statistic as high as, or higher than, the detected value

Sampling the Null Distribution

- 8500 such runs were performed
- The detected value in the injection run stands out from the background, *consistent* with the posterior probability approach that the the detected value has a very high statistical significance
- But it also means that it will be difficult to estimate the p-value accurately

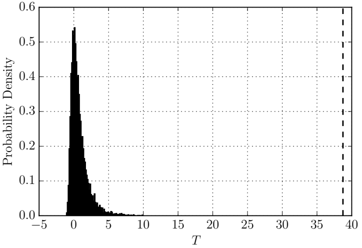


Figure: The histogram of the null distribution of T. The vertical dashline corresponds to the detected value

Gaussian KDE of the Null Distribution

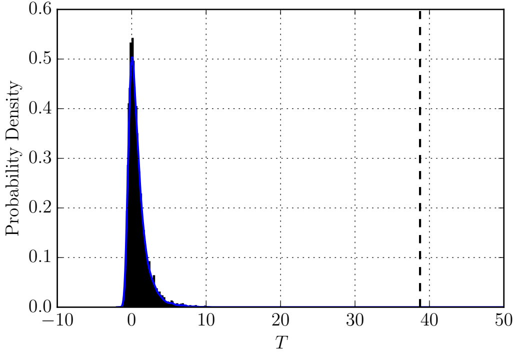


Figure: The estimated probability density function using Gaussian KDE is plotted in blue

Statistical Significance of the Detected Value

- The p-value obtained is

$$p = 9.52 \times 10^{-10}$$

- Which means that the detected value in the injection run has a statistical significance of

$$6.01\sigma$$

- Note that the injection is relatively **loud** and we are using *Gaussian noise*
- We don't expect such a high significance when we use real LIGO noise instead

Conclusions

- We successfully recovered the IMRE injection with colored Gaussian noise
- The recovered signal parameters were both accurate and precise
- We can estimate the background and hence the statistical significance of the detection
- This means that
  1. The analysis method can **find** an IMRE signal, and **infer** its parameters, if there is one in the real LIGO strain data
  2. Most importantly, we can report the statistical significance of a detection
  3. The pipeline was properly implemented

On-going Work and Future Work

- On-going:
  1. Estimate the background for GW150914 event
  2. Optimize and speed up the pipeline
- Future work:
  1. Perform the same analysis on LVT151012, GW151226, GW170104, ...
  2. Repeat the analysis using approximants by Mark et al. and Nakano et al.
  3. Implement the approximants in lalsuite such that we can utilize lalinference

Acknowledgement and References

Acknowledgement

My sincere gratitude to

- *Mentors*:
  - Alan Weinstein (Caltech)
  - Tjonnie Li (CUHK)
- *Zachary Mark* (Caltech), the first author of [3]
- *Rory Smith*
- *LIGO Laboratory*
- *LIGO SURF Program and Caltech SFP Office*
- Members of CUHK GW Group

Acknowledgement

The summer research would not be possible without

- National Science Foundation (NSF)
- Global Learning Award Scheme, Wu Yee Sun College, CUHK
- Charles K. Kao Research Exchange Scholarship
- CN YANG Scholarship
- Department of Physics, CUHK

References I

B. P. Abbott et al.
Observation of gravitational waves from a binary black hole merger.
Phys. Rev. Lett., 116:061102, Feb 2016.

J. Abedi, H. Dykaar, and N. Afshordi.
Echoes from the Abyss: Evidence for Planck-scale structure at black hole horizons.
ArXiv e-prints, December 2016.

Z. Mark, A. Zimmerman, S. M. Du, and Y. Chen.
A recipe for echoes from exotic compact objects.
ArXiv e-prints, June 2017.

Q&A

The End