

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY  
- LIGO -  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Technical Note	<b>LIGO-T1800042-v5</b>
<b>The A+ design curve</b>	
L. Barsotti, L. McCuller, M.Evans, P. Fritschel	

This is an internal working  
note of the LIGO project.

**California Institute of Technology**  
**LIGO Project, MS 18-34**  
**Pasadena, CA 91125**  
Phone (626) 395-2129  
Fax (626) 304-9834  
E-mail: [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)

**Massachusetts Institute of Technology**  
**LIGO Project, Room NW17-161**  
**Cambridge, MA 02139**  
Phone (617) 253-4824  
Fax (617) 253-7014  
E-mail: [info@ligo.mit.edu](mailto:info@ligo.mit.edu)

**LIGO Hanford Observatory**  
**Route 10, Mile Marker 2**  
**Richland, WA 99352**  
Phone (509) 372-8106  
Fax (509) 372-8137  
E-mail: [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)

**LIGO Livingston Observatory**  
**19100 LIGO Lane**  
**Livingston, LA 70754**  
Phone (225) 686-3100  
Fax (225) 686-7189  
E-mail: [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)

WWW: <http://www.ligo.caltech.edu/>

## Contents

1	Overview	3
2	Design curve	3
3	GWINC parameter file	6
4	Comparison with representative O2 noise curve and Advanced LIGO design	7
A	Range Integrand Plot	8

# 1 Overview

This document describes the A+ design curve and its parameters.

The design curve is based on a GWINC model<sup>1</sup> that includes calculations for the main fundamental noises of the interferometer, in particular: seismic noise, thermal noise and quantum noise. The comoving range values reported in the tables and in the plots are calculated with the GWINC `int73` function that includes cosmological effects, as documented in [T1500491](#). For a description of the February 2018 changes to GWINC incorporating the coating thermal noise parameters as reported by Gras and Evans in [P1700448](#), see [T1800044](#).

## 2 Design curve

The A+ model has been built by modifying the Advanced LIGO model (see [T1800044](#)) as follows:

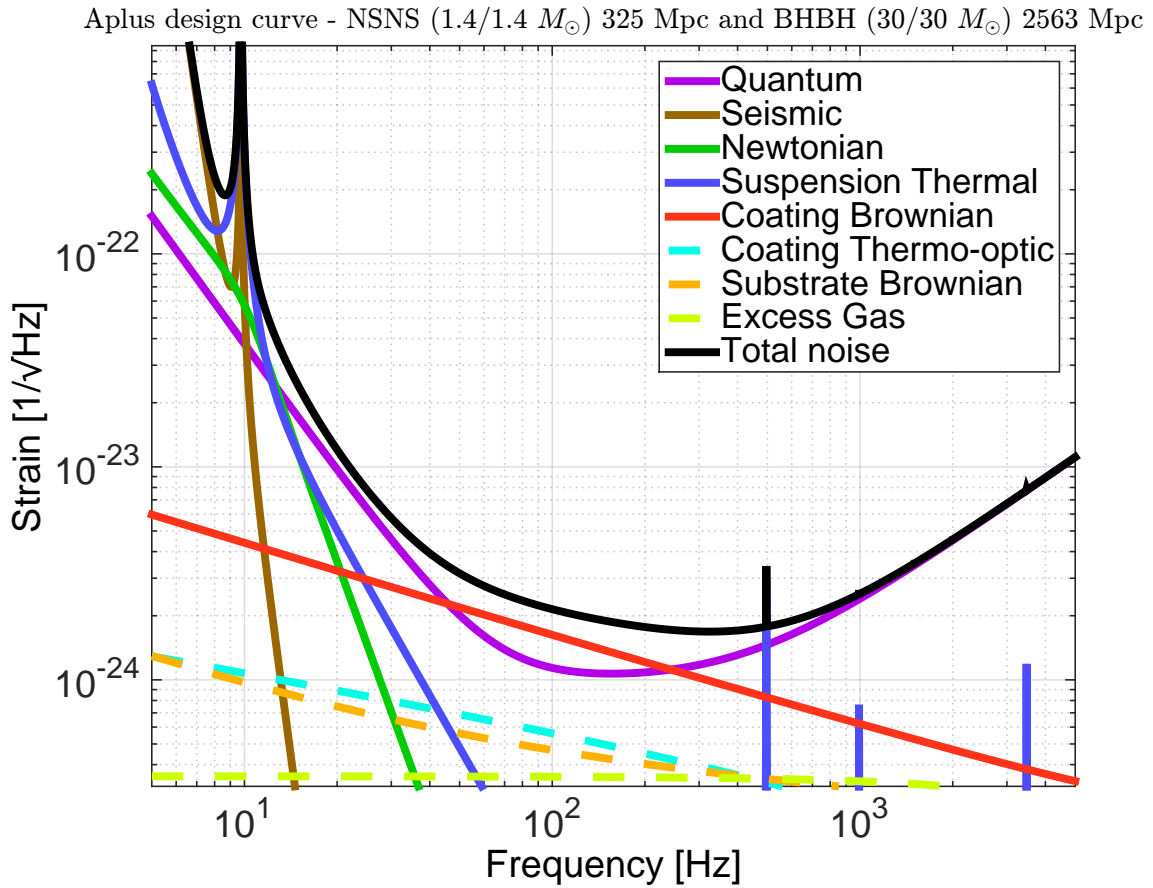
- coating thermal noise is a **factor of 2 lower** than the recent measurements on aLIGO samples reported in [P1700448](#). This is done by reducing the mechanical loss of the coating materials (tantala and silica) by a factor of 4;
- quantum noise has been calculated for 125W input power, 12 dB of squeezing injected with 10% loss in the readout and 5% loss in the squeezing input path, a 300m filter cavity with 60ppm round trip loss.

Figure 1 shows the A+ design curve with the main fundamental noise sources.

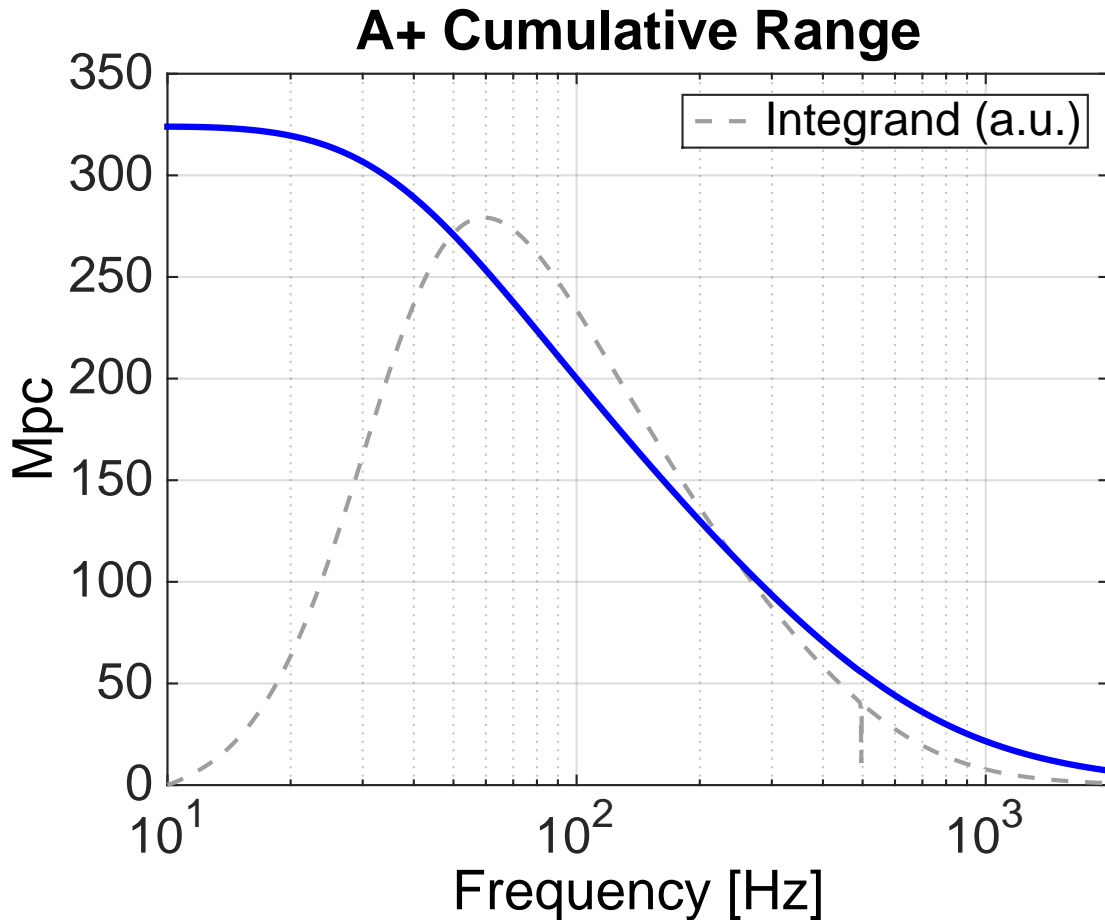
Figure 2 shows the corresponding cumulative range plot.

---

<sup>1</sup>GWINC svn revision 2894. GWINC can be downloaded from the [MIT svn](#).



**Figure 1:** Aplus design curve: coating thermal noise is a factor of 2 lower than in [P1700448](#); other relevant parameters are reported in table 1.



**Figure 2:** Cumulative range of the A+ design curve, that shows BNS range acquired as function of frequency. This plot answers the question: what is to be gained by including the integrand below a certain frequency? For example: assuming that we have achieved the A+ design sensitivity down to a given frequency  $f_0$ , let's say 30 Hz, how much more range can be acquired by reaching design sensitivity further down to, let's say, 10 Hz? In this particular example, the answer is: less than 20 Mpc. For a description of how this plot has been generated, see the appendix A.

### 3 GWINC parameter file

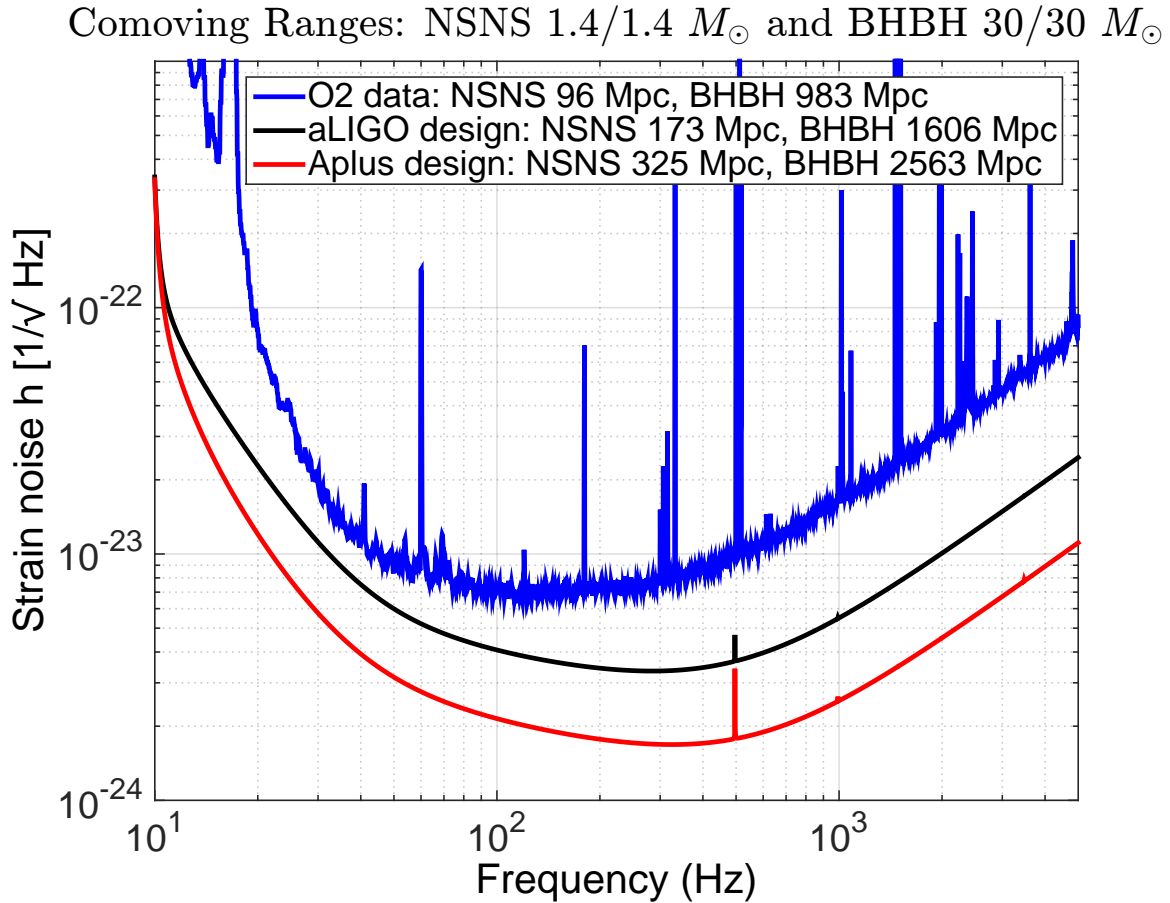
A list of parameters relevant for producing the A+ noise curve is presented in table 1.

**Table 1:** Gwinc parameters for the A+ design curve.

Gwinc Parameter	Value	Comment
ifo.Materials.Coating.Phihighn	$3.6 \times 10^{-4}/4$	tantala mechanical loss
ifo.Materials.Coating.Philown	$5 \times 10^{-5}/4$	silica mechanical loss
ifo.Laser.Power	125W	full power
ifo.Optics.Loss	37.5e-6	75 ppm round-trip
ifo.Optics.BSLoss	0.5e-3	
ifo.Optics.PhotoDetectorEfficiency	0.9	Improved readout loss
ifo.Optics.SRM.Transmittance	0.325	SRM transmission
ifo.Optics.SRM.Tunephase	0	SRM tuning
ifo.Optics.Quadrature.dc	$90 \cdot \pi / 180$	Readout phase
ifo.Squeezer.Type	'Freq Dependent'	Squeezing injection
ifo.Squeezer.AmplitudedB	12	SQZ amplitude [dB] (at the OPO)
ifo.Squeezer.SQZAngle	$0 \cdot \pi / 180$	SQZ phase [radians]
ifo.Squeezer.InjectionLoss	0.05	5% loss in SQZ path to IFO
fcParams.L	300m	filter cavity length
fcParams.Lrt	60e-6	round-trip loss in the cavity
fcParams.Te	1e-6	end mirror transmission
Gwinc Output	Value	Comment
Finesse	446	
Power Recycling Factor	43	
Arm power	750 kW	
Power on beam splitter	5.35 kW	
BNS range	325 Mpc	(comoving)
BBH range (30/30)	2.56 Gpc	(comoving, $z = 0.7$ )

## 4 Comparison with representative O2 noise curve and Advanced LIGO design

Figure 3 shows a comparison of the A+ design curve with a representative sensitivity measured during O2 (L1 interferometer, [G1701571](#)), and the updated Advanced LIGO design curve (see [T1800044](#)). The A+ BNS range is 1.9 times higher than the one achievable by Advanced LIGO operating at design sensitivity.



**Figure 3:** A+ design curve compared with the representative noise measured in O2 (L1 interferometer, [G1701571](#)), and an updated version of the Advanced LIGO design curve (see [T1800044](#) for details).

## A Range Integrand Plot

The computation of the cumulative range and range integrand show in figure 2 is not as obvious as one might expect, so we will show it explicitly here. The range integrand itself is straightforward

$$R_{\text{int}}(f) = \kappa \frac{f^{-7/3}}{\text{PSD}} \quad (1)$$

where  $\kappa$  is a constant and PSD is the interferometer **noise power spectrum** calibrated in  $(\text{strain})^2/\text{Hz}$ . This is defined such that the range is given by

$$R = \sqrt{\int_0^\infty R_{\text{int}}(f) df} \quad (2)$$

To produce a cumulative range plot, one must first consider in which direction the range accumulates. The mathematical answer to the question “**what is the range if the integrand below  $f_0$  is excluded**” is given by

$$R_{\text{low-}}(f_0) = \sqrt{\int_{f_0}^\infty R_{\text{int}}(f) df} \quad (3)$$

which clearly goes to  $R$  as  $f_0 \rightarrow 0$ . A mathematically similar and often more intuitive question is “**what is gained by including the integrand below  $f_0$** ” the answer to which is simply

$$R_{\text{low+}}(f_0) = R - R_{\text{low-}}(f_0) \quad (4)$$

which goes to zero as  $f_0 \rightarrow 0$  and to  $R$  as  $f_0 \rightarrow \infty$ .

The more mathematically obvious answer to this question might be

$$R_{\text{high-}}(f_0) = \sqrt{\int_0^{f_0} R_{\text{int}}(f) df}$$

but that is actually the answer to “what is the range if the integrand **above**  $f_0$  is excluded”, and is related to “what is gained by including the integrand **above**  $f_0$ ”, but is typically not as useful as  $R_{\text{low+}}$ .

The cumulative range plotted in figure 2 is  $R_{\text{low+}}(f)$ , and the expected use is to evaluate potential gains from improving low-frequency sensitivity. While the cumulative range can be useful, the range integrand itself also offers insight into the impact of potential sensitivity improvements and unlike the cumulative value it does not suffer from the need to consider carefully the question being addressed.

Making a useful plot of the integrand, however, can be a little tricky. The trouble arises from the need to use a log scale on the x-axis to make the low-frequency region visible; the log scale gives disproportionate area to low frequencies. Since the intent of plotting the integrand is to allow the reader to estimate the value of a sensitivity improvement in a given frequency band, the area under the integrand curve in that band should be proportional to



the potential available improvement. To satisfy the “area proportional to impact” criterion, when plotting with a logarithmic x-axis, the value plotted must have an additional factor of  $f$  since

$$\begin{aligned} R^2 &= \int_0^\infty f R_{\text{int}}(f) d(\log(f)) \\ &= \int_0^\infty R_{\text{int}}(f) df \end{aligned}$$

Thus, the integrand plotted in figure 2 is  $f R_{\text{low}+}(f)$ .