Searching for Lensed Gravitational Waves from Compact Binary Coalescences
Research Proposal

Li Ka Yue Alvin[1]
Mentor: Alan J. Weinstein[2]

[1]Department of Physics, The Chinese University of Hong Kong
[2]LIGO Laboratory, California Institute of Technology

LIGO SURF 2018 Program, California Institute of Technology
Date : 3rd May, 2018

Einstein's general relativity predicts the radiation of gravitational waves when masses move through the spacetime fabric. This was confirmed when LIGO (Laser-Interferometry Gravitational-wave Observatory) made the first detection of gravitational waves from a binary black hole merger on 14 September 2015. The success in gravitational waves detection opens a new window for scientists to study the Universe. In Einstein's general theory of relativity, it is also predicted that light rays bend when passing by masses in spacetime, which is known as gravitational lensing. Because of the similar nature of gravitational waves and light waves, gravitational waves will also be gravitationally lensed, resulting in multiple signals which differ in arrival times and amplitudes. Since the amplitudes of such signals may differ, there are cases that they are not identified as signals. In this research, we aim to search for auxiliary signals of existing LIGO events from compact binary coalescences. Our major objective is to propose and test a method to re-identify possible lensed signals which may have been insufficiently loud to be distinguishable from detector noise. We will further attempt to infer the intrinsic properties of the gravitational lenses from the lensed gravitational wave signals identified.

I. Introduction

A. Properties of Gravitational Waves

According to Albert Einstein's general theory of relativity in 1915 [1], the Universe can be perceived as a fabric of spacetime. Masses like black holes and neutron stars on this fabric produce spacetime curvature [2]. When masses move in spacetime, they cause ripples like water waves generated when one throws a stone into water. Such ripples are known as **gravitational waves**. General relativity predicts that there are four fundamental properties for gravitational waves, namely their speed, polarization, weak interaction with matter and ability to be lensed gravitationally.

The speed of gravitational waves is predicted by General Relativity to be the same as the speed of light c in vacuum [3]. This has been experimentally confirmed by the detection of gravitational wave from the neutron star inspiral GW170817 in 2017, which constrained the difference between the speed of light and the speed of gravitational wave to between -3×10^{-15} and $+7 \times 10^{-16}$ [4] times the speed of light c.

For the polarization of gravitational waves, as discussed in Ref. [5], we imagine placing a circular ring of test masses on the $x-y$ plane with its center coinciding with the origin. If we assume there exists a transverse - traceless (TT) gravitational wave propagating in the z-direction, then the effect of such gravitational wave is constrained to be on the $x - y$ plane only. Under the influence of the wave, the test masses ring can exhibit two orthogonal deformation modes.

As shown in Figure 1, when a **plus (+) polarized** gravitational wave passes through our ring of test masses, the ring is stretched along the y-direction and then along the x-direction into an ellipse of the same area as the original circle throughout one period. On the other hand, if the gravitational wave passing through is **cross** ($\times$) **polarized** instead, the ring will be stretched along the $y = x$ and $y = -x$ line in a similar way as for plus (+) polarized gravitational wave. We can see that gravitational waves can be polarized in two particular modes, namely the plus (+) polarization, and cross ($\times$) polarization. The effect of stretching and shrinking of proper lengths between test masses in the ring by polarized gravitational waves is applied to the detection of gravitational waves. In particular, detectors including LIGO and VIRGO detect gravitational waves using interferometry.

B. Detection of Gravitational Waves

A schematic overview of the gravitational wave detector used by LIGO is shown in Figure 2 [6]. It is a Michelson interferometer consisting of two arms, each of 4 km long. A laser beam is incident on a beam splitter, which splits the incident laser beam into two beams propagating along the two arms of the interferometer. At the end of the arm, a mirror reflects the beams which then rejoin at the beam splitter and is finally collected by a photodetector to observe the interference pattern.

The interference of the two laser beams is set to be destructive. Alternatively, when there are alterations to the

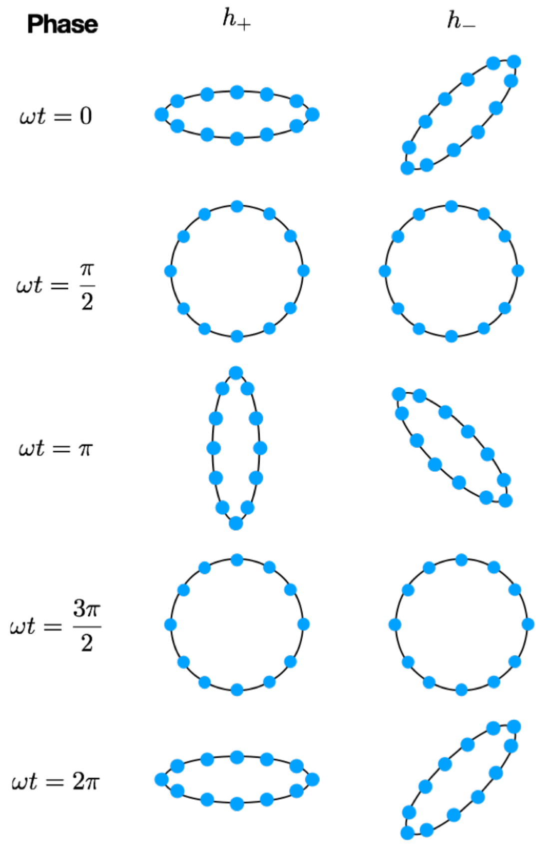

Fig. 1. Two orthogonal deformation modes within one period of the test mass ring in response to a TT-gravitational wave. The upper row refers to the plus polarization (denoted by +) and the lower row refers to the cross polarization (denoted by $\times$) of the gravitational wave. Image reproduced from [5].

lengths of the arms which cause a path difference between the two laser beams, a constructive interference pattern will be observed. The change in arm lengths is not necessarily caused by gravitational waves because there is also noise which can cause such effect. These noises include seismic noises, thermal noises, gravity-gradient noises and quantum noises [7].

In order to detect gravitational waves, we must constrain ourselves to those which have a sufficiently large perturbation to spacetime. Typically, we focus on four types of gravitational waves, namely Continuous Waves, Stochastic Waves, Bursts and Compact Binary Inspirals, which is the focus of this research. Compact Binary Inspirals are the signals generated when two compact objects, for instance, neutron stars and black holes, orbit about their common center-of-mass before they collide. Among the four mentioned types of gravitational waves, Compact Binary Coalescences are sources of gravitational waves with well modelled waveforms compared to other kinds of gravitational wave sources, and hence one can use a technique called **matched filtering** to search for such signals.

We now outline the major steps in analyzing gravitational-wave data [5]. Currently, matched filtering is a method to

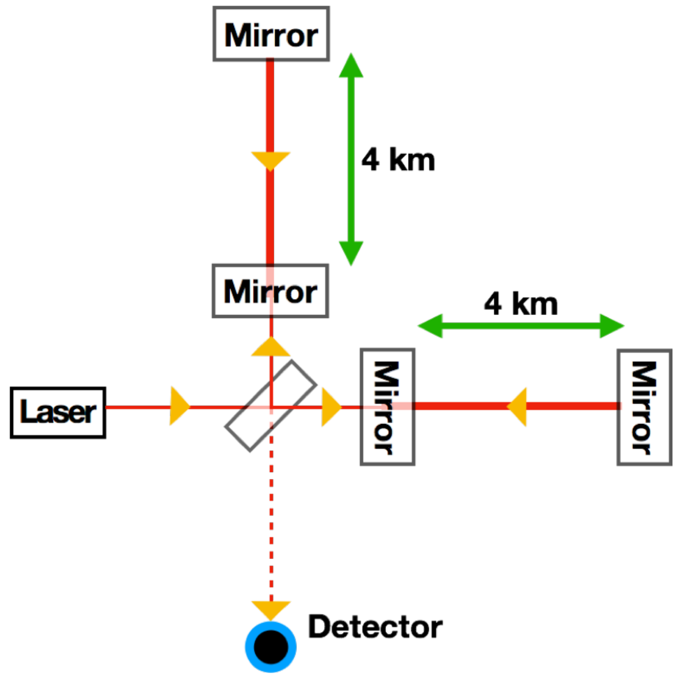

Fig. 2. A schematic overview of the gravitational wave detector used by LIGO. Image reproduced from [6].

search for gravitational waves. The principle of matched filtering is to slide templates of an expected waveform from an astrophysical event across the received data and compare the similarity between the two.

We denote $s(t)$ as the signal received from a detector, $n(t)$ as the background noise and $h(t)$ as the gravitational wave signal (if it exists). $s(t)$ is the sum of $n(t)$ and $h(t)$, that is

$$s(t) = n(t) + h(t). \tag{1}$$

If we have a filter $P(t)$, we may define

$$\hat{s} = \int s(t)P(t)dt. \tag{2}$$

We denote $\langle S \rangle$ and N as the expectation value and root mean square value of $\hat{s}$ if a gravitational wave signal is included in the data respectively. Then we have

$$\begin{aligned} \langle S \rangle &= \int \langle s(t)P(t) \rangle dt \\ &= \int \langle (n(t) + h(t))P(t) \rangle dt \\ &= \int \langle h(t)P(t) \rangle dt \\ &= \int h(\tilde{f})\tilde{P}^*(f)df, \end{aligned} \tag{3}$$

where we have taken $\langle n(t) \rangle = 0$ (since the noise is assumed to be random and gaussian) and the tilde $(\tilde{A})$ denotes the Fourier-transformed function of A.

Also, if $h(t) = 0$, we have

$$
\begin{aligned}
N^2 &= \langle \hat{s}^2 \rangle - \langle \hat{s} \rangle^2 \\
&= \langle \hat{s}^2 \rangle dt \\
&= \int\int P(t)P(t')\langle n(t)n(t')\rangle dt dt' \\
&= \frac{1}{2}\int S_n(f)|\tilde{P}(f)|^2 df,
\end{aligned} \tag{4}
$$

where $S_n(f)$ denotes the power spectral density. We can therefore define **signal-to-noise ratio (SNR)** as

$$
\begin{aligned}
\rho &= \frac{\langle S \rangle}{N} \\
&= \frac{\int \tilde{h}(f)\tilde{P}^*(f)df}{\sqrt{\int \frac{1}{2}S_n(f)|\tilde{P}(f)|^2 df}}.
\end{aligned} \tag{5}
$$

Furthermore, we define the inner product between two functions $x(t)$ and $y(t)$ to be:

$$
\langle x, y \rangle = \Re\Bigg(\int_{-\infty}^{\infty} \frac{\tilde{x}^*(f)\tilde{y}(f)}{\frac{1}{2}S_n(f)} \Bigg) df. \tag{6}
$$

Consequently, we can have equation (5) simplified as

$$
\rho^2 = \frac{\langle d, h \rangle^2}{\sqrt{\langle h, h \rangle}}, \tag{7}
$$

where d is the data received in the detector. With an optimal matched filter $P(t)$, and provided that the identical gravitational wave signal can be seen in coincidence between two or more detectors, LIGO detectors can detect inspiral signals with an SNR as low as 6 [8].

In reality, the rate of gravitational wave events occurring is expected to as low as about a few per year [7]. To avoid mistaking abnormal noise mimicking events as signals, we need to find out the **false alarm rate (FAR)**, which is how often an abnormal noise signal mimicking event can be measured. The smaller the FAR is, the more plausible the candidate is a real astrophysical event. FAR refers to the rate of detecting an unusual noise signal mimicking event. The FAR for any signal is estimated by [9]

$$
\text{FAR} = \frac{N}{\Sigma_i T_i}, \tag{8}
$$

where N is the total number of background triggers similar to the one which we consider as a real signal, and T_i is the analyzed time interval in the i^{th} background trial.

C. Gravitational Lensing

As predicted by General Relativity, since masses can curve spacetime, the path of a light ray from a source can be bent and deflected before reaching the observer (See Figure 3) Such effect is known as gravitational lensing, in the sense that it is similar to light rays being bent by optical lenses, but in this case the "lenses" are masses instead. In particular, since a source emits light rays in all direction, light rays propagating along different directions are bent differently and may, therefore, form multiple images. The images can vary in arrival time and amplitudes.

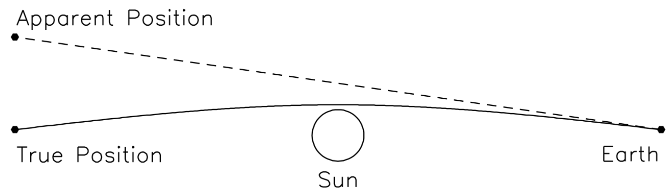

Fig. 3. Light rays from a source are bent because of a gravitational lens in between the source and the observer. Image from [10].

Due to lensing, there are time delays among the light waves of images. In the discussion for light, there are two major contributions to the delay, namely geometrical and gravitational time delay. Gravitational lensing occurs when light rays pass through spacetime perturbed by masses. This will form multiple signals which differ in amplitude and time of arrival. The difference in arrival time is due to 1) The path lengths travelled from the images to the observer vary, which is known as the gravitational factor, and 2) The effective speed of light can be different under the influence of gravitational lensing, resulting in arrival time delay. This is known as the geometrical factor. For the purpose of our research, we will focus on the contribution from the geometrical factor to avoid confusion. The geometrical factor for the time delay is about the path difference between the images and detectors under the influence of the gravitational lens. As shown in Figure 4, the angular diameter distances from the observer to the lens and the source are given by D_L and D_S respectively, and that from the source to the lens is D_{LS}. When we compare the path difference between the unperturbed ray (dotted line between the observer and the source in figure 4), that is when the lens is absent, and the lensed ray (solid lines between the observer and the source), we have [11]

$$
\vec{\xi} = \frac{D_L D_{LS}}{D_S}(\vec{\theta} - \vec{\theta_S}), \tag{9}
$$

where ξ is the separation between the two rays at the lens, $\vec{\theta}$ is the two-dimensional angle between the horizontal of the observer and the point where the gravitational waves strike the lens, and $\vec{\theta_S}$ is the angle between the horizontal of the observer and the source.

With this, we have the geometrical path difference $\Delta\lambda$ between the unperturbed ray and lensed ray is given by

$$
\Delta\lambda = \frac{\vec{\xi}(\vec{\theta} - \vec{\theta_S})}{2}. \tag{10}
$$

Finally, the geometrical time delay Δt due to gravitational lensing is given by

$$
\Delta t = (1 + z_d)\frac{D_L D_{LS}}{2 D_S c}(\vec{\theta} - \vec{\theta_S})^2, \tag{11}
$$

where z_d denotes the gravitational redshift. From the calculation of the time delay, we are able to infer the distance of the lens from the observer.

Light, as a kind of electromagnetic waves, can be deflected under gravitational lensing effect. However, the study of gravitational lensing of light encounters difficulties from the blocking of light by dust clouds in the Universe, as well as the large noise which screened the light signals [12]. General relativity also predicts that gravitational waves, having a similar nature as light, can also be lensed gravitationally, producing multiple signals, and same as light, is achromatic. In contrast to light, gravitational waves are not disturbed by the dust clouds between the source and observing point. Over the past two years, more than six gravitational wave detections have been successfully made [13], [14], [15], [16], [17], [18], which have confirmed the prediction of the existence of gravitational waves. Among the four predicted fundamental properties of gravitational waves, we are only left with the ability to be gravitational lensed unproved. Therefore, it is now the right time for us to start searching for lensed gravitational wave signals so as to test the final property of gravitational wave predicted from general relativity.

Gravitationally lensed gravitational wave signals may differ in amplitudes and time of arrival. In fact, the lensed waveform has an amplitude $h^{\text{lensed}}_{+,\,\text{x}}(f)$ given by [19] [20]

$$h^{\text{lensed}}_{+,\,\text{x}}(f) = F(\omega, y) h^{\text{unlensed}}_{+,\,\text{x}}(f), \tag{12}$$

where $h^{\text{unlensed}}_{+,\,\text{x}}(f)$ denotes the amplitude of the unlensed gravitational waves, and $F(\omega, y)$ is the amplification function given by

$$F(\omega, y) = \exp\Bigg[\frac{\pi\omega}{4} + i\frac{\omega}{2}\Bigg(\ln(\frac{\omega}{2}) - \frac{\sqrt{y^2+4}-y^{2}}{4} + \ln(\frac{\sqrt{y^2+4}+y}{2}\Bigg)\Bigg]\Gamma(1-\frac{i}{2}\omega) \times {}_1F_1(\frac{i}{2}\omega, 1; \frac{i}{2}\omega y^2), \tag{13}$$

where $h^{\text{lensed}}_{+,\,\text{x}}(f)$ is the waveform without lensing, Γ is the complex gamma function, ${}_1F_1$ is the confluent hypergeometric function of the first kind, $\omega = 8\pi M_{\text{Lz}} f$ is the dimensionless frequency, $M_{\text{Lz}} = M_{\text{L}}(1+z_L)$ is the redshifted lens mass, $y = \frac{D_L S}{\Xi_0 D_S}$ is the source position, $\Xi_0 = (\frac{4 M_L D_L D_{\text{LS}}}{D_S})^{\frac{1}{2}}$ is a normalisation constant, and M_L and z_L are the lens mass and redshift respectively. From finding the amplitude of the lensed gravitational waves, we can infer the mass of the lens M_L.

For a point mass lens, particularly for compact objects like black holes or stars, the time delay is typically $2 \times 10^3 s \times \left(\frac{M_L}{10^8 M_\odot}\right)$. Furthermore, for gravitational waves from coalescence of super massive black holes of mass $10^4 - 10^7 M_\odot$ under the lensing effect of a point mass lens of mass in the range $10^6 - 10^9 M_\odot$, then the typical time delay will be $10 - 14s$ [20]. Therefore, for gravitational waves from blackholes of masses lower than $10^4 M_\odot$, we would expect a time delay in the range $10^1 - 10^3$s.

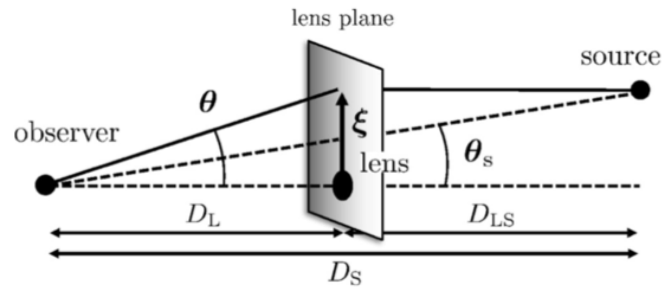

Fig. 4. In this figure, D_L denotes the distance between the lens and the observer, D_S denotes the distance between the observer and the image, D_{LS} denotes the distance between the lens and the image, $\vec{\theta}$ denotes the two dimensional angle between the observer and lensing point, and $\vec{\theta_s}$ denotes the two dimensional angle between the source and the observer. Image from [11].

II. Objectives

The main objective of this research is to re-identify lensed gravitational wave signals which may be originally regarded as background noise. We will attempt to do template searches on gravitational wave data to look for possible candidates of the lensed images of existing detections. After identification of lensed signals, we will attempt to infer the intrinsic properties of the gravitational lens.

III. Approach

The research can be classified into several steps. The first step is to design possible models for the candidates of the lensed gravitational waves. In order to do so, studies about gravitational lensing and the calculations of the effect of gravitational lens on the amplitude and arrival time of gravitational wave signals have to be made. Some of the members in the CUHK gravitational waves research group have already been doing related research [21], [22], which can provide relevant readings and advice on this project.

The next step is to develop a method to search for lensed gravitational waves. We attempt to follow the GstLAL search to look for lensed gravitational wave [23]. Figure 8 shows the schematic flow of the GstLAL search pipeline. For our search, we may have to make some modification to some of the internal steps in the search pipeline depending on the method used. After we identify the lensed signals, we will go on to infer the intrinsic physical properties of the gravitational lens by measuring the time delays and amplitude ratios of the lensed gravitational wave signals. We will be able to perform a parameter estimation on the mass of the gravitational lens, and also the distance between the lens and observer D_L, and that between the observer and the source D_S.

Typically, from the time delay we can infer the distance between the observer and the lens. And, from the amplitude of the lensed gravitational wave, we can, from equation [12] and [13], infer the mass of the lens M_L.

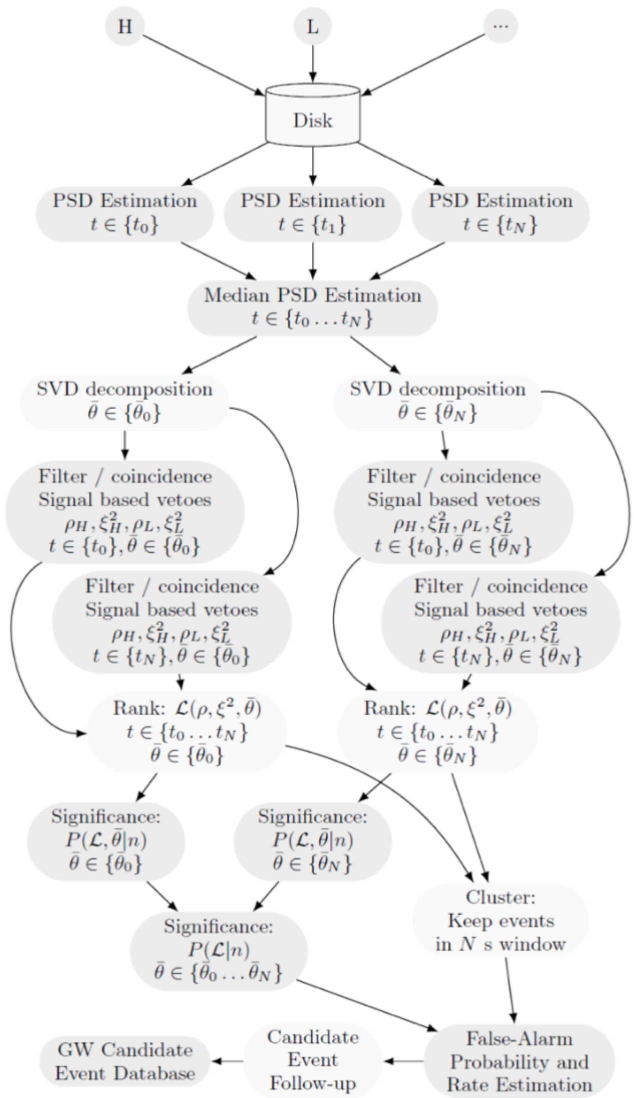

Fig. 5. A schematic flow of the GstLAL search pipeline. Image from [24].

IV. Work Plan

1) **Pre-Surf Period:**

- Literature review on gravitational lensing, gravitational wave data analysis and related mathematics
- Construct models for lensed gravitational wave signals
- Complete the mathematical formulation for the SNR, FAR and related data analysis for this research
- Learn how to run a GstLAL search
- Run template searches on the drafted models to observe the effect, and modify them in accordance.

2) **SURF Period:**

- 19 June, 2018 : Arrive at Caltech
- 19 June - 9 July, 2018 [Week 1-3] :
  (i) Complete the follow-up work during the Pre-Surf Period
  (ii) Rerun searches using the modified models
  (iii) Analyse the results and suggest further modifications on the models
- 10 July, 2018 : First Interim Report to be submitted
- 11 July - 20 July, 2018 [Week 4-5] :
  (i) Final run using the modified models
  (ii) Analyse the results and prepare data analysis for future inference of gravitational lens
- 21 July - 7 August, 2018 [Week 5-7] :
  (i) Attempt to infer the intrinsic properties of the gravitational lens
  (ii) Complete the data analysis for the research
  (iii) Start preparing the second interim report and the final paper
- 8 August 2018 : Second Interim Report to be submitted
- 9 July - 17 August, 2018 [Week 8-10] :
  (i) Data analysis
  (ii) Preparation of final presentation and final paper
- 23 August 2018 : Summer Seminar Day

3) **Post Surf Period:**

- 24 August 2018 : Returning to Hong Kong
- Complete final paper
- Follow-up work on the summer research
- Continue related research in gravitational wave science

REFERENCES

[1] Albert Einstein and M. Grossmann. Kovarianzeigenschaften der feldgleichungen der auf die verallgemeinerte relativitätstheorie gegründeten gravitationstheorie. *Zeitschrift für Mathematik und Physik*, 63:215–225, 1915.

[2] James Hartle. *Gravity - An Introduction to Einstein's General Relativity*. Pearson Education, 2003.

[3] Albert Einstein. Näherungsweise integration der feldgleichungen der gravitation. *Preussische Akademie der Wissenschaften, Sitzungsberichte*, pages 688–696, 1916.

[4] LIGO-Scientific-Collaboration, Virgo-Collaboration, Fermi-Gamma-Ray-Burst-Monitor, and INTEGRAL. Gravitational waves and gamma-rays from a binary neutron star merger: Gw170817 and grb 170817a. *The Astrophysical Journal Letters*, 848, 2017.

[5] Tjonnie G. F. Li. *Extracting Physics from Gravitational Waves*. Springer International Publishing, 2015.

[6] https://www.ligo.caltech.edu/.

[7] B.S. Sathyaprakash and B.F. Schutz. Physics, astrophysics and cosmology with gravitational waves. *Living Rev. Relativity*, 2, 2009.

[8] https://losc.ligo.org.

[9] Drew Garvin Keppel. *Signatures and dynamics of compact binary coalescences and a search in LIGO's S5 data*. PhD thesis, California Institute of Technology, 2009.

[10] Ramesh Narayan and Matthias Bartelmann. Lectures on gravitational lensing. -, 1997.

[11] Ryuichi Takahashi. Arrival time differences between gravitational waves and electromagnetic signals due to gravitational lensing. *ApJ*, 835, 2016.

[12] Steven T. Myers. Scaling the universe: Gravitational lenses and the hubble constant. *Proc. Natl. Acad. Sci. USA*, 1999.

[13] B.P. Abbott. Observation of gravitational waves from a binary black hole merger. *Phys. Rev. Lett.*, 166, 2016.

[14] B.P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration). Gw151226: Observation of gravitational waves from a 22-solar-mass binary black hole coalescence. *Phys. Rev. Lett.*, 116, 2016.

[15] B.P. Abbott et al. (LIGO Scientific and Virgo Collaboration). Gw170104: Observation of a 50-solar-mass binary black hole coalescence at redshift 0.2. *Phys. Rev. Lett.*, 118, 2017.

[16] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and V. B. Adya. Gw170608: Observation of a 19 solar-mass binary black hole coalescence. *The Astrophysical Journal Letters*, 851(2), 2017.

[17] B.P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration). Gw170814: A three-detector observation of gravitational waves from a binary black hole coalescence. *Phys. Rev. Lett.*, 119, 2017.

[18] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration). Gw170817: Observation of gravitational waves from a binary neutron star inspiral. *Phys. Rev. Lett.*, 119, 2017.

[19] Ehlers J. Falco E.E. Schneider, P. *Gravitational Lenses*. Springer International Publishing, 1992.

[20] Ryuichi Takahashi and Takashi Nakamura. Wave effects in gravitational lensing of gravitational waves from chirping binaries. *Astrophys.J.*, 593:1039–1051, 2003.

[21] Kwun-Hang Lai, Otto Hannuksela, and Tjonnie Li. Discovering intermediate-mass black hole lenses through gravitational wave lensing. *LIGO DCC*, 2018.

[22] Kwan-Yeung Ng, Wang-Kei Wong, and Tjonnie Li. Precise ligo lensing rate predictions for binary black holes. *LIGO DCC*, 2017.

[23] Lo Ka-Lok. Searching for gravitational waves from the coalescence of high mass black hole binaries. -, 2016.

[24] C. Messick, K. Blackburn, P. Brady, P. Brockill, K. Cannon, R. Cariou, S. Caudill, S. J. Chamberlin, J. D. E. Creighton, R. Everett, C. Hanna, D. Keppel, R. N. Lang, T. G. F. Li, D. Meacher, A. Nielsen, C. Pankow, S. Privitera, H. Qi, S. Sachdev, L. Sadeghian, L. Singer, E. G. Thomas, L. Wade, M. Wade, A. Weinstein, and K. Wiesner. Analysis framework for the prompt discovery of compact binary mergers in gravitational-wave data. , 95(4):042001, February 2017.