Calibrating semi-analytic VT's against reweighted injection VT's

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A tale of two VT's

- semi-analytic VT's
  - single-detector model using a reference PSD, operating for some time T, based on a detection threshold
  - gives us VT on a grid of intrinsic parameters, λ
    - e.g., $\lambda = (m_1^{\text{source}}, m_2^{\text{source}}, \chi_{1z}, \chi_{2z})$, other spins typically set to zero
- injection VT's
  - perform an injection campaign, and count the number of detections by your pipeline (e.g., pyCBC, GstLAL, cWB) to get an average VT for the injected population, Λ – $\langle VT \rangle(\Lambda)$
  - re-weigh injections to get alternate populations, $\langle VT \rangle(\Lambda^*)$ (Tiwari 2018)

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Technical issues with injection VT's

- Population inference ideally needs $VT(\lambda)$ – function of intrinsic parameters
- Injection code provides $\langle VT \rangle(\Lambda)$ – function of population hyperparameters
- Injection code limited to broad populations – lots of injections needed to cover all possible narrow populations, due to Monte Carlo error
- Reweighting code is rather slow, problematic for already slow population inference methods

Calibrating semi-analytic VT's to injection VT's

- Solution: assume a parameterized relationship between VT_{analytic} and VT_{inj}, and optimize for the free parameters ζ

$$VT_{\text{inj}}(\lambda) \approx VT_{\text{analytic}}(\lambda)\, f(\lambda; \zeta)$$

Solving for the calibration prescription

- basis functions $\{g_\alpha(\lambda)\}_\alpha$ with calibration coefficients $\{\zeta_\alpha\}_\alpha$ to be solved for

$$f(\lambda; \zeta) = \sum_\alpha \zeta_\alpha \, g_\alpha(\lambda)$$

- linear least squares problem:
  - Compute $y_k \equiv \langle VT \rangle_{\mathrm{inj}}(\Lambda_k)$ on a discrete grid of Λ_k's
  - Compute the "design matrix"

$$H_{k,\alpha} = \int p(\lambda \mid \Lambda_k) \, VT_{\mathrm{analytic}}(\lambda) \, g_\alpha(\lambda) \, \mathrm{d}\lambda$$

  - Compute the least squares solution to the coefficients

$$\boldsymbol{\zeta} = (\mathbf{H}^\top \boldsymbol{\gamma} \mathbf{H})^{-1} \mathbf{H}^\top \boldsymbol{\gamma} \mathbf{y}$$

($\boldsymbol{\gamma}$ is the inverse covariance matrix for $\langle VT \rangle_{\mathrm{inj}}$ estimates)

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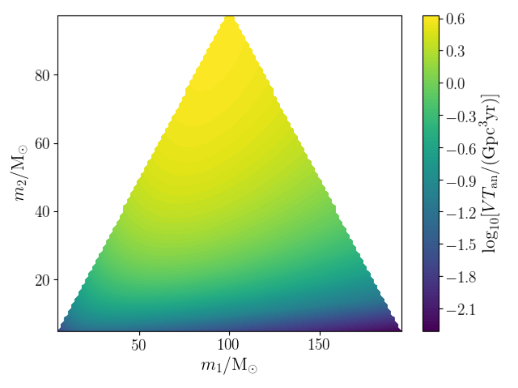
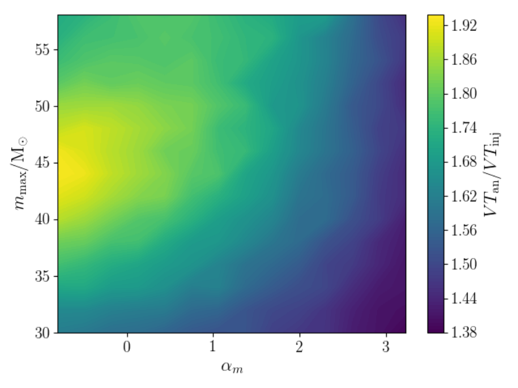
Comparison – no calibration





$$VT_{\rm inj}(m_1, m_2) \approx VT_{\rm an}(m_1, m_2)$$

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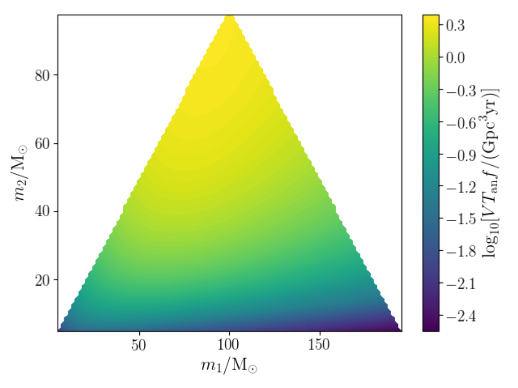
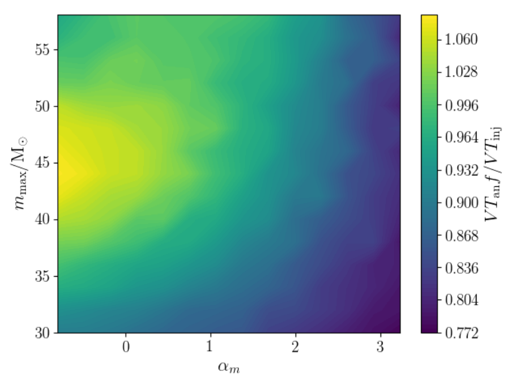
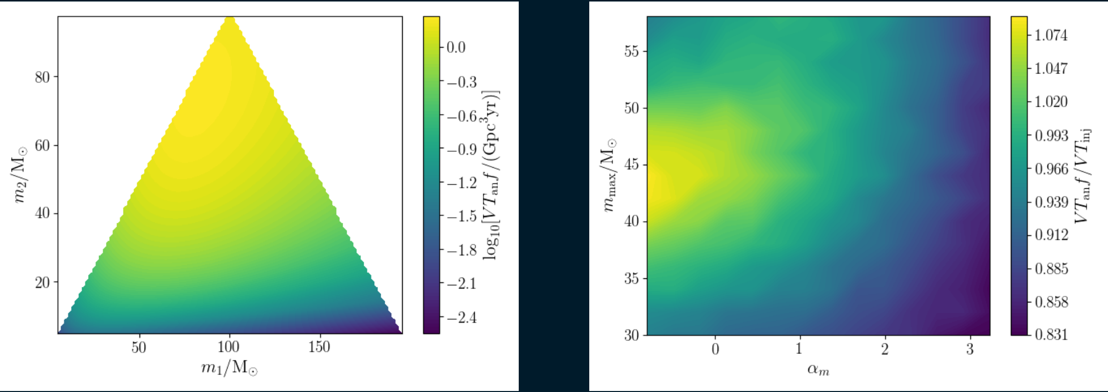
Comparison – scalar multiple calibration





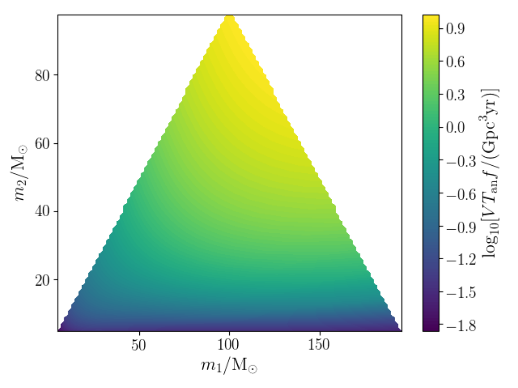
$$VT_{\mathrm{inj}}(m_1, m_2) \approx \zeta_0 VT_{\mathrm{an}}(m_1, m_2)$$

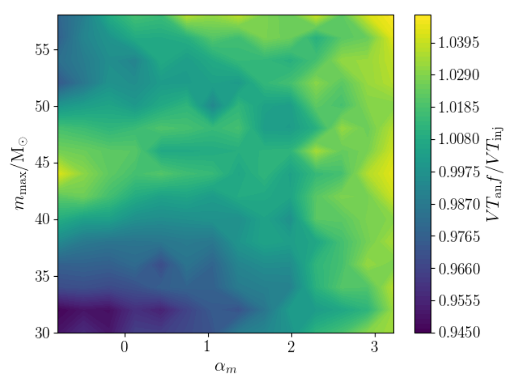
Comparison – linear calibration



$$VT_{\mathrm{inj}}(m_1, m_2) \approx (\zeta_0 + \zeta_1 m_1 + \zeta_2 m_2) VT_{\mathrm{an}}(m_1, m_2)$$

Comparison – quadratic calibration

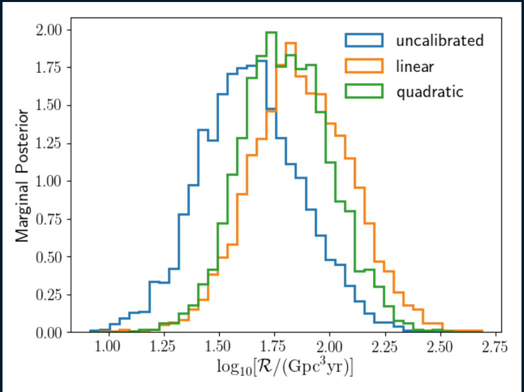


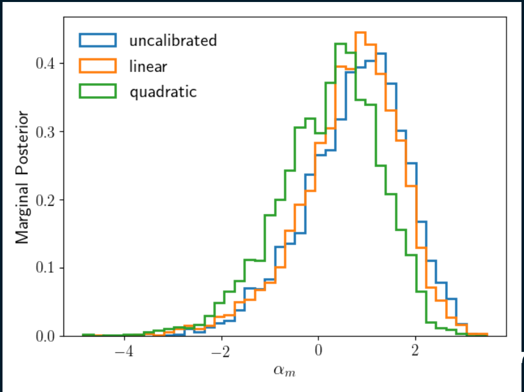


$$VT_{\mathrm{inj}}(m_1, m_2) \approx (\zeta_0 + \zeta_1 m_1 + \zeta_2 m_2 + \zeta_3 m_1 m_2 + \zeta_4 m_1^2 + \zeta_5 m_2^2) VT_{\mathrm{an}}(m_1, m_2)$$

Population inference comparisons

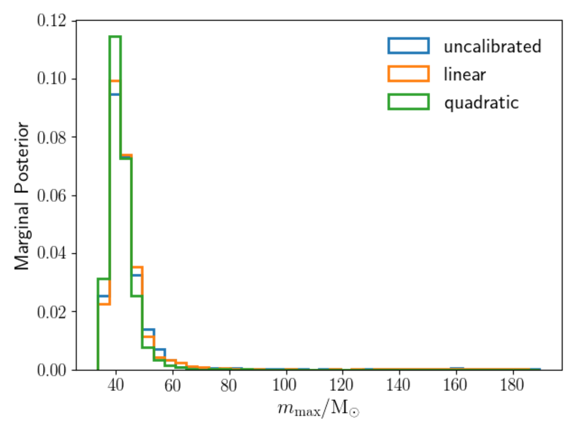
The scale of the change in results may be concerning for some parameters, most importantly the rate.





Population inference comparisons – maximum mass

Effect is still there but lesser for maximum mass.



Conclusions

- Semi-analytic VT's are still a necessary evil
- Can calibrate against injection VT's to improve accuracy significantly
- Can be applied to any $\langle VT \rangle$ table
  - (e.g., GstLAL; including spin dependence; redshift dependence; etc)
- allows cross-checking VT's between pipelines, and captures systematic effects across parameter space
- Disagreement reduced from biased high 39%–93% to symmetric 5%
- Easy fix – should definitely utilize this in O2 Populations Paper, runs ongoing now
- Note: we can easily choose any basis functions we want, these are just the first we tried

Generalizing method

- Method generalizes beyond just mass calibrations
- Can do spin and redshift calibrations as well
- Beyond calibration: idea of basis functions for VT can be used for efficient population estimation, including redshift distributions
    - Ongoing project, upcoming paper by Wysocki & O'Shaughnessy

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References I

Vaibhav Tiwari. Estimation of the sensitive volume for gravitational-wave source populations using weighted Monte Carlo integration. *Classical and Quantum Gravity*, 35:145009, 145009, July 2018. DOI: 10.1088/1361-6382/aac89d.

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